Incentives for Motivated Experts in a Partnership

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Abstract

A Principal needs two experts for production. Efficiency requires the experts to coordinate and each specializes on distinctive projects. The Experts are motivated. But the Principal is missing information about the projects' benefits and the experts' degrees of motivation. We show that the Principal can implement any project assignment between the two experts at the minimum cost by contracting with a profit-sharing Partnership formed by the experts. The Principal's single contract is linear in the experts' production costs. Moreover, the Principal need not concerned with how projects are processed and profits are shared within the Partnership.

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1 Introduction

In this paper, we study how a Principal contracts with experts for production. We have in mind a vertical structure. Suppose that a patient suffers from some skin problem. A primary-care physician (a generalist) and a dermatologist (a specialist) are the experts. The primary-care physician should provide care if the problem is not serious; otherwise, the dermatologist should do so. Similar vertical distinctions and matching issues are common among accounting, legal, and engineering professionals.

Suppose that these experts have private information about i) the Principal's benefits from various projects, and ii) their own preferences. Experts likely know more about clients' benefits from services, so the first source of asymmetric information is ubiquitous. Many professionals are motivated, which has been increasingly recognized in the economics literature.[1] Experts' payoffs are both motivational utilities and profits. Moreover, we hypothesize that the motivational utilities are privately known, so regard this as a second source of private information.[2] How can the Principal assign the low-cost-low-ability generalist to the low-benefit projects but the high-cost-high-ability specialist to the high-benefit projects?

We put forward a theory of the Principal contracting with an expert organization. Experts form a Partnership, which sets up a profit-sharing rule, and a project referral protocol between themselves. Experts share the same information within the organization. Each expert makes his own service decision. We show how the Principal can achieve the first best by a (quasi) linear contract to pay the Partnership, despite missing information about project benefits and experts' preferences.

We let the Principal's contract with the Partnership be based only on the number of services provided by each expert. The Principal then directs all projects to the Partnership. The Partnership has a gatekeeper. In what we call bottom-up referral, Expert 1 (the generalist) initially assesses all projects, and chooses between abandoning a project, providing service, and referring it to Expert 2 (the specialist). In top-down referral, Expert 2 is the gatekeeper, and Expert 1's role is also reversed. These are what we have called referral

[1] Benábou and Tirole (2003), Besley and Ghatak (2005), Delfgaauw and Dur (2007, 2008), Prendergast (2007), Khalil, Kim and Lawarrée (2013), and Kolstad (2013).

[2] Choné and Ma (2011), Frankel (2014), Jack (2005), Liu and Ma (2013), Markis and Siciliani (2013).

protocols.

The Partnership also decides on a sharing rule. We treat a Partnership as an accounting identity. Each expert incurs costs when services are provided, but receives payments from the Principal. The Partnership sharing rule splits the net proceeds and must be budget neutral; all net proceeds in all contingencies must be distributed among the experts themselves.

The Principal's objective is to abandon projects that have small benefits, get Expert 1 to provide service if the benefit is medium, and get Expert 2 to provide service if the benefit is large. Each expert is motivated, but has preferences that are a weighted sum of monetary profit, and the Principal's benefit, which we call motivation utility. Each expert's weight on motivation utility varies. The project potential benefits and experts' weights on motivation are unknown to the Principal.

We assume that a Partnership aims to maximize the experts' total surplus. For each of bottom-up and top-down referrals, the Partnership picks a sharing rule for surplus maximization subject to each expert earning his minimum profit. A sharing rule is also linear, and based on the number of services provided by each expert. Any project's potential benefit remains noncontractible, so it does not appear in the Principal's contract or a sharing rule. Experts' weights on motivation utility are assumed to be commonly known among themselves (but, again, unknown to the Principal). Hence, we allow a sharing rule to be based on the motivation parameters also (again linearly).

Our conception of a Partnership is that it is an organization where experts work. Within this organization, experts interact, often in a long-term basis; so to speak, they know each other.[3] The Principal is not part of this organization, has no control over the referral protocol or the corresponding sharing rule. We make the natural assumption that when an expert is to make a decision on a project, he knows that project's potential benefit. Each expert decides on his action: he is the sole decision maker on project termination, implementation, and referral (under the relevant protocol).

Each expert must earn a minimum profit. Although an expert also earns motivation utility when providing services to the Principal, we assume that this utility cannot be monetized to satisfy the minimum profit

[3] support this claim with cites

constraints. We adopt the common interpretation of intrinsic and extrinsic motivation, and the natural assumption that there is no capital market for these utilities. However, experts do trade-off between giving up profit and earning more motivation utility once they achieve their minimum profits.

The main result in our paper is this. The Principal can implement any assignment of projects among the two experts without paying them more than their minimum profits. In other words, the Principal achieves the first best—as if he knew about each project's potential benefit as well as each expert's degree of motivation. Even more striking, the Principal implements the first best by a single contract although he faces two dimensions of missing information.

The result can be explained as follows. If the Principal knew each project's potential benefit, he could simply assign them to experts to achieve any goal. Without this information, he needs to decentralize this assignment to a Partnership consisting of the two experts. The Principal also anticipates that, in turn, the Partnership will decentralize the assignment to the experts through a referral protocol and a sharing rule. We show that it is enough for the Principal to incentivize the Partnership for choosing his desired assignment, as if the Partnership were able to dictate each expert's decision.

Given the Principal's contract the Partnership attempts to maximize the experts joint surplus. This means achieving the appropriate tradeoff between motivation utility and profit. Generally, Partnerships that consist of highly motivated experts would like to provide more services and be more willing to give up profits. Hence, at some motivation level, the minimum profit constraints begin to bind. If there were more profits, the Partnership would like to provide more services at the expense of profit, but that would be impossible given the binding minimum profit constraint. Hence, once the minimum profit constraint bind for a Partnership at some motivation level, it continues to bind for higher levels. In other words, the more motivated Partnerships must choose the same allocation.

The Principal now exploits this monotonicity property. For his desired assignment, he simply offers a single contract; given this contract the least motivated Partnership finds that the desired assignment would maximize the experts' joint surplus and would just satisfy the minimum profit constraint. It follows that Partnerships with higher degrees of motivation must find the same assignment surplus-maximizing and satisfying the minimum profit constraint. If the Principal can rely on the Partnership to maximize the experts' joint surplus subject to the minimum profit constraint, he has achieved implementation at minimum expenses.

Can the Principal rely on the Partnership to do that? We show that he can! The Partnership has at its disposal two instruments to maximize its surplus subject to minimum profits. First, it can use a referral protocol, either bottom-up or top-down, to control when an expert gets to decide on a project. Second, for each referral protocol, it can design a sharing rule. Although the Principal's contract is independent of the experts' motivation parameters, the Principal realizes that the Partnership will adjust the sharing rule according to these parameters. For example, if Expert 1 is highly motivated to provide services to many projects, the Partnership will impose a high fee on him every time he provides service. In essence, the Partnership figures out how to align incentives to achieve the surplus-maximizing tradeoff between motivation and the net cost of providing service given the Principal's contract.

Our results extend to Seniority Partnership, in which one expert contracts with the Principal, and chooses a sharing rule under a referral protocol to maximize his own payoff. Here, the Principal's contract is a minimal and straightforward modification of the equilibrium contract under the symmetric Partnership considered above.

Our paper contributes to an emerging literature which studies how a principal should incentivize an agent with intrinsic motivation (See footnote 1). Most of these papers assume that the motivated agent's preference is observable to the principal except for Delfgaauw and Dur (2007), Jack (2005), Chone and Ma (2010) and Liu and Ma (2013). The main distinction between our paper and the literature is that we study the principal contracting with a Partnership consisting of multiple motivated experts.

Our paper is related to the literature on decentralization. Mookherjee (2006) provides an excellent survey on this topic. The main insight of the literature is that delegation has advantages over centralization in avoiding communication and information processing costs but incurs a problem of "loss of control". We show that by delegating the project to a Partnership, the principal does not suffer from loss of control even when experts' preferences are not perfectly aligned with the principal.

Garicano and Santos (2004) adopts a similar vertical structure and studies the matching between projects and experts under different organizations. In their model, projects arrives randomly at the marketplace and there does not exists a third party (principal) who can influence the match by contracting with the experts. Garichano and Santoz show that the efficient allocation can be achieved under top-down but not bottom-up referral. We find that the first best can be achieved both under bottom-up and top-down referral.

Our paper is also related to the literature on Partnership. While most of the papers study moral hazard(Holmstrong (1982), Legros and Matthews (1993), Strausz (1999)), we focus on adverse selection. Levin and Tadelis (2005) propose a selection-based theory of Partnership and explain why Parnterhip is a common organization in professional service industries. They show that Partnership can be regarded as a quality assurance mechanism. We take Partnership as one existing organization and analyze why a principal may benefit from contracting with a Partnership.

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2 The model

2.1 Principal and the first best

A Principal has a continuum of production projects with total mass normalized at 1. Each project is defined by a benefit index, b, a random variable distributed on a strictly positive support $[\underline{b}, \bar{b}]$ with distribution F and density f, so the total mass of projects with benefit less than b is $F(b)$. To proceed with production the Principal needs the service from one of two experts. We call these two experts *Expert 1* and *Expert 2*. Expert 1 has a productivity factor r_1, while Expert 2's productivity factor has a higher value r_2, so $0 < r_1 < r_2$. Expert 1's cost of providing service for the Principal is c_1, while Expert 2's cost is c_2, and $0 < c_1 < c_2$. If an expert works for a project with benefit index b, the Principal receives a utility or revenue $r_i b$, $i = 1, 2$. The productivity factor r_i can be thought of a (linear) rate of converting the benefit parameter b to utility or revenue. For a benefit index, Expert 2 generates a higher revenue for the Principal than Expert 1, but Expert 2 costs more. We assume $\frac{c_1}{r_1} < \frac{c_2}{r_2}$, which says that Expert 2's cost per productivity unit is higher than Expert 1; the assumption captures a notion of cost convexity for our discrete model.

An allocation is an assignment of an expert, or none at all, to each project. The first best is an allocation that maximizes the surplus $r_i b - c_i$, $i = 1, 2$, when the benefit index b is common knowledge. The first best assignment is: i) do not use any expert if $b < \frac{c_1}{r_1}$, ii) use Expert 1 if $\frac{c_1}{r_1} \le b < \frac{c_2}{r_2}$, and iii) use Expert 2 if $\frac{c_2}{r_2} \le b$. We define $b_1^* \equiv \frac{c_1}{r_1}$, $b_2^* \equiv \frac{c_2}{r_2}$, and assume that the support of b is wide enough to include both b_1^* and b_2^*. For future use, we summarize the first best as a lemma.[4]

Lemma 1 *If b is below b_1^*, it is not worthwhile for the Principal to use an expert. For b between b_1^* and b_2^*, it is optimal to use Expert 1's service, while for b higher than b_2^*, it is optimal to use Expert 2's service.*

The first best describes a hierarchical allocation. If the benefit index is low, no expert is used. As benefit increases, the low-productivity-low-cost expert will be used. At still higher benefits, the high-productivity-high-cost expert will be used. As we will see, hierarchical allocations are important, and we will write down a formal definition later.

2.2 Motivated experts

The experts are motivated, and enjoy utilities proportional to the Principal's revenue. If the Principal obtains a utility or revenue R from production, Expert 1 and Expert 2, respectively, receive utilities $\alpha_1 R$ and $\alpha_2 R$. Here, the parameters α_1 and α_2 are, respectively, Expert 1's and Expert 2's degrees of motivation, which are distributed on strictly positive supports $[\underline{\alpha}_1, \overline{\alpha}_1]$ and $[\underline{\alpha}_2, \overline{\alpha}_2]$ with some distributions and densities. We assume that each expert receives the utility due to motivation; the two experts work as a team, so each expert derives some satisfaction from production.

