Wealth and Volatility*

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Abstract

Between 2007 and 2013, U.S. households experienced a large and persistent decline in net worth. The objective of this paper is to study the business cycle implications of such a decline. We first develop a model of a monetary economy where households face idiosyncratic unemployment risk that can insure using their wealth. We show that low wealth opens the door to self-fulfilling fluctuations. If wealth-poor households expect high unemployment, they have a strong precautionary incentive to cut spending, which, through nominal rigidities, makes the expectation of high unemployment a reality. We also show that when wealth is high an aggressive monetary policy can prevent self-fulfilling fluctuations, but when wealth is low, monetary policy, because of the zero lower bound, cannot avoid recessions or long stagnations due to negative expectations. We finally document that during the Great Recession in U.S. wealth poor households cut spending much more sharply than richer households, supporting the importance of the precautionary motive for aggregate spending.

Keywords: Business cycles; Aggregate demand; Precautionary saving; Multiple equilibria, Self-fulfilling crises, Zero Lower Bound

JEL classification codes: E12, E21, E52

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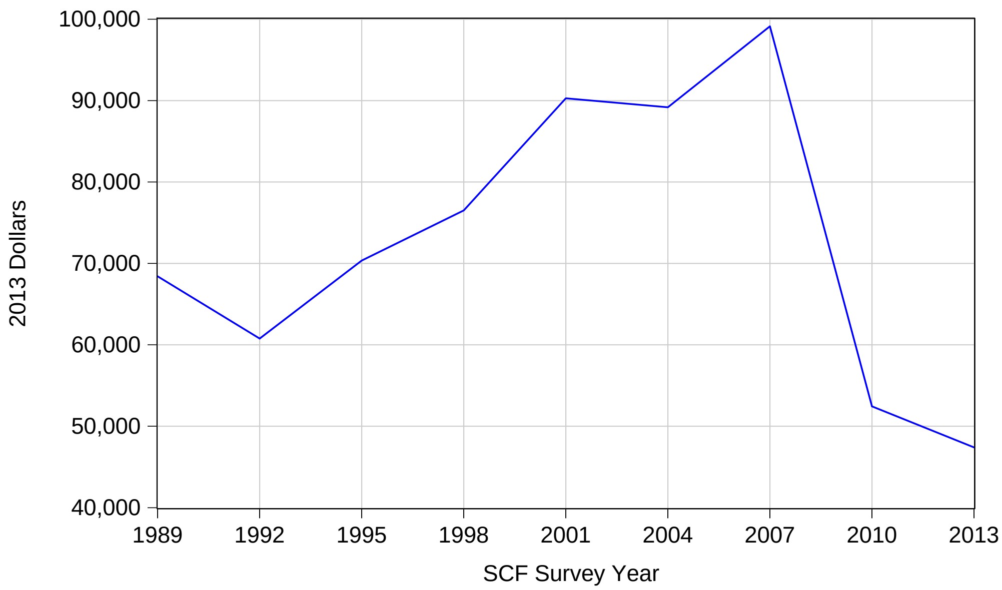
1 Introduction

Between 2007 and 2013, a large fraction of U.S. households experienced a large and persistent decline in net worth. Figure 1 plots median real net worth from the Survey of Consumer Finances (SCF), for the period 1989-2013, for households with heads between ages 22 and 60. Since 2007 median net worth for this group has roughly halved and shows no sign of recovery through 2013. In relation to income, the decline is equally dramatic: the median value for the net worth to income ratio fell from 1.58 in 2007 to 0.92 in 2013.

Figure 1: Median household net worth in the United States



Note: Sample includes households with heads between ages 22 and 60.

The objective of this paper is to study the business cycle implications of such a large and widespread fall in wealth. We will argue that falls in household wealth (driven by falls in asset prices) leave the economy more susceptible to confidence shocks that can increase macroeconomic volatility. Thus, policymakers should view low levels of household wealth as presenting a threat to macroeconomic stability. Figures 2 and 3 provide some motivating evidence for this message.

Figure 2 shows a series for the log of total real household net worth in the United States from 1920 to 2013, together with its linear trend. The figure shows that over this period there have been three large and persistent declines in household net worth: one in the early 1930s, one in the early 1970s, and the one that started in 2007. All three declines have marked the start of periods characterized by deep recessions and elevated macroeconomic volatility.[1]

Figure 2: Household net worth since 1920

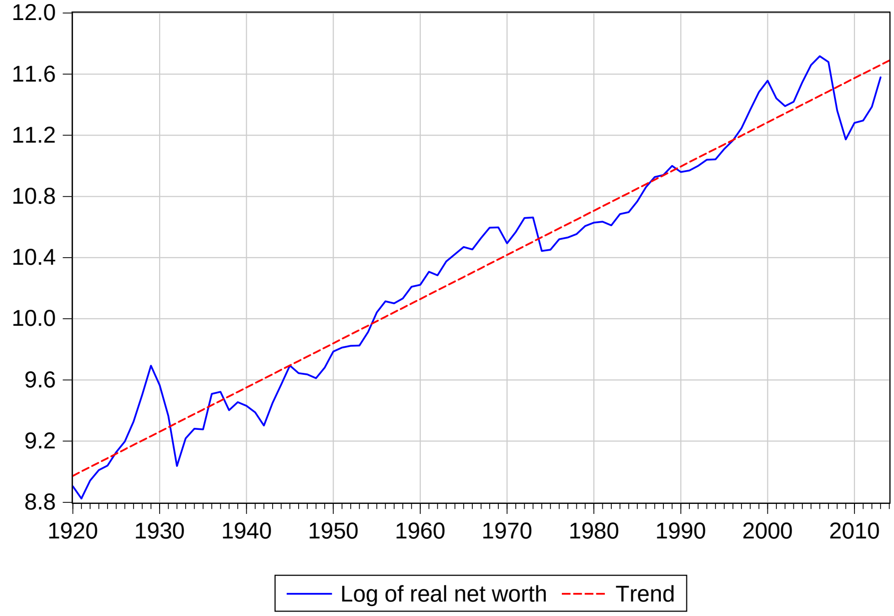
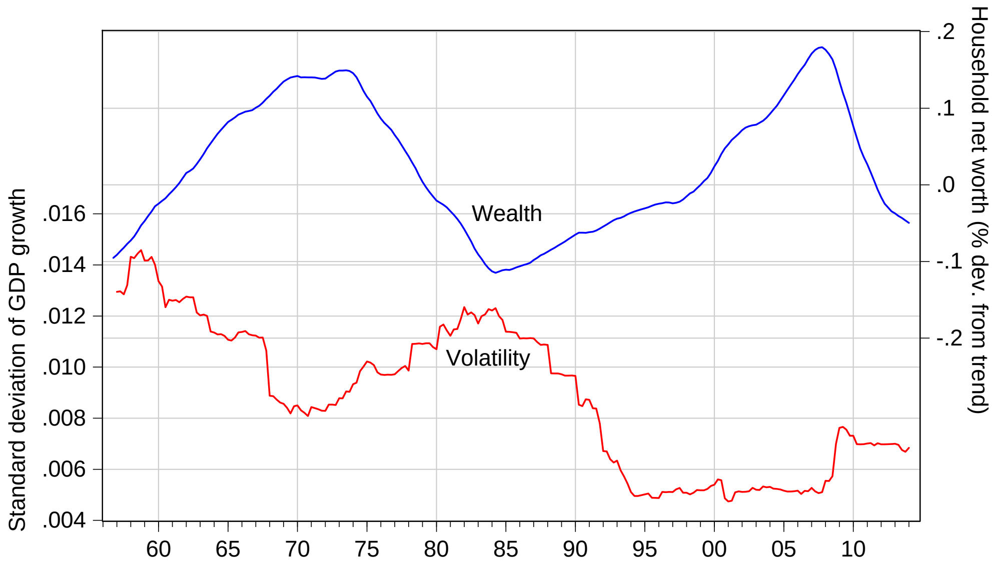


Figure 3 focuses on the postwar period, for which we can obtain a consistent measure of macroeconomic volatility. We measure volatility as the standard deviation of quarterly real GDP growth over a 10-year window. The figure plots this measure of volatility for overlapping windows starting in 1947.1 (the values on the x-axis correspond to the end of the window), together with wealth, measured as the deviation from trend (the difference between the solid and dashed lines in Figure 2) averaged over the same 10-year window. The figure reveals that periods when wealth is high relative to trend, reflecting high prices for housing and/or stocks, tend to be periods of low volatility in aggregate output (and hence employment and consumption). Conversely, periods in which net worth is below trend tend to be periods of high macroeconomic volatility. For example, during windows ending in the late 1950s and early 1980s, wealth is well below trend, and volatility peaks; conversely, in windows ending in the early 2000s and late 1960s, wealth is well above trend and volatility is low.

Why should wealth affect volatility? The novel idea of this paper is that the value of wealth in an economy determines whether or not the economy is vulnerable to economic fluctuations driven

[1] In order to construct a consistent series for net worth, we focus on three categories of net worth for which we can obtain consistent data throughout the sample: real estate wealth (net of mortgages), corporate securities, and government treasuries. See Appendix B for details on the construction of the series.

Figure 3: Wealth and volatility



Note: Standard deviations of GDP growth are computed over 40-quarter rolling windows. Observations for net worth are averages over the same windows.

by changes in household optimism or pessimism (animal spirits). When wealth is low, consumers are poorly equipped to self-insure against unemployment risk, and hence have a precautionary saving motive which is highly sensitive to unemployment expectations. Suppose households come to expect high unemployment. With low wealth, the precautionary motive to save will be strong, and households will cut desired expenditure sharply. In an environment in which demand affects output (because, say, of nominal rigidities), this decline in spending rationalizes high expected unemployment. Suppose, instead, that households in the same low wealth environment expect low unemployment. In this case, because perceived unemployment risk is low, the precautionary motive will be weak, consumption demand will be relatively strong, and hence equilibrium unemployment will be low. Thus, when asset values are low, economic fluctuations can arise due to self-fulfilling changes in expected unemployment risk.

In contrast, when the fundamentals are such that asset values are high, consumers can use wealth to keep their consumption smooth through unemployment spells, and thus the precautionary motive to save is weak irrespective of the expected unemployment rate. Thus, high wealth rules out a confidence-driven collapse in demand and output. One additional important issue is the role of monetary policy, and in particular whether monetary authority can, through changes in the nominal interest rate, stimulate household spending enough so to prevent these self-fulfilling crises. We will show that monetary policy can indeed prevent self-fulling crisis, but only when fundamentals are such that asset values and net worth are high.

This paper is broadly divided in two parts. In the first part we develop our theoretical analysis, while in the second we provide micro based empirical evidence supporting the importance of the precautionary motive for aggregate spending.

Theory Our contribution is to develop a simple model of a monetary economy in which precaution-driven changes in consumer demand can generate self-fulfilling aggregate fluctuations. The model contains three key ingredients. First, labor markets are frictional, so that unemployment can arise in equilibrium. Specifically, we use a standard Neo-Keynesian model of sticky wages (as in, for example, Rendahl, 2016), so that consumer demand, by affecting the price level, affects real wages, labor demand and thus the the unemployment rate. Second, unemployment risk is imperfectly insurable, so that anticipated unemployment generates a precautionary motive to save. Finally there is a monetary authority that can control the nominal interest rate, and through that can affect aggregate demand. Importantly though the monetary authority is limited in its action by the zero lower bound on interest rates.

It is important that idiosyncratic unemployment risk in the model is imperfectly insurable, so that a precautionary motive is active and consumption demand is sensitive to perceived unemployment risk. We therefore rule out explicit unemployment insurance, but assume that households own an asset (housing) that provides services and can also be used to smooth consumption in the event of an unemployment spell. We avoid the numerical complexity associated with standard incomplete markets models (e.g., Huggett 1993 or Aiyagari 1994) by assuming that individuals belong to large representative households.[2] However, the household cannot reshuffle resources from working to unemployed household members within the period. This preserves the precautionary motive, which is the hallmark of incomplete markets models. We will heavily exploit one property of the model: higher household wealth (i.e., high house prices) makes desired precautionary saving (and thus consumption demand) less sensitive to the level of unemployment risk. The intuition is simply that higher wealth permits higher consumption for unemployed household members, and thus better within-household risk sharing.

After describing the structure of model, we characterize equilibria under different fundamentals and under different monetary regimes. Note that we use a set-up where full employment is always

[2] Challe and Ragot (2012) show that an alternative way to preserve a low-dimensional cross-sectional wealth distribution while still admitting a precautionary motive is to assume that utility is linear above a certain consumption threshold.

an equilibrium and it is also the efficient one. The question is whether sometimes, due to pessimistic expectation the economy can end in an in efficient equilibrium with positive unemployment. When fundamentals are such that house prices (and hence net worth) are sufficiently high, then, provided that monetary policy is sufficiently aggressive, i.e. the monetary authority responds to positive unemployment by lowering the nominal rate, no equilibria with high unemployment are possible. The idea here is that high net worth provides households with good insurance against unemployment risk, and hence their precautionary saving motive is weak. Low demand for savings implies that interest rates will be relatively high, and thus the monetary authority can effectively stave off low demand and high unemployment by lowering nominal rates. Hence it can never be the case that households come to expect high unemployment and their expectations are realized.

