Optimal Regulation in a Two-Sided Model of Banking*

Preliminary Draft

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Abstract
I present a model of banking in which financial intermediation transmits the financial friction to the real economy. In the model the bank simultaneously raises capital from the households and lends to a firm. The bank can monitor the cash flows after the investment has been made but the households cannot. I show that it is optimal for the bank to finance itself by demand deposits. Bank runs have a strategic value as a disciplining device to induce the bank to truthfully report the cash flows to the households. However, bank runs change the incentives of the bank ex ante. The bank reduces its lending when it cannot capture the entirety of surplus from its lending decision. The financial friction, therefore, spills over to the real economy, leading to underinvestment. This main result implies that aiming at financial stability is necessary to improve welfare on the real side. I provide a welfare analysis of banking regulations such as capital, liquidity and reserve requirements and government bailouts, and evaluate their effectiveness in mitigating real inefficiency.

1 Introduction

Financial frictions can have large effects on the real economy. For example, Ivashina and Scharfstein (2010) document a dramatic decline in bank lending to corporations during the 2008 financial crisis. In response to the role the banks played in the credit crunch, regulators around the world have endorsed substantial financial reforms. Dodd-Frank Act and Basel III Accords are two such examples that promote financial stability by increasing oversight on the banking industry and the regulation of bank capital and liquidity. These policies presume a strong interaction between the solvency of the banks and the real sector. However, most research focuses on the bank’s financing and investment policies separately. In this paper I present a two-sided model in which the bank has to separately contract with the households to collect their savings and with the firm to finance

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a long term project. The model allows me to ask: (i) What are the real effects of the bank’s capital structure? (ii) Given these real effects and faced with a variety of prudential policy tools, how to choose an optimal regulation?

A banking equilibrium in this paper is defined by two simultaneously optimal contracts between the bank and the firm on the one hand, and the bank and the households on the other. The key friction in the model is that the households are small and dispersed and they cannot individually or collectively monitor the cash flows of the project once the investment has been made. This friction prevents direct financing between the firm and the households. The households delegate the monitoring of the project and renegotiating with the firm to the bank who specializes in them. However, the same friction prevents incentive compatible contracting between the two. The bank can solve this problem by issuing demandable claims. The households can force the bank into liquidation by running at the bank and demanding early payment whenever the bank reports low cash flows from the project. This induces the bank to truthfully report cash flows to households and acts as a disciplining device. I interpret the bank being the unique provider of demandable claims as if the firm draws a renegotiable credit line and delegates the financing of the credit to the bank. In the model the bank acts like a market-maker between the households and the firm, providing delegated monitoring for the households and delegated financing for the firm.

At the equilibrium a bank run occurs with certainty when the cash flow is lower than a threshold, which sometimes lead to inefficient liquidation of solvent projects. The bank anticipates that there are states of the world in which it cannot capture the total surplus from lending decisions made ex ante. This reduces the expected marginal return of the loan and consequently, the bank has less incentive to lend to the firm. That is, the friction on the bank-household contract spills over to the real economy and lead to underinvestment. The main result that the banking equilibrium does not achieve the socially desirable level of investment provides rationale for financial stability regulations. Since the real inefficiency is a negative externality stemming from the bank-household contract, mitigating it requires designing prudential policies that makes the bank financially stable ex post.

I analyze two sets of regulations aiming at this goal. The first is reminiscent of a narrow banking proposal but it does not restrict the bank from making risky loans. If the bank provides collateral to the households so that the liquidation value of the bank is high enough to guarantee the required return on savings, then the disciplinary role of bank runs is no longer needed. The bank’s collateral can take the form of shareholder equity, liquid reserves or lending to firms that can pledge additional collateral to the bank. Imposing a limit on debt-to-equity ratio forces the bank to finance the loan partly by shareholder equity which buffers the loss for the households in case the firm defaults on the loan. If there is an equity premium to be paid to the shareholders, it makes the capital requirement distortionary as the marginal cost of lending increases ex ante. That is, the policy fails to achieve the socially desirable investment level even though it makes the bank financially stable. However, a capital requirement is less distortionary than a liquidity requirement in which a fraction of the bank funds is invested in non-interest-bearing liquid reserves. The opportunity cost
of a liquidity requirement is the forgone required return on savings or equity, whereas it is only the equity premium over the required return on savings for the capital requirement. Deposit insurance and fractional reserve system are two prevalent examples of a liquidity requirement and a capital requirement improves on them both as a prudential policy. The capital of the borrowing firm that the bank can seize in the event of a default is a perfect substitute to shareholder equity. The more capitalized the firm is, the less regulatory burden there is on the bank. The optimal policy is a debt-to-equity ratio adjusted for the borrowing firm’s capital.

The second regulatory proposal is a lender of last resort intervention to bail out solvent but illiquid banks. A liquidity provision during bank runs allows the bank to capture the full surplus of its loan and pay the gains to the regulator who benevolently transfers it to the households1. The trade-off is the moral hazard it introduces to the bank. If the regulator does not commit to a strict foreclosure policy following a bailout, the bank has ample incentives to lie about the project payoff. This erodes the market discipline that is necessary for an incentive compatible bank-household contract. That is, the regulator has to appropriate the final payoff of every bank that receives lender of last resort funds to prevent the moral hazard while preserving the disciplinary function of runs. Comparing this policy to the capital requirement, it necessitates commitment on the regulator’s part which might fail due to political frictions. Moreover, capital requirement is more household-friendly as it offers complete insurance to savings, whereas the households bear the loss from an insolvent project and make the required return in expectation with the bailout policy.

This paper contrasts with Calomiris and Kahn (1991) and Diamond and Rajan (2001) who provide a similar disciplining role for bank runs. Both models abstract away from equilibrium level of investment. In the former bank runs occur at the equilibrium for a low signal of cash flows but their welfare effect is ex post only since the investment in the project is fixed. Diamond and Rajan provide a two-sided model in which the disciplinary role of bank runs is crucial in order for the bank to commit to making large payments to the households. However, the bank runs occur off-the-equilibrium and have no effect on returns from the project either ex post or ex ante. By contrast I take into account how the bank’s financing problem relates to determination of equilibrium level of investment and show that the bank runs reduce the loan supply. This implies that the disciplinary role of bank runs is necessary but at the same time very costly. The latter is what motivates the study of banking regulations, which is absent in both papers.

A bank run is a result of a coordination failure and a pure loss to the economy in the seminal model of Diamond and Dybvig (1983) in which the bank exists to provide liquidity insurance to the households. Allen and Gale (1998, 2004) challenge its regulatory implication that preventing bank runs is always welfare-improving. They show that a constrained-efficient solution of a general equilibrium model with incomplete markets may involve banking crisis. In my model the bank is constrained by the second-best in the bank-household contract. Interpreting the banking equilibrium as a constrained-efficient solution, I replicate the Allen and Gale critique that preventing bank runs is necessary but at the same time very costly. The latter is what motivates the study of banking regulations, which is absent in both papers.

1I do not model an interbank market, therefore a lender of last resort is the only source of liquidity. For earlier work on frictions leading to private liquidity shortages which justify government bailouts, see e.g. Gorton and Huang (2004) and Rochet and Vives (2004).
a bank run when it serves a disciplining role is not welfare-improving. However, Allen and Gale argument precludes the negative externality on the real economy as the financial market does not interact with the real sector. Internalizing the negative externality through capital requirement and bailout has distortionary effects on the economy but is welfare-improving if they outweight the welfare cost of bank runs ex ante and ex post.

The current study contributes to the growing literature on macroprudential policy. Recently, Admati et al. (2011), Brunnermeier et al. (2009), Hanson et al. (2010) and Morris and Shin (2008) proposed various capital and liquidity requirements. They are motivated by an intuitive notion that the financial system as a whole exhibits systemic risk factors that impede macroeconomic activity as the recent crisis have demonstrated. I formalize this negative externality argument in a representative bank economy without any reference to a financial contagion narrative that these papers have adopted. In a complex financial system with multiple layers of intermediaries, it is plausible to expect that the inefficiency is amplified through the layers. It is worth noting that the existence of a negative externality is what justifies prudential regulation and not how it is amplified. The latter is unambiguously important in targeting the parts of the financial system that contributes most to the underinvestment problem.

The rest of the paper is organized as follows: Section 2 introduces the model, section 3 derives the optimal bank-household contract and section 4 presents the main result. Section 5 and 6 discuss ex ante and ex policy policies respectively. Concluding remarks appear in section 7 and all proofs are relegated to the Appendix.

2 The Model

Consider an economy with a risky investment opportunity that takes two periods to complete. The two-period opportunity cost of capital is \( R = 1 + r > 1 \) and the return to holding cash for a single period is 0. There are three parties in the economy who have unique resources and abilities. The firm owns the project and has to be compensated by \( \Pi > 0 \) for its cost of effort. The firm has no funds of its own to finance the project. Large number of identical households have savings to finance it but they cannot monitor the project after the investment has been made. The households are small and dispersed so the monitoring cost is not affordable individually and the free-riding on other households’ monitoring effort prevents a collective solution. The bank specializes in overseeing projects with access to costless monitoring technology and renegotiation skills, if necessary. If the bank has no capital to contract with the firm directly, it needs to borrow from the households. In return the households have an incentive to hire the bank as their agent to oversee the investment financed by their savings.

Figure 1 illustrates the investment time line. The project has three dates indexed by \( t \in \{0, 1, 2\} \) and the investment has to be made at \( t = 0 \). The project has stochastic returns represented by \( z\phi(I) \). Here \( \phi(I) \) denotes the baseline production function and \( z \) the stochastic component with distribution \( F \). Its realization takes place at \( t = 2 \) but a perfectly informative signal is received.
at $t = 1$. The firm and the bank can observe this signal but the households cannot. The signal is not verifiable by a court. This makes the bank-firm contract incomplete. The project can be shut down at $t = 1$. In case of an early liquidation, the incomplete capital good is sold at a discounted price $\alpha < R$ per unit.