We provide two interpretations, which are now quite standard in the literature. First, the utilities $\alpha_1 R$ and $\alpha_2 R$ originate from the experts' *intrinsic* preferences. For example, physicians are altruistic towards their patients. Here, the Principal's utility R represents the gain from a medical treatment. The productivity factor is the success probability of a treatment. The issue concerns whether a (less expensive) primary care physician or a (more expensive) specialist should provide treatment. In any case, each physician enjoys some

[4] For project with benefit parameter b, the surplus from an expert is $r_i b - c_i$, $i = 1, 2$. Clearly, for $b < b_1$, $r_i b - c_i < 0$, so using an expert is optimal if and only if $b > b_1$. Now $(r_2 - r_1)b - (c_2 - c_1) > 0$ if and only if $b > b_2$, so Expert 2 generates more surplus if and only if $b > b_2$. This completes the proof of Lemma 1.

utility when the patient is cured. As another example, the Principal may be a charity, and the experts are fund raisers, who target different donors. The two experts may use different methods, say mail versus electronic, to solicit donations. Either mail or electronic solicitation may be used, but one method may be more efficient for a particular target population. Fund raisers may get some satisfaction from what they do for the charity.

Second, the utilities $\alpha_1 R$ and $\alpha_2 R$ may refer to *extrinsic* preferences. For example, two lawyers with different expertise may offer services to a Principal. Depending on the case at hand, either lawyer may be more suitable to represent the Principal. The success of a case with the current Principal may bring in more business in the future. Hence, the utilities $\alpha_1 R$ and $\alpha_2 R$ may indicate these future returns from the lawyers' current engagement.

In any case, utilities from motivation are to be distinguished from the monetary payoffs—profits—experts receive. In other words, $\alpha_1 R$ and $\alpha_2 R$ do not count towards monetary profit. In the case of $\alpha_1 R$ and $\alpha_2 R$ representing enjoyment (intrinsic preferences), this is a natural interpretation. In the case of $\alpha_1 R$ and $\alpha_2 R$ representing future earnings (extrinsic preferences), this requires that experts cannot borrow against them, which also seems natural. We assume that any monetary payoff will add onto the utility from motivation in a separable way.

Each expert must earn a nonnegative minimum profit.[5] These are $\overline{\pi}_1$ for Expert 1 and $\overline{\pi}_2$ for Expert 2. (Again, profit does not include the motivation enjoyment $\alpha_1 R$ and $\alpha_2 R$.) Each expert is an economic entity, and cannot afford to earn less than the market value of his expertise. This assumption is shared by almost all previous works on motivated agents, as we have noted in the Introduction.[6] We assume that $\overline{\pi}_1$ and $\overline{\pi}_2$ are sufficiently small, so it is efficient for the Principal to hire the experts. The minimum profits are also the experts' reservation utilities. If they did not contract with the Principal, they would not obtain any utilities from motivation.

[5]Many authors use the term "limited liability" for a situation such as ours. We find this term literally misleading for our model. We do not deal with liability issues.

[6]Jack (2005) is an exception. There, a provider earns a strictly negative profit for enjoying the higher quality it chooses for its services.

2.3 Partnership

The two experts form a Partnership. To begin, we can regard the Partnership as a fictitious player: Partnership preferences are the sum of the experts' payoffs. A Partnership will satisfy a number of properties. First, a Partnership is an accounting identity; it does not receive any new resource other than what the Principal pays the experts, and it cannot dispose of resources other than through the experts themselves. In other words, a Partnership must be budget balanced. The Partnership, however, can establish a sharing rule among the experts to split the (net) profits among themselves. For example, the Partnership can stipulate that each expert's profit is 50% of partnership profit, no matter what that is, but of course it can be more general.

Second, the Partnership and experts share some information. We assume that the two experts in the Partnership know each other's degrees of motivation; the values of α_1 and α_2 are common knowledge among the experts. What about projects' benefit indexes? We could proceed to assume that the two experts shared the benefit-index information of all projects. However, we do not need to make such a strong assumption. Instead, we let an expert observe a project's benefit index any time he is asked to consider providing service for it.

Third, the Partnership can decide on a screening or gatekeeping protocol, which sets up initial assignments of projects to experts. For example, the Partnership can decide that each expert will initially look at 50% of all projects. Then each expert decides whether to abandon, provide service, or refer a project to the other. Alternatively, the Partnership can decide on a referral protocol in which one expert acts as the gatekeeper, assesses all projects first, and then refers some of them to the other expert.

What the Partnership cannot do is to dictate whether an expert should or should not carry out production. Once an expert is put in charge of a project, it is up to him to decide if he should carry out production or not. Our main analysis concerns how, given the Principal's contract, the Partnership sets up a screening or gatekeeping protocol together and an associated sharing rule to implement various productive decisions.

Later in Section 5, we will dispense with the Partnership being a fictitious player. We will let an expert become a Senior Partner, who contracts with the Principal, designs sharing rules, and sets up referral or gatekeeping protocols. The Senior Partner respects the balanced-budget constraint, and cannot dictate the other expert's actions.

2.4 Principal's contract and Partnership sharing rule

The Principal does not know the benefit index, only its distribution. Neither does the Principal know the experts' motivation parameters. The contractible events are whether an expert has provided service, and if so, whether it is Expert 1 or Expert 2. Furthermore, the Principal contracts with the Partnership.

The Principal's contract to the Partnership consists of a lump sum payment Γ, a payment γ_1 if Expert 1 provides a service, and a payment γ_2 if Expert 2 provides it. We denote this quasi-linear contract by the triple $(\Gamma, \gamma_1, \gamma_2)$. If m_1 and m_2 denote the masses of projects that Expert 1 and Expert 2 have provided services respectively, the Partnership receives $\Gamma + m_1\gamma_1 + m_2\gamma_2$ from the Principal. More complicated contracts can be considered, but unnecessary for our purpose.

Given the Principal's contract, the Partnership designs a sharing rule after it learns experts' motivation parameters α_1 and α_2. The Partnership sharing rule is based on the same contractible events, namely m_1 and m_2. For each (m_1, m_2), the sharing rule specifies $S_1(m_1, m_2; \alpha_1, \alpha_2)$ and $S_2(m_1, m_2; \alpha_1, \alpha_2)$, the shares of profit received by Expert 1 and Expert 2, respectively. Each expert must earn a minimum profit in equilibrium, so $S_1(m_1, m_2; \alpha_1, \alpha_2) \geq \overline{\pi}_1$, and $S_2(m_1, m_2; \alpha_1, \alpha_2) \geq \overline{\pi}_2$ in an equilibrium.

We use the accounting rule that the Partnership bears the production costs incurred by the experts. Partnership budget balance requires that for all $m_1, m_2 \geq 0$, $0 \leq m_1 + m_2 \leq 1$, and α_1 and α_2:

$$\Gamma + m_1(\gamma_1 - c_1) + m_2(\gamma_2 - c_2) = S_1(m_1, m_2; \alpha_1, \alpha_2) + S_2(m_1, m_2; \alpha_1, \alpha_2). \tag{1}$$

In the budget-balance definition (1), the left-hand side is the revenue received from the Principal less the cost of providing m_1 and m_2 units of services by Expert 1 and Expert 2, respectively; note that this is independent of the motivation parameters. The right-hand side is the sum of the revenue received by the two experts, and they can be dependent on the motivation parameters. This budget-balance requirement is to be satisfied for any combination of service provisions, and for all motivation parameters.

2.5 Partnership protocol and extensive form

As we have said above, the Partnership can determine how projects are to be processed. We will consider two protocols, which we call *bottom-up referral* and *top-down referral*. In bottom-up referral, the Partnership stipulates that Expert 1 initially decides on whether a project is to be abandoned. If it is, then no production will take place. Otherwise, Expert 1 decides whether he wants to provide service or to refer the project to Expert 2. Upon receiving a referral, Expert 2 decides whether he will provide service. Under bottom-up referral, Expert 2 cannot provide service to those projects that have been rejected by Expert 1. This kind of gatekeeping is common in the health care market. In top-down referral, Expert 2 screens all projects initially, and the rest of the referral process is like bottom-up referral with the experts' roles and responsibilities reversed. This type of referral is common in legal service and consulting markets.

The following is the extensive form under bottom-up protocol.

Stage 1 Nature draws the Principal's project benefit indexes and the experts' motivation parameters, all of which are unknown to the Principal. The experts' motivation parameters are common knowledge among the experts. The Principal offers a contract to the Partnership.

Stage 2 If the Partnership rejects the contract, the game ends. If the Partnership accepts the Principal's contract, it sets up a profit sharing rule.

Stage 3 For each project, Expert 1 observes its benefit index b and decides whether to withhold service, provide service, or refer it to Expert 2.

Stage 4 If Expert 2 receives a referral, he observes the benefit index of the referred project and decides whether to withhold or provide service. The Partnership will be paid by the Principal according to the terms of the contract, and each expert will be paid according to the Partnership sharing rule.

The extensive form under top-down protocol has the same four stages, with the roles of Experts 1 and 2 in Stages 3 and 4 interchanged, and we do not rewrite it here.

2.6 Allocations and payoffs

For brevity, we define allocations and payoffs with reference to bottom-up referral. For a project with benefit index b, an allocation determines which expert, if any, will provide service, and is defined by the following three functions: $\sigma_1 : [\underline{b}, \overline{b}] \to [0,1]$, $\rho_1 : [\underline{b}, \overline{b}] \to [0,1]$ and $\sigma_2 : [\underline{b}, \overline{b}] \to [0,1]$. The function σ_1 specifies the probability that Expert 1 implements a project with benefit index b, whereas the function ρ_1 is the probability that he refers the project to Expert 2. Finally, the function σ_2 specifies the probability that Expert 2 implements the project upon a referral. We require $0 \le \sigma_1 + \rho_1\sigma_2 \le 1$.

At b, the Principal's (expected) revenue is $\sigma_1(b)r_1 b + \rho_1(b)\sigma_2(b)r_2 b$. Given an allocation, the Principal's payoff is

$$\int_{\underline{b}}^{\overline{b}} \left[\sigma_1(b)r_1(b-\gamma_1) + \rho_1(b)\sigma_2(b)r_2(b-\gamma_2)\right] f(b)\mathrm{d}b - \Gamma. \tag{2}$$

This is simply the Principal's expected revenue less the cost paid to the Partnership. Similarly, we write down the Partnership payoff:

$$\Gamma + \int_{\underline{b}}^{\overline{b}} \left\{\sigma_1(b)[(\alpha_1+\alpha_2)r_1 b + \gamma_1 - c_1] + \rho_1(b)\sigma_2(b)[(\alpha_1+\alpha_2)r_2 b + \gamma_2 - c_2]\right\} f(b)\mathrm{d}b, \tag{3}$$

which is the sum of the experts' utilities given an allocation. The masses of projects that Expert 1 and Expert 2 provided services, respectively m_1 and m_2, are:

$$m_1 = \int_{\underline{b}}^{\overline{b}} \sigma_1(b)f(b)\mathrm{d}b \quad \text{and} \quad m_2 = \int_{\underline{b}}^{\overline{b}} \rho_1(b)\sigma_2(b)f(b)\mathrm{d}b. \tag{4}$$

Given an allocation and a sharing rule that specifies $S_1(m_1, m_2; \alpha_1, \alpha_2)$ and $S_2(m_1, m_2; \alpha_1, \alpha_2)$, Expert 1's and Expert 2's payoffs are, respectively:

$$S_1(m_1, m_2; \alpha_1, \alpha_2) + \alpha_1 \int_{\underline{b}}^{\overline{b}} \left[\sigma_1(b)r_1 b + \rho_1(b)\sigma_2(b)r_2 b\right] f(b)\mathrm{d}b, \tag{5}$$

$$S_2(m_1, m_2; \alpha_1, \alpha_2) + \alpha_2 \int_{\underline{b}}^{\overline{b}} \left[\sigma_1(b)r_1 b + \rho_1(b)\sigma_2(b)r_2 b\right] f(b)\mathrm{d}b. \tag{6}$$

For each expert, the payoff is the sum of profits and the motivation utilities.