When fundamentals are such that net worth is low, in contrast, high unemployment can be an equilibrium outcome, even if the central bank is very aggressive. To see this imagine that households come to expect high unemployment. In this case, because of the low net worth, the precautionary motive to save strengthens, aggregate demand would fall, unemployment would increase and the interest rates would fall. The monetary authority will try to increase aggregate demand by lowering the nominal rate, but if the increase in pessimism is large enough the precautionary motive will drive the nominal rates to 0, the actions of the central bank will then be limited, and the initial expectation of high unemployment will be validated.

It is important at this point to distinguish our theory from an alternative theory of self-fulfilling fluctuations in which declines in output are coincident with declines in asset values (see, e.g., Farmer 2013). In fluctuations of this type, the narrative goes as follows. Households expect asset prices to collapse, which makes them feel poorer, and a standard wealth effect channel induces them to cut spending. Low demand and the associated fall in output then rationalizes the expected fall in asset prices.

Although the two theories are related, there is a crucial difference between them. In our theory, the primary factor that drives the reduction in spending, and hence the recession, is the expected increase in risk. In the alternative theory, it is the expected fall in asset prices. We do not view the two theories as mutually exclusive, but we note that if the main driver of reduced spending during the Great Recession was falling asset prices, then high wealth households (who suffered the largest wealth losses) should have exhibited the largest spending declines. Instead, the data show that low wealth households were the ones who reduced consumption most, suggesting an important role for our precautionary mechanism.

After characterizing equilibria in the model, we show that the theory can be applied to help us better understand features of U.S. Great Recession of 2007-2008.

Micro Empirical Evidence In the second part of the paper we use micro data from the Consumer Expenditure Survey (CES) and the Panel Study of Income Dynamics (PSID) to document that, around the onset of the Great Recession, low net worth households increased their saving rates (i.e., cut their expenditures) by significantly more than high net worth households. This pattern is especially remarkable when considered alongside a second finding, which is that low wealth households suffered much smaller wealth losses during the recession. This new evidence indicates that the precautionary motive, in the context of sharply eroded home equity wealth and rising unemployment risk, was a key driver of consumption dynamics during the recession.

The paper is organized as follows. Section 2 contain our theory and its application to the Great Recession. Section 3 presents the evidence on households' expenditures and wealth during the Great Recession. Section 4 discusses the related literature, and Section 5 concludes.

2 Theory

The model we will develop is stylized, and has three key ingredients. First, unemployment risk is imperfectly insurable. This implies that when wealth is low, fear of high unemployment generates a precautionary motive to save, and thus a reduction in consumer demand. Second, nominal wages are rigid. This implies that a reduction in consumer demand will potentially drive a fall in output and validate the initial fear of unemployment. Third, there is a central bank that can adjust the nominal interest rate to try to stabilize the economy, subject to a constraint that the nominal rate cannot be negative.

There are two goods in the economy: a perishable consumption good, c, produced by a continuum of identical competitive firms using labor, and housing, h, which is durable and in fixed supply. There is a continuum of identical households, each of which contains a continuum of measure one of potential workers. Households and firms share the same information set and have identical expectations. Let s_t denote the current state of the economy and s^t denote the history up to date t.

2.1 Firms

Firms are perfectly competitive, and the representative firm produces using indivisible labor according to the following technology:

$$y(s^t) = n(s^t)^{\frac{1}{1+\sigma}} \tag{1}$$

where $y(s^t)$ is output and $n(s^t)$ is the number of workers hired. The curvature parameter $\sigma > 1$ determines the rate at which the marginal product of labor declines as additional workers are hired. Firms take as given the price of output $p(s^t)$ and must pay workers a fixed nominal wage w. These prices are both relative to a nominal numeraire (money). Thus, firms solve a static profit maximization problem:

$$\max_{n(s^t)\geq 0} \left\{p(s^t)y(s^t) - wn(s^t)\right\} \tag{2}$$

subject to eq. 1. The first order condition to this problem implies a mechanical link between employment and the price level:

$$\frac{1}{1+\sigma} n(s^t)^{\frac{-\sigma}{1+\sigma}} = \frac{w}{p(s^t)} \tag{3}$$

Thus, because firms are always on their labor demand curve, higher model employment $n(s^t)$ (lower unemployment) will always correspond to a lower real wage $w/p(s^t)$ and hence a higher price level p. The representative firm's profits, which we denote $\varphi(s^t)$, can be interpreted as the returns to a fixed non-labor factor.[3]

2.2 Households

Households are infinitely-lived. They can save in the form of housing and government bonds. At the start of each period, the head of the representative household sends out its members to look for jobs in the labor market and to purchase consumption. If the representative firm's labor demand $n(s^t)$ is less than the unit mass of workers looking for jobs in the representative household, then jobs are randomly rationed, and the probability that any given potential worker finds a job is $n(s^t)$. Let

$$u(s^t) = 1 - n(s^t) \tag{4}$$

denote the unemployment rate. Because each household has a continuum of members, this is both the fraction of unemployed workers in any given household, and the aggregate unemployment rate.

Within the period, it is not possible to transfer wage income from household members who find a

[3] The technology can be re-interpreted as Cobb-Douglas, $y(s^t) = k^{\frac{\sigma}{1+\sigma}} n(s^t)^{\frac{1}{1+\sigma}}$, where the fixed factor k is equal to one.

job to those who do not. Thus, unemployed members must rely on savings to finance consumption. If wealth is low or illiquid, it will not be possible to equate consumption between employed and unemployed household members. At the end of the period, all the household members regroup, pool resources, and decide on savings to carry into the next period.

More precisely, the representative household seeks to maximize

$$\sum_{t=0}^{\infty t} \left(\frac{1}{1+\rho}\right)^t \sum_{s^t} \pi(s^t) \left\{ \left[1-u(s^t)\right] \log c^w(s^t) + u(s^t) \log c^u(s^t) + \phi \log h(s^{t-1}) \right\}, \tag{5}$$

where ρ is the household's rate of time preference, and $\pi(s^t)$ is the probability of history s^t as of date 0. The values $c^w(s^t)$ and $c^u(s^t)$ denote household consumption choices that are potentially contingent on whether an individual household member is working (w) or unemployed (u) following history s^t. The parameter ϕ defines the utility from housing consumption, which is common across all household members. Note that utility is effectively Cobb-Douglas between housing and non-housing consumption, a specification consistent with Davis and Ortalo-Magne (2001).

Within the period, when intra-period transfers are ruled out, household members face budget constraints specific to their employment status:

$$p(s^t)c^u(s^t) \leq \psi p^h(s^t)h(s^{t-1}) + b(s^{t-1}) \tag{6}$$

$$p(s^t)c^w(s^t) \leq \psi p^h(s^t)h(s^{t-1}) + b(s^{t-1}) + w \tag{7}$$

where $h(s^{t-1})$ and $b(s^{t-1})$ denote the household's holdings of housing and nominal one-period government bonds. Bonds are assumed to be perfectly liquid, so they can be used dollar-for-dollar to finance consumption. Housing is imperfectly liquid within the period, so any household member can only use a fraction $\psi \in (0,1)$ of home value to finance current consumption. The simplest interpretation of ψ is that it captures the maximum loan-to-value ratio for home equity loans. The only difference between the within-period constraints for unemployed versus employed workers is that the employed can also access wage income w. Assets are (optimally) identically distributed between working and unemployed household members because unemployed is randomly allocated within the period.

The household budget constraint at the end of the period takes the form

$$\begin{aligned} & \left[1-u(s^t)\right] p(s^t)c^w(s^t) + u(s^t)p(s^t)c^u(s^t) + p^h(s^t)h(s^t) + \frac{1}{1+i(s^t)}b(s^t) \\ \leq \; & \left[1-u(s^t)\right] w + \varphi(s^t) + p^h(s^t)h(s^{t-1}) + b(s^{t-1}) \end{aligned} \tag{8}$$

The left-hand side of eq. (??) captures total household consumption and the cost of housing and bond purchases. The nominal price per unit of housing is $p^h(s^t)$, while the price of bonds is $(1 + i(s^t))^{-1}$, where $i(s^t)$ is the nominal interest rate. The first term on the right-hand side is earnings for workers w, the second is nominal firm profits, and the last two reflect the nominal values of housing and bonds purchased in the previous period.

Note that each household solves an identical problem, and therefore chooses the same asset portfolio. The equilibrium cross-household wealth distribution is therefore degenerate. Thus, this model of the household is a simple way to introduce idiosyncratic risk and a precautionary motive, without having to keep track of the cross-sectional distribution of wealth as in standard incomplete-markets models.

2.3 Monetary Authority

The monetary authority sets the nominal interest rate $i(s^t)$ paid on government bonds, which are in zero net supply. It follows a simple rule of the form

$$i(s^t) = \max\left\{\rho - \kappa u(s^t), 0\right\} \tag{9}$$

The parameter κ defines how aggressively the monetary authority cuts nominal rates in response to unemployment. A small value for κ defines a relatively passive monetary authority, while a large value defines an aggressive reaction function. Note that the zero lower bound constraint rules out negative rates.[4]

One way to micro-found the assumption that the monetary authority can impose a rule of the form (9) is to explicitly model money, and derive a mapping from changes in the money supply to changes in the nominal rate. In Section XX we develop this extension formally, introducing money in the utility function and as an additional source of liquidity in households' budget constraints. The baseline model described above can be interpreted as the "cashless limit" of the underlying monetary economy.

2.4 Household Problem

The household's problem is to choose $\left\{c^w(s^t), c^u(s^t), b(s^t), h(s^t)\right\}$ for all t and s^t in order to maximize eq. 5 subject to eqs. 6, 7, 8 and $\left\{c^w(s^t), c^u(s^t), h(s^t)\right\} \geq 0$, taking as given $\left\{u(s^t), p(s^t), p^h(s^t)\right\}$

[4] One could enrich the policy rule to allow the monetary authority to respond to fluctuations in the inflation rate, $p(s^{t+1})/p(s^t)$. However, our assumption of rigid nominal wages implies that inflation must be zero in any steady state.

and the policy rule for $i(s^t)$ in eq. 9.

The first-order conditions (FOCs) that define the solution to this problem can be combined to give two inter-temporal conditions, one for bonds, and one for stocks. The condition for bonds is

$$\frac{1}{c^w(s^t)}\frac{1}{1+i(s^t)} = \frac{1}{1+\rho}\sum_{s^{t+1}}\pi(s^{t+1}|s^t)\frac{p(s^t)}{p(s^{t+1})}\left[\frac{1-u(s^{t+1})}{c^w(s^{t+1})}+\frac{u(s^{t+1})}{c^u(s^{t+1})}\right], \tag{10}$$

where

$$c^u(s^{t+1}) = \begin{cases} c^w(s^{t+1}) & \text{if} \quad c^w(s^{t+1}) \leq \psi\frac{p^h(s^{t+1})}{p(s^{t+1})}h(s^t)+\frac{b(s^t)}{p(s^{t+1})} \\ \psi\frac{p^h(s^{t+1})}{p(s^{t+1})}h(s^t)+\frac{b(s^t)}{p(s^{t+1})} & \text{if} \quad c^w(s^{t+1}) > \psi\frac{p^h(s^{t+1})}{p(s^{t+1})}h(s^t)+\frac{b(s^t)}{p(s^{t+1})}. \end{cases} \tag{11}$$

This condition is easy to interpret. The real return on the bond (gross real interest rate) is the gross nominal rate divided by the inflation rate between s^t and s^{t+1}. The marginal value of an extra real unit of wealth at s^{t+1} is the average marginal utility of consumption within the household, i.e., the unemployment-rate-weighted average of workers' and unemployed members' marginal utilities. Eq. 11 indicates that these two marginal utilities will be equal if household liquidity is sufficient to equate consumption within the household (so that eq. 6 is not binding). Otherwise, unemployed workers will consume as much as possible, but within-household insurance will be imperfect. Note that if $c^u(s^{t+1}) = c^w(s^{t+1})$ for all s^{t+1}, then the FOC looks just as it would in a representative agent model. In contrast, if $c^u(s^{t+1}) > c^w(s^{t+1})$ for some histories s^{t+1} in which $u(s^{t+1}) > 1$ then households have a stronger incentive to save. In particular, there is then an active precautionary motive: higher next period wealth loosens the liquidity constraint for the unemployed, and improves insurance within the household.

