Soft Transactions*

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Abstract

This paper postulates that when help is provided without contract, a soft debt is tacitly accrued by the beneficiary to the provider. It introduces a two-player stochastic game with random benefits and costs, focusing on strategies that rely on the soft debt balance. It shows the existence of stationary Markov equilibriums in which players trade favors whenever the resulting soft debt balance is within a certain limit. It also demonstrates that the equilibriums could exist when costs are the provider's private information.

KEYWORDS: Stochastic games; Reciprocity; Transaction costs

JEL Classifications: C73, D2

1 Introduction

Typical models of repeated games (e.g. repeated prisoner's dilemma) feature fixed payoff matrices for each stage. Players may choose to cooperate or defect in each round, trading off between potential gains from opportunistic actions and risks of jeopardizing future co-operation. In reality, however, payoffs in repeated interactions are seldom fixed. Players

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often face a new and unique situation each time they meet. Even between close partners there can be variations among established routines. In addition, in typical repeated games players can possibly achieve mutual benefits simultaneously in each round. But in reality each interaction is often characterized by one player helping another unilaterally, resulting in a lopsided payoff. Mutual benefits are possible only when their roles alternate over repeated interactions.

In the absence of contracts, how is cooperation possible under varying and lopsided payoffs? Why would a player voluntarily help another without explicit promise of return of favor large enough to justify her cost? This paper proposes soft debt as a social institution that makes such kind of reciprocity possible. More specifically, when help is rendered, a soft debt is tacitly accrued by the beneficiary to the provider. The agents make decisions on soliciting and offering help based on the soft debt balance between them. The notion of soft debt is evident from the daily use of words like "owe", "repay", "price", "indebted" between acquaintances even when no contract is involved. Like a seller in the market, the provider voluntarily offers help if and only if the the soft price she earns (the increase in her value in the relationship due to higher debt holding) outweighs her cost of help. On the other hand, the beneficiary, like a buyer, accepts help if and only if the benefit is large enough to cover his soft cost (the drop in value). I call such voluntary favor trading without contracts "soft transactions", as opposed to the familiar "hard transactions" carried out in compliance with contracts. Similarly, the beneficiary and helper are called "soft buyer" and "soft seller" respectively.

There are two main advantages of using the soft transaction approach to model cooperation. First, along with hard transactions, soft transactions constitute a unified approach to analyzing a host of economic activities. When individuals have needs (e.g. to move to a new home), often they can compare the hard (hire a mover) and soft (ask a friend for help) alternatives side-by-side.[1] Conversely when they have goods or services to offer (e.g. baby-sitting), again they can opt for either hard (get paid) or soft (help friends for "free") transactions. Both types of transactions stem from similar cost and benefit analysis medi-

[1] Of course, self production is usually another alternative.

ated by prices, be they hard or soft, although in the latter case the prices and terms are not specified. Second, while favor trading models based on fixed payoffs could generate interesting results (e.g. Möbius (2001) and Hauser and Hopenhayn (2008) take the net number of times a favor is transferred as the debt), allowing random benefits and costs would support a richer variety of results and better capture the nuances of long term relationships. For instance, potential trades can be inefficient under random benefits and costs. Assuming fixed benefits and costs is akin to fixing the value and cost in hard transactions. With randomness, players consider not only the number of times favors are traded, but also the particular benefits and costs both in the past and the expected future.

The availability of both hard and soft transactions begs the question of which one is chosen under what situations. The key advantage of soft transactions over the hard alternatives is their saving in transaction costs. By their very nature, soft transactions involve no (or minimal) negotiation of price and conditions. (The absence of formal contracts also means no formal record keeping and no taxes.) On the other hand, the lack of formal enforcement mechanism means that they are harder to start between strangers. It is therefore unsurprising that soft transactions are preferred when the needs are personal and specific, for which customization would be valuable but contracting would be costly. They are well suited for interactions between acquaintances, especially when repeated interactions would further lower the costs as players learn more about each others' tastes, habits and costs. Also, since the favors are often non-standardized, they tend to be personal services rather than tangible goods. In contrast, when the need is general and standardized, hard transactions gain the advantage by exploiting the economies of scale through mass production and routine transactions.[2]

A second factor that distinguish between the two alternatives concerns the medium of exchange. While hard transactions enjoy the benefits of having money as the medium of exchange, soft transactions are essentially tacit barters that occur over time, which require both parties to have something that the other wants. Even if multilateral favor trading is

[2]Economists have long recognized the differences between personal interactions between acquaintances and anonymous market transactions. Adam Smith examined the two types of social interactions in *The Theory of Moral Sentiments* and *The Wealth of Nations* respectively.

allowed in a group through indirect favors, as in Möbius (2001), trading in soft debts still cannot match the flexibility and convenience of trading in money. Therefore soft transactions are more viable when there are common interest between the parties. As a result of these two factors, soft transactions are pervasive everywhere from the family to the neighborhood to the workplace.[3] Spouses share household duties; neighbors trade favors such as baby-sitting and house-sitting; research collaborators take turns in contributing to their projects, all probably without formal contracts. The reliance on personal contacts also suggests that soft transactions play a particularly strong role in the social fabric of developing countries. Their lack of formal records makes it difficult to compare their size with the hard sector, but a moment of casual observation would reveal that they are pervasive in essentially everyone's daily economic activities.

Soft transactions may also be advantageous in situations where silent mural understanding is preferred to explicit agreement (e.g. tacit collusion between businesses). In some other cases, hard transactions are simply unavailable for legal and ethnic reasons. For instance, researchers can contract out tasks only to a certain extent.