It is clear that we can define allocations and payoffs with reference to top-down referral similarly. We will need to replace Expert 1's referral function ρ_1 by Expert 2's referral function ρ_2. The expressions from (2) to (6) will be rewritten correspondingly.

The Partnership is regarded as a fictitious player. It sets up the sharing rule, as prescribed by Stage 2 of the extensive form, and must obey the budget-balance condition (1). It must also ensure that each expert makes his respective minimum profit. Finally, Partnership preferences are the total surplus, the sum of the experts' payoffs (5) and (6).

3 Hierarchical allocation and Partnership surplus

In this section, we will be concerned with Partnership surplus given a contract from the Principal. We defer to the next section for the design of sharing rule and each expert's strategies. For this reason, we only need to consider functions σ_1 and σ_2, so will ignore experts' referral functions ρ_1 and ρ_2. Also, we can think of a Partnership's degree of motivation as the sum of that of the experts, $\alpha_1 + \alpha_2$.

3.1 Surplus maximization

To begin, we say that an allocation is *hierarchical* if for b_1 and b_2 where $\underline{b} \leq b_1 \leq b_2 \leq \overline{b}$,

$$\begin{aligned} \sigma_1(b) &= 1 \text{ for } b \in [b_1, b_2] \text{ and } 0 \text{ otherwise} \\ \sigma_2(b) &= 1 \text{ for } b \in [b_2, \overline{b}] \text{ and } 0 \text{ otherwise.} \end{aligned}$$

In a hierarchical allocation, a project is abandoned when its benefit index is below b_1, taken on by Expert 1 when the benefit index is between b_1 and b_2, and taken on by Expert 2 when it is higher than b_2. A hierarchical allocation is therefore characterized by the cutoffs b_1 and b_2, so we use (b_1, b_2) as a short-hand for a hierarchical allocation. The first best is the hierarchical allocation (b_1^*, b_2^*).

Given any contract $(\Gamma, \gamma_1, \gamma_2)$, the Partnership surplus (again ignoring referral function ρ_1) is

$$\Gamma + \int_{\underline{b}}^{\overline{b}} \{\sigma_1(b)[(\alpha_1 + \alpha_2) r_1 b + \gamma_1 - c_1] + \sigma_2(b)[(\alpha_1 + \alpha_2) r_2 b + \gamma_2 - c_2]\} f(b) \mathrm{d}b. \tag{7}$$

We first ignore the minimum profit constraint and characterize the allocation that maximizes the Partnership's surplus for any contract $(\Gamma, \gamma_1, \gamma_2)$.

Lemma 2 *Given any contract $(\Gamma, \gamma_1, \gamma_2)$, the Partnership's surplus is maximized by a hierarchical allocation.*

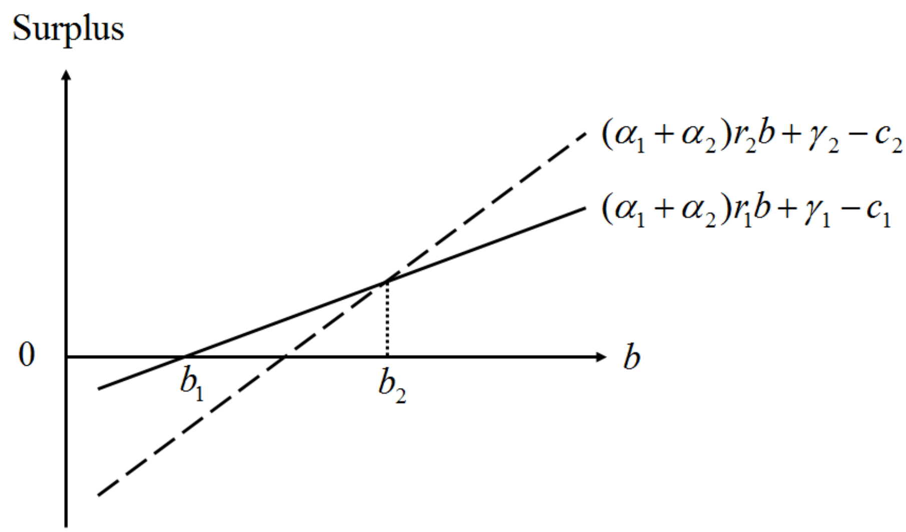

Figure 1: Experts' contribution to Partnership's surplus

To maximize the Partnership's surplus, differentiate (7) with respect to $\sigma_1(b)$ and $\sigma_2(b)$. The first-order derivatives give experts' contribution rates to surplus: $(\alpha_1+\alpha_2)r_1b+\gamma_1-c_1$ and $(\alpha_1+\alpha_2)r_2b+\gamma_2-c_2$ from Expert 1 and Expert 2, respectively. Because the objective function is quasi-linear in $\sigma_1(b)$ and $\sigma_2(b)$, the experts' contribution rates do not contain $\sigma_1(b)$ or $\sigma_2(b)$, so we must have corner solutions. If an expert's contribution at b is negative, the expert is not used. If at least one expert has a positive return at b, use the one that yields the higher return.

Figure 1 illustrates experts' contributions. In the diagram, the solid curve and dashed curve, respectively, plot Expert 1's and Expert 2's contributions. Each of the two curves is quasi-linear in the benefit index b, but $r_2 > r_1$ so Expert 2's contribution increases faster than Expert 1's as b increases. For any positive support $[\underline{b}, \overline{b}]$ in Figure 1, the surplus-maximizing allocation is hierarchical.

However, hierarchical allocations that use at most one expert are uninteresting. For example, if $b_1 = \overline{b}$ the Partnership will not provide any service at all. Similarly, if $b_2 = \underline{b}$ the Partnership will always use Expert 2. The implementation of such allocations is trivial (simply do not contract with one or both of the experts). From now on we will focus on *nontrivial* hierarchical allocations, those in which $\underline{b} < b_1 < b_2 < \overline{b}$, with optimal cutoffs in the interior of the benefit-index support.

To characterize optimal allocations, we use the first-order derivatives of (7) with respect to σ_1 and σ_2

(see also (28) and (29) in the Appendix). The first cutoff b_1 is the solution of $(\alpha_1+\alpha_2)r_1b_1+\gamma_1-c_1=0$, while the second cutoff b_2 is the solution of $(\alpha_1+\alpha_2)r_1b_2+\gamma_1-c_1=(\alpha_1+\alpha_2)r_2b_2+\gamma_2-c_2$:

$$b_1 \equiv \frac{c_1-\gamma_1}{(\alpha_1+\alpha_2)r_1} \quad \text{and} \quad b_2 \equiv \frac{(\gamma_1-c_1)-(\gamma_2-c_2)}{(\alpha_1+\alpha_2)(r_2-r_1)}. \tag{8}$$

A hierarchical allocation requires $\underline{b}<b_1<b_2<\overline{b}$, so from (8), we have $\gamma_2-c_2<\gamma_1-c_1<0$. In other words, the Principal must make the Partnership be responsible for some costs, and incur a higher net cost from Expert 2's service.[7] We summarize the comparative statics in the next result (with the proof omitted).

Lemma 3 *Suppose the surplus-maximizing allocation is a nontrivial hierarchical so that $\gamma_2-c_2<\gamma_1-c_1<0$. The cutoff b_1 is decreasing in γ_1, independent of γ_2, and decreasing in $\alpha_1+\alpha_2$. The cutoff b_2 is increasing in γ_1, and decreasing in both γ_2 and $\alpha_1+\alpha_2$.*

Lemma 3 presents the usual benefit-cost trade-off. If an expert's net cost is lower, that expert will be used more often. Because of the hierarchical structure of the solution, an increase in payment γ_1 will lead Expert 1 to provide services to more projects, both previously abandoned as well as those previously taken on by Expert 2. Finally, the Partnership obtains a higher motivation benefit from each project when $\alpha_1+\alpha_2$ increases, and it will lower both experts' cutoffs. The joint degree of motivation $\alpha_1+\alpha_2$ also determines the Partnership's surplus and profit, as the next lemma shows.

Lemma 4 *The maximized surplus of a Partnership increases in $\alpha_1+\alpha_2$, but the corresponding profit decreases in $\alpha_1+\alpha_2$.*

Because the surplus-maximizing allocation is hierarchical, we rewrite the Partnership surplus in (7) as

$$\Gamma+\int_{b_1}^{b_2}[(\alpha_1+\alpha_2)r_1b+\gamma_1-c_1]f(b)\mathrm{d}b+\int_{b_2}^{\overline{b}}[(\alpha_1+\alpha_2)r_2b+(\gamma_2-c_2)]f(b)\mathrm{d}b. \tag{9}$$

The Partnership's profit is

$$\Gamma+\int_{b_1}^{b_2}(\gamma_1-c_1)f(b)\mathrm{d}b+\int_{b_2}^{\overline{b}}(\gamma_2-c_2)f(b)\mathrm{d}b. \tag{10}$$

[7] To see this, first suppose either $\gamma_1-c_1\geq 0$ or $\gamma_2-c_2\geq 0$. Then at least one expert always makes a positive contribution to suplus, so $b_1=\underline{b}$ or $b_2=\underline{b}$. Next, if $\gamma_1-c_1\leq\gamma_2-c_2<0$, Expert 2 is more productive but has a lower net cost than Expert 1, so $b_2=b_1$.

First consider Partnership surplus. Since (b_1, b_2) is chosen to maximize (9) the Envelope Theorem applies. The indirect effect of $\alpha_1 + \alpha_2$ on (9) through b_1 and b_2 must sum to zero. The direct effect of $\alpha_1 + \alpha_2$ is strictly positive because the Partnership enjoys a higher motivation benefit from each project as $\alpha_1 + \alpha_2$ increases. To see that Partnership profit decreases in $\alpha_1 + \alpha_2$, observe that by Lemma 3, both b_1 and b_2 decrease in $\alpha_1 + \alpha_2$, so a more motivated Partnership will substitute Expert 2 for Expert 1 for projects with high benefits, and let Expert 1 take on more projects with low benefits. Such actions result in a higher net cost given that $\gamma_2 - c_2 < \gamma_1 - c_1 < 0$, so Partnership profit decreases.

3.2 Partnership surplus and minimum profit

In the previous subsection, we have studied the Partnership's surplus maximization given a contract from the Principal. Now we study how this is affected by the Partnership's need to earn a minimum profit. In this subsection, we impose the minimum profit constraint

$$\Gamma + \int_{b_1}^{b_2} (\gamma_1 - c_1) f(b) \mathrm{d}b + \int_{b_2}^{\bar{b}} (\gamma_2 - c_2) f(b) \mathrm{d}b \geq \overline{\pi}_1 + \overline{\pi}_2. \tag{11}$$

That is, an allocation must also allow the Partnership to achieve each expert's minimum profit. Given a contract, what is the Partnership's optimal allocation and surplus when it must also make the profit $\overline{\pi}_1 + \overline{\pi}_2$? The next Proposition presents a striking result: once a Partnership makes only the minimum profit, its optimal allocation must remain constant even when its degree of motivation increases, so any incremental surplus is due to higher motivation.