The first-order condition for housing is

$$\frac{p^h(s^t)}{p(s^t)c^w(s^t)} = \frac{1}{1+\rho}\sum_{s^{t+1}}\pi(s^{t+1}|s^t)\frac{p^h(s^{t+1})}{p(s^{t+1})}\left[\frac{1-u(s^{t+1})\psi}{c^w(s^{t+1})}+\frac{u(s^{t+1})\psi}{c^u(s^{t+1})}\right]+\frac{1}{1+\rho}\frac{\phi}{h(s^t)}. \tag{12}$$

Here $p^h(s^t)/p(s^t)$ is the price of housing relative to consumption. The real financial return on housing is the change in this real price. In addition, an additional unit of housing delivers additional marginal utility $\phi/h(s^t)$ to all household members. Similarly to the bond, an additional unit of housing is differentially valued by employed versus unemployed household members. However, because housing imperfectly liquid, an extra real unit of housing wealth can only be used to finance an additional ψ units of consumption by unemployed workers.

2.5 Equilibrium

An equilibrium in this economy is a fixed nominal wage w, a process for the state s_t (which later will be a sunspot), and associated quantities and prices $u(s^t)$, $n(s^t)$, $y(s^t)$, $\varphi(s^t)$, $c^w(s^t)$, $c^u(s^t)$, $h(s^t)$, $b(s^t)$, $i(s^t)$, $p(s^t)$, $p^h(s^t)$ that satisfy, for all t and for all s^t, eqs. 4, 1, 2, 3, 9, 10, 12, 11, and the following three market clearing conditions

$$\left[1 - u(s^t)\right] c^w(s^t) + u(s^t)c^u(s^t) = y(s^t), \tag{13}$$

$$h(s^t) = 1, \tag{14}$$

$$b(s^t) = 0. \tag{15}$$

The second of these reflects an assumption that the aggregate supply of housing is equal to one, while the third reflects the fact that government bonds are in zero net supply.

3 Characterizing Equilibria

In this section of the paper, we show that the number of model steady states and their stability properties depend on the level of liquid household wealth, defined by the parameters ϕ and ψ, and on the aggressiveness of monetary policy, defined by the parameter κ. To preview the key results, when liquidity is high, and the precautionary motive to save is therefore relatively weak, an aggressive monetary policy ensures that full employment is the unique model steady state. Furthermore, this steady state this steady state is locally unstable, in the sense that it is not possible to construct sunspot shocks that feature temporary deviations from full employment. When liquidity is low, in contrast, richer equilibrium dynamics arise, and no value for the monetary policy κ ensures full employment. When policy is aggressive, the model features multiple steadys states, including one in which the interest rate is zero and unemployment is strictly positive. When policy is passive, full employment is the unique steady state, but this steady state is locally stable, so that non-fundamental shocks to confidence can induce temporary recessions.

3.1 Steady States: General Properties

We start by describing some general properties of model steady states. Steady states are equilibrium in which all model variables are constant. Thus, in any steady state the inflation rate will be zero, and there will be no distinction between real and nominal interest rates. See the Appendix for detailed derivations of the following results.

3.1.1 Full Employment Steady State

Result 1: Irrespective of parameter values, the model always features a full employment steady state in which

$$\begin{aligned} u &= 0, \\ y &= 1, \\ i &= \rho, \\ \frac{p^h}{p} &= \frac{\phi}{\rho}. \end{aligned}$$

This is the only efficient allocation, given that utility is strictly increasing in consumption, and there is no utility cost from working. Note that the real interest is simply the household's rate of time preference, and the price of housing is the present value of full employment implicit rents, ϕ/ρ.

3.1.2 Steady States with Perfect Risk Sharing

Result 2: If $\psi\phi/\rho \geq 1$ then risk-sharing is perfect in any steady state, so that $c^u = c^w = y$. If, in addition, $\kappa > 0$ then full employment is the unique steady state. If $\kappa = 0$, then there is a continuum of steady states, one for each $u \in [0, 1]$. In each such steady state

$$\begin{aligned} y &= (1-u)^{\frac{1}{1+\sigma}}, \\ i &= \rho, \\ \frac{p^h}{p} &= \frac{\phi}{\rho} y. \end{aligned}$$

The intuition for the parametric condition $\psi\phi/\rho \geq 1$ is simple. With perfect risk-sharing, the model collapses to a representative agent environment. Given Cobb-Douglas preferences, the real house price in a steady state with output y is proportional to the representative agent's consumption, $p^h/p = (\phi/\rho)\, y$. The maximum an unemployed worker can consume is $\psi p^h/p = (\phi/\rho)\, y$, which is larger than per capita output y if and only if $\psi\phi/\rho > 1$. Note that perfect risk sharing can be achieved either because the fundamental value of housing is high (i.e., ϕ/ρ is high), or because it is easy to borrow against housing (i.e., ψ is high).

It is also easy to see why $\kappa > 0$ guarantees that full employment is the unique steady state. With perfect risk sharing, the only interest rate consistent with households optimally choosing constant consumption is $i = \rho$. By promising $i < \rho$ whenever the unemployment rate is positive, the central bank can rule out steady states with $u > 0$.

3.1.3 Steady States with Imperfect Risk Sharing

Result 3: If $\psi\phi/\rho < 1$ then risk-sharing is imperfect in any steady state, so that $c^u < y < c^w$.

For the rest of the paper, we will focus on this region of the parameter space. We start our analysis by exploring how imperfect risk sharing affects asset pricing, taking as given a constant unemployment rate u. We will then move to ask which values for u are consistent with the central bank's policy rule.

Result 4: Given $\psi\phi/\rho < 1$, the household FOC for housing implies the following steady state relationship between the unemployment rate u and the real housing price p^h/p :

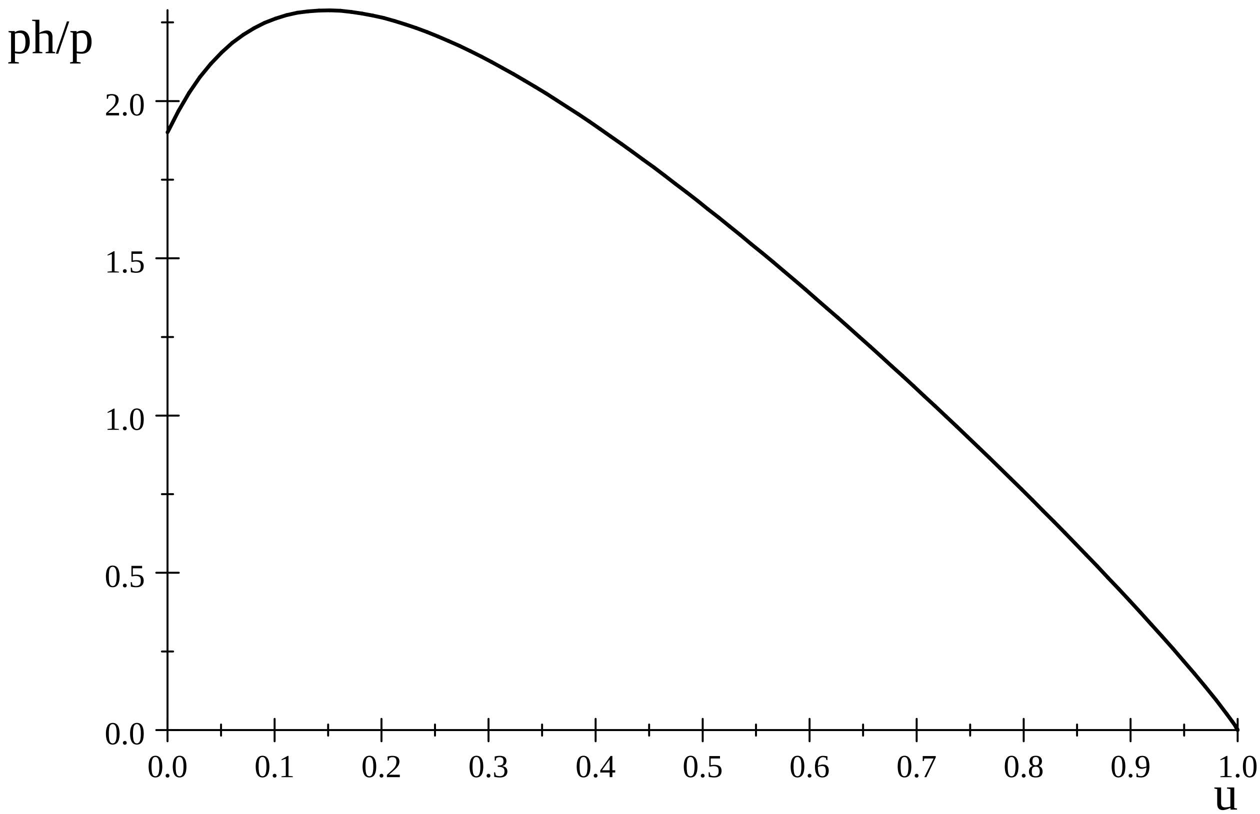
$$\frac{p^h}{p} = \underbrace{\frac{\phi}{\rho}(1-u)^{\frac{1}{1+\sigma}}}_{\text{fundamental component}} \times \underbrace{\frac{u+\phi}{\frac{\psi\phi}{\rho}u + \left(1 + \left(\frac{\psi\phi}{\rho} - 1\right)u\right)\phi}}_{\text{liquidity component}} \tag{16}$$

The first term in this expression is the "fundamental" component of house value, defined as the market clearing price $(\phi/\rho)\, y$ in a representative agent version of the model. This fundamental value declines linearly with steady state output $y = (1-u)^{\frac{1}{1+\sigma}}$. The second term, which is larger than one given $\psi\phi/\rho < 1$, reflects the "liquidity" premium embedded in equilibrium house prices. House prices exceed their fundamental value because housing serves a role in providing insurance within the household. The liquidity term is always increasing in the unemployment rate given $\psi\phi/\rho < 1$.

At $u = 0$ the steady state house price is increasing in u if $\frac{\psi\phi}{\rho} < \frac{1+\frac{\sigma}{1+\sigma}\phi}{1+\phi}$. Thus, if liquidity is sufficiently low, a marginal increase in unemployment risk at $u = 0$ increases households' willingness to pay for housing because the marginal additional liquidity value of housing wealth outweighs the marginal loss in fundamental value. For higher values for unemployment, the fundamental component of home value comes to dominate, and house prices decline in the unemployment rate.

As $u \to 1$, the steady state real house price converges to zero. The real house price value implied by eq. 16 is plotted below for $\sigma = 0.1$, $\rho = 0.05$, $\phi = 0.1$, and $\psi = 0.3$.

Figure 4: Real house prices as a function of unemployment



Result 5: Given $\psi\phi/\rho < 1$, the household FOCs for bonds and housing imply the following steady state relationship between the unemployment rate u and the interest rate i :

$$i = i(u) \equiv \rho \frac{\psi\phi + u\left(\psi + \frac{\psi\phi}{\rho} - 1\right)}{\psi\phi + u\left(\psi - \rho\left(\frac{\psi\phi}{\rho} - 1\right)\right)} \tag{17}$$

This equation can be derived starting from the steady state version of the household FOC for bonds, recognizing that a binding liquidity constraint implies $c^u = \psi p^h/p$, and then substituting in the steady state expression for p^h/p in eq. 16. The function $i(u)$ describes the interest rate at which households will optimally choose zero bond holdings given an unemployment rate u. Implicit in this expression is that for each value for u the corresponding constant real house price clears the market for housing. The interest rate varies with unemployment because the unemployment rate determines the strength of the household's precautionary motive. In fact, it does so through two channels. First, the unemployment rate mechanically determines the fraction of household members who will be liquidity constrained. Second, the unemployment rate also affects the steady state house price, and thus the consumption differential between employed and unemployed household members.

Result 6: The function $i(u)$ is equal to ρ at $u = 0$ and is declining and convex in u.

We conclude that a higher unemployment rate always translates into a stronger precautionary motive to accumulate bonds.

A steady state is a pair (i, u) that satisfies eq. 17 and also satisfies the steady state version of the policy rule eq. 9.

$$i = i^{CB}(u) = \max\{\rho - \kappa u, 0\}$$

The set of model steady states can thus be visualized by plotting eqs. 17 and 9 on the same graph.

Consider the following illustrative parameterization in which $\rho = 0.05$, $\phi = 0.1$, $\psi = 0.3$, and $\kappa = 0.35$. Note that this parameterization satisfies the condition $\psi\phi/\rho < 1$. The red line in figure 5 is the policy rule, which kinks at $u = \rho/\kappa$ where the nominal rate hits the zero lower bound. The black line plots eq. 17, i.e., the interest rate implied by the household's first order condition.

A steady state is a point at which these two lines intersect, so that the household's intertemporal first-order condition is satisfied at exactly the interest rate dictated by the central bank's policy rule. In the graphical example, there are three steady states: one at $u = 0$, and two with positive unemployment rates.