![Investment Timeline Diagram](image)

Figure 1: The Investment Timeline

I make the following assumption on $\phi(I)$:

**Assumption 1** $\phi$ is a strictly concave and increasing function that satisfies the Inada conditions:

$$
\lim_{I \to 0} \phi'(I) = \infty, \lim_{I \to \infty} \phi'(I) = 0.
$$

Furthermore, $\phi'(I)$ is convex.

In addition to the standard assumptions on a production function, $\phi(I)$ has diminishing marginal returns at an increasing rate. Two common functions $I^\beta$ with $\beta < 1$ and $\ln I$ satisfy Assumption 1.

Let $f$ denote the continuous density function of $z$ on $(0, \infty)$. The distribution of $z$ satisfies:

**Assumption 2** The survival function $1 - F$ is log-concave; that is the hazard rate $h(z) = \frac{f(z)}{1 - F(z)}$ is continuous and increasing.

The increasing hazard rate family includes commonly used distributions such as normal, logistic, exponential, chi-squared and certain parametrizations of gamma and beta.

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2Tirole (1999) surveys various other motivations for contractual incompleteness put forward in the literature. Maskin and Tirole (1999) show that they are not sufficient to prevent contract completeness. As Maskin (2001) notes however, the Maskin and Tirole construction does not apply to the situation in which the state is not verifiable to a third party. Hart and Moore (1988) and (1999) are seminal examples of incomplete contracts based on non-verifiability assumption.

3Bagnoli and Bergstorm (2005) survey the properties and implications of log-concavity in economic models.


2.1 The First-Best Problem

Suppose first that there is an hypothetical social planner who runs the firm and can directly allocate household savings to the project. The planner chooses $I$ ex ante and decide when to shut down the project at $t = 1$. Proceeding by backward induction, for any given level of investment $I$ the planner liquidates the project if $z < \tilde{z}(I)$ where $\tilde{z}(I)$ is defined by:

$$
\tilde{z}\phi(I) = \alpha I
$$

For notational convenience, in what follows I suppress the dependence of $\tilde{z}$ on $I$. $\tilde{z}$ gives the liquidation rule which is strictly increasing in $I$ by strict concavity of $\phi$. The latter also implies that $\tilde{z}\phi'(I) < \alpha$ so that the marginal return on the last completed project is in fact lower than its marginal liquidation payoff $\alpha$.

The social planner’s ex ante problem is formulated as:

$$
\max I \int_\tilde{z}^{\infty} z\phi(I)dF(z) + F(\tilde{z})\alpha I - RI
$$

The first order condition is:

$$
\int_\tilde{z}^{\infty} z\phi'(I)dF(z) + \alpha F(\tilde{z}) - R = 0
$$

The first two terms represent the expected marginal return on investment and $R$ is its marginal cost. As the investment goes up, the expected marginal return from completed projects falls but there is a gain on the margin by being able to liquidate $\tilde{z}$ since the new marginal completed project is higher. The proof of the following proposition shows that under Assumption 1 and 2, the diminishing marginal returns outweighs the positive effect.

**Proposition 1** There exists a unique first-best investment level $I^F \in (0, \infty)$ such that

$$
\int_\tilde{z}^{\infty} z\phi'(I^F)dF(z) + \alpha F(\tilde{z}) - R = 0
$$

As for comparative statics, the first-best investment falls in $R$ and rises in $\alpha$. A hazard rate dominant shift in distribution, a distribution with pointwise lower hazard rate, also increases $I^F$. Throughout the paper the ex ante efficiency refers to $I^F$ and the ex post efficiency refers to liquidation threshold $\tilde{z}$ for any $I$. Even though they are independent by definition, the main result of the paper in section 4 establishes a direct connection between the two.

2.2 Bank-Firm Contract

Absent such a social planner, the firm approaches the bank for funds. To develop a benchmark for the financial frictions involving households, assume that the bank has an arbitrarily large capital
of its own with an opportunity cost $R$. The firm’s cost of effort $\Pi > 0$ denotes its reservation payoff. If $\Pi$ is too large, the first-best may not be a feasible outcome. I assume that if the bank gives the first-best loan and leaves all the surplus to the firm, the firm strictly prefers to participate.

**Assumption 3** \( \int_{-\infty}^{\infty} (z\phi(I^F) - \alpha I^F) dF(z) > \Pi \)

The subgame outcome in Figure 1 is either continuation or liquidation of the firm’s project, so the ex ante contract should specify payments for each event. Non-verifiability of the signal at \( t = 1 \) implies that the liquidation payment cannot be contingent on \( z \). For now, restrict attention to debt contracts in which the bank takes the full liquidation value if the project shuts down\(^4\) and the final payment is not contingent on \( z \). The loan contract is \((I, D)\) where \( I \) is the loan amount, \( D \) is the face value of debt and \( \alpha I \) is the liquidation payment. At the end of this section I show that the main result remains invariant to contracts other than debt as long as there is no cost to renegotiation of the contract and the bargaining power is correctly assigned.

The loan contract is implemented as follows: the firm chooses whether to complete or shut down the project at \( t = 1 \). The payment to the bank is \( D \) at \( t = 2 \) if the project is completed. If the bank disagrees with the firm’s decision at \( t = 1 \), it can attempt to renegotiate the contract by making a take-it-or-leave-it offer to the firm. If the firm rejects, then the original contract is executed. Therefore the contract assigns all bargaining power to the bank. When the project shuts down, \( \alpha I \) is paid at \( t = 1 \).

The firm follows a threshold rule \( \tilde{z}(I) \) defined by:

\[
\tilde{z}\phi(I) = D
\]

In words the firm wants to liquidate the project if \( z < \tilde{z} \). In this case the bank can accept a lower payment from the firm by offering a lower payment \( z\phi(I) \), which leaves the firm just indifferent between continuation and liquidation. If \( z\phi(I) > \alpha I \), the bank is willing to write off some of the firm’s debt and allow it continue. The marginally completed project after renegotiation is \( \tilde{z}\phi(I) = \alpha I \), which corresponds to the ex post efficient threshold.

Given this unique subgame outcome, the bank makes the ex ante contract offer at \( t = 0 \) solving:

\[
\max_{(I, D)} (1 - F(\tilde{z}))D + \int_{\tilde{z}}^{\infty} z\phi(I) dF(z) + F(\tilde{z})\alpha I - RI
\]

subject to the participation constraint of the firm \( PCF \):

\[
\int_{-\infty}^{\tilde{z}} (z\phi(I) - D) dF(z) \geq \Pi
\]

\(^4\) More generally, the bank can choose a liquidation payment bounded above by \( \alpha I \). The reader can verify that this constraint becomes redundant in the construction that leads to Proposition 2. The main result remains valid with two payments not uniquely identified.
The next proposition shows that at the equilibrium, the bank provides the first-best loan to the firm.

**Proposition 2** The bank gives the first-best loan $I^F$ to the firm and $D^F$ is set to make $PCF$ bind:

$$\int_\tilde{z}^\infty (z\phi(I^F) - D^F) dF(z) = \Pi$$

The crucial step that leads to Proposition 2 is to show that renegotiation guarantees that $PCF$ binds. Suppose the constraint slacks and the bank raises $D$ to $D'$ for a given $I$. This increases $\tilde{z}$ to $\tilde{z}'$ and forces more types of the project into liquidation. Changing $D$ leaves the marginally completed project $\tilde{z}$ unchanged so the bank’s payoff remains the same for $z < \tilde{z}$. Each project $\tilde{z} \leq z < \tilde{z}'$ that previously paid $D$ and now wants to liquidate, the firm ends up paying more than $D$ to the bank after renegotiation. Finally, the projects $z > \tilde{z}'$ pays $D'$. Accounting for all the types, the bank is strictly better off and hence the constraint needs to bind at the optimum.

Now substitute $D$ from (2.4) into (2.3). The bank’s chooses $I$ according to:

$$\max \int_\tilde{z}^\infty z\phi(I)dF(z) + F(\tilde{z})\alpha I - \Pi - RI$$

(2.5)

Comparing this to (2.1), the bank acts like a social surplus maximizer leaving only the reservation payoff to the firm. As $\Pi$ does not depend on the choice of $I$, the solution corresponds to $I^F$. Moreover, Assumption 3 guarantees that $I^F$ is feasible and hence $D^F$ is pinned down by (2.4). The intuition behind obtaining the first-best is similar to Hart and Moore (1988)’s general result. The incomplete contract signed at $t = 0$ creates a friction at $t = 1$ as some projects the firm would otherwise continue have to be liquidated. Renegotiation at $t = 1$ remedies this ex post inefficiency. It is only the bank’s ex ante decisions, $(I, D)$, that matter for the subgame payoffs. The bank captures the entirety of the surplus from its lending decision owing to two facts: it retains the bargaining power and renegotiation is costless. A friction in renegotiation, either because the firm can appropriate some of the surplus or because it is costly to write a new contract would lead to a different ex ante outcome. Both of these frictions are exogenous, I explore endogenous frictions arising from the bank-household contract in section 3 and 4.

Let $V^F$ denote the maximized value of (2.5) which I interpret as the first-best value of the bank charter. The first three terms represent the net project payoff to the bank after the firm is compensated for its reservation payoff. If this net payoff is larger than the cost of capital $RI^F$, then it is individually rational for the bank to lend. For the remainder of the paper I assume that $V^F > 0$ or alternatively, the social planner’s surplus is larger than firm’s reservation payoff so that the bank is willing to participate.