Proposition 1 *Consider a contract $(\Gamma, \gamma_1, \gamma_2)$. Suppose that a Partnership with a degree of motivation $\widetilde{\alpha}_1 + \widetilde{\alpha}_2$ chooses an allocation $(\widetilde{b}_1, \widetilde{b}_2)$ to maximize its surplus (9) subject to the binding minimum profit constraint (11). Then a Partnership with a degree of motivation $\alpha_1 + \alpha_2 > \widetilde{\alpha}_1 + \widetilde{\alpha}_2$ chooses the same allocation $(\widetilde{b}_1, \widetilde{b}_2)$ to maximize its surplus subject to the binding minimum profit constraint.*

How does a Partnership's surplus-maximizing allocation and optimal surplus change as its degree of motivation, $\alpha_1 + \alpha_2$ change when the Partnership must also earn a minimum profit $\overline{\pi}_1 + \overline{\pi}_2$? At low motivation levels (small $\alpha_1 + \alpha_2$), the Partnership is less concerned with project benefits, so profits are higher than the minimum, and results in the previous subsection apply. As motivation $\alpha_1 + \alpha_2$ increases, according to Lemma

4, the minimum profit constraint (11) binds at some motivation level. We have, therefore,

$$\Gamma + \int_{b_1}^{b_2} (\gamma_1 - c_1) f(b) \mathrm{d}b + \int_{b_2}^{\bar{b}} (\gamma_2 - c_2) f(b) \mathrm{d}b = \overline{\pi}_1 + \overline{\pi}_2.$$

The surplus of a Partnership in (9) can then be simplified to

$$(\alpha_1 + \alpha_2) \left[\int_{b_1}^{b_2} r_1 b f(b) \mathrm{d}b + \int_{b_2}^{\bar{b}} r_2 b f(b) \mathrm{d}b \right] + \overline{\pi}_1 + \overline{\pi}_2. \tag{12}$$

From Lemma 4, we know that, for a given contract $(\Gamma, \gamma_1, \gamma_2)$, if the minimum profit constraint binds for a Partnership with a degree of motivation $\widetilde{\alpha}_1 + \widetilde{\alpha}_2$, it also binds for a Partnership with motivation $\alpha_1 + \alpha_2 > \widetilde{\alpha}_1 + \widetilde{\alpha}_2$. Therefore, the objective function of a Partnership with a degree of motivation higher than $\widetilde{\alpha}_1 + \widetilde{\alpha}_2$ must have the form in (12). When the minimum profit constraint binds, the degree of motivation, $\alpha_1 + \alpha_2$, is inessential for surplus maximization subject to the minimum profit constraint (11). Any Partnership that makes only the minimum profit must choose the same allocation to maximize its surplus.

An immediate implication of Proposition 1 is the following (and its proof is omitted).

Corollary 1 *Suppose that for the contract* $(\Gamma, \gamma_1, \gamma_2)$ *the Partnership with motivation* $\underline{\alpha}_1 + \underline{\alpha}_2$ *makes only the minimum profit when it maximizes its surplus subject to the minimum profit constraint by an allocation* (b_1, b_2)*, then any Partnership with motivation* $\alpha_1 + \alpha_2$ *will choose* (b_1, b_2) *to maximize its surplus subject to the minimum profit constraint, and also makes only the minimum profit.*

Corollary 1 allows the Principal to implement any hierarchical allocation if a Partnership could be entrusted to maximize its surplus subject to minimum profit. All the Principal has to do is to make the Partnership with the least motivation to optimally choose that hierarchical allocation and make only the minimum profit. Any Partnership with a higher motivation would have preferred to undertake more projects than the least motivated Partnership, but the minimum profit constraint prevents them from doing so. They all end up choosing the same allocation, and also make only the minimum profit. We state this result as a proposition.

Proposition 2 *Consider a hierarchical allocation* $(\widehat{b}_1, \widehat{b}_2)$*. If the Principal offers a contract*

$$\widehat{\Gamma} = (\underline{\alpha}_1 + \underline{\alpha}_2) \left\{ r_1 \widehat{b}_1 \left[1 - F(\widehat{b}_1) \right] + (r_2 - r_1) \widehat{b}_2 \left[1 - F(\widehat{b}_2) \right] \right\} + \overline{\pi}_1 + \overline{\pi}_2 \tag{13}$$

$$\widehat{\gamma}_1 = c_1 - (\underline{\alpha}_1 + \underline{\alpha}_2) r_1 \widehat{b}_1 \tag{14}$$

$$\widehat{\gamma}_2 = c_2 - (\underline{\alpha}_1 + \underline{\alpha}_2) \left[r_1 \widehat{b}_1 + (r_2 - r_1) \widehat{b}_2 \right], \tag{15}$$

then a Partnership of any degree of motivation maximizes its surplus and makes the minimum profit by choosing the same hierarchical allocation $(\widehat{b}_1, \widehat{b}_2)$*.*

Proposition 2 is a straightforward application of results in the previous and the current subsections. From Corollary 1, if the least motivated Partnership earns only minimum profit when it chooses an allocation to maximize surplus, all other, more motivated Partnerships will choose the same allocation. Therefore we only have to consider the constrained surplus maximization of the least motivated Partnership.

For surplus maximization without any concern for minimum profit, condition (8) gives a Partnership's best response against any contract. Equations in (8) are linear. We simply invert them to find the cost shares γ_1 and γ_2 against which the allocation $(\widehat{b}_1, \widehat{b}_2)$ is optimal. The Partnership must bear some net cost for undertaking some projects, so we make sure that the transfer is sufficient to let the Partnership make $\overline{\pi}_1 + \overline{\pi}_2$ if it chooses the allocation $(\widehat{b}_1, \widehat{b}_2)$. These steps yield the contract $(\widehat{\Gamma}, \widehat{\gamma}_1, \widehat{\gamma}_2)$ in Proposition 2.

It is important to understand the scope of Proposition 2. If a Partnership chooses an allocation to maximize its surplus subject to obtaining minimum profits for experts, then any hierarchical allocation, including the first best, can be implemented by the Principal. This requires no knowledge of the experts' motivation distributions.

Partnership surplus maximization is more a mathematical construct than an analysis of strategic interactions. The strategic roles played by Experts 1 and 2 have been sidestepped so far. In the next section, we analyze the extensive form introduced in section 2.

4 Partnership sharing rule and implementation

We now return to the extesnive-form games defined in Subsection 2.5. Our results in the previous section characterize Partnership surplus maximization subject to minimum profits, and we now explain how those results will fit into the analysis of the strategic interactions between the two experts.

From Proposition 2, given the Principal's contract $(\widehat{\Gamma}, \widehat{\gamma}_1, \widehat{\gamma}_2)$, any Partnership, irrespective of its motivation parameter, maximizes its surplus by the hierarchical allocation $(\widehat{b}_1, \widehat{b}_2)$ and earns the minimum profits $\overline{\pi}_1 + \overline{\pi}_2$. If a Partnership could dictate experts' actions, it would prescribe the allocation $(\widehat{b}_1, \widehat{b}_2)$. The Partnership, however, cannot dictate actions. It only can prescribe a referral protocol and a sharing rule. We consider bottom-up and top-down referrals. Can the Partnership construct a sharing rule, under each referral protocol, to implement $(\widehat{b}_1, \widehat{b}_2)$?

4.1 Bottom-up referral

In this subsection, we present a sharing rule to implement allocation $(\widehat{b}_1, \widehat{b}_2)$ under bottom-up referral, given the Principal's contract $(\widehat{\Gamma}, \widehat{\gamma}_1, \widehat{\gamma}_2)$ in Proposition 2. It is more convenient to write the sharing rule also in terms of the allocation $(\widehat{b}_1, \widehat{b}_2)$ as well as the Principal's contract $(\widehat{\Gamma}, \widehat{\gamma}_1, \widehat{\gamma}_2)$. (Do note that $(\widehat{b}_1, \widehat{b}_2)$ maximizes Partnership surplus given the contract $(\widehat{\Gamma}, \widehat{\gamma}_1, \widehat{\gamma}_2)$.) The sharing rule is

$$S_1^B(m_1, m_2; \alpha_1, \alpha_2) = \Delta - \alpha_1 r_1 \widehat{b}_1 m_1 + \alpha_2 r_2 \widehat{b}_2 m_2 + (\widehat{\gamma}_2 - c_2) m_2 \quad (16)$$

$$S_2^B(m_1, m_2; \alpha_1, \alpha_2) = \widehat{\Gamma} - \Delta - \alpha_2 r_2 \widehat{b}_2 m_2 + \alpha_1 r_1 \widehat{b}_1 m_1 + (\widehat{\gamma}_1 - c_1) m_1, \quad (17)$$

where Δ is some constant, and where, again, $\widehat{b}_1$ and $\widehat{b}_2$ satisfy (14) and (15) (or equivalently (8)). We note that $(\widehat{b}_1, \widehat{b}_2)$ can be any (nontrivial) hierarchical allocation, and that the contract $(\widehat{\Gamma}, \widehat{\gamma}_1, \widehat{\gamma}_2)$ in Proposition 2 uses the lower bounds of the experts' motivation parameters.

How does this sharing rule work? First, according to the contract $(\widehat{\Gamma}, \widehat{\gamma}_1, \widehat{\gamma}_2)$ the Partnership receives a net, negative payment $\widehat{\gamma}_i - c_i$ for a project taken on by Expert i, $i = 1, 2$. In the sharing rule, each expert is responsible for the net payment due to the other expert's service; these are the last terms in (16) and (17), $\widehat{\gamma}_2 - c_2$ and $\widehat{\gamma}_1 - c_1$. Second, from the terms involving $\widehat{b}_1$ and $\widehat{b}_2$ in (16) and (17), if Expert 1 provides service to one more project, he pays Expert 2 $\alpha_1 r_1 \widehat{b}_1$, and if Expert 2 provides service to one more project, he pays

Expert 1 $\alpha_2 r_2 \widehat{b}_2$. Each expert pays more if he is more motivated: these payments, $\alpha_1 r_1 \widehat{b}_1$ and $\alpha_2 r_2 \widehat{b}_2$, are increasing in the motivation parameters α_1 and α_2. Finally, the lump sum Δ will ensure that each expert makes his minimum profit.

If Expert 1 were the only decision maker (so that m_2 was fixed), then his incremental payoff for providing service to a project with benefit index b would be $\alpha_1 r_1 (b - \widehat{b}_1)$, which is positive when $b > \widehat{b}_1$. Similarly, consider S_2^B in (17). Again, suppose that Expert 2 were the only decision maker (so that m_1 was fixed), then his incremental payoff for providing service to a project with benefit index b would be $\alpha_2 r_2 (b - \widehat{b}_2)$, which is positive when $b > \widehat{b}_2$.

Expert 2's decision in the subgame in Stage 4 is quite straightforward. Because Expert 1's service decision has already been taken in Stage 3, the value of m_1 is determined. According to (17), Expert 2 pays $\alpha_2 r_2 \widehat{b}_2$ for rendering service to a project. Now for a project with benefit index b, Expert 2 earns a motivation utility $\alpha_2 r_2 b$. To decide whether he provides service or not, Expert 2 compares $\alpha_2 r_2 b$ and $\alpha_2 r_2 \widehat{b}_2$. Clearly, we can state the following (with proof omitted):

Lemma 5 *In the subgame in Stage 4, Expert 2 provide service to a project with benefit index b if and only if $b > \widehat{b}_2$.*

We next consider the subgame at Stage 3 given Expert 2's equilibrium strategy in Stage 4. Here, if Expert 1 provides service to a project with benefit b, he earns motivation benefit $\alpha_1 r_1 b$ and incurs the cost $\alpha_1 r_1 \widehat{b}_1$. We consider two cases. First, suppose that $b < \widehat{b}_2$, then Expert 2 will not take on this project even if Expert 1 refers it to him. Therefore, Expert 1 will render service to this project if and only if $b > \widehat{b}_1$. Because by assumption $\widehat{b}_1 < \widehat{b}_2$, we have derived Expert 1's equilibrium decision for $b < \widehat{b}_2$: take on the project if the benefit is between $\widehat{b}_1 < \widehat{b}_2$, and abandon otherwise.