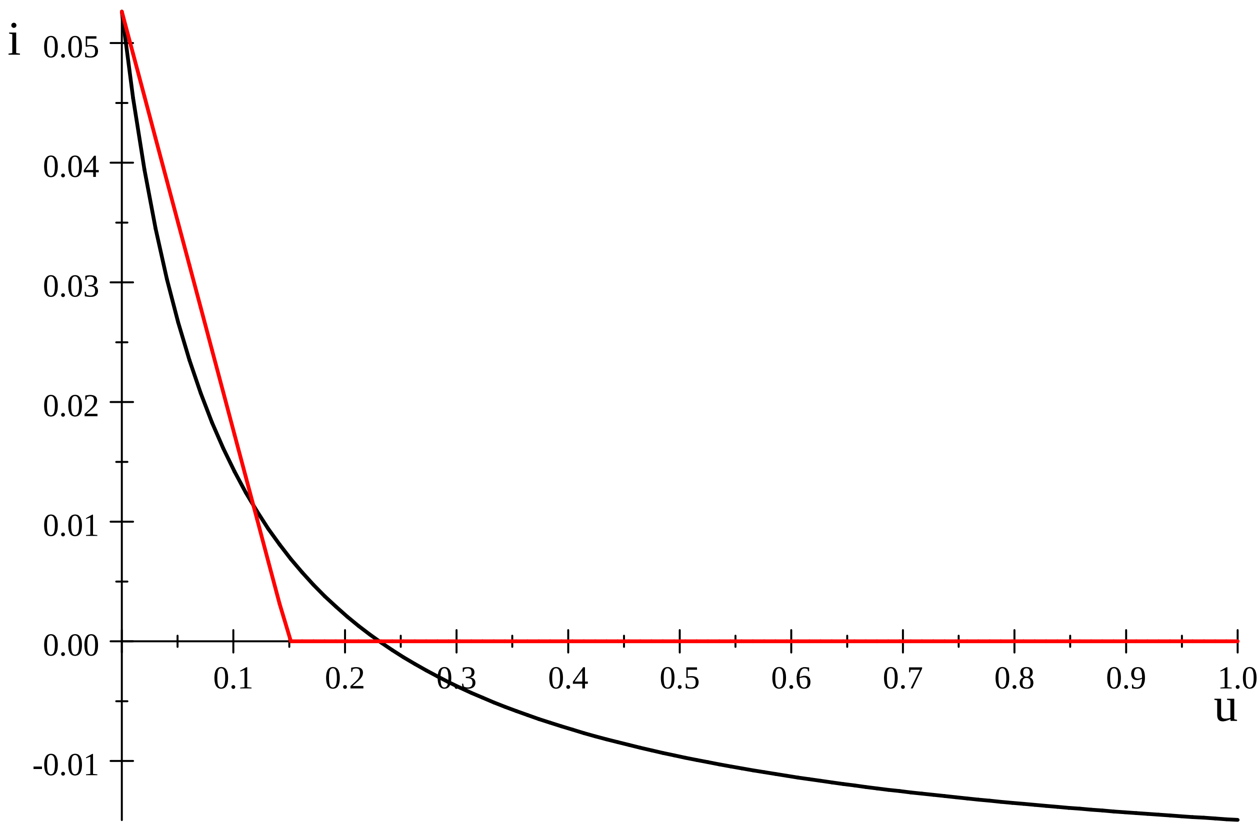
The tractability of the model can be exploited to offer a complete characterization of how many model steady states exist for different regions of the parameter space, and to characterize in closed form equilibrium prices and quantities in each of those steady states. In particular, we will partition the parameter space in two dimensions, based on the level of liquidity and on the aggressiveness of monetary policy.

Definition: Liquidity is high if $\frac{\psi\phi}{\rho} > \frac{(1-\psi)}{1+\rho}$ and is low otherwise.

Definition: Monetary policy is aggressive if $\kappa > \frac{(\rho-\phi\psi)(1+\rho)}{\phi\psi}$ and is passive otherwise.

These two definitions correspond to simple properties of the functions $i(u)$ and $i^{CB}(u)$ plotted in Figure XX. In particular, the high liquidity definition ensures that $i(u) > 0$ at $u = 1$ and thus at

Figure 5: Steady States



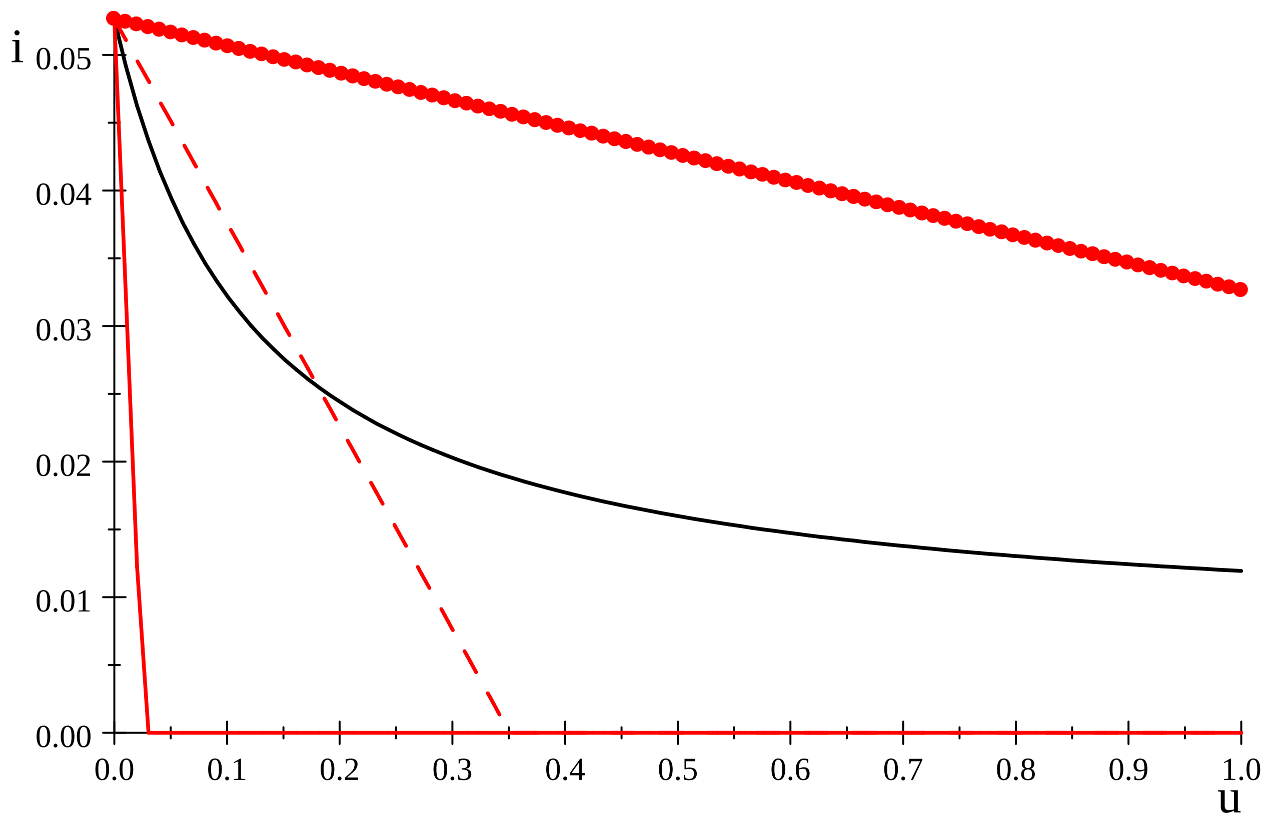
all $u \in [0, 1]$. The aggressive monetary policy definition ensures that the policy rate $i^{CB}(u)$ declines more rapidly in unemployment than does the steady state market clearing rate $i(u)$ at $u = 0$.

3.2 High Liquidity Steady States

If liquidity is high, the precuationary motive to save is relatively weak, and $i^{EQ}(u)$ is positive for any unemployment rate u. Three examples of high liquidity economies, corresponding to different monetary policy parameters κ, are illustrated in Figure 6 below. Parameter values are as in the previous example, except $\phi = 0.13$, so that liquidity is high according to the definition and the steady state interest rate consistent with household optimization (the black line) is always positive. Three policy rules are plotted: $\kappa = 2$ (solid line), $\kappa = 0.15$ (dashed line), and $\kappa = 0.02$ (dotted line).

Result 7: If liquidity is high and monetary policy is aggressive, then full employment is the

Figure 6: High Liquidity Equilibria



only steady state.

Graphically, if liquidity is high and monetary policy is aggressive, the black line lies always above the red in the above figure, so that full employment is the only steady state. The first policy rule plotted ($\kappa = 0.4$) satisfies the definition of aggressive monetary policy. In this case the figure indicates a unique steady state with full employment. What is the economic interpretation? With high household liquidity, the precautionary motive to save is relatively weak, and the real interest rate that satisfies the household's inter-temporal condition is positive for any unemployment rate. With an aggressive monetary authority, the policy rate therefore always falls below the rate defined by the household first-order condition, except at full employment. Thus an aggressive Fed effectively steers the economy away from any candidate positive unemployment steady state. Mechanically, in any candidate steady state with positive unemployment, the promise of very low interest rates would induce agents to borrow and spend, putting upward pressure on the price level, and downward pressure on the real wage and unemployment. This would lead the candidate positive unemployment steady state to unravel.

From the figure, it is clear that the high liquidity parameterization of the model also has a unique steady state with full employment when monetary policy is sufficiently passive. In particular, because $i(u)$ is convex while $i^{CB}(u)$ is linear, a sufficient condition for full employment to be the unique steady state is $i^{CB}(u) > i(u)$ at $u = 1$, or, equivalently, $\kappa < \frac{(\rho-\psi\phi)(1+\rho)}{\psi+\rho}$. However, as we will show later, while a passive monetary policy ensures that full employment is the unique steady state, a passive policy also introduces the possibility of temporary confidence driven recessions, while an aggressive policy rules out such fluctuations.

3.3 Low Liquidity Steady States

Figure 7 describes the set of possible equilibria when liquidity is low ($\phi = 0.1$). Now the household's precautionary motive is relatively strong, and the real interest rate that satisfies the household's inter-temporal condition falls below zero for a sufficiently high unemployment rate. The three policy rules plotted correspond to $\kappa = 2$ (solid line), $\kappa = 0.35$ (dashed line), and $\kappa = 0.02$ (dotted line).

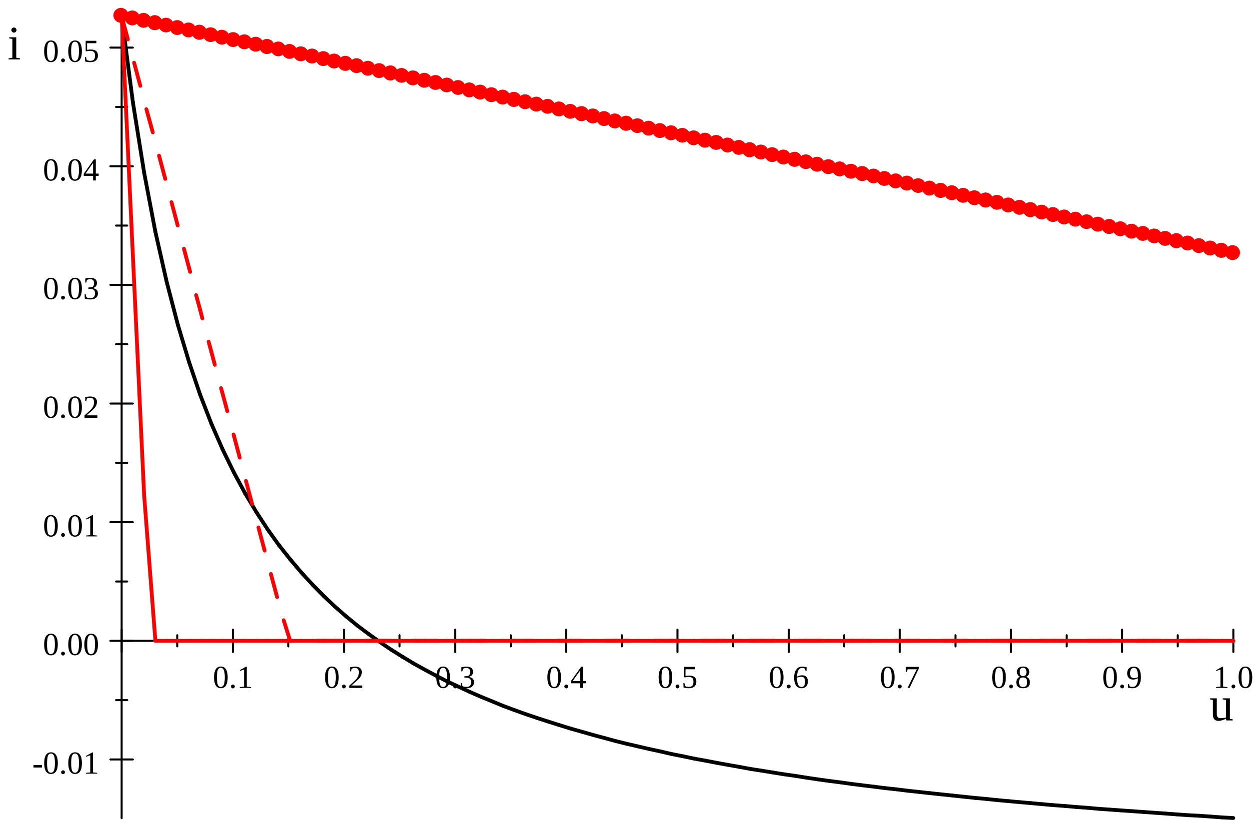
The key difference here, relative to the low liquidity case, is that an aggressive monetary policy no longer guarantees steady state uniqueness.