To conclude this section I illustrate why the restriction to a simple debt contract is without loss of generality. Under any payment function $D(z)$, the bank ultimately maximizes the total
surplus leaving the firm its reservation payoff alone. If the final payoff $z\phi(I)$ is also non-verifiable, then debt is the unique optimal contract as the contracts $(I, D(z))$ are not enforceable through a court. Otherwise several optimal contracts exist and they are welfare-neutral to the debt contract. If there were exogenous frictions in the renegotiation stage, the bank would gain from being able to write a renegotiation-proof contract that locks into the same payoffs underlying Proposition 2. Absent these ex post frictions, renegotiation acts as a substitute for contractual completeness and the debt contract implements the first-best.

3 Bank-Household Contract

There is a continuum of identical small households uniformly distributed on $[0, 1]$. Any amount can be raised from the households as long as they make the required gross rate of return $R$ on their savings. Fix any loan $(I, D)$ the bank might give to the firm. What is the optimal way to finance $I$ from the households who cannot monitor the project after the investment has been made? Assume that the bank does not hold reserves and has no access to additional credit at $t = 1$.

Consider first the possibility of direct finance from the households leaving the intermediary out. The firm observes $z$ at $t = 1$ and reports to the households. Since the households cannot verify the report, the firm will always report $\tilde{z}$ capturing all the surplus from continuation for itself. The households make $\alpha I$ for all states of the world which is lower than their reservation payoff $RI$. Therefore without an intermediary, there is no lending or investment at the equilibrium with direct finance. As in Diamond (1984), the households potentially gain from delegating monitoring to the bank. However, a similar problem exists in the bank-household contracting problem since the bank has superior information at $t = 1$ and the households cannot monitor the bank either. Suppose the bank grants the households the right to withdraw at will at $t = 1$. I show in this section that the right to withdraw acts as a disciplining device on the bank and allows it to write an incentive compatible contract that can overcome the monitoring friction. Therefore I extend the definition of the bank in the earlier section that it is also the unique provider of demandable claims$^5$.

At $t = 1$ the bank learns $z$ and announces the cash flow $Y$ that the bank will generate at $t = 2$ if the project is completed. The bank-household contract specifies a payment function $P(Y)$ with the payment occurring at $t = 2$. Since $(I, D)$ is fixed, $Y$ and $P(Y) \in [\alpha I, D]$ and $Y \geq P(Y)$ by limited liability. These cash flows correspond to the projects $[\tilde{z}, \bar{z}]$ some of which will be liquidated at $t = 1$ unless the bank forgives debt. So the bank has an outside option to reduce the cash flow to $\alpha I$ collectible at $t = 1$. The distribution of $z$ induces a cash flow distribution with two atoms at the end points. Let $f_Y$ stand for the pdf of the bank cash flow distribution and $E_Y$ the expectation operator with respect to $f_Y$.

The bank-household contract gives a withdrawal right to the households that they choose to exercise at $t = 1$. Each identical household decides $\sigma(Y)$; the probability of withdrawal upon

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$^5$Borrowing short term to finance long term illiquid assets has long been recognized as a key role for banking. Diamond and Dybvig (1983) is a seminal example. Here I do not assume that the households face consumption uncertainty or receive liquidity shocks.
hearing the report $Y$. If every household withdraws with this probability then a fraction $\sigma(Y)$ of the households demand payment at $t = 1$. Constrained by liquidity demand, the bank has to forgo the payoff from renegotiation and liquidate the firm to collect $\alpha I$. Let $T_{\sigma(Y)} \in \{0, 1\}$ denote the indicator function taking value 1 if $\sigma(Y) = 0$ and 0 if the withdrawals force the bank to early liquidation. The literature following Diamond and Dybvig (1983) analyze the coordination game among households which has an undesirable equilibrium in which they fail to coordinate on the right action. I aim to isolate the game between the households and the bank by simplifying the game among households. When the bank reports a cash flow $Y > \alpha I$ such that the promised payment is more than the liquidation payoff, $P(Y) > \alpha I$, each household strictly prefers that no one withdraws. Yet it is a best-response if a fraction of households does, even though they do not face a sequential service constraint. To rule out this inefficient equilibrium of the subgame at $t = 1$, I restrict the aggregate behavior so that:

$$\forall Y : P(Y) > \alpha I \Rightarrow \sigma(Y) = 0$$  \hfill (3.1)

Equivalently $T_{\sigma(Y)} = 0$ is possible only when $P(Y) = \alpha I$ so that the households are indifferent between withdrawing and waiting.

To make it incentive compatible for the bank to truthfully report the cash flow, $P(Y)$ and $\sigma(Y)$ need to satisfy:

$$\forall Y, Y' : T_{\sigma(Y)}(Y - P(Y)) \geq T_{\sigma(Y')}(Y - P(Y'))$$  \hfill (3.2)

Since the number of households is large, the competition drives their equilibrium payoff to the required return $R$. The contract $(P(Y), \sigma(Y))$ is chosen ex ante to maximize the bank’s payoff subject to the households’ participation constraint and incentive compatibility. Choosing $\sigma(Y)$ at $t = 0$ can be interpreted as the households ex ante committing to play according to (3.1) at $t = 1$.

$$\max_{(P(Y), \sigma(Y))} E_Y [T_{\sigma(Y)}(Y - P(Y))$$

subject to (3.1), (3.2) and $PCH$:

$$E_Y [T_{\sigma(Y)}P(Y) + (1 - T_{\sigma(Y)})\alpha I] \geq RI$$

**Proposition 3** The unique optimal bank-household contract is demand deposit such that $\exists \hat{P} > \alpha I$:

$$P(Y) = \begin{cases} \hat{P} & \text{if } Y \geq \hat{P} \\ \alpha I & \text{if } Y < \hat{P} \end{cases}$$

The households liquidate the bank by withdrawing $\sigma(Y) > 0$ whenever $Y < \hat{P}$.

The flat payment $\hat{P}$ is an implication of incentive compatibility. Suppose there are two types $Y, Y'$ such that $P(Y) > P(Y') > \alpha I$. Both types know that the households will not withdraw when
the waiting yields more than liquidation. So \( Y \) reports \( Y' \) to take advantage of the lower payment.

To understand the disciplining role of the withdrawals, consider the type \( Y \) such that \( P(Y) = \alpha I \) and \( Y' \) such that \( P(Y') = \hat{P} \). For \( Y' \) to truthfully report its type, it must expect liquidation if it reports \( Y \) instead. Therefore \( T_{\sigma(Y)} = 1 \) whenever \( P(Y) = \alpha I \). Notice that the households can withdraw with any mixed strategy \( \sigma(Y) > 0 \). The fact that the bank loan cannot be partially liquidated, \( T_{\sigma(Y)} \in \{0, 1\} \), works analogous to the deterministic audits assumption in Townsend (1979).

To revert to the original notation, notice that \( \hat{P} = \hat{z}\phi(I) \) for some \( \hat{z} \in (\tilde{z}, \bar{z}) \). With the demand deposit form as given, the contract choice boils down to picking the threshold \( \hat{P} \) or equivalently \( \hat{z} \).

\[
\max_{\hat{z}} (1 - F(\hat{z}))D + \int_{\tilde{z}}^{\hat{z}} z\phi(I)dF(z) - (1 - F(\hat{z}))\hat{z}\phi(I)
\]

subject to \( PCH \)

\[
F(\hat{z})\alpha I + (1 - F(\hat{z}))\hat{z}\phi(I) \geq RI
\]

If follows immediately by differentiating (3.3) that the bank’s payoff falls strictly in \( \hat{z} \). So the optimal \( \hat{z} \) is the smallest threshold that makes \( PCH \) bind. The proof in the appendix shows that it exists and it is unique.

In the seminal models of Townsend (1979) and Gale and Hellwig (1985) similar debt contracts arise at the equilibrium. In Costly State Verification games, the principal penalizes false reports by taking all the surplus when it monitors the agent. The welfare loss is the exogenous fixed cost of monitoring. When monitoring is infeasible, demand deposit allows the households to make false reports costly for the bank by burning surplus. Figure 2 illustrates the equilibrium payoffs. The horizontally shaded area is the bank’s payoff and the area under the dashed line is the payoff to the households. For \( z \leq \tilde{z} \) there is no welfare loss from withdrawals as the bank also prefers these projects be shut down. The withdrawals when \( z \in (\tilde{z}, \hat{z}) \) are inefficient. If the households put pressure on the bank to satisfy the liquidity demand, debt renegotiation cannot take place as it did in the benchmark model to ensure the efficient continuation of the project. I refer to this event as a bank run and it creates an endogenous welfare loss to the economy. In Figure 2 this welfare loss corresponds to the vertically shaded area. For \( [\tilde{z}, \bar{z}) \) the renegotiation takes place as usual since the bank can extract more than what it owes to the households.

The bank run can alternatively be interpreted as a failure of a bank-household renegotiation at \( t = 1 \). At the equilibrium the bank truthfully reports the cash flow from a completed project. The households would benefit from forgiving the bank’s debt in return for the cash flow the loan would generate if the bank had the opportunity to renegotiate with the firm. This mechanism might fail because it is too costly to renegotiate with the households individually. I demonstrate in section 6 that even if it were costless, debt forgiveness coupled by a monitoring friction at \( t = 2 \) erodes the disciplining mechanism behind the bank run and leads to a collapse of the financial contract.

Proposition 3 has intuitive resemblance to Calomiris and Kahn (1991)’s main result. They analyze the bank’s moral hazard problem when it can secretly abscond cash flows. The depositors
value demandable claims as it allows them to penalize low signals that might indicate fraud by forcing the bank into liquidation. However, they do not analyze what implications the bank run has on the bank’s investment policy. I provide a simple model that captures the same intuition that can be embedded into an economy with a real sector. The premise of the next section is showing that this financing structure changes the ex ante incentives to lend and leads to an inefficient level of investment.