Second, suppose that $b > \widehat{b}_2$. Here, again, Expert 1 knows that he can get motivation benefit $\alpha_1 r_1 b$ and incurs the cost $\alpha_1 r_1 \widehat{b}_1$ for a net payoff

$$\alpha_1 r_1 b - \alpha_1 r_1 \widehat{b}_1. \tag{18}$$

However, he can refer the project to Expert 2, whose best response is to render service to it. Now this yields

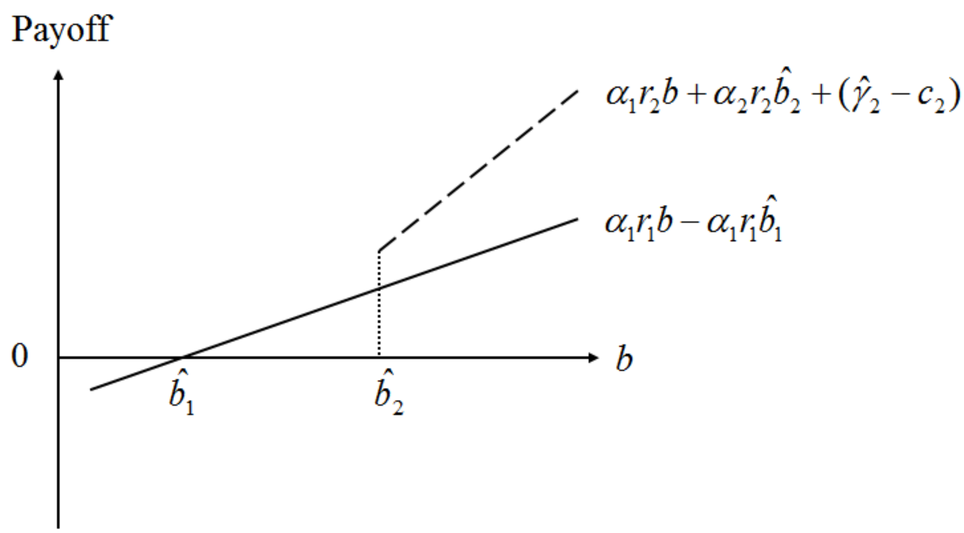

Figure 2: Expert 1's payoff in Stage 3

a motivation benefit $\alpha_1 r_2 b$ to Expert 1, and a revenue $\alpha_2 r_2 \widehat{b}_2 + (\widehat{\gamma}_2 - c_2)$. Expert 1's payoff from referral is

$$\alpha_1 r_2 b + \alpha_2 r_2 \widehat{b}_2 + (\widehat{\gamma}_2 - c_2). \tag{19}$$

How does the payoff from providing service in (18) compare with referring in (19)?

Lemma 6 *In the subgame in Stage 3, Expert 1 provides service to a project if and only if $\widehat{b}_1 < b < \widehat{b}_2$ and refers a project to Expert 2 if and only if $b \geq \widehat{b}_2$.*

What should Expert 1 do when he has a project with a benefit index $b > \widehat{b}_2$? Expert 1 can render service to this project, but also knows that Expert 2 will also take it as a referral. Expert 1's payoff is $\alpha_1 r_1 b - \alpha_1 r_1 \widehat{b}_1$ from providing service to the project, and $\alpha_1 r_2 b +$ $\alpha_2 r_2 \widehat{b}_2 + (\widehat{\gamma}_2 - c_2)$ from referral. In Figure 2, the solid line plots Expert 1's payoffs from providing service to a project, and the dashed line plots his payoffs from referral. Because Expert 2 will not provide service to a project with benefit $b < \widehat{b}_2$, Expert 1's payoff from referral is discontinuous at $b = \widehat{b}_2$; it drops to 0 for any $b < \widehat{b}_2$. Because $r_2 > r_1$, Expert 1's payoff from referral increases faster than taking on a project as b increases, so the dashed line is steeper than the solid line. Figure 2 shows that Expert 1 strictly prefers referral to taking on the project himself when the project benefit index is $\widehat{b}_2$. We now state the main result in this subsection.

Proposition 3 *Given a hierarchical allocation* $(\widehat{b}_1, \widehat{b}_2)$*, suppose that the Principal offers the contract in Proposition 2. In a subgame-perfect equilibrium of the bottom-up referral extensive form in Subsection 2.5, 1) the Partnership accepts the contract and chooses the budget-balanced sharing rule defined by (16) and (17); 2) the hierarchical allocation* $(\widehat{b}_1, \widehat{b}_2)$ *is implemented by the two experts; and 3) Expert 1's profit is* $\overline{\pi}_1$*, and Expert 2's profit is* $\overline{\pi}_2$*.*

We have shown in Lemmas 5 and 6 that given the sharing rule, $(\widehat{b}_1, \widehat{b}_2)$ is implemented by the two experts in the continuation equilibria in Stages 3 and 4. By Proposition 2, given the Principal's contract $(\widehat{\Gamma}, \widehat{\gamma}_1, \widehat{\gamma}_2)$, the Partnership makes profit $\overline{\pi}_1 + \overline{\pi}_2$ by implementing $(\widehat{b}_1, \widehat{b}_2)$. We then pick the value of Δ in (16) so that Expert 1 makes his minimum profit $\overline{\pi}_1$ in equilibrium (see equation (33) in the appendix). Because the sharing rule is always budget balanced, Expert 2 must also make his minimum profit $\overline{\pi}_2$ in equilibrium. These minimum profits ensure that accepting the Principal's contract in Proposition 2 and forming the sharing rule in (16) and (17) under bottom-up referral is a continuation equilibrium in Stage 2.

Because the first best allocation in Lemma 1 is a hierarchical allocation, the following follows from Proposition 3 immediately.

Corollary 2 *The first-best allocation* (b_1^*, b_2^*) *can be implemented as a subgame-perfect equilibrium outcome of the bottom-up referral game defined in Subsection 2.5.*

4.2 Top-down referral

Now we turn to top-down referral. Here, Expert 2 screens all projects first, and refers some to Expert 1. Let the Principal offer the same contract in Proposition 2. Can the Partnership use a top-down referral protocol and a sharing rule to implement $(\widehat{b}_1, \widehat{b}_2)$? Our next result gives an affirmative answer.

Proposition 4 *Given a hierarchical allocation* $(\widehat{b}_1, \widehat{b}_2)$*, suppose that the Principal offers the contract in Proposition 2. In a subgame-perfect equilibrium of the top-down referral extensive form in Subsection 2.5,* *1) the Partnership accepts the contract and chooses the budget-balanced sharing rule*

$$S_1^T(m_1, m_2; \alpha_1, \alpha_2) = \Lambda + \alpha_2 r_1 \widehat{b}_1 m_1 + \alpha_2 \left[r_1 \widehat{b}_1 + (r_2 - r_1)\widehat{b}_2 \right] m_2 + (\widehat{\gamma}_1 - c_1) m_1 + (\widehat{\gamma}_2 - c_2) m_2 \quad (20)$$

$$S_2^T(m_1, m_2; \alpha_1, \alpha_2) = \widehat{\Gamma} - \Lambda - \alpha_2 r_1 \widehat{b}_1 m_1 - \alpha_2 \left[r_1 \widehat{b}_1 + (r_2 - r_1)\widehat{b}_2 \right] m_2, \quad (21)$$

for some constant Λ; 2) the hierarchical allocation $(\widehat{b}_1, \widehat{b}_2)$ is implemented by the two experts; and 3) Expert 1 and Expert 2's profits are $\overline{\pi}_1$ and $\overline{\pi}_2$, respectively.

The sharing rule in (20) and (21) makes Expert 1 bear the net costs $c_1 - \widehat{\gamma}_1$ and $c_2 - \widehat{\gamma}_2$ for each project implemented by himself and Expert 2, respectively. But this sharing rule also specifies that Expert 2 pays Expert 1 a positive transfer $\alpha_2 r_1 \widehat{b}_1$ for every project Expert 1 takes on, and $\alpha_2[r_1\widehat{b}_1 + (r_2 - r_1)\widehat{b}_2]$ for every project that Expert 2 takes on.

Now consider Expert 1's decision in the continuation game in Stage 4. If he provides service to a project with benefit index b, his payoff is $\alpha_1 b r_1 + \alpha_2 r_1 \widehat{b}_1 + \widehat{\gamma}_1 - c_1$. By the definition of $\widehat{\gamma}_1$ in Proposition 2 (see (14)), we have $\widehat{\gamma}_1 - c_1 = -(\underline{\alpha}_1 + \underline{\alpha}_2) r_1 \widehat{b}_1$. Hence, Expert 1 provides service to a project with benefit index b if and only if $\alpha_1 b r_1 + \alpha_2 r_1 \widehat{b}_1 - (\underline{\alpha}_1 + \underline{\alpha}_2) r_1 \widehat{b}_1 \geq 0$, which is rewritten as

$$b \geq \left(\frac{\underline{\alpha}_1 + \underline{\alpha}_2 - \alpha_2}{\alpha_1} \right) \widehat{b}_1 = \left(\frac{\underline{\alpha}_1}{\alpha_1} - \frac{\alpha_2 - \underline{\alpha}_2}{\alpha_1} \right) \widehat{b}_1 \equiv \widetilde{b}_1. \quad (22)$$

For future use, we note that because $\underline{\alpha}_1 \leq \alpha_1$ and $\underline{\alpha}_2 \leq \alpha_2$, it follows that $\widetilde{b}_1 \leq \widehat{b}_1$.

Given Expert 1's equilibrium strategy in Stage 4, what does Expert 2 do in Stage 3? If Expert 2 takes on a project with benefit index b, he receives a motivation benefit $\alpha_2 r_2 b$ but pays an amount $\alpha_2[r_1\widehat{b}_1 + (r_2 - r_1)\widehat{b}_2]$ to Expert 1. Expert 2's payoff from providing services to the project is $\alpha_2[r_2(b - \widehat{b}_2) + r_1(\widehat{b}_2 - \widehat{b}_1)]$, which is illustrated by the dashed line in Figure 3. Alternatively, if Expert 2 refers the project, Expert 1 will render services if and only if the project's benefit index is at least $\widetilde{b}_1$. Hence, Expert 2 receives a motivation benefit $\alpha_2 r_1 b$ and pays an amount of $\alpha_2 r_1 \widehat{b}_1$ to Expert 1 per referral for projects with $b \geq \widetilde{b}_1$. If Expert 2 refers a project with benefit index below $\widetilde{b}_1$, the project will be abandoned by Expert 1. Consequently, Expert 2 receives zero payoff. The solid line in Figure 3 represents Expert 2's payoffs from referral and it is discontinuous at $b = \widetilde{b}_1$.