Result 8: When liquidity is low and monetary policy is aggressive, there are two steady states: full employment, and a second steady state in which

$$\begin{aligned} u &= u^{+} = \frac{\phi}{\frac{1-\psi}{\psi} - \frac{\phi}{\rho}} \\ y &= y^{+} = (1-u^{+})^{\frac{1}{1+\sigma}} \\ i &= 0 \\ \frac{p^h}{p} &= \frac{\phi}{\rho} y^{+} \times \frac{1}{1-\psi-\phi\psi} \end{aligned}$$

Why does an aggressive monetary policy no longer guarantee steady state uniqueness when liquidity is low? The logic is that when liquidity is low, the precautionary motive to save increases strongly in the unemployment rate, such that for a sufficiently large unemployment rate, the household is indifferent about saving in bonds at a zero real interest rate. The central bank responds aggressively to unemployment, but the zero lower bound prevents it from setting a negative rate.

Figure 7: Low Liquidity Equilibria



Thus it cannot rule out the steady state in which the economy is depressed, unemployment is high, and the interest rate is zero.

One might wonder whether in this case a passive Fed could do better. From the figure, it is visually clear that there are two types of passive policy. First, if monetary policy is sufficiently passive, the policy rule $i^{CB}(u)$ will lie everywhere above the pricing function $i(u)$, and thus full employment will be the unique steady state: see, for example, the example with $\kappa = 0.02$ above. However, if the policy rule is passive according to the definition, but hits zero when the pricing function is still positive (i.e., if $i(u)$ is positive at $u = \rho/\kappa$) then the model has three steady states: $u = 0$, $u = u^+$, and a third steady state in which the unemployment rate is zero while the interest rate is strictly positive: see the example with $\kappa = 0.35$.[5]

Again, however, we will now show that while a sufficiently passive policy ensures that full employment is the unique steady state, the problem is that the full employment steady state is vulnerable to sunspot shocks under a passive policy.

3.4 Stability of Steady States

Result 9: If monetary policy is aggressive, then the full employment steady state is locally unstable, in the sense that there are no perfect foresight equilibrium paths that converge to this steady state in which the initial unemployment rate is positive. If monetary policy is passive, then the full employment steady state is stable, and sunspot shocks are possible.

Thus, when monetary policy is passive, one can construct an equilibrium path in which unemployment is initially zero, but where agents collectively co-ordinate expectations on a path in which unemployment jumps today, and gradually converges to zero looking forward. A passive central bank cuts the interest rate only modestly when this confidence shock hits, and this rate cut is insufficient to offset the increase in the precautionary motive to save, driving a period a weak demand and rationalizing positive unemployment as an equilibrium outcome. Because agents expect a gradual recovery along the perfect foresight transition back to full employment, positive expected income growth counter-balances the precautionary motive to save, and asset markets do clear even though interest rates remain relatively high.

To recap, in a low liquidity passive central bank environment, a confidence-driven recession can be a self-fulfilling prophecy, because agents anticipate that if unemployment jumps, the central bank will not work aggressively to stimulate demand, and agents will cut back spending sharply given a strong precautionary motive. In contrast, if the central bank is expected to respond aggressively, then agents expect any recession to trigger very low interest rates. With very low rates, agents would want to consume today, if they expected the recession to be temporary, and thus there are no equilibrium paths in which the economy converges back to full employment.

[5] In this steady state

$$\begin{aligned} u &= u^{\$} = \frac{\rho(1+\rho) - \phi\psi\,(1+\kappa+\rho)}{\kappa\,(\rho+\psi(1-\phi))} \\ y &= y^{S} = (1-u^{S})^{\frac{1}{1+\sigma}} \\ i &= \frac{\rho\psi - (\rho - \phi\psi) + \kappa\phi\psi}{\rho+\psi(1-\phi)} \end{aligned} \tag{18}$$

However, if a recession today is expected to be followed by a further decline in output looking foward, then low demand today is potentially consistent with low interest rates.

3.4.1 Policy Takeaway

The key conclusions from this section are two. First, if household liquidity is high, monetary policy is simple. An aggressive Fed can keep the economy stuck at full employment, the efficient allocation. Second, if liquidity is low, monetary policy is much more difficult. In particular, the economy will be vulnerable to confidence driven crises regardless of whether monetary policy is aggressive or passive.

It follows that if liquidity is high, advising the model central bank is simple. Following an aggressive interest rate rule ensures permanent full employment, whereas if the central bank is passive it risks confidence-driven recessions.

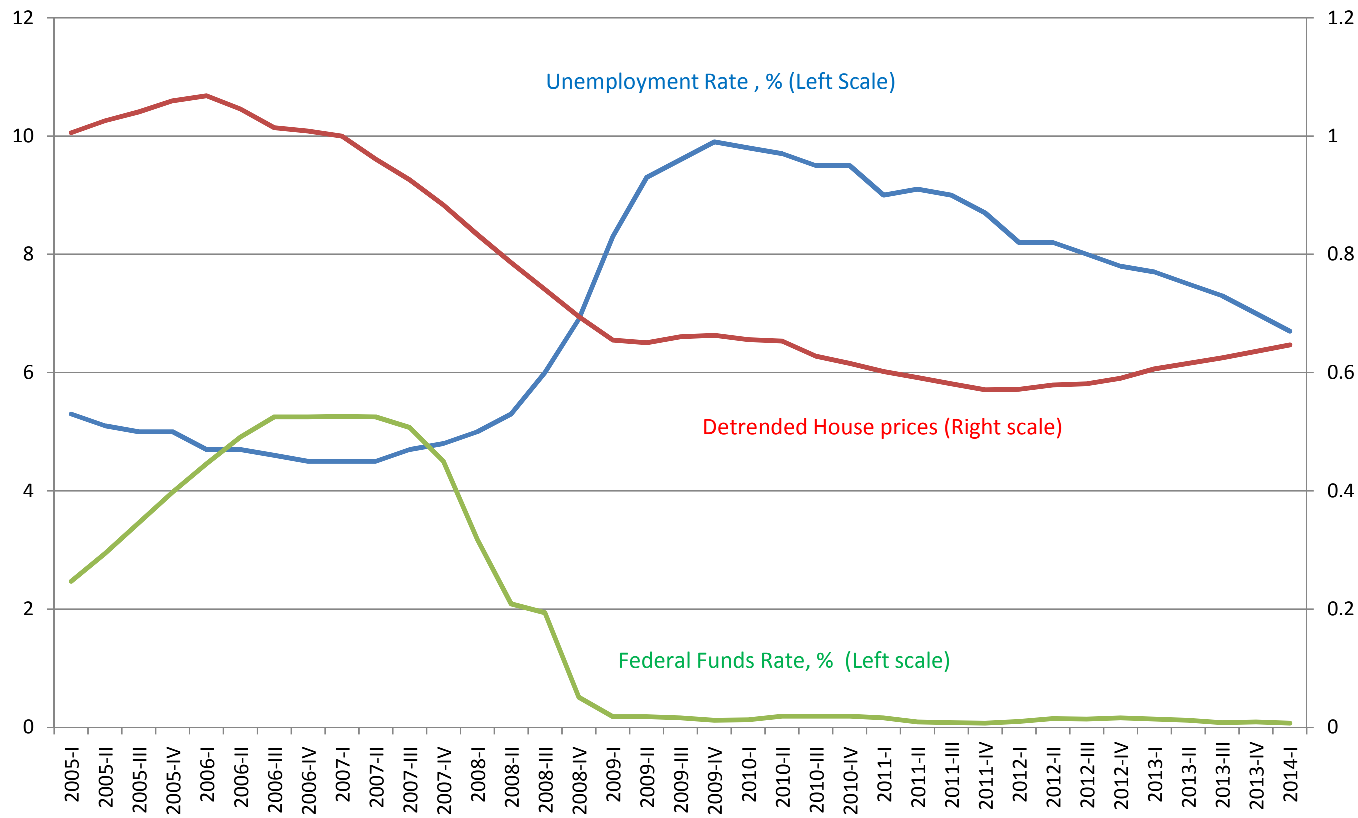
3.5 Application: The Great Recession

We now use model to interpret the time path for the unemployment rate in the United States over the course of the Great Recession. The key finding is that our model can generate dynamics for unemployment and interest rate that are qualitatively similar to those experienced by the United States over the course of the Great Recession. Figure 8 shows time paths for the unemployment rate, the federal funds rate and for house prices in the United States between the first quarter of 2005 and the first quarter of 2014. The house price series plotted is the Case-Shiller U.S. National Home Price Index, deflated by the GDP deflator, and relative to a 2 percent trend growth rate for the real price.[6] Between the start of 2007 and the end of 2008, house prices fell by 30 percent relative to trend, largely accounting for the sharp fall in median net worth documented in Figure 1. The rise in the unemployment rate was concentrated in the second half of 2008 and the first half of 2009. Thus, the fall in house prices began well before the most severe portion of the recession. We interpret this fall in housing prices as a change in fundamentals that moved the economy from the high liquidity region (the region with a unique full employment equilibrium) into the low liquidity region, and thus makes high unemployment equilibria like the one described above possible

We can then interpret the collapse of Lehman Brothers in the fall of 2008 as a sunspot that moved the economy from the full employment equilibrium to the high unemployment steady state.

[6]This is the average growth rate for real GDP per capita between 1947 and 2007. It is also close to the average growth rate for real house prices between 1975 and 2006 (see Figure 1 in Davis and Heathcote, 2007).

Figure 8: House prices, interest rates and unemployment



In the high unemployment equilibrium households cut back consumption – thereby rationalizing the surge in unemployment – because they now expect persistently high unemployment and therefore have a strong precautionary motive to save. The central bank cuts the nominal rate to 0 (which also implies a zero real rate) but, because of the high precautionary motive a 0 real rate is not enough to stimulate demand and to reduce the unemployment rate.

Although this model is simple, it can replicate some key features of the Great Recession. The negative shock to expectations generates a deep and rapid contraction, followed by a very slow recovery. Indeed our basic version of the model generate no recovery, while in the data obviously we observe that unemployment slowly falls. Although it is possible to extend the basic version so that the model displays a slow recovery, here we want to stress that our mechanism has the potential of generating very long lasting recessions.

4 Microeconomic Evidence

The key idea of this paper is that when an household has little wealth, its desired consumption becomes sensitive to perceived unemployment risk. When this risk rises, wealth-poor households reduce consumption sharply, which translates into lower employment, and rationalizes ex-post the fear of high unemployment. Is this mechanism empirically relevant for understanding the decline in aggregate consumption during the Great Recession? An alternative – possibly complementary – hypothesis is that negative wealth effects associated with sharp declines in asset prices played the dominant role. In this Section we offer microeconomic evidence that can help discriminate between these two hypotheses. The key insight is that the precautionary mechanism should be quantitatively more important for the consumption behavior of low wealth households, while declines in asset values should matter more for high wealth households. Thus, if the precautionary mechanism is more important in aggregate we should expect to see relatively sharp declines in consumption for low wealth households, while if traditional wealth effects played the dominant role, we should expect to see wealthier households cutting consumption disproportionately more than poorer ones.

In this section we use provide novel evidence, based on data from the Consumer Expenditure Survey (CES) and the Panel Study of Income Dynamics (PSID) to show that at the onset of the recession lower wealth households exhibit systematically larger declines in their consumption rates. This evidence as broadly consistent with Mian et al. (2013), who find that zip codes in the United States with poorer and more levered households experienced the sharpest consumption declines during the Great Recession. Collectively, this evidence lends support to demand-driven theories of the Great Recession, and to the importance of wealth in understanding demand dynamics.

4.1 Empirical Strategy

Our goal is to compare changes in consumption rates during the course of the Great Recession for wealth rich versus wealth poor households. In each data set we rank households by net worth and compute changes over time in the consumption rates of households in different groups of net worth distribution. It is important that the set of households in each wealth group is fixed when we measure the change in the consumption rate between t and $t+1$, so that the change in the measured consumption rate reflects a true change in savings behavior, and is not an artifact of a change in the composition of the groups. Fortunately both the PSID and CES data sets have a panel dimension: in the PSID, households are re-interviewed every two years, while in the CES they are interviewed for four consecutive quarters, and are asked about income in their first and last interviews.