4 Banking Equilibrium

A Banking Equilibrium is a triplet \((I^B, D^B, P^B)\) such that \((I^B, D^B)\) is the solution to bank-firm contracting problem and \((I^B, P^B)\) is the solution to the bank-households contracting problem. \(PCF\) does not change when the bank is financed by demand deposits. Even though the marginally competed project is different, the firm still makes the liquidation payoff for the projects below \(\bar{z}\). The bank’s problem is a slight modification of (3.3):

\[
\max_{(I,D,P)} (1 - F(\bar{z}))D + \int_{\bar{z}}^{\tilde{z}} z\phi(I)dF(z) - (1 - F(\bar{z}))P
\]

subject to (2.4) and (3.4). The part of the proof in Proposition 2 that \(PCF\) binds also remains identical. Changing \(D\) when the constraint slacks does not affect \(\hat{z}\), therefore the same line of argument is valid. Proposition 3 shows that choosing \(P\) for any \((I, D)\) implies a binding \(PCH\). Put together, the bank chooses investment by maximizing:

\[
\max_{I} \int_{\hat{z}}^{\infty} z\phi(I)dF(z) + F(\hat{z})\alpha I - \Pi - RI - \int_{\hat{z}}^{\tilde{z}} (z\phi(I) - \alpha I)dF(z) \tag{4.1}
\]

The interpretation of (4.1) is similar to (2.5). The first two terms represent the payoff from
investment out of which $\Pi$ and $RI$ are paid to the firm and the households respectively. Different than (2.5), the bank is constrained by the second-best in the bank-household contract. The last term captures the surplus that has to be burned to discipline the bank. This welfare loss acts like an implicit cost of financial intermediation. Let $I^B$ denote the investment made at the banking equilibrium as a solution to (4.1). If the marginal welfare loss is positive, then the investment made in the firm has effectively a higher marginal cost than $R$ to the bank. A general analysis of the marginal welfare loss is technically cumbersome but it suffices to do it around the first-best $I^F$. I show in the appendix that the marginal welfare loss is positive at the first-best when $\alpha$ is sufficiently small, for which a rigorous criteria is presented in the proof. Then (4.1) is not maximized at $I^F$, in fact it is negatively sloped at that point. Then the maximum occurs at a lower value and there is underinvestment at the equilibrium.

**Proposition 4** For a sufficiently small $\alpha$, a banking equilibrium $(I^B, D^B, P^B)$ exists. In any banking equilibrium $I^B < I^F$ and the payments are uniquely given by:

$$
\int_{\hat{z}}^{\infty} (z\phi(I^B) - D^B) dF(z) = \Pi \\
F(\hat{z})\alpha I^B + (1 - F(\hat{z}))\hat{P} = RI^B
$$

where $P^B = \hat{z}\phi(I^B)$. Let $V^B$ denote the equilibrium payoff to the bank. $V^B < V^F$.

Unless the welfare loss term has an erratic behavior beyond $I^F$, this equilibrium is unique. I omit the technical details of uniqueness and focus on the equilibrium described in Proposition 4. The negative externality of bank fragility on the real economy is the main result of the paper. To understand this, compare the banking equilibrium to Proposition 2 when the financial friction is absent. It is only the bank who has an ex ante action and the contract renegotiation works in a way that the bank captures the surplus. This leads to a special case in Hart and Moore (1988) that ex post efficiency begets ex ante efficiency. A variation of this result is the hold-up problem in which one party has to make a relationship-specific investment ex ante, knowing that the other party can appropriate some of the gains ex post. Williamson (1979) shows that this leads to underinvestment. On the bank’s financing problem, the bank has to forgo surplus from renegotiation to obtain the second-best outcome. This creates an endogenous friction in the bank-firm contract. The bank lends to the firm knowing that a banking crisis will dissipate the surplus from its lending decision for some states of the world. Mechanically, this works the same way as in the hold-up problem in reducing the bank’s willingness-to-lend ex ante. The second-best in the financing problem effectively reduces the loan supply compared to the first-best.

I showed in section 2 that the restriction to debt contracts is without loss of generality as long as the renegotiation is costless and the bank retains all the bargaining power. Confronted with inefficiencies, it is curious to ask whether any improvement can be made by allowing contingent payments $D(z)$ at $t = 2$. Since ex post inefficiency leads to underinvestment, the bank might consider offering a contract $(I, D(z))$ that locks into efficient continuation of the project i.e. all
$z > \tilde{z}$ chooses to continue under the original contract. The bank can still exploit its superior information ex post by reporting $\tilde{z}$ all the time. So the withdrawal mechanism has to work exactly in the same way to deter underreporting the true state. However, this time the bank faces a liquidity demand with a loan that it cannot liquidate at will. Put simply, the firm with project $z > \tilde{z}$ has bargaining power in renegotiation and the bank needs to lower the liquidation payment below $\alpha I$ to encourage the firm to shut down and pay at $t = 1$. With some liquidating projects paying less than the full liquidation value, more than $\hat{z}$ have to be shut down to compensate the households. Hence this new contract exacerbates underinvestment by implicitly transferring bargaining power to the firm. Regardless how $D(z)$ is set, the projects below $\tilde{z}$ needs to be liquidated for an incentive compatible bank-household contract. One alternative is to offer $(I, D(z))$ so that the projects $z < \hat{z}$ chooses liquidation under the current contract. It follows immediately that the bank solves (4.1) again therefore this contract is welfare-equivalent to the debt contract. If allowing richer contracts does not recover ex ante efficiency, there is room for regulation.

5 Ex Ante Policy

So far I have implicitly assumed that the bank has an owner-manager. Suppose instead that the bank is owned by a large group of shareholders that hold inside equity in the bank. The shareholders are different than the households; they can monitor the bank activities and have no conflict of interest with the management. The only agency problem exists between the bank and the households. The bank can borrow any amount by issuing equity, $E$, whose required return is $R_E \geq R$. The equity premium $R_E - R$ can be interpreted as a compensation for the cost of bank governance that only the shareholders incur. As long as the bank charter does not require capital and the bank is protected by limited liability, the shareholders do not need to risk their endowment in the bank.

Proposition 4 implies that the underinvestment problem cannot be overcome if there is a welfare loss ex post i.e. $\hat{z} > \tilde{z}$ so that the marginally completed project is higher than the ex post efficient threshold. The households have a strategic gain from bank runs since they make $\alpha < R$ when they are lax at disciplining the bank. However, if the bank is required to finance and invest in a way that it is capable of compensating the households even if the firm is insolvent for $z \leq \tilde{z}$, then there is no need for burning surplus. Put simply, the bank runs are avoided if the household savings are fully insured. To distinguish the notation from earlier sections, let $B$ denote the borrowing from households as deposit, $L$ denote the liquid reserves of the bank with zero nominal return and $I$ is the illiquid investment in the firm. The ex ante balance sheet of the bank is:

$$L + I = B + E$$

**Proposition 5** Absent any ex post intervention, there is no welfare loss at $t = 1$ if and only if

$$L + \alpha I \geq RB$$

(5.1)
The banking equilibrium in section 4 corresponds to the case $L = 0$ and $I = B$. If (5.1) is satisfied, the bank's payoff always exceeds the required return on senior claimant deposits, therefore it is feasible for the bank to offer $P(Y) = RB$ for all $Y$ and report whether the firm is solvent at $t = 1$. Conversely, when (5.1) is not satisfied and the bank reports $\tilde{z}$, the households make $L + \alpha I < RB$ if they do not run. Proposition 3 implies that the households need to extract more from the high types to compensate the difference and to make it incentive compatible for them to report the true cash flow, some projects have to be forced into liquidation. Thus there has to be some welfare loss at the equilibrium.

Figure 3 illustrates the payoffs with insured savings. The households have a constant payoff $RB$ and the shareholders lose everything if the project efficiently shuts down but capture the surplus when $z > \tilde{z}$. In the following subsections, I analyze various implementations of (5.1) and their effect on ex ante investment.

![Bank Payoff Figure 3: Payoffs with Insured Savings](image)

### 5.1 Capital Requirement

A capital requirement regulates only the liability side of the bank balance sheet. As the deposits are senior claimants in case of a default, equity buffers the loss allowing the households make $R$ in these states of the world. Setting $L = 0$ in (5.1), the implied debt-to-equity ratio $\kappa$ that guarantees ex post efficiency is:

$$\kappa = \frac{\alpha}{R - \alpha} \quad (5.2)$$

which is equivalent to a capital ratio $\frac{R - \alpha}{R}$.

**Proposition 6** The debt-to-equity ratio $\kappa$ in (5.2) implements the first-best investment level if $R_E = R$. If $R_E > R$, then the equilibrium investment $I^B$ is unique and less than $I^F$.

The shareholders' profit is the shaded area in Figure 3 and the equity costs $R_E E$ to the bank. Since the entirety of surplus is captured by some party ex post, the bank's objective is to maximize
the total surplus by choosing $B$ and $E$ which boils down to:

$$\max_I \int_{\tilde{z}}^{\infty} z\phi(I) dF(z) + F(\tilde{z})\alpha I - \Pi - R'I \quad (5.3)$$

where the ex ante marginal cost of investment, $R'$, is

$$R + (R_E - R) \left( \frac{R - \alpha}{R} \right)$$

The marginal cost goes up since a fraction $\frac{R - \alpha}{R}$ of the investment is financed by shareholders who have an equity premium $R_E - R$. Admati et al. (2012) argues that this premium is negligible in which case the bank solves (2.5) and the capital requirement implements the first-best i.e. $I^B = I^F$. Otherwise (5.3) has a unique solution $I^B < I^F$ by Proposition 1.