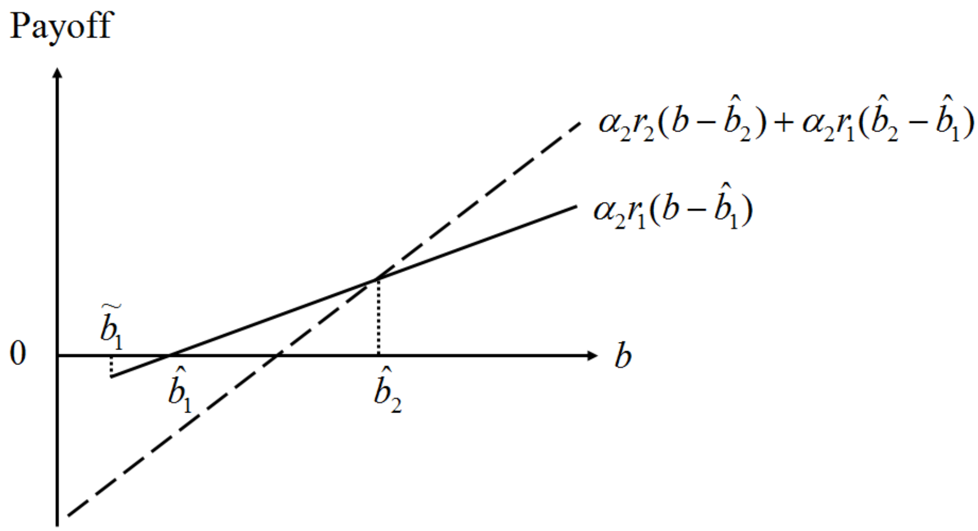

Figure 3: Expert 2's payoff in Stage 3

The diagram shows that in Stage 3 of the top-down referral game, Expert 2 maximizes his payoff by providing services if and only if $b \geq \widehat{b}_2$, making a referral if and only if $\widehat{b}_1 \leq b < \widehat{b}_2$, and abandoning a project if and only if $b < \widehat{b}_1$. In the equilibrium, even though Expert 1 may be willing to render services to projects with benefit below $\widehat{b}_1$, Expert 2 will never send him those projects.

The equilibrium sharing rule for top-down referral in (20) and (21) differs markedly from that for bottom-up referral in (16) and (17). In bottom-up referral, each expert is made to bear the net cost of his partner's services. By contract, in top-down referral, Expert 1 is asked to bear the net costs of both experts' services.

Nevertheless, each sharing rule uses net payments and transfers to achieve the same equilibrium outcome, experts implementing the hierarchical allocation $(\widehat{b}_1, \widehat{b}_2)$ in the referral protocol under consideration. The sharing rules, however, may lead to different off-equilibrium moves. Under bottom-up referral, the sharing rule in (16) and (17) ensures that Expert 2 will take on a project in Stage 4 *if and only if* the hierarchical allocation requires him to do so. If Expert 1 did refer a project with benefit less than $\widehat{b}_2$, Expert 2 would reject it; see Figure 2. The sharing rule under top-down referral provides somewhat different financial incentives. Given (20) and (21), Expert 1 would take on more projects in Stage 4 than the hierarchical allocation prescribe; see Figure 3. Expert 2 would rather implement some of these projects himself, and optimally refers a project *if and only if* $\widehat{b}_1 \leq b < \widehat{b}_2$.

For both bottom-up and top-down referrals, given the Principal's contract in Proposition 2, the continuation equilibria in Propositions 3 and 4, respectively under bottom-up and top-down referrals, implement the hierarchical allocation $(\widehat{b}_1, \widehat{b}_2)$ at minimum profits to the experts. The Principal does not have to be concerned with whichever of these two referral protocols the Partnership happens to use. However, we do not provide an exhaustive investigation on whether other protocols (and their corresponding sharing rules) may also implement the allocation at minimum profits to the experts..

5 Seniority Partnerships

The Partnership has been regarded is a fictitious player who has inherited the two experts' payoffs, and who is in a position to set up details for implementation. This modeling method is a reduced-form version for how partnerships set up their charter and governance: members of a partnership decide on these details when the organization is set up. Now, we consider an alternative partnership design in which one member is the Senior Partner while the other is the Junior Partner.

The Senior Partner and the Junior Partner may be joint owners of the firm. Alternatively, the Senior partner may be the owner, and the Junior Partner may be an employee. For our purpose, we use the assumption that the Senior Partner is the person who contracts with the Principal, and who designs any details of implementation. It is the Senior Partner who sets up the sharing rule. We let the Senior Partner choose a sharing rule to maximize his own utility (financial gains and motivation utility), but he must always respect minimum profit constraints.

There are two cases. First, Expert 1 is the Senior Partner; second, Expert 2 is the Senior Partner. Again, In each case, we consider bottom-up and top-down referrals. The Senior Partner contracts with the Principal, and designs sharing rules to maximize his own payoff. Otherwise, we maintain the other assumptions: experts share the same information about motivation, and each expert chooses his actions.

The extensive form for Expert 1 being the Senior Partner under bottom-up referral is as follows.

Stage 1 Nature draws the Principal's project benefit indexes and the experts' motivation parameters, all of which being unknown to the Principal. Experts' motivation parameters are common knowledge among themselves. The Principal offers a contract to the Senior Partner Expert 1.

Stage 2 If Expert 1 rejects the contract on behalf of the Partnership, the game ends. If Expert 1 accepts the Principal's contract, he sets up a profit sharing rule.

Stage 3 For each project Expert 1 observes its benefit index b and decides whether to withhold service, provide service, or refer it to Expert 2.

Stage 4 If Expert 2 receives a referral, he observes the benefit index of the referred project and decides whether to withhold or provide service. The Partnership will be paid by the Principal according to the terms of the contract, and each expert will be paid according to the Partnership sharing rule set up in Stage 2.

The extensive form for Expert 1 being the Senior Partner under top-down referral can be written down analogously, as can those for Expert 2 being the Senior Partner. For brevity, we have omitted them here. Can the Partnership continue to implement any hierarchical allocation $(\widehat{b}_1, \widehat{b}_2)$ as before? How would the change in partnership governance affect the Principal's contract and the Partnership's sharing rules in equilibrium?

5.1 Expert 1 as Senior Partner

We begin by assuming that Expert 1 uses bottom-up referral. Consider the hierarchical allocation $(\widehat{b}_1, \widehat{b}_2)$ just like before. We begin by presenting the following Principal's contract $(\Gamma, \gamma_1, \gamma_2)$:

$$\Gamma = \underline{\alpha}_1 \left\{ r_1 \widehat{b}_1 \left[1 - F(\widehat{b}_1) \right] + (r_2 - r_1) \widehat{b}_2 \left[1 - F(\widehat{b}_2) \right] \right\} + \overline{\pi}_1 + \overline{\pi}_2 \tag{23}$$

$$\gamma_1 = c_1 - \underline{\alpha}_1 r_1 \widehat{b}_1 \tag{24}$$

$$\gamma_2 = c_2 - \underline{\alpha}_1 \left[r_1 \widehat{b}_1 + (r_2 - r_1) \widehat{b}_2 \right]. \tag{25}$$

This new contract is obtained by replacing $\underline{\alpha}_1 + \underline{\alpha}_2$ in the one in Proposition 2 by $\underline{\alpha}_1$; this is the only difference.[8] We now show that this new contract implements $(\widehat{b}_1, \widehat{b}_2)$.

Given the Principal's contract $(\Gamma, \gamma_1, \gamma_2)$ in (23) to (25), Expert 1 chooses a sharing rule in Stage 2 to maximize his payoff in the continuation subgame. We show implementation in two steps. First, we let

[8] For this reason, we use almost the same notation $(\Gamma, \gamma_1, \gamma_2)$ for the Principal's contract.

Expert 1 fully control Expert 2's decisions (as well as his own). In this auxiliary game, there is no need for a sharing rule. Expert 1 only has to guarantee Expert 2 (and himself) minimum profits. Second, we show implementation of the equilibrium in the auxiliary game by a sharing rule.

Clearly, Expert 1's equilibrium payoff in the auxiliary game must not be lower than in the extensive form defined earlier. Any equilibrium in the extensive form can be mimicked by Expert 1's choice in the auxiliary game. Now we prove that in fact Expert 1 chooses the hierarchical allocation $(\widehat{b}_1, \widehat{b}_2)$ in the auxiliary game.

Lemma 7 *Suppose that Expert 1 chooses an allocation to maximize his own payoff subject to minimum profits. If the Principal offers the contract $(\Gamma, \gamma_1, \gamma_2)$ in (23) to (25), Expert 1 of any degree of motivation maximizes his payoff by choosing the hierarchical allocation $(\widehat{b}_1, \widehat{b}_2)$.*

The intuition behind Lemma 7 is similar to that in Propositions 1 and 2. In the auxiliary game, Expert 1 can dictate all decisions, and only attempts to maximize his own payoff. To do so, it is best to use Expert 2's and his own services, and then pay Expert 2 the required minimum profit. Under the contract $(\Gamma, \gamma_1, \gamma_2)$ in (23) to (25), the allocation $(\widehat{b}_1, \widehat{b}_2)$ maximizes the joint surplus of a Partnership with motivation $\underline{\alpha}_1$. The minimum profit constraint deters a motivated Expert 1 from rendering services more than the allocation $(\widehat{b}_1, \widehat{b}_2)$ prescribes.

What if Expert 1 cannot dictate Expert 2's decisions? Can Expert 1 design a sharing rule in Stage 2 to implement allocation $(\widehat{b}_1, \widehat{b}_2)$. The following proposition gives an affirmative answer. More surprising, the sharing rule for implementation is essentially identical to (16) and (17) in Subsection 4.1 under bottom-up referral.

Proposition 5 *Given the Principal's contract $(\Gamma, \gamma_1, \gamma_2)$ in (23) to (25), it is an equilibrium for Senior Partner Expert 1 to accept the contract and in Stage 2 offer the budget-balanced sharing rule*

$$S_1(m_1, m_2; \alpha_1, \alpha_2) = \Gamma - \Upsilon - \alpha_1 r_1 \widehat{b}_1 m_1 + \alpha_2 r_2 \widehat{b}_2 m_2 + (\gamma_2 - c_2) m_2 \quad (26)$$

$$S_2(m_1, m_2; \alpha_1, \alpha_2) = \Upsilon - \alpha_2 r_2 \widehat{b}_2 m_2 + \alpha_1 r_1 \widehat{b}_1 m_1 + (\gamma_1 - c_1) m_1, \quad (27)$$

for some constant Υ, and for the two experts to implement the hierarchical allocation $(\widehat{b}_1, \widehat{b}_2)$ under bottom-up *referral in Stages 3 and 4. In equilibrium, Expert 1 and Expert 2 earn their minimum profits, respectively,* $\overline{\pi}_1$ *and* $\overline{\pi}_2$*.*

The Principal's contract $(\Gamma, \gamma_1, \gamma_2)$ in (23) to (25) for the Senior Partner Expert 1 is obtained from the one for the symmetric Partnership, $(\widehat{\Gamma}, \widehat{\gamma}_1, \widehat{\gamma}_2)$ in (13) to (15) by reducing the minimum Partnership motivation $\underline{\alpha}_1 + \underline{\alpha}_2$ to Expert 1's minimal motivation $\underline{\alpha}_1$. Nevertheless, Lemma 7 establishes that Expert 1 wants to implement the same allocation $(\widehat{b}_1, \widehat{b}_2)$. For implementation, Expert 1 simply uses the sharing rule (16) and (17) in the bottom-up referral of a symmetril Partnership, replacing term-by-term the values of $\widehat{\Gamma}$, $\widehat{\gamma}_1$, and $\widehat{\gamma}_2$ by Γ, γ_1, and γ_2. It is as if the motivation parameters have gone down to the minimum of Expert 1's.

The proof of Proposition 5 mirrors that of Proposition 3 in Subsection 4.1. It simply verifies that given the sharing rule, Expert 1 rejects all projects with benefit indexes below $\widehat{b}_1$, refers those with indexes between $\widehat{b}_1$ and $\widehat{b}_2$, while Expert 2 will provide services to projects with indexes above $\widehat{b}_2$. This similarity is to be expected. The sharing rules in both Propositions 5 and 3 are essentially the same, so the experts play the same game.