4.2 Aggregates

Before contrasting consumption behavior across wealth groups, we first explore the dynamics of aggregate consumption, income and wealth in our cross sectional data, in order to verify that the micro data captures the broad contours of the Great Recession. Panel A of Figure 9 shows the dynamics of average per capita expenditures in the PSID and the CES against the equivalent measure in the National Income and Product Accounts (NIPA). Panel B shows average per capita disposable income in the PSID and the CES versus NIPA personal disposable income. Panel C shows median household net worth in the PSID and the CES versus median net worth in the Survey of Consumer Finances (SCF). Our consumption concept includes all categories except expenditure on housing and on health. Net worth includes net financial wealth plus housing wealth net of all mortgages (including home equity loans).[7] The key message from the figure is that the dynamics of consumption, income and wealth are broadly comparable across data sets. In particular, both micro data sets exhibit a marked reduction in consumption expenditure during the recession.[8]

4.3 Measurement

We now describe precisely how we define and compute changes in consumption rates for rich and poor households in the PSID. The procedure for the CES is very similar, adapted to the slightly different panel structure of the survey (see Appendix *B*).

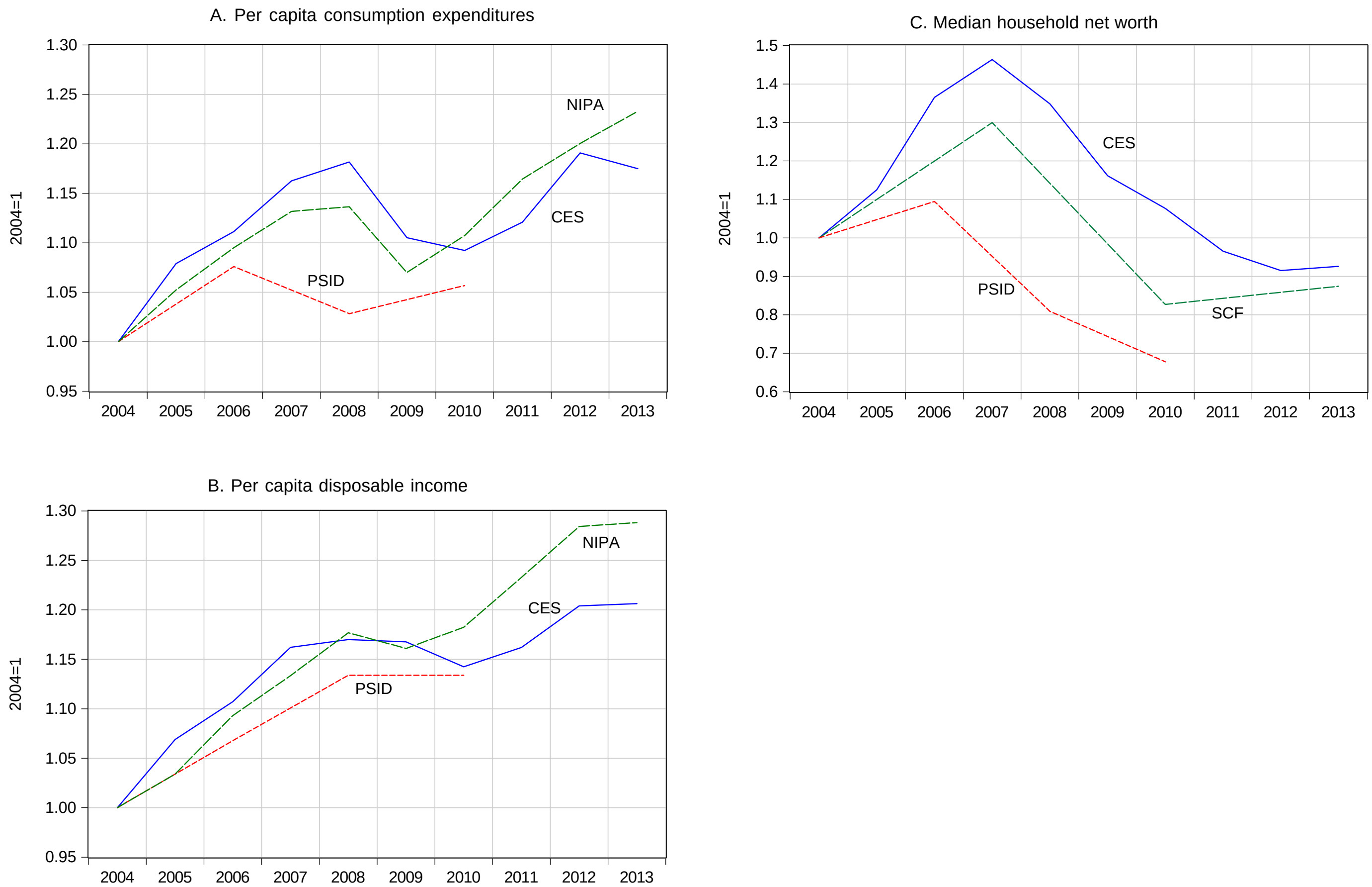
First, for any year t, we construct the sample we use to measure changes in consumption rates between year t and year $t+2$. We select all households with a head or spouse aged between 22 and 60, and which report income, consumption and wealth in both the t and $t+2$ waves. We focus on households of working age, since unemployment risk is most relevant for this group.

Second, we rank households by net worth in year t relative to the average of consumption expenditures in years t and $t+2$. We then divide the sample into two equal size subgroups, rich and poor, where the dividing line is (weighted) median net worth relative to consumption. We

[7] Appendix *B* reports more details of how we measure each variable. We do not impose any sample selection when constructing the PSID, CES and SCF series in Figure 9.

[8] One discrepancy is that consumption expenditures decline somewhat earlier in the PSID than in the CES or the NIPA. Note, however, that due to the bi-annual nature of the PSID we have no observation for 2007. In addition, it is difficult to date consumption precisely in the PSID because some of the survey questions ask explicitly about spending in the previous year – the year to which we attribute consumption – while others ask about current consumption. In Appendix *B* we discuss how excluding the latter consumption categories reduces the difference in dynamics between the PSID and the other two sources.

Figure 9: Comparing aggregates across micro data sets



measure household wealth relative to consumption since the strength of a household's precautionary motive to save is likely to be more closely connected to wealth relative to permanent income (for which average consumption is a proxy), rather than to absolute wealth.

Third, for each group we compute consumption rates in years t and $t+2$, where the consumption rate is defined as the average consumption of the group divided by the average disposable income of the group. The change in the consumption rate between t and $t+2$ for each group is simply the $t+2$ rate minus the t rate.

We then move to compute the change in the consumption rate from $t+2$ to $t+4$. This involves constructing a new sample, following the same procedure described in the first step, ranking households in the new set to construct new rich and poor groups, and constructing new measures of consumption rates for $t+2$ and $t+4$.

4.4 Descriptive Statistics

Table 1 reports characteristics of the rich and poor groups in both the PSID and the CES, for the year 2006. Differences between rich and poor are very similar across the two data sets. With respect to demographics, the wealth poor group tends to be younger and less educated. The most striking difference between the rich and poor groups, not surprisingly, is in terms of wealth. Median net worth for the poor group is near zero, while for the rich group it is around $265,000 in the PSID and around $187,000 in the CES.[9] This dramatic difference suggests that the precautionary saving motive for the poor group should be much stronger than for the rich group, while the rich group likely experiences much larger capital losses when housing and stock prices fall. The wealth-poor group has a little more than half the average income of the rich group, but has a much higher consumption rate, so that differences in consumption between the two groups are quite small.

TABLE 1. CHARACTERISTICS OF THE WEALTH RICH AND THE WEALTH POOR, 2006

	PSID		CES	
	Poor	Rich	Poor	Rich
Sample size	3446	2523	1915	1960
Mean age of head	37.9	47.1	40.2	46.4
	(0.21)	(0.21)	(0.25)	(0.24)
Heads with college (%)	21.3	36.5	24.8	39.4
	(0.86)	(1.1)	(1.1)	(1.2)
Mean household size	2.45	2.72	2.84	2.79
	(0.04)	(0.03)	(0.04)	(0.04)
Mean household net worth (current $)	11,931	619,831	11,967	338,535
	(879)	(49,388)	(1,155)	(12,644)
Median household net worth	5,000	265,000	1,800	187,102
	(476)	(6,602)	(294)	(4,893)
Per capita disposable income	15,028	28,475	18,739	30,184
	(256)	(667)	(334)	(593)
Per capita consumption expenditure	9,831	13,101	9,185	10,858
	(177)	(250)	(232)	(188)
Consumption rate (%)	65.8	46.0	49.0	36.0
	(0.90)	(0.86)	(1.18)	(0.66)

Note: Bootstrapped standard errors are in parentheses.

[9] One reason for the difference between the two data sets is that the measure of net worth in the PSID includes more assets, such as individual retirement accounts, vehicles, and family businesses. See Appendix B for details.

4.5 Changes in Consumption Rates: Rich versus Poor Households

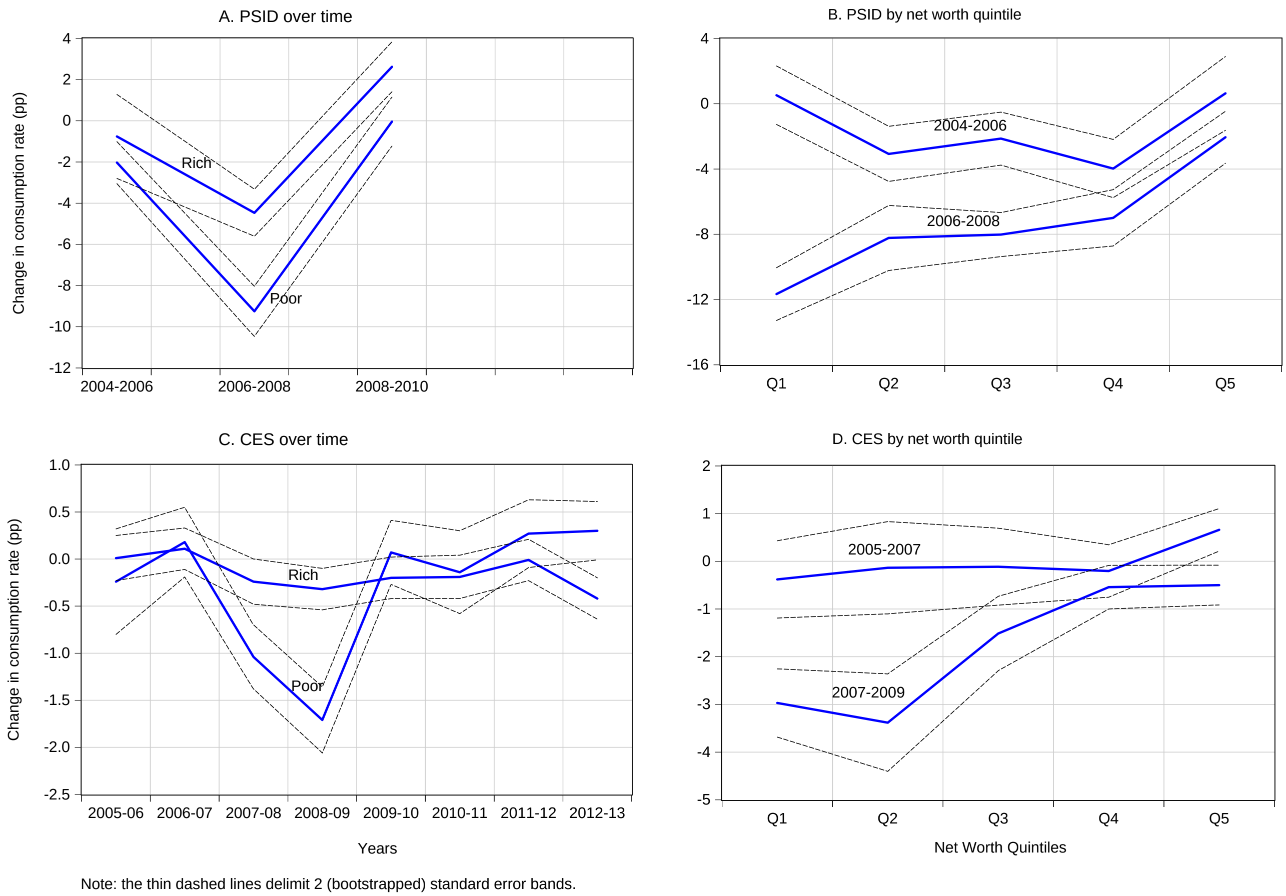
Figure 10 contains the key findings of this section. The figure plots changes in consumption rates computed from the PSID (top panels) and from the CES (bottom panels). The two panels on the left (A and C) show the evolution over time of the changes in consumption rates of the bottom 50% of the net worth distribution (labeled "Poor") against the same changes for the top 50% (labeled "Rich"). Both panels show that in "normal" times, there are no significant differences in consumption rate changes between the two wealth groups, while at the onset of the recession low net worth households reduce consumption rates significantly more than high net worth households.[10]

The two panels on the right (B and D) disaggregate more finely by net worth, and report the changes in consumption rates for the five quintiles of the net worth distribution. Here we focus on just two intervals: a pre-recession period, and the period when the economy enters recession. The plots show that as the economy moves into recession, consumption rates fall for all wealth quintiles, but the consumption rate drop is a declining function of initial net worth, with the bottom quintiles experiencing the largest consumption rate drops, and the top quintiles experiencing much smaller falls.