In the latter case even though the equilibrium is ex post efficient with the capital requirement, it is ex ante inefficient. I refer to this as a distortionary effect of a regulation since achieving ex post efficiency distorts the marginal cost of investment ex ante. Let $V^{CR}$ denote the maximized value of (5.3). Even though $V^{CR} < V^F$, the capital requirement improves welfare if $V^{CR} > V^B$. Proposition 6 highlights the trade-off between the increasing marginal cost of investment and the insurance against bank run. A precise policy recommendation requires an empirical evaluation of this trade-off.

### 5.2 Liquidity Requirement and Deposit Insurance

The bank can be required to hold liquid reserves ex ante to pay back the households in case the firm defaults. In this subsection I analyze shareholder-financed liquidity which corresponds to $L = E$ and $I = B$. That is, the loan is financed by deposits and equity is invested in liquid reserves. Using (5.1), the optimal amount of liquid reserves is given by:

$$L = (R - \alpha)I \quad (5.4)$$

The simplest implementation of (5.4 is a shareholder-financed deposit insurance. $R - \alpha$ is the insurance fee that the bank is required to pay to the regulator per a dollar deposit. The regulator holds liquid reserves on behalf of the bank and compensates the households if the firm defaults. Alternatively, for every dollar the bank borrows from the households, the shareholders inject $R - \alpha$ dollars into bank capital. This gives a debt-to-equity ratio $\frac{1}{R - \alpha}$ different than (5.2). Moreover, the ratio of liquid reserves to total assets\(^6\) need to be $\frac{R - \alpha}{1 + R - \alpha}$. So it is possible to interpret this liquidity requirement as a capital requirement with an additional regulation on the asset side.

**Proposition 7** The equilibrium investment $I^B$ with (5.4) is unique and less than the first-best $I^F$. Moreover, it is also less than the equilibrium investment with the capital requirement (5.2) for any $R_E$.

\(^6\)This is known as statutory liquidity ratio.
The straight line in Figure 2 is the loan payoff and the liquid reserves shift this payoff up by 
\((R - \alpha)I\) resulting in Figure 3. The bank maximizes the shaded area minus the cost of liquidity
\(R_EL\) which results in (5.3) but \(R'\) is given by:

\[ R + r_E(R - \alpha) \]

The marginal cost of investment goes up by the forgone return \(r_E\) to liquid reserves \(R - \alpha\) per dollar deposit. Different than the capital requirement, this distortion term is always positive. More importantly, it is larger than the distortion \((r_E - r)(\frac{R - \alpha}{R})\) the capital requirement creates. Liquidity requirement not only uses proportionally larger amount of shareholder equity in relation to the investment in the firm, but also forgoes a larger return by investing in liquid reserves rather than the firm’s project. The two policies can be Pareto-ranked; a capital requirement is strictly preferred to a liquidity requirement or a deposit insurance.

### 5.3 Fractional Reserve Requirement

An alternative liquidity requirement is to save a fraction of household deposits as reserves. When \(\alpha \geq 1\) reserve holding is dominated by investing everything in the firm. I assume \(\alpha < 1\) so that the liquid reserves create collateral.

Let the required reserve ratio be \(\rho \in (0, 1)\). The required reserves do not suffice to fully insure the deposits. To see this, set \(E = 0, B = I\) and \(L = \rho I\) in (5.1) so that the investment in the firm is \((1 - \rho)I\). The liquidation value of the bank is \((\rho + (1 - \rho)\alpha)I\) which is maximized as \(\rho \to 1\) and still strictly lower than \(RI\) for any \(I\). Suppose the regulator imposes a capital requirement on the fractional reserve system. To isolate the welfare effect of required reserves, assume that \(R_E = R\) so that the capital requirement is non-distortionary. (5.1) implies a debt-to-equity ratio:

\[ \kappa(\rho) = \frac{\alpha}{R - \alpha - (1 - \alpha)\rho} \]  

(5.5)

The reserves and the shareholder equity are substitutes for buffering the loss for the households. Higher the required reserve ratio, the lower the need for shareholder’s equity and hence higher the debt-to-equity ratio. This explains \(\kappa(\rho)\) increasing in \(\rho\). Nonetheless, they are not perfect substitutes as the fractional reserve requirement is always distortionary even if the capital requirement is not.

**Proposition 8** For all \(\rho\), the equilibrium investment level \(I_B\) under \(\kappa(\rho)\) in (5.5) is unique and less than the first-best \(I_F\).

Following a similar line of argument as the capital requirement, the bank’s problem can be written as (5.3) with a marginal cost of investment \(R'\) strictly larger than \(R\). For every dollar lent to the firm, the bank borrows \(\frac{\alpha}{R - \rho}\) dollars from the households of which a fraction \(\rho\) is not invested in the firm but a positive return \(r\) is promised to the households. The marginal cost of investment
has to account for the forgone but insured payoff to reserves. In particular I derive in the appendix that:

\[ R' = R + \frac{\rho \alpha R}{R - \rho} \]

Setting \( \rho = 0 \) and using capital requirement alone improve on a fractional reserve system. The only advantage of fractional reserve system as a prudential policy is relaxing the burden on the shareholders, which might be useful if the shareholder wealth is finite so that Proposition 6 may not apply. In the last subsection I show that the firm capital serves this purpose better than the fractional reserves.

## 5.4 Firm Capital

So far the firm has no capital of its own. Let \( K \) denote firm’s cash that it can borrow against at \( t = 0 \) and \( K < I^F \) so that it has an incentive to borrow. The bank-firm contract is a triplet \((I - K, D, \alpha I)\) where \( I - K \) is the amount the firm borrows from the bank, whenever positive. When the financing friction is absent, \( K \) has no meaningful impact on the equilibrium outcome. Since \( K \) serves as additional collateral when the project shuts down, it contributes to insuring the household deposits. \( K \) is not contingent on \( I \) so it cannot insure any loan but it suffices that it insures the first-best loan \( I^F \). The latter requires that \( \alpha I^F \geq R(I^F - K) \). A firm is secure if:

\[ K \geq \left( \frac{R - \alpha}{R} \right) I^F \tag{5.6} \]

If the firm can self-finance at least \( \frac{R - \alpha}{R} \) fraction of the first-best investment, if not all, then lending to this firm guarantees \( R \) to the households for all states of the world. Notice also (5.6) is the capital ratio implied by (5.2). The firm capital is a perfect substitute to bank capital inasmuch as there is a sufficient collateral to ensure the required return for households. In fact when the bank capital is distortionary, lending to a secure firm is even preferable.

When the firm has some capital but not secure in the sense of (5.6), the underinvestment problem persists absent regulation. Yet the magnitude of the capital requirement is lower. The effective equity the bank needs to issue is:

\[ E = \min \left( \left( \frac{R - \alpha}{R} \right) I - K, 0 \right) \]

Implementing the effective capital requirement that allow the bank to substitute the firm capital with the shareholder’s equity implicitly requires that the regulator not only monitors the bank balance sheet but also examines how secure the borrower is. This can be done by imposing the requirement as it is but allowing deductions based on the bank’s report after the loan contract is signed. Since the loan \( I - K \) and the collateral \( \alpha I \) are both stated in the contract, the regulator can verify \( K \) and make the deduction accordingly. This mechanism gives an incentive to the bank to prioritize financing the secure firms as it reduces the regulatory burden on its shareholders.
6 Ex Post Policy

The regulator can allow the bank operate with uninsured household savings but step in at \( t = 1 \) on behalf of the households to alleviate the welfare loss. In Figure 2, the bank is solvent but illiquid for \( z \in [\bar{z}, \hat{z}] \). Rather than forcing the firm to shut down the project inefficiently for its liquidation value \( \alpha I \), the regulator can give this amount as credit at \( t = 1 \) to the bank. Injecting more than the liquidation value is risky; as it does not observe the cash flow signal at \( t = 1 \), the regulator might bail out the bank for more than its fundamental value. To isolate the welfare effect of the bailout from regulatory uncertainty, suppose at \( t = 0 \) the regulator announces an explicit promise \( \Gamma \), the highest cash flow type of the bank who will get a bailout. The regulator injects \( \alpha I \) to the bank if it reports lower than \( \Gamma \) and choosing \( \Gamma < \alpha I \) is interpreted as not bailing out anyone. Effectively the regulator plays a lender-of-last-resort role during the bank run. Assume that the regulator does not charge interest rate on a single period credit and wants all surplus to be transferred to the households at \( t = 2 \).

Suppose the regulator cannot observe the bank’s final payoff either. \( Y \) denotes the bank’s cash flow report and let \( \Phi(Y) \) be the repayment function on the credit due at \( t = 2 \). By construction, the regulator chooses \( \Phi(Y) = Y \). \( P(Y) \) and \( \sigma(Y) \) are defined as in section 4. Whenever \( Y \leq \Gamma \) is reported, the households can secure the full cash flow \( Y \) by withdrawing and letting the regulator step in. Therefore the bank-household contract should make them at least indifferent. Since by limited liability \( P(Y) \leq Y \) as well, the contract offers \( P(Y) = Y \) for all \( Y \leq \Gamma \). Thus all types that are promised a bailout will report \( \alpha I \) to take advantage of the lower payment.

Consider the incentives of a type \( Y' > \Gamma \). If it reports \( \alpha I \), it pays only \( \alpha I \) regardless whether the households withdraw or not. The promised bailout introduces a moral hazard problem to the subgame at \( t = 1 \). Absent the lender-of-last-resort, underreporting its true type would imply a forced liquidation on the bank. Whereas now this disciplining device is inutile since the regulator injects liquidity on the bank’s behalf but it cannot extract the bank’s surplus either. With incentive compatibility not satisfied, the households make \( \alpha I \) for all states of the world and simply do not lend to the bank ex ante. Hence the bailout policy not only fails to deliver the first-best but in fact has an adverse effect. This analysis remains valid when the regulator announces the lowest type to bailout instead. All types in the bailout region pretend to be the lowest type who is promised a bailout and the others are inefficiently liquidated. Proposition 3 shows that \( \hat{P} \) is the threshold that makes \( PCH \) bind, therefore any promise less than \( \hat{P} \) will lead to a lower expected payment than the required return.