Soft transactions could also play a role in property rights allocation problems. For example, although auctioning would be an efficient way for a family to allocating the right to choose TV programs, apparently it is seldom adopted in reality. Instead family members would make compromises, probably without negotiating explicit terms of compensation to the conceder, who nevertheless expects to be compensated (or have some of his soft debt offset). The same kind of tacit give-and-take reciprocity is as well practiced between friends, neighbors and coworkers, etc.

Despite the pervasiveness of favor trading, not many studies are done directly on the topic. Möbius (2001) is perhaps the first to study the topic, allowing indirect favors to be made within a group. Hauser and Hopenhayn (2008) follow the model and allow exchange rate between current and future favors to deviate from one. Nayyar (2009) studies a discrete time version of the Hauser and Hopenhayn (2008) model. Kalla (2010) examines the the

[3]Although a marriage is often accompanied by a contract, the "contract" is probably far too vague to make the marriage a hard transaction.

effects of incomplete information and concave utility function. All of these papers assume fixed benefits and costs.

The rest of the paper is organized as follow. Section 2 presents a general set-up for a two-player stochastic game of soft transactions, and formally defines soft debts and soft prices. Section 3 focuses on a discrete benefits model where the cost to benefit ratio is constant. Under the model, there always exists stationary Markov equilibria where the players trade if and only if the debt is within certain limits, as long as the costs meet certain conditions. Section 4 modifies the model by assuming the cost associated with each benefit is random and observable only to the seller. The random cost may exceed the benefit. Stationary Markov equilibria could exists where additional requirements are imposed by the seller. Finally, the conclusion lays out some potential interplays between hard and soft transactions.

2 General Model

2.1 Game structure

Two players, player 1 and player 2, are randomly matched from a large population to play an infinitely repeated game, starting in round 1. At the beginning of each round of stage game, one player is assigned the role of (soft) buyer, who needs help from the other player. The other player is assigned as the (soft) seller, who is the only one in the population that can render the help. In each round, player i ($i = 1, 2$) is assigned as the buyer with probability π^i, which is fixed across rounds, so that $\pi^1 + \pi^2 = 1$. The probabilities are common knowledge.

In round t, if the help is rendered, the buyer will receive a benefit of $b_t > 0$ and the seller (player j) will incur a cost of $c_t > 0$. The joint distribution of $\left(b_t^i, c_t^i\right)$ is independent and identically distributed (i.i.d.), which is again common knowledge. The role assignment, b_t and c_t are revealed to both players once drawn.

After the drawing, the buyer and seller simultaneously decide whether to buy and sell respectively. This is the only decision the players need to make in each round. The decisions are then revealed to both players. A soft transaction occurs (i.e. help is rendered) if and only if *both* the buyer buys and the seller sells. If help is not rendered, both players receive zero payoffs for this round. The game proceeds to the next round.

Unlike typical repeated games, the rendering of help requires the consent of not just the provider (the seller), but also the beneficiary (i.e. the buyer). The reason for this requirement is that the help, as in the case of hard transactions, does not come free. As explained in the next subsection, there is a (soft) price in terms of soft debt to be paid. Even if the seller finds it profitable to sell, the buyer may not want to buy.

In the model the buyer and seller are treated as if they make decisions simultaneously. In practice, since the soft transaction occurs only if both players agree, it makes no difference if one of them initiate the offer, and then the other respond, as long as one's decisions does not depend on the other's.

When deciding whether to trade or not in round τ, player i's objective is to maximize the expectation of discounted sum of current and future payoffs:

$$E_\tau \sum_{t=\tau}^{\infty} \left(\delta^i\right)^{t-\tau} \left(b_t^i - c_t^i\right)$$

where E_τ denotes the expectation operator conditional on player i's information set in round τ, and $\delta^i \in (0,1)$ represents player i's discount factor.[4] Note that in each round, one or both of b_t^i and c_t^i will be zero, depending on what role he is assigned and whether the soft transaction goes through. Both discount factors are common knowledge and reflect the chance of future meetings as well as the patience levels of individual players. This completes the specification of the stochastic game with complete information and perfect monitoring (both observe the full history of all actions).

[4] There is no player superscript for the expectation operator because the players posses identical information relevant to decision making.

2.2 Strategy

Following the notion of soft transactions, I focus on strategies that prescribe actions based on the soft debt.

In each round, if soft transaction occurs, then the buyer accrue a debt of $d(b, c)$ to the seller, where b is the benefit to the buyer and c is the cost to the seller. Assume $d > 0$ for all b and c; and d is increasing in both b and c. Then the net soft debt balance owed by player j to the other player i at the beginning of round τ is

$$D_\tau^i = \sum_{t=1}^{\tau-1} \left[d\left(b_t^j, c_t^i\right) - d\left(b_t^i, c_t^j\right) \right]$$

with $D_1^i = 0$. In other words, D_τ^i is the (net) debt holding of player i at the beginning of round τ. Obviously, $D_\tau^1 = -D_\tau^2$. Note also that for simplicity no "interest" (positive or negative) is charged on the debt.[5]

Now suppose player 2's decisions depend on the entire history of actions and information only through the current debt level. Since player 1's payoff depends on player 2's decisions, at the beginning of the round player 1's expected future value of the relationship (derived from his best response to her strategy) also depends only on the debt level (in addition to the parameters). Let $V^1\left(D^1\right)$ be player 1's expected future value at the beginning of the round. After the drawing player 1 will make his decision based on D^1 together with the role assignment, benefit and cost. Therefore player 1 decision, like his counterpart's, also depend on the entire history only through the current debt level. Reciprocally let $V^2\left(D^2\right)$ be player 2's expected future value at the beginning of the round. V^1 is determined by V^2, and vice versa. The more generous player 1 is, the more likely he will help and therefore player 2's future value is also higher. This in turn would make player 2 more generous. Since the probability distributions for role assignment, benefits and costs are all i.i.d. across time,

[5] Unlike the nominal interest rate in hard transactions, the interest rate here can be negative, which probably reflects fading of memories.

the value functions are stationary. An equilibrium will be a stationary Markov equilibrium, with state variables being the debt level and the drawn assignment, benefit and cost.