The disproportionate expenditure decline for wealth-poor households during the recession points toward an important precautionary motive in the face of rising unemployment risk. Moreover, the observed differential consumption decline is especially remarkable considering that it occurs at a time of sharply falling asset prices which, *ceteris paribus*, should disproportionately hurt the rich and lead them to reduce consumption more than the poor. To quantify the differential impact of asset price declines, we exploit the fact that households in the PSID report wealth at each interview, allowing us to compute changes in net worth over time for both the rich and poor groups. Table 2 reports the changes in consumption rates plotted in Figure 10 for each wealth group as defined above (lines 1 and 4), alongside changes in wealth (as a fraction of group-specific average income) over the same period (lines 2 and 5). Consider the period 2006–2008. Line 5 of the table shows that wealth-rich households experience a decline in net worth equivalent to 137 percent of their annual income over this period, while wealth-poor households actually see net worth increase (line 2). A conventional wealth effects story would therefore predict larger declines in consumption rates for the rich, contrary to the pattern observed. Note also that households in the poor group increase their net worth by 83 percent of their income during the recession. This provides independent evidence that this group really is increasing saving. In addition, the net worth data confirm the popular

[10] Consumption rate declines in the CES appear to be smaller than in the PSID. We conjecture that this primarily reflects the fact that the CES consumption rate changes are computed over 9-month intervals, while the PSID changes are recorded over 2-year intervals.

Figure 10: Changes in consumption rates for rich and poor



narrative of deleveraging during the Great Recession: low wealth households cut expenditure to reduce debts and rebuild eroded balance sheets.

The long panel dimension of the PSID also allows us to compute future income growth for the two groups, which we take as a measure of income prospects. Table 2 reports the growth rates of income over the two-year period following the measured consumption rate changes (lines 3 and 6).[11] As the economy enters the recession, expected future income growth declines for both the rich and poor groups, perhaps accounting for some of the decline in the aggregate consumption rate. However, the change in expected income growth is similar across wealth groups: the poor income growth changed by -8.8% (5.6-14.4) while the rich income growth changed by -7.6% (-0.2-

[11]Thus, future income growth for 2004-2006 refers to income growth of the group between 2006 and 2008, and analogously for the 2006-2008 period. We cannot compute future income growth for the 2008-2010 period because data for 2012 are not yet available.

7.4), suggesting that differential income prospects are not the primary factor behind the especially sharp decline in the consumption rate of the poor.

Table 2. Consumption, wealth & future income, PSID

		2004-2006	2006-2008	2008-2010
POOR				
1.	Δ consumption rate (pp)	-2.0	-9.3	-0.0
		(1.2)	(1.2)	(1.2)
2.	Δ net worth (% of income)	113	83	61
		(15)	(32)	(11)
3.	Δ future income (%)	14.4	5.6	
		(1.6)	(1.3)	
RICH				
4.	Δ consumption rate (pp)	-0.8	-4.6	2.6
		(1.8)	(1.1)	(1.1)
5.	Δ net worth (% of income)	189	-137	-22
		(71)	(50)	(23)
6.	Δ future income (%)	7.4	-0.2	
		(1.8)	(1.9)	

Note: Bootstrapped standard errors are in parentheses.

Overall, the evidence in Table 2 supports our hypothesis that the differential consumption rate changes shown in Figure 10 reflect a strong precautionary motive to save on the part of wealth-poor households, in the face of rising unemployment risk, and, more broadly, that the precautionary motive played a major role in accounting for the decline in aggregate consumption during the Great Recession. The magnitudes of the observed changes in consumption rates are economically relevant. For example, in the PSID over the period 2006-2008, poor households reduced their consumption rate by about 4 percentage points more than rich households. If we attribute this difference entirely to a stronger precautionary motive, then given that the poor account for about 1/3 of total disposable income (see Table 1), we can conclude that increased precautionary saving by the poor reduced aggregate consumption by $1/3 \times 4\% \simeq 1.3\%$ of aggregate disposable income.

This evidence shows that a rising precautionary motive to save may have played an important role in reducing consumer demand during the Great Recession. In the next section, we develop a simple macro model with a precautionary motive, and show that when wealth is sufficiently low, changes in precautionary saving can make perceived changes in unemployment risk self-fulfilling.

5 Related Literature

On the theory side, there is a long tradition of models in which self-fulfilling changes in expectations generate fluctuations in aggregate economic activity (see Cooper and John, 1988, for an overview). A classic early contribution is Diamond (1982), who constructs a model in which the expected presence of more trading partners makes trade easier, thereby stimulating production and generating the existence of more trading partners. In Farmer (2013, 2014) the labor market features search and matching frictions. Rather than assuming Nash bargaining over wages, he assumes that households form expectations – tied to asset prices – about the level of output, and that wages then adjust to support the associated level of hiring. Chamley (2014) constructs a model in which different equilibria are supported by differences in the strength of the precautionary motive to save, as in our model. In the low output equilibrium, individuals are reluctant to buy goods because they are pessimistic about their future opportunities to sell goods and because credit is restricted. In Kaplan and Menzio (2014), multiplicity is driven by a shopping externality: when more people are employed, the average shopper is less price sensitive, thereby increasing firms' profits and spurring vacancy creation. Bacchetta and Van Wincoop (2013) note that with strong international trade linkages, expectations-driven fluctuations will necessarily tend to be global in nature.

Perhaps the most important difference between all these papers and ours is that we focus on the role of household wealth in determining when self-fulfilling fluctuations can arise. In most models that admit nonfundamental-driven fluctuations, the theory has little to say about when fluctuations should occur. In contrast, we have argued that a precondition for a confidence-driven recession is a low level of household wealth.

Guerrieri and Lorenzoni (2009), Challe et al. (2015), den Haan et al. (2016), and Ravn and Sterk (2014) all emphasize the role of precautionary savings as a mechanism that amplifies fundamental shocks, but none of these papers consider the possibility of self-fulfilling precaution-driven fluctuations. In Beaudry et al. (2014) the precautionary savings channel amplifies a negative demand shock – via higher unemployment risk – but in their model, the impetus to low demand is excessively high past wealth accumulation, whereas we emphasize vulnerability when wealth is low.

Our emphasis on the role of asset values in shaping the set of possible equilibrium outcomes is shared by the literature on bubbles in production economies. Martin and Ventura (2014) consider an environment in which credit is limited by the value of collateral. Alternative market expectations can give rise to credit bubbles, which increase the credit available for entrepreneurs and therefore generate a boom (see also Kocherlakota, 2009). Hintermaier and Koeniger (2013) link the level of wealth to the scope for equilibrium multiplicity in an environment in which sunspot-driven fluctuations correspond to changes in the equilibrium price of collateral against consumer borrowing.

In other papers that emphasize a link between asset values and volatility, causation generally runs from volatility to asset prices. For example, Lettau et al. (2008) point out that higher aggregate risk should drive up the risk premium on risky assets relative to safe assets. Lower prices for risky assets like housing and equity then just reflect higher expected future returns on these assets. In our model, asset prices are the primitive, and the level of asset prices determines the possible range of equilibrium output fluctuations (i.e., macroeconomic volatility).

Our emphasis on the role of confidence is also a feature of Angeletos and La'O (2013), in which sentiment shocks (i.e., shocks to expectations about other agents behavior) can lead to aggregate fluctuations. Angeletos et al. (2014) develop a quantitative dynamic business cycle model that builds on similar ideas.

A general challenge in constructing models in which a notion of demand plays an important role in driving fluctuations is that many forces that tend to reduce desired consumption (such as lower asset values or greater idiosyncratic risk) also tend to increase desired labor supply. For this reason, models that emphasize the demand channel – including ours – need to ensure that increased desired labor supply does not automatically increase equilibrium output. Hall (2005), Farmer (2013), Michaillat (2012), and Shimer (2012) all exploit the fact that there are different possible ways to split the match surplus in search matching models, and negative aggregate shocks to the economy can be amplified if wages do not decline much in response. An alternative is to simply assume that real wages are sticky (see, for example, Midrigan and Philippon, 2011). Our approach blends both approaches: wages do not respond to unemployment in equilibrium precisely because the labor market is frictional.

On the empirical side, our model is related to a large literature which relates individual expenditures to labor income risk and to wealth, in order to assess the importance of the precautionary motive for consumption dynamics. Using British micro data, Benito (2006) finds that more job insecurity (using both model-based and self-reported measures of risk) translates into lower consumption. Importantly for the mechanism in our model, he finds that this effect is stronger for groups that have little household net worth. Engen and Gruber (2001) exploit state variation in unemployment insurance (UI) benefit schedules and estimate that reducing the UI benefit replacement rate by 50 percent for the average worker increases gross financial asset holdings by 14 percent. Carroll (1992) argues that cyclical variation in the precautionary savings motive explains a large fraction of cyclical variation in the savings rate.

Carroll et al. (2012) find that increased unemployment risk and direct wealth effects played the dominant roles in accounting for the rise in the U.S. savings rate during the Great Recession. Mody et al. (2012) similarly conclude that the global decline in consumption was largely due to an increase in precautionary saving. Alan et al. (2012) exploit age variation in savings responses in U.K. data to discriminate between increased precautionary saving driven by larger idiosyncratic shocks versus the direct effects of tighter credit. They conclude that a time-varying precautionary motive was the key factor: tighter credit, in their model, mostly affects the young, whereas all age groups increased saving. Mian and Sufi (2010 and 2015) and Baker (2015) use, respectively, county and household-level data to show that consumption declines during the Great Recession were larger for units with lower initial net worth, evidence again consistent with a heightened precautionary motive. Kaplan et al. (2014) argue that the number of households for whom the precautionary motive is strong might be much larger than would be suggested by conventional measures of net worth, since there is a large group of households with highly illiquid wealth.

6 Conclusions

The message of this paper is that when household wealth is low, a decline in consumer confidence can be self-fulfilling, because with low wealth, higher unemployment risk implies a large increase in the precautionary motive to save, rationalizing low equilibrium consumption and output. We argued that the decline in U.S. house prices in 2007 and 2008 reduced U.S. household net worth and left the U.S. economy vulnerable to just such a self-fulfilling wave of pessimism.

We developed a simple model to characterize the conditions under which confidence-driven fluctuations can arise and to better understand the link between the level of wealth on the one hand and the volatility and persistence of fluctuations on the other. Precautionary motives that vary with wealth and unemployment risk play the key role in these links. These precautionary motives are central in standard intertemporal consumption theory and are consistent with consumption patterns observed in micro data during the course of the Great Recession.

An obvious project for future work would be to develop a richer, more quantitative version of the model. One interesting extension would be to introduce household heterogeneity in net worth, to better understand the implications for aggregate precautionary demand of the extremely uneven distribution of net worth in the United States. A richer model of labor markets, in which desired labor supply plays a role in the long-run adjustment process, is another direction for future research. Finally, it would be interesting to introduce capital in the environment, since an increase in perceived unemployment risk would both increase the supply of saving and reduce the demand for investment, thereby changing the contours of the macroeconomic response.

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Appendix A. Empirical Analysis

Total household net worth

Total net worth of U.S. households is computed as the sum of the following components of the Financial Accounts of the United States (Z1 release): (i) Households and nonprofit organizations; real estate at market value minus home and commercial mortgages, (ii) Households and nonprofit organizations; corporate equities, (iii) Households and nonprofit organizations; treasury securities, including U.S. savings bonds. For the period 1920-1944, these series are not available from the source above so we extend them as follows. The value of real estate is backcast using the growth rate of the value of total residential non-farm wealth in Grebler et al. (1956). Home mortgages are backcast using the growth rate of nonfarm residential mortgage debt from Grebler et al. (1956) and commercial mortgages are backcast using the growth rate of nonfarm commercial mortgage debt from the same source. The value of Treasury securities is backcast using the growth rate of the amount of public debt outstanding from the Treasury Department, and finally the value of corporate equities is backcast using the historical growth of the S&P 500 price index.

Micro Data

For each data set our key variables are net worth, disposable income and consumption expenditures. Below we first briefly describe the data sets, and then discuss the construction of these variables. The micro data used for the analysis are available on the authors' websites.