I interpreted the welfare loss as a failure of bank-household renegotiation in section 3. In this case it is possible to interpret the regulator as a representative household renegotiating with the bank if it reports less than \( \Gamma \) instead of immediately running at it. Then \( \Phi(Y) \) is the new payment to the households after renegotiation. The construction above implies that the households cannot make it incentive compatible for the bank to report truthfully if they are lax on their threat of termination. This completes the proof of an earlier claim that the households cannot capture the gains from renegotiation by themselves. However, it is plausible to think that the regulator has
Proposition 9 If the regulator verifies and appropriates the bank’s final payoff, there exists a unique \( \Gamma \in (\alpha I, \hat{P}) \) such that bailing out the solvent banks with cash flow less than \( \Gamma \) implements the first-best investment level.

The moral hazard problem can be overcome if the regulator verifies \( z \) at \( t = 2 \). As the spot contracts with repayment function \( \Phi(Y, z) \) are now feasible, the regulator can offer \( \Phi(Y, z) = z\phi(I) \) for any report \( Y \leq \Gamma \). A more practical interpretation of this offer is that the regulator buys a senior claim on the bank payoff than the existing shareholders at a price of \( \alpha I \) and ignores the cash flow report. By committing to close every bank that received aid at \( t = 1 \), no types of the bank can gain strategically by lying to the households. Figure 4 illustrates the equilibrium payoffs for a given \( \Gamma \). The area under the thick line is the household payoff. For types \( Y > \Gamma \), incentive compatibility requires a flat payment that all such types can afford, which is \( \Gamma \). To deter high types from cheating, the households force all solvent but illiquid types into receivership. Let \( \Gamma = \gamma \phi(I) \), the types \( z \in [\tilde{z}, \gamma] \) makes zero profit ex post due to the bailout policy, thus the contract can offer \( P(Y) = Y \) without loss of generality. The bank’s payoff is the shaded area for a given \( \Gamma \). The regulator chooses \( \Gamma \) to satisfy \( PCH \) and let the bank keep the surplus after compensating the households. \( \Gamma \) solves:

\[
F(\tilde{z})\alpha I + \int_{\tilde{z}}^{\gamma} z\phi(I) \, dF(z) + (1 - F(\gamma))\Gamma = RI
\]

(6.1)

The appendix proves that there is a unique \( \Gamma \) that satisfies (6.1) and \( \Gamma < \hat{P} \) i.e. the spread between the rate the bank borrows and lends increases with the policy. Since there is no welfare loss in the economy, the bank chooses \( I \) to maximize (2.5) which leads to the first-best investment \( IF \). \( D \) and \( \Gamma \) are uniquely determined by \( PCF \) and \( PCH \) respectively.
7 Conclusion

In this model the bank runs have strategic value as they are substitute to monitoring the bank. However, they create welfare loss ex post which is a negative externality to the credit channel that determines the equilibrium investment level. I analyze two regulatory approaches to overcome the underinvestment problem: regulating bank capital and liquidity to prevent the bank run from happening, and a lender of last resort policy designated to make the bank runs less costly.

The former policy has distortionary effects on the economy as it increases the marginal cost of lending. However, a capital requirement is less distortionary than a liquidity requirement that regulates both the asset and the liabilities of the bank. Recent economic research on prudential policy takes the deposit insurance and fractional reserve banking as given, which are both liquidity regulations, and analyzes new regulatory frameworks built on top of existing regulations. The results of the current study challenges this status quo and imply that gearing towards a single capital regulation and replacing the old prudential regime can be welfare-improving. It is worth noting that this paper studies a representative bank economy that excludes interbank lending where liquidity might be more valuable than the bank capital. Future research that extends the current framework to banking competition and interbank market may deliver such comparisons.

The success of the lender of last resort policy depends critically on the regulator’s commitment to writing off the bank shareholders following a bailout. This paper derives the optimal policy but does not shed light on what frictions can compromise its implementation. The simplest of such frictions can be the commitment itself, a more interesting scenario would be an hold-up problem between the bank and the regulator. Suppose extracting the firm’s cash flow after the bank run requires costly effort and the regulator commits to foreclosing the bank after the project is completed. Then the bank has no incentive to exert effort unless the regulator allows the bank to capture some of the surplus. Put simply, there is room for moral hazard even though the regulator can solve its commitment problem. A more detailed modeling of the policy environment can address these issues, along with a comparison to capital regulation where the regulator’s involvement is minimal ex post.

References


A Appendix

The first lemma is known results in stochastic orders, a short proof is presented for convenience. See Shaked and Shanthikumar (2007) ch.2.

**Lemma 1** \( \forall \bar{z}; \frac{\partial}{\partial \bar{z}} E(z - \bar{z}|z > \bar{z}) = h(\bar{z})E(z - \bar{z}|z > \bar{z}) - 1 \leq 0 \)

**Proof.**
Integrating by parts:

\[
E(z - \bar{z}|z > \bar{z}) = \left. \frac{1}{1 - F(\bar{z})} \int_{\bar{z}}^{\infty} (z - \bar{z}) dF(z) \right|_{\bar{z}}^{\infty} = \frac{\int_{\bar{z}}^{\infty} (1 - F(z)) dz}{1 - F(\bar{z})}
\]

Log-concavity is preserved under partial integration: if \( 1 - F \) is log-concave then \( \int_{\bar{z}}^{\infty} (1 - F(z)) dz \) also is. By definition of log-concavity of \( \int_{\bar{z}}^{\infty} (1 - F(z)) dz \):

\[
(\ln \int_{\bar{z}}^{\infty} (1 - F(z)) dz)' = \left( -\frac{1 - F(\bar{z})}{\int_{\bar{z}}^{\infty} (1 - F(z)) dz} \right)' \leq 0
\]

Equivalently \( \left( \frac{\int_{\bar{z}}^{\infty} (1 - F(z)) dz}{1 - F(\bar{z})} \right)' \leq 0 \). Now compute

\[
\left( \frac{\int_{\bar{z}}^{\infty} (1 - F(z)) dz}{1 - F(\bar{z})} \right)' = h(\bar{z})E(z - \bar{z}|z > \bar{z}) - 1 \leq 0
\]

**Proof of Proposition 1.**

Divide the left hand side of (2.2) by \( 1 - F(\bar{z}) \); the probability that the project is completed. Rearrange terms using \( R = 1 + r \) to get:

\[
\phi'(I)E(z|z > \bar{z}) - 1 - \frac{1}{1 - F(\bar{z})} ((1 - \alpha)F(\bar{z}) + r) = 0 \tag{A.1}
\]

Now \( \phi'(I)E(z|z > \bar{z}) - 1 \) is the marginal return from investing a dollar in the project at \( t = 0 \) conditional on the project being completed. The term \( \frac{1}{1 - F(\bar{z})} ((1 - \alpha)F(\bar{z}) + r) \) can be interpreted as the conditional marginal cost.

Evaluate (A.1) as \( I \to 0 \) and hence \( \bar{z} \to 0 \). By Inada condition:

\[
\lim_{I \to 0} \phi'(I)E(z|z > \bar{z}) - 1 - \frac{1}{1 - F(\bar{z})} ((1 - \alpha)F(\bar{z}) + r) = \infty
\]

By Lemma 1

\[
E(z|z > \bar{z}) \leq \frac{1}{h(\bar{z})} + \bar{z}
\]
Multiply both sides by $\phi'(I) > 0$. As $\tilde{z}\phi'(I) < \alpha < R$

$$\phi'(I)E(z|z > \tilde{z}) - 1 < \frac{\phi'(I)}{h(\tilde{z})} + r$$

Since $\lim_{I \to \infty} \frac{\phi'(I)}{h(\tilde{z})} = 0$, the conditional marginal return as $I \to \infty$ is bounded above by $r$. $\lim_{I \to \infty} \tilde{z}(I) = \infty$ and $\lim_{\tilde{z} \to \infty} F(\tilde{z}) = 1$ imply that

$$\lim_{I \to \infty} \phi'(I)E(z|z > \tilde{z}) - 1 - \frac{1}{1 - F(\tilde{z})}((1 - \alpha)F(\tilde{z}) + r) = -\infty$$

All the terms in (A.1) are continuous in $I$ therefore by the Intermediate Value Theorem the first order condition is satisfied at least once. To show uniqueness, analyze the conditional marginal return and cost separately. The conditional marginal cost is increasing:

$$\frac{\partial}{\partial I} \left( \frac{1}{1 - F(\tilde{z})}((1 - \alpha)F(\tilde{z}) + r) \right) = (R - \alpha)h(\tilde{z}) \frac{1}{1 - F(\tilde{z})} > 0$$

Differentiate the conditional marginal return to get:

$$\phi''(I)E(z|z > \tilde{z}) + \phi'(I) \frac{\partial}{\partial \tilde{z}} E(z|z > \tilde{z})\tilde{z}'(I)$$