Suppose that in the current round, player i is assigned as the buyer and player j the seller. For simplicity, drop time subscripts. Player i will ask for help if the benefit outweighs the reduction in the value of his future payoff. That is, :[6]

$$b^i > p_b^i$$

where $p_b^i \equiv \delta^i \left[V^i\left(D^i\right) - V^i\left(D^i - d\left(b^i, c^j\right)\right)\right]$ is the soft buying price player i would pay if the soft transaction goes through.

Player j will offer help if the cost is more than covered by the increase in the value of her future payoffs:

$$c^j < p_s^j$$

where $p_s^j \equiv \delta^j \left[V^j\left(D^j + d\left(b^i, c^j\right)\right) - V^j\left(D^j\right)\right]$ is the soft selling price player j would earn from the soft transaction.

The soft price is the increase (for the seller) or decrease (for the buyer) in the future value of the relationship resulting from the change in the debt position. Unlike hard prices, the soft price paid by the buyer and that received by the seller can be different because there is no money or other exchange media that defines a single price.

In general, the value function is composed of four components that correspond to the following scenarios: player i is the buyer and he is helped; he is the buyer but is not helped; he is the seller and he helps; he is the seller and does not help. Denote by $H^i(D)$ the event that player i is helped given that he is the buyer with debt holding D, then (tilde is added to emphasize random variables):

[6] Assume that players do not buy or sell if they are indifferent.

$$\Pr\left(H^i(D)\right) = \Pr\left(\widetilde{b}^i > \widetilde{p}_b^i(D) \text{ and } \widetilde{c}^j < \widetilde{p}_s^j(-D)\right)$$

where $\widetilde{p}_b^i(D) \equiv \delta^i\left[V^i(D) - V^i\left(D - d\left(\widetilde{b}^i, \widetilde{c}^j\right)\right)\right]$ and $\widetilde{p}_s^j(-D) \equiv \delta^j\left[V^j\left(-D + d\left(\widetilde{b}^i, \widetilde{c}^j\right)\right) - V^j(-D)\right]$ are the random buying price and selling price respectively.

The value function of player i at the beginning of a round can be formulated recursively:

$$\begin{aligned} & V^i(D) \\ = \quad & \pi^i \left\{ \begin{array}{cl} & \Pr\left(H^i(D)\right) E\left[\widetilde{b}^i + \delta^i V^i\left(D - d\left(\widetilde{b}^i, \widetilde{c}^j\right)\right) \mid H^i(D)\right] \\ + & \left(1 - \Pr\left(H^i(D)\right)\right) \delta^i V^i(D) \end{array} \right\} \\ & + \pi^j \left\{ \begin{array}{cl} & \Pr\left(H^j(-D)\right) E\left[-\widetilde{c}^i + \delta^i V^i\left(D + d\left(\widetilde{b}^j, \widetilde{c}^i\right)\right) \mid H^j(-D)\right] \\ + & \left(1 - \Pr\left(H^j(-D)\right)\right) \delta^i V^i(D) \end{array} \right\} \end{aligned}$$

I further simplify the notations by defining the following conditional probability and conditional mean benefit and cost:

$P^i(D) \equiv \Pr\left(H^i(D)\right)$, $\overline{b}^i(D) \equiv E\left(\widetilde{b}^i \mid H^i(D)\right)$, $\overline{c}^i(-D) \equiv E\left(\widetilde{c}^i \mid H^j(-D)\right)$, for $i = 1, 2$, $j \neq i$. Then

$$\begin{aligned} V^i(D) = \quad & \pi^i \quad \left\{P^i(D)\left[\overline{b}^i(D) + \delta^i E\left[V^i\left(D - d\left(\widetilde{b}^i, \widetilde{c}^j\right)\right) \mid H^i(D)\right]\right] + \left(1 - P^i(D)\right)\delta^i V^i(D)\right\} \\ & + \pi^j \quad \left\{P^j(-D)\left[-\overline{c}^i(-D) + \delta^i E\left[V^i\left(D + d\left(\widetilde{b}^j, \widetilde{c}^i\right)\right) \mid H^j(-D)\right]\right] + \left(1 - P^j(-D)\right)\delta^i V^i(D)\right\} \end{aligned} \tag{1}$$

Upon rearranging terms,

$$V^i(D) = \tfrac{1}{1-\delta^i} \left\{ \begin{array}{c} \pi^i\left\{P^i(D)\left[\overline{b}^i(D) - \overline{p}_b^i(D)\right]\right\} \\ + \pi^j\left\{P^j(-D)\left[-\overline{c}^i(-D) + \overline{p}_s^i(D)\right]\right\} \end{array} \right\} \tag{2}$$

where

$$\overline{p}_b^i(D) \equiv E\left[\widetilde{p}_b^i(D) \mid H^i(D)\right],$$

$$\overline{p}_s^i(D) \equiv E\left[\widetilde{p}_s^i(D) \mid H^j(-D)\right], \text{ for } i = 1, 2$$

Some key terms are interpreted as follow (compare to p_b and p_s above):

- $\overline{p}_b^i(D)$: player i's conditional mean buying price with debt holding D
- $\overline{p}_s^i(D)$: player i's conditional mean selling price with debt holding D
- $\overline{b}^i(D) - \overline{p}_b^i(D)$: player i's conditional mean buying surplus with debt holding D
- $-\overline{c}^i(-D) + \overline{p}_s^i(D)$: player i's conditional mean selling surplus with debt holding D

(2) shows that a player's future value equals the discounted sum of surpluses weighted by the chances of buying and selling.