What if Senior Partner Expert 1 uses top-down referral under the same contract from the Principal? Here, the sharing rule for implementation of $(\widehat{b}_1, \widehat{b}_2)$ can be obtained from Proposition 4 in Subsection 4.2. We adjust the sharing rule in the same way that we have done so under bottom-up referral in Proposition 5. Expert 1 replaces, respectively, the values of $\widehat{\Gamma}$, $\widehat{\gamma}_1$, and $\widehat{\gamma}_2$ in (20) and (21) by Γ, γ_1, and γ_2. In particular, these changes do not affect the profit share received by Expert 2. Figure 3 in Subsection 4.2 continues to illustrates Expert 2's payoff comparisons in Stage 3.

Senior Partner Expert 1's sharing rule incentivies Expert 2 to provide services to projects with benefit indexes higher than ,$\widehat{b}_2$, and refers those that have indexes between $\widehat{b}_1$ and $\widehat{b}_2$. In turn, Expert 1 will provide services on referred projects. the Principal can always implement $(\widehat{b}_1, \widehat{b}_2)$ when he contracts with Senior Partner Expert 1. As in the case of symmetric Partnership, the Principal can leave the Partnership to determine the details for implementation.

5.2 Expert 2 as Senior Partner

Now Expert 2 is the Senior Partner, and he contracts with the Principal and sets up a sharing rule. The analysis is so similar to the previous subsection of Expert 1 as Senior Partner that we can be brief. First, the Principal replaces $\underline{\alpha}_1$ in the contract $(\Gamma, \gamma_1, \gamma_2)$ in (23) to (25) by $\underline{\alpha}_2$. Lemma 7 directly applies. Senior Partner Expert 2 of any degree of motivation chooses $(\widehat{b}_1, \widehat{b}_2)$ to maximize his payoff and the Partnership will make the minimum profit.

Sharing rules that implement allocation $(\widehat{b}_1, \widehat{b}_2)$ exist under both bottom-up and top-down referrals. The key of implementation remains the same as when Expert 1 is the Senior Partner. Expert 2 starts with the sharing rules in either bottom-up or top-down referrals in a symmetric Partnership in Subsections 4.1 and 4.2. Expert 2 then replaces the values of $\widehat{\Gamma}$, $\widehat{\gamma}_1$, and $\widehat{\gamma}_2$ by Γ, γ_1, and γ_2 in the new Principal's contract.

We have shown that the same allocation $(\widehat{b}_1, \widehat{b}_2)$ can be implemented under both Seniority Partnerships. The derivations demonstrate that the two sharing rules in Section 4 easily accommodate differences in partnership governance. From the Principal's perspective, it is sufficient to know who has the authority to decide the details for implementation. The Principal can then adjust his contract according to the decision maker's least degree of motivation. The single contract will allow the Partnership to implement $(\widehat{b}_1, \widehat{b}_2)$ at the minimum profit.

6 Conclusion

Appendix

Proof of Lemma 2: At each b, the first-order derivatives of (7) with respect to σ_1 and σ_2 are, respectively,

$$(\alpha_1 + \alpha_2)r_1 b + \gamma_1 - c_1, \tag{28}$$

$$(\alpha_1 + \alpha_2)r_2 b + \gamma_2 - c_2. \tag{29}$$

There are also the boundary conditions $0 \leq \sigma_1(b), \sigma_2(b) \leq 1$, and $\sigma_1(b) + \sigma_2(b) \leq 1$. Because (28) and (29) do not contain $\sigma_1(b)$ and $\sigma_2(b)$, we must have corner solutions (generically), so at (almost) each b, either $\sigma_i(b) = 0$, or $\sigma_i(b) = 1$, $i = 1, 2$.

We show by contradiction that the surplus-maximizing allocation is hierarchical. Suppose the allocation is not hierarchical. The allocation must involve at least one of the following two cases: there exist $b, b' \in [\underline{b}, \bar{b}]$, $b < b'$, for which either 1) $\sigma_2(b) = 1$ and $\sigma_1(b') = 1$; or 2) $\sigma_1(b) = 1$ or $\sigma_2(b) = 1$, and $\sigma_1(b') = \sigma_2(b') = 0$. Case 1 results in a contradiction. Because $r_1 < r_2$, if (29) is at least as large as (28) at b, (29) must be strictly larger than (28) at b', which implies that $\sigma_1(b') = 0$, a contradiction. Case 2 results in a contradiction, too. Both (28) and (29) are strictly increasing in b, so if any of them is positive at b, it is strictly positive at b', which implies that either $\sigma_1(b') > 0$ or $\sigma_2(b') > 0$, a contradiction.■

Proof of Lemma 4: First, according to the Envelope Theorem, the derivative of the maximized surplus with respect to $(\alpha_1 + \alpha_2)$ is the partial derivative of (9) with respect to $(\alpha_1 + \alpha_2)$ evaluated at the solution. Hence, this is

$$\int_{b_1}^{b_2} r_1 b f(b) \mathrm{d}b + \int_{b_2}^{\bar{b}} r_2 b f(b) \mathrm{d}b > 0$$

which says the maximized surplus increases in $(\alpha_1 + \alpha_2)$. Second, after using (8) to substitute for b_1 and b_2 we obtain the derivative of (10) with respect to $(\alpha_1 + \alpha_2)$:

$$[(\gamma_2 - c_2) - (\gamma_1 - c_1)] \frac{b_2 f(b_2)}{(\alpha_1 + \alpha_2)} + (\gamma_1 - c_1) \frac{b_1 f(b_1)}{(\alpha_1 + \alpha_2)} < 0$$

where the inequality follows because $\gamma_2 - c_2 < \gamma_1 - c_1 < 0$.■

Proof of Proposition 1: Given the contract $(\Gamma, \gamma_1, \gamma_2)$, the Partnership with a degree of motivation

$\widetilde{\alpha}_1 + \widetilde{\alpha}_2$ optimally chooses $(\widetilde{b}_1, \widetilde{b}_2)$ to maximize surplus

$$\Gamma + \int_{b_1}^{b_2} [(\widetilde{\alpha}_1 + \widetilde{\alpha}_2) r_1 b + \gamma_1 - c_1] f(b) \mathrm{d}b + \int_{b_2}^{\overline{b}} [(\widetilde{\alpha}_1 + \widetilde{\alpha}_2) r_2 b + (\gamma_2 - c_2)] f(b) \mathrm{d}b$$

subject to the minimum profit constraint

$$\Gamma + \int_{b_1}^{b_2} (\gamma_1 - c_1) f(b) \mathrm{d}b + \int_{b_2}^{\overline{b}} (\gamma_2 - c_2) f(b) \mathrm{d}b \geq \overline{\pi}_1 + \overline{\pi}_2. \tag{30}$$

Because the constraint binds by assumption, we use it as an equality and rewrite the objective function as

$$(\widetilde{\alpha}_1 + \widetilde{\alpha}_2) \left[\int_{b_1}^{b_2} r_1 b f(b) \mathrm{d}b + \int_{b_2}^{\overline{b}} r_2 b f(b) \mathrm{d}b \right] + \overline{\pi}_1 + \overline{\pi}_2. \tag{31}$$

Now consider a Partnership with a degree of motivation $\alpha_1 + \alpha_2 > \widetilde{\alpha}_1 + \widetilde{\alpha}_2$. It chooses a hierarchical allocation to maximize its surplus

$$\Gamma + \int_{b_1}^{b_2} [(\alpha_1 + \alpha_2) r_1 b + \gamma_1 - c_1] f(b) \mathrm{d}b + \int_{b_2}^{\overline{b}} [(\alpha_1 + \alpha_2) r_2 b + (\gamma_2 - c_2)] f(b) \mathrm{d}b$$

subject to the same minimum profit constraint (30). We first claim that the minimum profit constraint must bind for this Partnership. Indeed, according to Lemma 4, a Partnership's profit, when its surplus is maximized, decreases in its degree of motivation. Therefore, if (30) did not bind at $\alpha_1 + \alpha_2 > \widetilde{\alpha}_1 + \widetilde{\alpha}_2$, then the profit would have been even higher at $\widetilde{\alpha}_1 + \widetilde{\alpha}_2$, which would contradict the assumption that (30) was binding at $\widetilde{\alpha}_1 + \widetilde{\alpha}_2$.

Next, we again use the binding minimum profit constraint to rewrite the objective function of the Partnership with motivation $\alpha_1 + \alpha_2$ as

$$(\alpha_1 + \alpha_2) \left[\int_{b_1}^{b_2} r_1 b f(b) \mathrm{d}b + \int_{b_2}^{\overline{b}} r_2 b f(b) \mathrm{d}b \right] + \overline{\pi}_1 + \overline{\pi}_2. \tag{32}$$

Obviously $(\widetilde{b}_1, \widetilde{b}_2)$ maximizes (32) subject to (30) if and only it $(\widetilde{b}_1, \widetilde{b}_2)$ maximizes (31) subject to (30). ■

Proof of Proposition 2: By Corollary 1, it is sufficient to show that a Partnership with motivation $\underline{\alpha}_1 + \underline{\alpha}_2$ will optimally choose the hierarchical allocation $(\widehat{b}_1, \widehat{b}_2)$ and make the minimum profit.

Consider the Partnership with motivation $\underline{\alpha}_1 + \underline{\alpha}_2$. Ignore the minimum profit constraint for now. Equations in (8) define a Partnership's optimal allocation given a contract. We invert these linear equations to find the values of $\widehat{\gamma}_1$ and $\widehat{\gamma}_2$ in terms of $\widehat{b}_1$ and $\widehat{b}_2$. They are those values of γ_1 and γ_2 in (14) and (15) of the Proposition.

Next, we write the Partnership's profit (10) as $\widehat{\Gamma} + (\widehat{\gamma}_1 - c_1)\left[F(\widehat{b}_2) - F(\widehat{b}_1)\right] + (\widehat{\gamma}_2 - c_2)\left[1 - F(\widehat{b}_2)\right]$. Then we substitute Γ, γ_1, and γ_2 in the Proposition into the above. Upon simplification, the Partnership's profit turns out to be $\overline{\pi}_1 + \overline{\pi}_2$. Because the minimum profit constraint holds as an equality, the Partnership's constrained maximization problem must have the same solution.■