By Lemma 1, $\frac{\partial}{\partial \tilde{z}} E(z|z > \tilde{z}) \leq 1$ and by the chain rule $\tilde{z}'(I) = \frac{\alpha - \tilde{z}\phi'(I)}{\phi(I)} > 0$. Therefore

$$\phi''(I)E(z|z > \tilde{z}) + \phi'(I) \frac{\partial}{\partial \tilde{z}} E(z|z > \tilde{z})\tilde{z}'(I) \leq \phi''(I)E(z|z > \tilde{z}) - \tilde{z} \frac{\phi'(I)^2}{\phi(I)} + \alpha \frac{\phi'(I)}{\phi(I)}$$

$$\leq \tilde{z}(\phi''(I) - \frac{\phi'(I)^2}{\phi(I)}) + \alpha \frac{\phi'(I)}{\phi(I)}$$

$$= \left( \frac{\phi'(I)}{\phi(I)} \right)' \tilde{z}\phi(I) + \alpha \frac{\phi'(I)}{\phi(I)}$$

$$= \alpha \left\{ \left( \frac{\phi'(I)}{\phi(I)} \right)' I + \frac{\phi'(I)}{\phi(I)} \right\}$$

$$\frac{\phi'(I)}{\phi(I)}$$ is a strictly convex decreasing function. It follows from by Assumption 1 that both $\frac{1}{\phi}$ and $\phi'$ are convex (the former is strict) and decreasing. $\frac{\phi'}{\phi}$ is a multiplication of two such functions so it is also strictly convex and decreasing. A known property of strictly convex functions is:

$$\left| \left( \frac{\phi'(I)}{\phi(I)} \right)' \right| > \frac{1}{I} \left( \frac{\phi'(I)}{\phi(I)} \right)$$

Rearranging the terms shows that the slope of the conditional marginal return is negative. Together with increasing conditional marginal cost, the second order condition to (2.1) is negative for all $I$. This suffices to conclude that $I^F$ is unique.
As for comparative statics, differentiating (2.2) yields the result for $R$ and $\alpha$. For a shift in distribution, the conditional marginal return goes up pointwise if $E(z|z > \tilde{z})$ goes up. This requires Mean Residual Life Order. The conditional marginal cost falls if $\frac{F(\tilde{z})}{1-F(\tilde{z})}$ falls. This requires First Order Stochastic Dominance. Hazard Rate Order is the simplest stochastic order that jointly implies the two. For a detailed relationship among these orders, the reader can refer to Shaked and Shanthikumar (2007) ch.1.

Proof of Proposition 2.

To formalize the argument made in the text that PCF binds at the optimum, define $D' = D + \delta$ and $\tilde{z}' = \tilde{z} + \frac{\delta}{\phi(I)}$. The payoff to the bank is $\alpha I$ if $z < \tilde{z}$ and $z\phi(I)$ if $z \in [\tilde{z}, \bar{z})$. For $z \in [\bar{z}, \bar{z}')$ the firm chooses liquidation as $z\phi(I) < D'$. The bank makes a take-it-or-leave-it offer $z\phi(I) \geq D$ and hence the payoff to the bank increases. Since $z \geq \bar{z}'$ pays $D'$, increasing $D$ whenever the constraint slacks unambiguously increases the bank’s payoff.

Substituting $D$ in (2.4) into (2.3) gets (2.5) which has a unique solution $I^F$ by Proposition 1.

Let $G(D) : [\alpha I^F, \infty] \to R$ be defined by:

$$G(D) = \int_{\tilde{z}}^{\infty} (z\phi(I^F) - D)dF(z) - \Pi$$

$G(D^F) = 0$ corresponds to a binding PCF. For $D \to \alpha I^F$, $\tilde{z} \to \bar{z}$ by definition and Assumption 3 implies:

$$\lim_{D \to \alpha I^F} G(D) > 0$$

To evaluate $\lim_{D \to \infty} G(D)$, make a change of variable from $D$ to $\bar{z}$ and integrate by parts:

$$\lim_{D \to \infty} G(D) = \phi(I^F) \lim_{\bar{z} \to \infty} \int_{\tilde{z}}^{\infty} (1 - F(z))dz - \Pi < 0$$

as the first limit is 0 and $\Pi > 0$. Lastly

$$G'(D) = -(1 - F(\bar{z})) \frac{\partial\bar{z}}{\partial D} < 0$$

This concludes that there exists a unique $D^F$ such that $G(D^F) = 0$. ■

Proof of Proposition 3.

Group the states $Y \in [\alpha I, D]$ into two sets: $S = \{Y : P(Y) = \alpha I\}$ and $S^C = \{Y : P(Y) > \alpha I\}$. (3.1) implies that $\sigma(Y) = 0$ on $S^C$ whereas $\sigma(Y)$ is unconstrained on $S$. $S \neq \emptyset$ as $\alpha I \in S$. Even though the contract can leave positive surplus to the bank by specifying $P(Y) < \alpha I$, the reader can verify that such contracts are no optimal.

Take any $Y, Y' \in S^C$:

$$Y - P(Y) \geq Y - P(Y')$$

Reverse the roles of $Y, Y'$: $P(Y) = \hat{P}$ for all $Y \in S^C$. Moreover $\hat{P} \leq \inf_{S^C} Y$ so that all types in $S^C$ can afford it.
Take any \( Y, Y' \in S \). (3.2) implies:

\[
T_{\sigma(Y)}(Y - \alpha I) \geq T_{\sigma(Y')}(Y - \alpha I)
\]

Reverse the roles for \( Y, Y' \) it must be that \( T_{\sigma(Y')} = T_{\sigma(Y)} \) on \( S \).

Take \( Y \in S^C \) and a deviation to \( Y' \in S \):

\[
Y - \hat{P} \geq T_{\sigma(Y')}(Y - \alpha I)
\]

implies \( T_{\sigma(Y')} = 0 \) on \( S \) otherwise \( Y - \hat{P} < Y - \alpha I \) and all types in \( S^C \) reports a type in \( S \).

Lastly take \( Y \in S \) and a deviation to \( Y' \in S^C \):

\[
0 \geq Y - \hat{P}
\]

Hence the payment \( \hat{P} \) should satisfy \( \hat{P} \geq \sup_{S} Y \). Put together \( \inf_{S^C} Y = \hat{P} = \sup_{S} Y > \alpha I \).

This completes the argument that (3.1) and (3.2) imply the demand deposit form with withdrawals whenever a cash flow in \( S \) is reported since \( T_{\sigma(Y)} = 0 \) implies \( \sigma(Y) > 0 \) for any \( Y \in S \).

Given this contract form, the ex ante problem is formulated in the text as maximizing (3.3) subject to (3.4) by choosing \( \hat{P} \). \( \hat{P} \equiv \hat{z}\phi(I) \) for some \( \hat{z} \in (\tilde{z}, \overline{z}] \) and (3.3) is decreasing in \( \hat{z} \), a corner solution that satisfies \( PCH \) is optimal. Define \( H(\hat{z}) \) by:

\[
H(\hat{z}) = F(\hat{z})\alpha I + (1 - F(\hat{z}))\hat{z}\phi(I) - RI
\]

I claim that there exists a unique \( \hat{z} \) such that \( H(\hat{z}) = 0 \). At the lower bound \( \hat{z} \), it is immediate that \( H(\hat{z}) < 0 \). Differentiate \( H(\hat{z}) \) to get

\[
H'(\hat{z}) = (\phi(I) - (\hat{z}\phi(I) - \alpha I)h(\hat{z}))(1 - F(\hat{z}))
\]

The sign of \( H'(\hat{z}) \) is given by \( \hat{z}\phi(I) - (\hat{z}\phi(I) - \alpha I)h(\hat{z}) \) as the last term is always positive. Since \( \hat{z}\phi(I) = \alpha I \), \( \lim_{\hat{z} \to \hat{z}} H'(\hat{z}) > 0 \). Ignore first that \( \hat{z} \) has an upper bound \( \hat{z} \). \( h(\hat{z}) \) is increasing by Assumption 2, \( \phi(I) - (\hat{z}\phi(I) - \alpha I)h(\hat{z}) \) is decreasing with limit \(-\infty\) as \( \hat{z} \) goes to infinity. Therefore \( H(\hat{z}) \) has a unique interior maximum \( \hat{z}_{\text{max}} > \hat{z} \).

Look at the maximum of \( H \) restricted to \([\tilde{z}, \hat{z}]\) which is \( \min(\hat{z}_{\text{max}}, \hat{z}) \). If \( H(\min(\hat{z}_{\text{max}}, \hat{z})) \leq 0 \) then the loan will not be financed at all. If positive, there exists a unique \( \hat{z} \in (\tilde{z}, \hat{z}) \) such that \( H(\hat{z}) = 0 \) at which \( H \) is positively sloped i.e. \( H'(\hat{z}) > 0 \).

**Proof of Proposition 4.**

The first order condition to (4.1) is:

\[
\int_{\tilde{z}}^{\hat{z}} z\phi'(I)dF(z) + F(\hat{z})\alpha - R - \int_{\tilde{z}}^{\hat{z}} (z\phi'(I) - \alpha)dF(z) - \frac{\partial}{\partial I}(\hat{z}\phi(I) - \alpha I)f(\hat{z}) = 0 \quad (A.2)
\]
The first three terms correspond to (2.2) and Proposition 1 shows that its limit is $\infty$ as $I \to 0$. The last two terms are the marginal welfare loss. $\hat{z}$ is implicitly defined by (3.4). Both $\hat{z}$ and $\tilde{z}$ go to 0 for $I \to 0$ and the marginal welfare loss disappears. Thus the left hand side of (A.2) evaluated as $I \to 0$ is $\infty$.