For the general model, the following observations can be made (all proofs are relegated to the Appendix).

Lemma 1. *For stationary Markov equilibriums in which the strategies depend on the entire history only through the soft debt level,*

(i) the first best allocation can never be achieved; and

(ii) an autarky equilibrium always exists.

Part (i) of the Lemma states that players who reduce the history to the single state variable of soft debt level can never attain first best allocation. Contrast this result with subgame-perfect equilibria that allows the strategy to depend on the entire history, for which first best allocation is possible. According to the folk's theorem the first best outcome can be achieved as long as the discount factors are high enough. The simplest example is the "single defection trigger strategy" that prescribes the player to (1) buy or sell iff the transaction is efficient as long as both players keep doing so; (2) refuse to buy or sell forever once (1) is violated by either player. Then the first best allocation is achieved if the cost to the seller is never high enough to justify foregoing the future value of the relationship:

$$c^i_{max} < \frac{1}{1/\delta^i - 1}\left[\pi^i E\left(\widetilde{b}^i \mid \widetilde{b}^i > \widetilde{c}^j\right) - \pi^j E\left(\widetilde{c}^j \mid \widetilde{b}^j > \widetilde{c}^i\right)\right] \text{ for } i = 1, 2,\ j \neq i$$

where c^i_{max} is an supremum of the support of the distribution of $\widetilde{c}^i$.

While part (ii) of the Lemma confirms that a stationary Markov equilibrium always exists, finding equilibria with trade would require some simplifying assumptions to the general model, as the next section demonstrates.

3 Discrete Benefit Model with Proportional Cost

Assume $\delta^1 = \delta^2 = \delta$, $\pi^1 = \pi^2 = \frac{1}{2}$, (b, c) are identically distributed for the two players, and focus on symmetric equilibria where V^1 and V^2 coincide. Consider the case where b follows a uniform discrete distribution, realizing each outcome of $1, .., M$ with probability $\frac{1}{M}$. The cost corresponding to each benefit of m is fixed at αm where$\alpha \in (0, 1)$ is constant, so the first best outcome is for the players to trade in all rounds. Although b is not continuous, in practice the benefit can be measured in units as small as one wants.

Suppose $d(\cdot, \cdot)$ is homogenous of degree 1, then $d(m, \alpha m) = m \cdot d(1, \alpha)$, and $D^i_\tau = \left[\sum\limits_{t=1}^{\tau-1}\left(m^j_t - m^i_t\right)\right] \cdot d(1, \alpha)$. Therefore we can use the cumulative net benefit as an index for the debt balance. Denote by V_k the expected future value when the cumulative net benefit is k. Define $p_{k+m,k} \equiv \delta\left(V_{k+m} - V_k\right)$, which is the soft price received by the seller holding k debt for providing a benefit of m (or equivalently the soft price paid by the buyer for a benefit of m when his debt holding is $k + m$).

The following proposition shows that when the costs are low enough, simple "debt limit strategies" would constitute equilibria with trade.

Proposition 1. *There exist stationary Markov equilibriums (called "debt limit equilibriums") in which both players follow the "debt limit strategy", i.e. the buyer buys and the seller sells iff the resulting debt level does not exceed a limit* $L \leq \frac{M}{2}$, $L \in \{1, 2, ..\}$ *if*

$$\alpha < \frac{1}{2M\left(1/\delta - 1\right) + 1} \tag{3}$$

L represents the range of soft debt that the players engage in. A higher L means more opportunity for soft transactions. Therefore L can be regarded as depth of relationship.

Obviously the lower M is, or the higher δ is, the easier it is for α to satisfy the condition. A higher M means a higher potential limit L. Then the cost ratio needs to be low enough to entice the players to engage in a high debt limit equilibrium. On the other hand, if δ is high, which means the players are patient and meet frequently, then a higher cost ratio can be supported in the equilibrium. The cost ratio can be arbitrarily close to 1 as δ approaches 1.

For deep relationships like those between family members, δ is high because they meet often, and α is low as the cost of providing a certain amount of benefit tends to be lower if the recipient is a familiar acquaintance. Therefore a high M, and thus L can be supported. Conversely, between strangers δ is low and α is high, so only a low L is possible.

Lemma 1 states that the stationary Markov equilibrium based on soft debt cannot achieve first best allocation. Proposition 1 in particular limits the trade to within a debt limit of $M/2$. On the other hand, if the simple single defection strategy is used, it is straightforward to show that full trade equilibrium is obtained when

$$\alpha < \frac{1}{4\left(1/\delta - 1\right) + 1}$$

Not only is the requirement on the cost ratio less stringent than in the debt limit equilibrium, the trade is also not subject to limits.

It is obvious from Proposition 1 that multiple debt limit equilibria with different limits can exist, as long as (3) is satisfied. In addition there always exists the no-trade equilibrium.At first glance the equilibrium seems indeterminate. However, a closer examination would reveal that players will always incline to reach the highest equilibrium limit supported by the cost structure. First I state the following proposition that will be useful for the argument.