The Panel Study of Income Dynamics (PSID) is a panel of U.S. households, selected to be representative of the U.S. population, collected (starting from 1997) at a bi-annual frequency. Starting in 2004 the PSID reports, for every household in the panel, comprehensive consumption expenditure information, alongside information on income and wealth. Our panel includes all households which have at least one member aged between 22 and 60, which report yearly consumption expenditure of at least $1,000, and which are in the panel for at least two consecutive interviews.

The Consumer Expenditure Survey (CES) is a rotating panel of U.S. households, selected to be representative of the U.S. population, collected at a quarterly frequency. Households in the CES report information for a maximum of four consecutive quarters. Households report consumption expenditures in all four interviews, income information in the first and last interview, and wealth information in the last interview only. We use CES data from the first quarter of 2004 to the last quarter of 2013, and include all households which have at least one member aged between 22 and 60, which report yearly consumption expenditure of at least $1,000, and which report consumption and income in the first and last interviews.

The Survey of Consumer Finance (SCF) is triennial survey of US households. The survey collects information on household income but focuses primarily on detailed information about household financial and non-financial assets and debts.

Net Worth In all three data sets we construct net worth by summing all categories of financial wealth (i.e. bank accounts, bonds, stocks) plus real estate wealth minus the value of any household debt (including mortgages, home equity loans and other debts). The PSID and SCF have a more accurate record of wealth, and report also the values of individual retirement accounts (IRAs), of family businesses, and of vehicles. Our measure of net worth in PSID and SCF also includes these variables.

Disposable Income In both data sets we construct disposable income by summing all money income received by all members of the household, including transfers, and then subtracting taxes. In the PSID we compute taxes using the NBER TAXSIM utility, while in the CES we use taxes paid as reported by the household.

Consumption Expenditure In both data sets we construct expenditure by summing the value of the purchases of: new/used cars and other vehicles, household equipment (including major appliances), goods and services used for entertainment purposes, food and beverages (at home and out), clothing and apparel (including jewelry), transportation services (including gasoline and public transportation), household utilities (including communication services such as telephone and cable services), education, and child care services. The two major categories that are excluded from our analysis are health expenditures and housing services. We exclude these categories to enable better comparison with NIPA data. Our key result regarding the differential behavior of consumption rates between rich and poor (shown in Figure 10) survives with consumption measures that include these two categories. We also experimented with a narrower consumption measure which excludes food, transportation services and utilities. The reason to consider excluding these categories is that households in the PSID are asked how much they spent on these in a typical week and not explicitly for the whole year (as for the other consumption categories). For this narrower consumption definition the discrepancy between aggregate consumption expenditures in the PSID and the other two data sources (Panel A of Figure 9) is much smaller. We have also reproduced Figure 10 with this narrower consumption measure, and found that the patterns of changes in consumption rates are very similar.

Measuring changes in consumption rates in the CES

We now outline the procedure used to produce Figure 10, Panel B.

1. Select households that (i) contain a head or spouse aged between 22 and 60, (ii) are interviewed for the first time in year t (e.g. the first year in the sample), and (iii) report annual income and quarterly consumption in their first and last interviews, and report wealth in their last interview.

2. Rank these households by net worth in their last interview (the only time wealth is reported) relative to the average of consumption reported in the first and last interview, and divide the sample into two equal (weighted) size subgroups: the rich and the poor.

3. For each group, compute the consumption rate in the first interview (in year t) and the last interview. Note that because the first and last interviews are 9 months apart, for some households the last interview is at the end of year t while for the rest it is in year $t + 1$. The consumption rate is the average (annualized) consumption of the group in the quarter divided by the average disposable income of the group in the year.

4. Record for year t changes in consumption rates from the first to the last interview.

5. Move to year $t+1$, and repeat Steps 1 through 4 to construct new rich and poor samples and to compute changes in consumption rates for year $t + 1$.

To produce Figure 10, Panel D, we follow the same procedure, except that to compute consumption rate changes over a two-year window, for a given wealth quintile, we add together the quintile-specific changes in the consumption rate over each of the two years in the window. Note that, in contrast to the PSID, the set of households used for a given quintile differs across the two years of a particular two-year window.

Appendix B. Proofs

Steady State Results

Consider a steady state with a constant unemployment rate u. The market clearing conditions are

$$h = 1 \tag{19}$$

$$b = 0 \tag{20}$$

and

$$c^w(u) = \frac{y(u) - uc^u(u)}{1-u} \tag{21}$$

where

$$y(u) = (1-u)^{\frac{1}{1+\sigma}}$$

Given the market clearing conditions eqs. **??** and 20 the steady state versions of the household's first-order conditions for houses and bonds are

$$\frac{p^h(u)}{p(u)c^w(u)} = \frac{1}{1+\rho}\frac{p^h(u)}{p(u)}\left(\frac{1-\psi u}{c^w(u)} + \frac{\psi u}{c^u(u)}\right) + \frac{\phi}{1+\rho} \tag{22}$$

$$\frac{1}{p(u)c^w(u)}\frac{1}{1+i(u)} = \frac{1}{1+\rho}\frac{1}{p(u)}\left(\frac{1-u}{c^w(u)} + \frac{u}{c^u(u)}\right) \tag{23}$$

where

$$c^u(u) = \max\left\{c^w(u), \psi\frac{p^h(u)}{p(u)}\right\}.$$

The steady state version of the firm's FOC is

$$p(u) = w(1+\sigma)(1-u)^{\frac{\sigma}{1+\sigma}}$$

while the central bank policy rule is

$$i(u) = \max\{\rho - \kappa u, 0\}.$$

Result 1: Full Employment Steady State

At full employment, the policy rule implies $i = 0$. It is immediate that at this interest rate and $u = 0$, the bond FOC is satisfied. Then, from the housing FOC, the real house price is given by $p^h/p = \phi/\rho$.

Result 4: Steady State House Price

Assuming imperfect risk sharing,

$$c^u(u) = \psi \frac{p^h(u)}{p(u)} \tag{24}$$

The steady state house price equation eq. 22 can then be written as

$$\frac{p^h(u)}{p(u)c^w(u)} = \frac{\phi + u}{\rho + \psi u}$$

Nominal spending by workers, using eqs. **??** and 21, is

$$\begin{aligned} p(u)c^w(u) &= (1+\sigma)w\,(1-u)^{\frac{\sigma}{1+\sigma}} \left(\frac{(1-u)^{\frac{1}{1+\sigma}} - u\psi \frac{p^h(u)}{p(u)}}{1-u} \right) \\ &= (1+\sigma)w - u\frac{\psi p^h(u)}{1-u} \end{aligned} \tag{25}$$

Substituting this into the previous equation, one can solve out for $p^h(u)$:

$$p^h(u) = \frac{(1+\sigma)w}{\frac{\rho+\psi u}{\phi+u} + \frac{\psi u}{1-u}}$$

Now, using **??**, we get the following real house price expression.

$$\begin{aligned} \frac{p^h(u)}{p(u)} &= \frac{1}{(1+\sigma)w(1-u)^{\frac{\sigma}{1+\sigma}}} \frac{(1+\sigma)w}{\frac{\rho+\psi u}{\phi+u} + \frac{\psi u}{1-u}} \\ &= (1-u)^{\frac{-\sigma}{1+\sigma}} \frac{1}{\frac{\rho+\psi u}{\phi+u} + \frac{\psi u}{1-u}} \\ &= \frac{\phi}{\rho} \times (1-u)^{\frac{1}{1+\sigma}} \times \frac{u+\phi}{\frac{\psi\phi}{\rho}u + \left(1 + \left(\frac{\psi\phi}{\rho} - 1\right)u\right)\phi} \end{aligned} \tag{26}$$

Result 3: If $\psi\phi/\rho < 1$ then risk-sharing is imperfect.

Proof by contradiction. Assume perfect insurance within the household and $\psi\phi/\rho < 1$. For any constant unemployment rate u, the implicit rental rate for housing is the ratio of marginal utilities of housing to consumption. Given the goods and housing market clearing conditions, eqs. **??** and 20, this ratio is $\phi y(u)$. The steady state real house price $p^h(u)/p(u)$ is the present value of these implicit rents, $\phi y(u)/\rho$. The maximum feasible consumption for unemployed household members is therefore $\psi\phi y(u)/\rho$. Perfect risk-sharing requires $\psi\phi y(u)/\rho \geq y(u)$, which contradicts $\psi\phi/\rho < 1$.

Result 2: If $\psi\phi/\rho \geq 1$ then risk sharing is perfect.

Proof by contradiction. Assume $\phi\psi/\rho \geq 1$ and that risk-sharing is imperfect, so that $c^u(u) = \psi p^h(u)/p(u) < y(u)$.

The steady state real price of housing (Result 4) is.then given by eq. 26

$$\frac{p^h(u)}{p(u)} = \frac{\phi}{\rho} \times (1-u)^{\frac{1}{1+\sigma}} \times \frac{u+\phi}{\frac{\psi\phi}{\rho}u + \left(1 + \left(\frac{\psi\phi}{\rho} - 1\right)u\right)\phi} \tag{27}$$

Now, for any constant u, if $\psi\phi/\rho \geq 1$ the third term is smaller than one, which implies $p^h(u)/p(u) < (\phi/\rho)y(u)$, which implies $c^u(u) = \psi p^h(u)/p(u) = \psi(\phi/\rho)y(u) > y(u)$. Contradiction.

Result 5: Steady State Bond Price

The steady state version of the bond first-order condition, can be written, after substituting in market clearing conditions, as

$$\begin{aligned} \frac{1}{p(u)c^w(u)}\frac{1}{1+i(u)} &= \frac{1}{1+\rho}\left(\frac{1-u}{p(u)c^w(u)} + \frac{u}{p(u)c^u(u)}\right) \\ \frac{1}{1+i(u)} &= \frac{1}{1+\rho}\left((1-u) + u\frac{p(u)c^w(u)}{p(u)c^u(u)}\right) \end{aligned}$$

Now substitute in 25 and 24

$$\frac{1}{1+i(u)} = \frac{1}{1+\rho}\left((1-u) + \frac{u\left((1+\sigma)w - u\frac{\psi p^h(u)}{1-u}\right)}{\psi p^h(u)}\right) \tag{28}$$

$$\frac{1}{1+i(u)} = \frac{1}{1+\rho}\left(\frac{(1-2u)}{(1-u)} + u\frac{(1+\sigma)w}{\psi p^h(u)}\right) \tag{29}$$

Subsituting eq. 26 into eq. 29 gives the expression in the text:

$$i(u) = \rho\frac{\psi\phi + u\left(\psi + \frac{\psi\phi}{\rho} - 1\right)}{\psi\phi + u\left(\psi - \rho\left(\frac{\psi\phi}{\rho} - 1\right)\right)} \tag{30}$$

Result 6: The function $i(u)$ is equal to ρ at $u = 0$ and is declining and convex in u.

It is immediate that either $u = 0$ or $\psi\phi/\rho = 1$ imply $i(u) = \rho$. Differentiating eq. 30 with respect to u, it is straightforward to show that the equilibrium interest rate $i^{EQ}(u)$ is declining in u given $\psi\phi/\rho < 1$. Similarly, $\psi\phi/\rho < 1$ ensures that the second derivative of $i^{EQ}(u)$ with respect to u is positive, so $i^{EQ}(u)$ is a convex function of u.

Result 7: If liquidity is high and monetary policy is aggressive, then full employment is the only steady state.

Aggressive monetary policy ensures $\kappa > -\frac{\partial i(u)}{\partial u}$ at $u = 0$. Because $i(u)$ is convex, this implies $i(u) > i^{CB}(u)$ for all $u \in (0, \rho/\kappa]$ (the point where $i^{CB}(u)$ kinks). The condition defining high liquidity ensures that $i(u) > 0$ at $u = 1$ and thus for all $u \leq 1$ (given $\frac{\partial i(u)}{\partial u} < 0$). Thus, $i(u) > i^{CB}(u) = 0$ for all $u \in (\rho/\kappa, 1]$. Combining these results, the only value for u at which $i(u) = i^{CB}(u)$ is $u = 0$.

Result 8: When liquidity is low and monetary policy is aggressive, there are two steady states: full employment, and a second steady state in which

$$\begin{aligned} u &= u^+ = \frac{\phi}{\frac{1-\psi}{\psi} - \frac{\phi}{\rho}} \\ y &= y^+ = (1-u^+)^{\frac{1}{1+\sigma}} \\ i &= 0 \\ \frac{p^h}{p} &= \frac{\phi}{\rho} y^+ \times \frac{1}{1-\psi-\phi\psi} \end{aligned}$$

This solution for u^+ comes from setting $i = 0$ in eq. 30 and solving for u. The solution for p^h/p then follows from substituting the solution for u into eq. 26.

Result 9: If monetary policy is aggressive, then the full employment steady state is locally unstable, in the sense that there are no perfect foresight equilibrium paths that converge to this steady state in which the initial unemployment rate is positive. If monetary policy is passive, then the full employment steady state is stable, and sunspot shocks are possible.