Evaluate the left hand side at $I^F$. By construction, the first three terms are 0. Analyze the marginal welfare loss term by term. Define $\psi(\alpha)$ as:

$$\psi(\alpha) = \int_{\hat{z}}^{\tilde{z}} (z \phi'(I^F(\alpha)) - \alpha) dF(z)$$

Proposition 1 implies that for $\alpha \in [0, R]$ there exists a unique $I^F(\alpha)$ increasing in $\alpha$. Consider its limit as $\alpha \to 0$. $I^F(0)$ is given by $E(z \phi'(I^F)) - R = 0$, $\hat{z}(0) > 0$ is implicitly defined by $(1 - F(\hat{z})) \hat{z} \phi(I^F) = R I^F$ and $\tilde{z}$ is 0. This implies

$$\lim_{\alpha \to 0} \psi(\alpha) > 0$$

Now evaluate $\alpha \to R$. By (3.4), $\hat{z}(R)$ satisfies $\hat{z} \phi(I^F) = R I^F$ and hence $\hat{z} = \tilde{z}$. The latter implies that $\psi(R) = 0$. Lastly $\psi(\alpha)$ is continuous in $\alpha$ on $[0, \tilde{\alpha}]$ so there exists an interval $[0, \tilde{\alpha}]$ such that:

$$\forall \alpha \leq \tilde{\alpha} : \psi(\alpha) \geq 0 \quad (A.3)$$

(A.3) is the sense in which I refer to $\alpha$ being sufficiently small. For $\alpha < \tilde{\alpha}$, the first term in the marginal welfare loss at the first-best is positive. I now claim that $\hat{z}$ increases in $I$ which concludes that the second term is also positive. By Implicit Function Theorem:

$$\frac{\partial \hat{z}}{\partial I} = - \frac{F(\hat{z}) \alpha + (1 - F(\hat{z})) \hat{z} \phi(I) - R}{H'(\hat{z})}$$

Using (3.4) and strict concavity of $\phi(I)$, the nominator is negative. Proposition 4 concludes that the denominator is positive. Hence $\hat{z}$ increases in $I$.

Put together the left hand side of (A.2) evaluated at $I^F$ is negative. Then there exists some $I^B < I^F$ such that (A.2) is satisfied. Fix $I^B$, the payments $(D^B, P^B)$ solve $PCF$ and $PCH$. Two equations with two unknowns imply a unique solution. \hfill \blacksquare

**Proof of Proposition 5.**

Suppose (5.1) is satisfied for a given $(I, L, B, E, D)$. $H(\hat{z})$ is defined as before:

$$H(\hat{z}) = L + F(\hat{z}) \alpha I + (1 - F(\hat{z})) \hat{z} \phi(I) - RB$$

and $H(\tilde{z}) \geq 0$. Proposition 3 proves that $\lim_{\hat{z} \to \tilde{z}} H'(\hat{z}) > 0$ and this suffices to show that the corner solution is $\tilde{z}$ and the welfare loss term in (4.1) disappears.

Conversely if the banking equilibrium is restricted to be ex post efficient, then $PCH$ evaluated at $\hat{z} = \tilde{z}$ needs to be satisfied. That is, even if the bank reports the lowest cash flow the households
make the required return. This is equivalent to \( H(\tilde{z}) \geq 0 \) or:

\[
L + F(\tilde{z})\alpha I + (1 - F(\tilde{z}))\tilde{z}\phi(I) \geq RB
\]

Since \( \tilde{z}\phi(I) = \alpha I \), it must be that \( L + \alpha I \geq RB \). At the equilibrium the bank always reports \( \tilde{z} \) and keeps the surplus for itself. All that the bank needs to report is whether the firm is liquidating the project. For solvent reports the households wait, otherwise the bank pays at \( t = 1 \).

**Proof of Proposition 6.**

Apply Proposition 5, the bank’s payoff is the shaded area in Figure 3.

\[
(1 - F(\bar{z}))D + \int_{\tilde{z}}^{\bar{z}} z\phi(I)dF(z) - (1 - F(\tilde{z}))RB
\]

Add and subtract \( F(\tilde{z})RB \) and use \( \alpha I = RB \) to write:

\[
(1 - F(\bar{z}))D + \int_{\tilde{z}}^{\bar{z}} z\phi(I)dF(z) + F(\tilde{z})\alpha I - RB
\]

The bank maximizes the payoff minus the cost of equity \( R_EE \) subject to a binding \( PCF \) or equivalently:

\[
\max_{B,E} \int_{\tilde{z}}^{\infty} z\phi(I)dF(z) + F(\tilde{z})\alpha I - \Pi - RB - R_EE
\]

Lastly, use \( I = B + E \) and (5.2) to replace \( B, E \) with \( I \):

\[
\max_{I} \int_{\tilde{z}}^{\infty} z\phi(I)dF(z) + F(\tilde{z})\alpha I - \Pi - \left( R + (R_E - R) \left( \frac{R - \alpha}{R} \right) \right) I
\]

Proposition 1 implies that the unique solution is \( I^F \) if \( R_E = R \) and \( I^B < I^F \) otherwise.

**Proof of Proposition 7.**

Apply Proposition 5, the shaded area in Figure 3

\[
(1 - F(\bar{z}))(D + L) + \int_{\tilde{z}}^{\bar{z}} (z\phi(I) + L)dF(z) - (1 - F(\tilde{z}))RI
\]

Rearranging terms using \( L = (R - \alpha)I \)

\[
(1 - F(\bar{z}))D + \int_{\tilde{z}}^{\bar{z}} z\phi(I)dF(z) + F(\tilde{z})\alpha I - \alpha I
\]

The bank maximizes the payoff minus the cost of liquidity \( R_EL \) subject to a binding \( PCF \) or equivalently:

\[
\max_{I} \int_{\tilde{z}}^{\infty} z\phi(I)dF(z) + F(\tilde{z})\alpha I - \Pi - (R + r_E(R - \alpha)) I
\]
Since $r_E(R - \alpha) > 0$, Proposition 1 implies that the unique solution is $I^B < I^F$. To compare the distortion term in Proposition 6, it follows immediately that

$$r_E(R - \alpha) > (r_E - r)\left(\frac{R - \alpha}{R}\right)$$

as $r_E > \frac{r_E - r}{R}$. □

**Proof of Proposition 8.**

Apply Proposition 5, the shaded area in Figure 3:

$$(1 - F(\bar{z}))D + \int_{\bar{z}}^{\tilde{z}} z\phi(I) + F(\tilde{z})\alpha I - (R - \rho)B$$

The bank maximizes the payoff minus the cost of equity $RE$ as $R_E = R$ by assumption, subject to a binding PCF:

$$\max_{(B,E)} \int_{\bar{z}}^{\tilde{z}} z\phi(I)dF(z) + F(\tilde{z})\alpha I - \Pi - (R - \rho)B - RE$$

Now write $(R - \rho)B + RE$ in terms of $I$. The balance sheet identity is $I = (1 - \rho)B + E$ and (5.5) implies $E = \left(\frac{R - \rho - (1 - \rho)\alpha}{\alpha}\right)B$.

$$(R - \rho)B + RE = RI + r\rho B = \left(R + \frac{r\rho\alpha}{R - \rho}\right)I$$

Let $R'$ denote the term in front of $I$. The bank chooses $I$ to maximize:

$$\max_{I} \int_{\bar{z}}^{\tilde{z}} z\phi(I)dF(z) + F(\tilde{z})\alpha I - \Pi - R'I$$

Proposition 1 implies that the unique solution is $I^B < I^F$. □

**Proof of Proposition 9.**

Suppose that the regulator can verify the realization of $z$. Whenever the bank reports $\alpha I \leq Y \leq \Gamma$, the regulator makes a take-it-or-leave-it offer $\Phi(Y, z) = z\phi(I)$ due at $t = 2$ in return for a credit $\alpha I$ at $t = 1$. Let $S = \{Y : Y \leq \Gamma\}$ denote the set of types that are promised a bailout and $S^C$ is its complement.

Take any $Y, Y' \in S^C$. Withdrawing for these reports violates the aggregate behavior constraint (3.1) since the types in $S^C$ are not promised a bailout and bank run reduces their payoff to $\alpha I$. Therefore $T_{\sigma(Y)} = 1$ and incentive compatibility implies:

$$Y - P(Y) \geq Y - P(Y')$$

Reversing the roles of $Y, Y'$, it follows that $P(Y) = \Gamma$ on $S^C$.

Consider any $Y \in S^C$ deviating to $Y' \in S$. As $P(Y') \leq \Gamma$, the high types have an incentive to report low. If the households withdraw by choosing $\sigma(Y') > 0$, the bank loses everything after
being bailed out by the regulator. So incentive compatibility implies that

\[ Y - \Gamma \geq T_{\sigma(Y')}(Y - P(Y')) \]

Hence \( T_{\sigma(Y')} = 0 \) on \( S \).

Lastly, any type \( Y \) in \( S \) gets 0 as it is forced into receivership to deter high types from lying. They have no incentive to misreport the cash flow regardless \( P(Y) \), so the contract can as well offer \( P(Y) = Y \) to leave the households indifferent. This concludes that the optimal contract given \( \Gamma \) is:

\[
P(Y) = \begin{cases} 
\Gamma & \text{if } Y \geq \Gamma \\
z\phi(I) & \text{if } \alpha I \leq Y < \Gamma \\
\alpha I & \text{if } Y < \alpha I
\end{cases}
\]

for \( \Gamma = \gamma\phi(I) \).

\( PCH \) is given in the text (6.1). The left hand side is less than \( RI \) for \( \gamma = \bar{z} \), greater than \( RI \) for \( \gamma = \hat{z} \) and is increasing in \( \gamma \). Therefore there exists a unique \( \gamma \in (\bar{z}, \hat{z}) \) and a corresponding \( \Gamma \) that satisfies \( PCH \).

Now the bank’s problem ex ante is:

\[
\max_{(I,D)} (1 - F(\bar{z}))D + \int_{\gamma}^{\bar{z}} z\phi(I) dF(z) - (1 - F(\gamma))\Gamma
\]

subject to (2.4) and (6.1) which are both binding. After substituting \( D \) and \( \Gamma \), the bank’s problem in choosing \( I \) corresponds to the frictionless benchmark (2.5). Thus the banking equilibrium investment is \( I^F \) and \((D^B, \Gamma^B) \) are uniquely pinned down by two participation constraints. ■