Proposition 2. *Denote by V_k^L the expected future value in debt limit equilibrium with limit L when the debt holding is k. Then $V_k^{L'} > V_k^L$ iff $L' > L$ for all $k = 0, \pm 1, .., \pm L$.*

In any round, the remaining candidates of equilibrium limits are bounded between the one immediately above the maximum debt attained thus far at the lower end, and the one immediately below the lowest debt that could have attained but did not at the upper end. Now suppose in the current round the benefit and cost are drawn, and if transaction occurs the debt will exceed some remaining limits but not the maximum one. The buyer in this round would have no reservation in buying. If the seller sells, not only that the buyer will gain from the transaction, the transaction will also signify a higher limit that brings more mutually beneficial trades in the future (as shown in Proposition 2). If the seller refuses to sell, there is no harm in buying anyway. Conversely, the same logic applies to the seller. Thus choosing higher limit will be strictly more profitable to each player given that all other players also follow. Being aware of each other's calculation, the players will always agree to trade within the maximum limit.

4 Discrete Benefit Model with Random Private Cost

In Section 3, the equilibrium outcome of the single deviation trigger strategy dominates that of the debt limit strategy in terms of efficiency. This section modify the discrete benefit model by assuming instead of a fixed cost to benefit ratio, that the cost is random and only observable to the seller. In particular, for each benefit $m \in \{1, 2, .., M\}$, the corresponding cost $\widetilde{c}_m$ is uniformly distributed over $(0, 2\overline{c}_m)$, so that $E(\widetilde{c}_m) = \overline{c}_m$. The highest cost $2\overline{c}_m$ can be greater than m, so that potential transactions can be inefficient. The distribution of

$\widetilde{c}_m$ is common knowledge. Since the actual costs is unobservable to the buyer, costs do not enter the soft debt formula and the cumulative net benefit is taken as the soft debt level, i.e., $D^i_\tau = \left[\sum_{t=1}^{\tau-1}\left(m^j_t - m^i_t\right)\right]$. Like before, V_k denotes the expected future value when the soft debt holding is k, and $p_{k+m,k}$ denotes the soft price $p_{k+m,k} \equiv \delta\left(V_{k+m} - V_k\right)$.

When cost is unobservable to the buyer, strategies that rely on complete full history including costs are not readily applicable. In particular, trigger strategies that prescribes punishment can no longer rely on efficiency as the criterion for defection. On the other hand, equilibrium with trade is possible with strategies that rely on the soft debt, as the following proposition illustrates.

Proposition 3. *(i) There exists stationary Markov equilibrium (called "restricted debt limit equilibrium") in which both players follow the "restricted debt limit strategy":*

- the buyer buys iff the resulting debt level does not exceed a limit $L \leq \frac{M}{2}$, $L \in \{1, 2, ..\}$,

- the seller sells iff the resulting debt level does not exceed L *and* $c_m < \overline{c}_m$

(ii) all transactions that occur under the equilibrium are efficient, i.e. $c_m < m$, $m = 1, 2, .., 2L$

if $\overline{c}_m$ *satisfies the following condition:*

$$\overline{c}_m < p_{k+m,k} = \frac{m\left(L + k + \frac{m+1}{2}\right) + \frac{1}{2}\sum_{r=0}^{m-1}\overline{c}_{L-k-r}}{4M\left(1/\delta - 1\right) + 2L + 1}\ ,\ k = -L, -L+1, .., L-m \qquad (4)$$

Moreover, there always exists some set of $\{\overline{c}_m > 0,\ m = 1, .., 2L\}$ *that satisfies this condition.*

Compared to the debt limit strategy, the restricted debt limit strategy imposes an additional requirement for the seller to trade. Specifically, the seller sells only if the cost drawn is below the mean value. The additional requirement is imposed because it can be shown that for the players to play the (unrestricted) debt limit strategy even if the highest costs are drawn, the IC (seller) will require the highest costs to be below the corresponding benefits. Then the more interesting case where potential trades can be inefficient is ruled out.

Again L measures the depth of the relationship. The more frequently the players meet (higher δ), or the more efficient they are in help each other (lower $\bar{c}_m$'s in general), the higher L can be sustained. Therefore the debt limit is highest in closely knit groups where members understand each other's needs well, such as the family. At the other extreme, the debt limit would be very low between strangers.

The restriction $c_m < \bar{c}_m$ together with (4) form the incentive compatibility (IC) condition for the seller. As shown in the proof, the IC for seller guarantees the IC for buyer as well as the individual rationality (IR) condition. To understand the formula for the soft price, it would be easier to start with the one-step soft price (see the proof for its derivation):

$$p_{k+1,k} = \frac{L + k + 1 + {}^{\bar{c}_{L-k}}/_{2}}{4M\,({}^{1}/_{\delta} - 1) + 2L + 1}$$

This is the increase in future value when the debt holding increases from k to $k+1$. The value increase for two reasons. First, the higher debt holding opens up the opportunity to receive a maximum benefit of $L + k + 1$ (which will bring his debt position to the lowest limit). Second, at the same time he owe his counterpart one less (or she owes him one more), so he will not help if the benefit drawn is $L - k$ or more (the ${}^{\bar{c}_{L-k}}/_{2}$ term). These factors are adjusted by the denominator for realizing these additional values under different possibilities in different future time. The higher M is, the lower the chance that the future benefits and costs fall within the debt limit and hence the less likely that the values can be realized soon. On the other hand, the higher δ is, either because the agents meet more frequently or they are more patient, the higher the values will be. The m-step soft price $p_{k+m,k}$ is just the summation of m number of one-step soft price.

As in Section 3, although multiple equilibrium may exist, the players will gravitate toward the highest limit $L = \frac{M}{2}$.

The existence of restricted debt limit equilibrium highlights the difference between the soft transaction approach and that based on full history and threat of punishment. In soft transactions, the players rely on each other's profit motivation to carry out trades. Even if the seller's cost is unobservable, the buyer does not need to guess whether the seller has "cheated" by not helping him. The seller will help if she finds it profitable. In the current model, since costs do not enter the formula for soft debt, there is no reason for the seller to fake costs.

5 Conclusion

There are several directions that the soft transaction can be extended. First, complete information is assumed in the general model in order to focus on the essence of soft transactions. More sophisticated information structure could be investigated. For instance, in the model benefits and costs are observable by both players. More realistically, the benefits (costs) are at best only imperfectly observed by the seller (buyer), making the soft price "fuzzy". More guesswork would be required on the part of the players.

Second, the paper focuses on isolated bilateral relationship and avoids mentioning soft markets. The model assumes the players are randomly matched and only the seller can help the buyer. But often the buyer would be able to choose from different sellers, and vice versa. Just like in the hard market, the soft prices will be the driving force behind the player's choices. However, here whether a relationship is exclusive (e.g. marriage) or non-exclusive (e.g. friends) would affect the player's decisions since breaking up an exclusive relationship can be costly. Another possibility is to consider multilateral relationship in a group, where indirect favors could be granted (as in Möbius (2001)).

Third, the paper presumes a simple dichotomy of hard and soft transactions, while in reality there would be many "hybrid" transactions. For example, an employment contract contains both the hard (employment contract) and soft (vague duties within "reasonable" bounds). The relational contract literature (e.g. Bull (1987), Levin (2003)) is pertinent to the subject since it is concerned with combining explicit and implicit incentives in repeated long term relations. In fact, to the extent that a contract is incomplete, there is always some "softness" in it and hence there is potential for long term relationship. In purely hard transactions which allow no room for ambiguity, if they ever exist, the parties would simply trade and part.

The presence of hybrid transactions may also help explain the puzzle that providing rewards and punishments sometimes has perverse effects (see Bénabou and Tirole (2006) for a collection of examples). These phenomena suggest that analysis based purely on the hard elements are incomplete. For example, Gneezy and Rustichini (2000) reports that fining late parents in an Israeli day-care center actually resulted in *more* late arrivals. Before the fine system, when the parents are late they may expect to pay a soft price in one form or another in the future. They would avoid being late if there are limited means to repay the soft debt. But with the fines in place, they may take the hard prices as substitutes for the soft prices, and therefore less worry about being late.

Lastly, a note on motivation of individuals is due. As in standard neo-classical economics, this paper takes profit maximization as individuals' only objective. It actually even portraits what normally perceived as the most intimate relationships as results of self-interest maximization. But undeniably altruism plays an important role in human relationship. Evolutionary biology (e.g. Trivers (1971)) provides the theoretical basis of altruism. However, the two perspectives need not be mutually exclusive. People are willing to sacrifice for no return, especially for loved ones and sometimes even for strangers. But even between the closest relationships, there are often give-and-take interactions that are better described by soft transactions.

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Appendix

Proof of Lemma 1

Proof. (i) Assume on the contrary that the first best allocation is attained in the stationary Markov equilibrium. Since help will be rendered iff the benefit exceeds the cost, the expected future value stays unchanged regardless of the soft debt level (recall that the role assignment, benefit and cost are i.i.d.). Let V^i be the constant expected future value be for player i. Then the selling price will be invariably zero for both players:

$$\widetilde{p}_s^i\left(D\right) \equiv \delta^i \left[V^i\left(D + d\left(\widetilde{b}^i, \widetilde{c}^j\right)\right) - V^i\left(D\right)\right] = \delta^i\left(V^i - V^i\right) = 0 \text{ , } i = 1, 2$$

But then the seller will never sell since cost is positive, so no transactions will occur. Therefore first best allocation can never be achieved.

(ii) If one player's strategy is to never buy or sell regardless of the debt level, the best response of the the other player is to follow the same strategy. In terms of future values, this means settingV^1 and V^2 to zero invariably. Trade never occurs in this equilibrium. □

Proof of Proposition 1

Proof. I first compute the future values assuming that both players follows the debt limit strategy. Next by verifying the incentive compatibility (IC) and individual rationality (IR) conditions using (3), I argue that in any round the best response to the debt limit strategy is indeed to adopt the same strategy.

1. Computation of value functions

Starting with a debt level between $-L$ and L in any arbitrary round, since $M \geq 2L$, the debt level after a transaction (if it occurs) can be any integer within the same range. Given that all transactions go through if the debt after transaction falls within $[-L, L]$, then for $-L \leq k \leq L$,

$$\begin{aligned} V_k = \quad & \frac{1}{2}\left[\frac{1}{M}\sum_{m=1}^{L+k}(m+\delta V_{k-m}) + \left(1-\frac{L+k}{M}\right)\delta V_k\right] \\ + \quad & \frac{1}{2}\left[\frac{1}{M}\sum_{m=1}^{L-k}(-\alpha m+\delta V_{k+m}) + \left(1-\frac{L-k}{M}\right)\delta V_k\right] \end{aligned} \tag{5}$$

In the first bracket the player is assigned as the buyer. The summation term in the bracket refers to the benefits and resulting future values if transaction occurs (when the resulting debt is within L), while the next term captures the no transaction case. Similarly, in the second bracket the player is drawn as the seller.

Group all V_k terms to the left hand side,

$$2\left[M(1-\delta)+L\delta\right]V_k = \sum_{m=1}^{L+k}(m+\delta V_{k-m}) + \sum_{m=1}^{L-k}(-\alpha m+\delta V_{k+m}) \tag{6}$$

where $\sum_{l=1}^{0} \equiv 0$.

Iterate k forward to $k+1$, then for $-L-1 \le k \le L-1$,

$$2\left[M\left(1-\delta\right)+L\delta\right]V_{k+1}=\sum_{m=1}^{L+k+1}\left(m+\delta V_{k-m+1}\right)+\sum_{m=1}^{L-k-1}\left(-\alpha m+\delta V_{k+m+1}\right) \tag{7}$$

Subtract (6) from (7), for $-L \le k \le L-1$,

$$2\left[M\left(1-\delta\right)+L\delta\right]\left(V_{k+1}-V_k\right)=\left(L+k+1\right)+\delta V_k+\alpha\left(L-k\right)-\delta V_{k+1}$$

$$\left[2M\left({}^{1}\!/\!_{\delta}-1\right)+2L+1\right]\delta\left(V_{k+1}-V_k\right)=L+k+1+\alpha\left(L-k\right)$$

Recall the definition of $p_{k+1,k} \equiv \delta\left(V_{k+1}-V_k\right)$,

$$p_{k+1,k}=\frac{L+k+1+\alpha\left(L-k\right)}{2M\left({}^{1}\!/\!_{\delta}-1\right)+2L+1} \tag{8}$$

Since $p_{k+m,k}=\sum_{r=0}^{m-1} p_{k+r+1,k+r}$ for $m=1,..,L-k$,

$$p_{k+m,k}=\frac{L+k+\frac{m+1}{2}+\alpha\left(L-k-\frac{m-1}{2}\right)}{2M\left({}^{1}\!/\!_{\delta}-1\right)+2L+1} \tag{9}$$

2. Verification of IC and IR

Now I show that under (3) the players indeed find it profitable to trade whenever the debt level will remain within L. For $k=0,\pm1,..,\pm L$, the IC and IR conditions are as follow:

$$\text{IC (buyer): } p_{k,k-m} < m \text{ if } m=1,..,L+k$$

$$\text{IC (seller): } p_{k+m,k} > \alpha m \text{ if } m = 1, .., L - k$$

$$\text{IR: } V_k > 0$$

By (9), IC (seller) can be rewritten as:

$$\alpha < \frac{L + k + \frac{m+1}{2}}{2M\left(1/\delta - 1\right) + L + k + \frac{m+1}{2}}$$

Since k is lowest at $k = -L$ and m is lowest at $m = 1$, therefore $L + k + \frac{m+1}{2} \geq 1$. By substituting this lowest value in IC (seller), we can obtain a sufficient condition for IC (seller):

$$\alpha < \frac{1}{2M\left(1/\delta - 1\right) + 1}$$

which is (3) in the proposition.

I now show that IC (buyer) is guaranteed by IC (seller). First note that IC (buyer) is equivalent to:

$$p_{k+1,k} < 1 \ , \ k = -L, -L+1, .., L-1$$

Use (8) and rearrange terms, IC (buyer) becomes:

$$\alpha < \frac{2M\left(1/\delta - 1\right) + L - k}{L - k}$$

which must hold because $\alpha < 1$.

To verify IR, since V_k is increasing in k (as $p_{k+1,k} > 0$), it is sufficient to show that $V_{-L} > 0$.

By (6), when $k = -L$,

$$\begin{aligned}
2\left[M\left(1-\delta\right)+L\delta\right]V_{-L} &= \sum_{m=1}^{2L}\left(-\alpha m+\delta V_{-L+m}\right) \\
&= \sum_{m=1}^{2L}\left(-\alpha m+\delta V_{-L}+p_{-L+m,-L}\right) \\
2M\left(1-\delta\right)V_{-L} &= -\sum_{m=1}^{2L}\alpha m+\sum_{m=1}^{2L}p_{-L+m,-L}
\end{aligned}$$

which is positive according to IC (seller). Therefore IR also holds given (3).

To summarize, for each player, given that his counterpart buys and sells iff the resulting debt level is within L, he will gain if he follow the same strategy, and he always wants the relationship to persist. $\square$

Proof of Proposition 2

Proof. Restate (5) for V_k^L:

$$\begin{aligned}
V_k^L = \quad & \frac{1}{2}\left[\frac{1}{M}\sum_{m=1}^{L+k}\left(m+\delta V_{k-m}^L\right)+\left(1-\frac{L+k}{M}\right)\delta V_k^L\right] \\
+ \quad & \frac{1}{2}\left[\frac{1}{M}\sum_{m=1}^{L-k}\left(-c_m+\delta V_{k+m}^L\right)+\left(1-\frac{L-k}{M}\right)\delta V_k^L\right]
\end{aligned}$$

where $c_m = \alpha m$

Compare it to $V_k^{L'}$, written as:

$$
\begin{aligned}
V_k^{L'} = \quad & \frac{1}{2}\left\{\left[\frac{1}{M}\sum_{m=1}^{L+k}\left(m+\delta V_{k-m}^{L'}\right)+\left(1-\frac{L+k}{M}\right)\delta V_k^{L'}\right]+\frac{1}{M}\sum_{m=L+k+1}^{L'+k}\left(m+\delta V_{k-m}^{L'}-\delta V_k^{L'}\right)\right\} \\
+ \quad & \frac{1}{2}\left\{\left[\frac{1}{M}\sum_{m=1}^{L-k}\left(-c_m+\delta V_{k+m}^{L'}\right)+\left(1-\frac{L-k}{M}\right)\delta V_k^{L'}\right]+\frac{1}{M}\sum_{m=L-k+1}^{L'-k}\left(-c_m+\delta V_{k+m}^{L'}-\delta V_k^{L'}\right)\right\}
\end{aligned}
$$

Note that the two terms outside the brackets are both positive because they are the surpluses from trades. Besides those two positive terms, the only difference between the two formulas is that the discountedV^L terms in the first are replaced by discounted $V^{L'}$terms in the second. But we can expand the V^L and $V^{L'}$ terms again in the same fashion as above. $V^{L'}$ again has two extra positive terms over V^L. Repeat the process, the difference between the $V^L$and $V^{L'}$terms tend to zero because $\delta < 1$. Due to the two extra positive terms that accumulate in each iteration, $V_k^{L'} > V_k^L$.

When there arise a round where the drawing reveals that a transaction would push the debt level over k for the first time but remain below k'. Each player will be better-off by trading given the other player will trade too. Their future value for the same debt level will be higher, and they will gain from the transaction. The equilibrium with the highest limit dominates all the rest. □

Proof of Proposition 3

Proof. Like in the proof for Proposition 2, I first compute the future values assuming that both players follows the debt limit strategy, and then verify the IC and IR conditions. In verifying the IC conditions, I also show that all transactions that occur are efficient. Lastly I confirm that the (4) is always met by some set of costs.

1. Computation of value functions

Given that all transactions go through if the debt after transaction falls within $[-L, L]$ and $c_m < \bar{c}_m$, then for $-L \le k \le L$,

$$
\begin{aligned}
V_k = \quad & \frac{1}{2}\left[\frac{1}{2M}\sum_{m=1}^{L+k}\left(m+\delta V_{k-m}\right)+\left(1-\frac{L+k}{2M}\right)\delta V_k\right] \\
+ \quad & \frac{1}{2}\left[\frac{1}{2M}\sum_{m=1}^{L-k}\left(-\frac{\overline{c}_m}{2}+\delta V_{k+m}\right)+\left(1-\frac{L-k}{2M}\right)\delta V_k\right]
\end{aligned}
$$

Compared to the V_k in (5) of the proof of Proposition 1, $1/M$ is replaced by $1/2M$ since the chance of transaction is halved, and the expected cost αm is replaced by $\frac{\overline{c}_m}{2}$.

Following similar steps as in Proposition 1, we get:

$$
p_{k+1,k} = \frac{L+k+1+\overline{c}_{L-k}/2}{4M\left(1/\delta-1\right)+2L+1} \tag{10}
$$

Again, since $p_{k+m,k} = \sum_{r=0}^{m-1} p_{k+r+1,k+r}$ for $m = 1,..,L-k$,

$$
p_{k+m,k} = \frac{m\left(L+k+\frac{m+1}{2}\right)+\frac{1}{2}\sum_{r=0}^{m-1}\overline{c}_{L-k-r}}{4M\left(1/\delta-1\right)+2L+1} \tag{11}
$$

2. Verification of IC and IR

Now I show that under (4) the players indeed find it profitable to trade whenever the debt level will remain within L. For $k = 0, \pm 1, .., \pm L$, the IC and IR conditions are as follow:

IC (buyer): $p_{k,k-m} < m$ if $m = 1,..,L+k$

IC (seller): $p_{k+m,k} > c_m$ if $m = 1,..,L-k$

IR: $V_k > 0$

IC (seller) is equivalent to the restriction $c_m < \bar{c}_m$ plus (4) in the proposition. I now show that IC (buyer) is guaranteed by IC (seller). First note that IC (buyer) is equivalent to:

$$p_{k+1,k} < 1 \ , \ k = -L, -L+1, .., L-1$$

Use (10) and rearrange terms, IC (buyer) becomes:

$$\bar{c}_{L-k}/2 < 4M\left(1/\delta - 1\right) + L - k$$

Substitute m for $L-k$, IC (buyer) can be rewritten as:

$$\bar{c}_m/2 < 4M\left(1/\delta - 1\right) + m \ , \ m = 1, .., 2L$$

Therefore showing that all transactions that occur are efficient (i.e. $\bar{c}_m < m$) is sufficient to prove that IC (buyer) holds.

I will prove the efficiency by induction. From (4) and (10), pick $k = L - m$, when $m = 1$,

$\bar{c}_1 < p_{L,L-1} = \frac{2L+\bar{c}_1/2}{4M(1/\delta-1)+2L+1} \implies \bar{c}_1 < \frac{2L}{4M(1/\delta-1)+1/2+2L} < 1$. So all transactions of single-unit benefits are expected to be efficient.

Next, assume efficiency ($\bar{c}_j < j$) holds for for $j = 1, .., m-1$, $m \leq L$, then from IC (seller) and (11),

$\bar{c}_m < p_{L,L-m} = \frac{m\left(2L+\frac{1-m}{2}\right)+\frac{1}{2}\sum_{r=0}^{m-1}\bar{c}_{m-r}}{4M(1/\delta-1)+2L+1} < \frac{m\left(2L+\frac{1-m}{2}\right)+\frac{1}{2}\sum_{r=0}^{m-1}(m-r)}{4M(1/\delta-1)+2L+1} = \frac{\left(2L+\frac{3-m}{4}\right)m}{4M(1/\delta-1)+2L+1} < m$.

The second inequality holds as a result of the above assumption. So given (4), all transactions that occur are efficient. IC (buyer) is satisfied. Since IC (buyer) is less stringent than IC (seller), transactions above the debt limit could be profitable to the buyer. But it makes no difference for him to buy or not because the seller will not sell anyway.

IR can be verified in similar manner as in Proposition 1.

3. Existence of costs that satisfy (4)

To show that (4) can always be satisfied by some set of $\{\overline{c}_m > 0,\ m = 1, .., 2L\}$ so that the equilibrium exists, consider the case where $\overline{c}_m = \overline{c}$ for all m. In this case if (4) is satisfied for $m = 1$, then it is for all m because $p_{k+m,k}$ is increasing in m. This condition simply requires $\overline{c} < \frac{L+k+1+\overline{c}/2}{4M(1/\delta-1)+2L+1} \Leftrightarrow \overline{c} < \frac{L+k+1}{4M(1/\delta-1)+2L+1/2}$. Since $k \geq -L$, picking a c smaller than $\frac{1}{4M(1/\delta-1)+2L+1/2}$ guarantees (4) . $\square$