Proof of Lemma 6: In the discussion before the statement of the Lemma, we have already established Expert 1's equilibrium decision when $b < \widehat{b}_2$. Expert 1's payoffs from rendering service to a project with benefit index $b > \widehat{b}_2$ and referring it to Expert 2 are in (18) and (19) respectively. We subtract (18) from (19):

$$
\begin{aligned}
& \alpha_1 r_2 b + \alpha_2 r_2 \widehat{b}_2 + (\widehat{\gamma}_2 - c_2) - \alpha_1 r_1 (b - \widehat{b}_1) \\
= \quad & \alpha_1 r_2 b + \alpha_2 r_2 \widehat{b}_2 + \{c_2 - (\underline{\alpha}_1 + \underline{\alpha}_2)[r_1 \widehat{b}_1 + (r_2 - r_1)\widehat{b}_2] - c_2\} - \alpha_1 r_1 (b - \widehat{b}_1),
\end{aligned}
$$

where the equality follows from our using Proposition 2 for the substitution of $\widehat{\gamma}_2$. We continue to simplify this difference to

$$
\begin{aligned}
& \alpha_1 (r_2 - r_1) b + \alpha_2 r_2 \widehat{b}_2 + \alpha_1 r_1 \widehat{b}_1 - (\underline{\alpha}_1 + \underline{\alpha}_2)[r_1 \widehat{b}_1 + (r_2 - r_1)\widehat{b}_2] \\
> \quad & \alpha_1 (r_2 - r_1) \widehat{b}_2 + \alpha_2 r_2 \widehat{b}_2 + \alpha_1 r_1 \widehat{b}_1 - (\underline{\alpha}_1 + \underline{\alpha}_2)[r_1 \widehat{b}_1 + (r_2 - r_1)\widehat{b}_2] \\
= \quad & \alpha_1 [r_1 \widehat{b}_1 + (r_2 - r_1)\widehat{b}_2] + \alpha_2 r_2 \widehat{b}_2 - (\underline{\alpha}_1 + \underline{\alpha}_2)[r_1 \widehat{b}_1 + (r_2 - r_1)\widehat{b}_2] \\
= \quad & \alpha_1 [r_1 \widehat{b}_1 + (r_2 - r_1)\widehat{b}_2] + \alpha_2 [r_1 \widehat{b}_1 + (r_2 - r_1)\widehat{b}_2] + \alpha_2 r_1 (\widehat{b}_2 - \widehat{b}_1) - (\underline{\alpha}_1 + \underline{\alpha}_2)[r_1 \widehat{b}_1 + (r_2 - r_1)\widehat{b}_2] \\
= \quad & [\alpha_1 + \alpha_2 - \underline{\alpha}_1 - \underline{\alpha}_2][r_1 \widehat{b}_1 + (r_2 - r_1)\widehat{b}_2] + \alpha_2 r_1 (\widehat{b}_2 - \widehat{b}_1) > 0,
\end{aligned}
$$

where the first inequality follows from $b > \widehat{b}_2$, and the second inequality follows from $\alpha_1 + \alpha_2 > \underline{\alpha}_1 + \underline{\alpha}_2$ and $\widehat{b}_2 > \widehat{b}_1$. We have shown that referring the project to Expert 2 is the equilibrium decision at $b > \widehat{b}_2$.■

Proof of Proposition 3: We have already shown in Lemmas 5 and 6 that in Stages 3 and 4, Expert 1 takes on a project if and only if $\widehat{b}_1 < b < \widehat{b}_2$ and refers a project if and only if $b \geq \widehat{b}_2$, and Expert 2 renders service if and only if $b \geq \widehat{b}_2$. It remains to show that the sharing rule in (16) and (17) is budget balanced and that each expert makes his minimum profit in equilibrium.

For budget balance, simply add together (16) and (17) to get

$$S_1^B(m_1, m_2; \alpha_1, \alpha_2) + S_2^B(m_1, m_2; \alpha_1, \alpha_2) = \widehat{\Gamma} + (\widehat{\gamma}_1 - c_1)m_1 + (\widehat{\gamma}_2 - c_2)m_2.$$

Next we define $\widehat{m}_1 \equiv F(\widehat{b}_2) - F(\widehat{b}_1)$, $\widehat{m}_2 \equiv 1 - F(\widehat{b}_2)$, and set

$$\Delta \equiv \alpha_1 r_1 \widehat{b}_1 \widehat{m}_1 - (\gamma_2 + \alpha_2 r_2 \widehat{b}_2 - c_2)\widehat{m}_2 + \overline{\pi}_1. \tag{33}$$

Expert 1's equilibrium profit is

$$\begin{aligned} S_1^B(\widehat{m}_1, \widehat{m}_2; \alpha_1, \alpha_2) &= \Delta - \alpha_1 r_1 \widehat{b}_1 \widehat{m}_1 + \alpha_2 r_2 \widehat{b}_2 \widehat{m}_2 + (\widehat{\gamma}_2 - c_2)\widehat{m}_2 \\ &= \alpha_1 r_1 \widehat{b}_1 \widehat{m}_1 - (\gamma_2 + \alpha_2 r_2 \widehat{b}_2 - c_2)\widehat{m}_2 + \overline{\pi}_1 - \alpha_1 r_1 \widehat{b}_1 \widehat{m}_1 + (\gamma_2 + \alpha_2 r_2 \widehat{b}_2 - c_2)\widehat{m}_2 \\ &= \overline{\pi}_1. \end{aligned}$$

Because the Partnership's equilibrium profit is $\overline{\pi}_1 + \overline{\pi}_2$, Expert 2's equilibrium profit must be $\overline{\pi}_2$.■

Proof of Proposition 4: In the second paragraph after the statement of the Proposition, we have shown that in the continuation game in Stage 4 Expert 1 provides service to a project if and only if inequality (22) holds.

Now consider Expert 2's payoffs in the continuation game in Stage 3. If $b < \widehat{b}_1$, both $\alpha_2 r_1(b - \widehat{b}_1)$ and $\alpha_2[r_1(b - \widehat{b}_1) - (r_2 - r_1)\widehat{b}_2]$ are negative, so Expert 2's best response is to abandon a project with $b < \widehat{b}_1$ whether or not Expert 1 will take on the project upon a referral in Stage 4. Next suppose $b \geq \widehat{b}_1$. By inequality (22) Expert 1 will accept the project upon a referral. Now $\alpha_2 r_1(b - \widehat{b}_1)$ is always nonnegative if $b \geq \widehat{b}_1$, and $\alpha_2[r_1(b - \widehat{b}_1) - (r_2 - r_1)\widehat{b}_2] \geq \alpha_2 r_1(b - \widehat{b}_1)$ if and only if $b \geq \widehat{b}_2$. It follows that Expert 2 provides service to a project if and only if $b \geq \widehat{b}_2$ and refers a project to Expert 1 if and only if $\widehat{b}_1 \leq b < \widehat{b}_2$. We conclude that $(\widehat{b}_1, \widehat{b}_2)$ is implemented in the continuation equilibria in Stages 3 and 4.

By Proposition 2, given the contract $(\widehat{\Gamma}, \widehat{\gamma}_1, \widehat{\gamma}_2)$ the Partnership's profit from the allocation $(\widehat{b}_1, \widehat{b}_2)$ is $\overline{\pi}_1 + \overline{\pi}_2$. Because the sum of (20) and (21) is $\widehat{\Gamma} + (\widehat{\gamma}_1 - c_1)m_1 + (\widehat{\gamma}_2 - c_2)m_2$, the sharing rule is budget balanced. There must exist a constant Λ so that the equilibrium profits of Experts 1 and 2 are $\overline{\pi}_1$ and $\overline{\pi}_2$, respectively.■

Proof of Lemma 7: Given the Principal's contract, for a project with benefit index b, Expert 1's payoffs from his own and Expert 2's services are $\alpha_1 r_1 b + \gamma_1 - c_1$ and $\alpha_1 r_2 b + \gamma_2 - c_2$, respectively. If $\sigma_1(b)$ and

$\sigma_2(b)$ are the probabilities that the service will be provided by Expert 1 and Expert 2, respectively, Expert 1's total payoff is

$$(\Gamma - \overline{\pi}_2) + \int_{\underline{b}}^{\overline{b}} \{\sigma_1(b)[\alpha_1 r_1 b + \gamma_1 - c_1] + \sigma_2(b)[\alpha_1 r_2 b + \gamma_2 - c_2]\} f(b)\mathrm{d}b. \tag{34}$$

The term $-\overline{\pi}_2$ appears in Expert 1's total payoff because this is the minimum profit he has to give Expert 2. By replacing $\alpha_1 + \alpha_2$ in (28) and (29) with α_1, we obtain the first-order derivatives of (34) with respect to $\sigma_1(b)$ and $\sigma_2(b)$. Using the same argument in the proof of Lemma 2, we conclude that Expert 1's payoff is maximized by a hierarchical allocation.

Because Expert 1's payoff-maximizing allocation is hierarchical, we can write the equilibrium in the auxiliary game as (b_1, b_2) that maximize

$$(\Gamma - \overline{\pi}_2) + \int_{b_1}^{b_2} [\alpha_1 r_1 b + \gamma_1 - c_1] f(b)\mathrm{d}b + \int_{b_2}^{\overline{b}} [\alpha_1 r_2 b + \gamma_2 - c_2] f(b)\mathrm{d}b$$

subject to the Partnership's minimum profit constraint

$$\Gamma - \overline{\pi}_2 + \int_{b_1}^{b_2} (\gamma_1 - c_1) f(b)\mathrm{d}b + \int_{b_2}^{\overline{b}} (\gamma_2 - c_2) f(b)\mathrm{d}b \geq \overline{\pi}_1.$$

This program is identical to the Partnership's surplus-maximization problem in Propositions 1 and 2 (up to a constant $-\overline{\pi}_2$ in the objective functions) if we replace $\alpha_1 + \alpha_2$, $\widetilde{\alpha}_1 + \widetilde{\alpha}_2$, and $\underline{\alpha}_1 + \underline{\alpha}_2$ in Propositions 1 and 2 by α_1, $\widetilde{\alpha}_1$, and $\underline{\alpha}_1$, respectively. Therefore, we conclude that given the Principal's contract in (23) to (25), Expert 1 of any degree of motivation maximizes his surplus and the Partnership makes the minimum profit by choosing the same allocation $(\widehat{b}_1, \widehat{b}_2)$.■

Proof of Proposition 5: Consider first the subgame in Stage 4. According to (27), Expert 2 pays $\alpha_2 r_2 \widehat{b}_2$ for providing service to a project. Suppose that Expert 2 receives a referred project with benefit index b, his payoff from rendering service to the project is $\alpha_2 r_2 b - \alpha_2 r_2 \widehat{b}_2$. Clearly, Expert 2 provides service to a project if and only if $b \geq \widehat{b}_2$.

Next, consider the subgame in Stage 3. Suppose that Expert 1 has a project with benefit index less than $\widehat{b}_2$. Expert 2 will abandon it if he is referred the project. If Expert 1 provides service himself, his payoff is $\alpha_1 r_1 b - \alpha_1 r_1 \widehat{b}_1$. Hence, Expert 1 renders service to a project if and only if $\widehat{b}_2 > b \geq \widehat{b}_1$. Now suppose $b \geq \widehat{b}_2$. Expert 1 knows that if Expert 2 is referred this project, it will be accepted, so Expert

1's payoff is $\alpha_1 r_2 b + \alpha_2 r_2 \widehat{b}_2 + (\gamma_2 - c_2)$. Alternatively, if Expert 1 takes up the project himself, his payoff is $\alpha_1 r_1 b - \alpha_1 r_1 \widehat{b}_1$. The difference is $\alpha_1 (r_2 - r_1) b + (\gamma_2 - c_2) + \alpha_2 r_2 \widehat{b}_2 + \alpha_1 r_1 \widehat{b}_1$. From (25), we have $(\gamma_2 - c_2) = -\underline{\alpha}_1 \left[r_1 \widehat{b}_1 + (r_2 - r_1) \widehat{b}_2 \right]$. Using this expression, we express the difference in Expert 1's payoff between referring the project and providing service:

$$\begin{aligned}
& \alpha_1 (r_2 - r_1) b + (\gamma_2 - c_2) + \alpha_2 r_2 \widehat{b}_2 + \alpha_1 r_1 \widehat{b}_1 \\
= \; & \alpha_1 (r_2 - r_1) b - \underline{\alpha}_1 \left[r_1 \widehat{b}_1 + (r_2 - r_1) \widehat{b}_2 \right] + \alpha_2 r_2 \widehat{b}_2 + \alpha_1 r_1 \widehat{b}_1 \\
= \; & \alpha_1 \left[r_1 \widehat{b}_1 + (r_2 - r_1) b \right] - \underline{\alpha}_1 \left[r_1 \widehat{b}_1 + (r_2 - r_1) \widehat{b}_2 \right] + \alpha_2 r_2 \widehat{b}_2 > 0,
\end{aligned}$$

where the inequality follows from $\underline{\alpha}_1 < \alpha_1$ and $\widehat{b}_2 \leq b$. We conclude that Expert 1 gets a higher payoff referring the project to Expert 2 when the benefit is higher than $\widehat{b}_2$.

We have shown that the hierarchical allocation $(\widehat{b}_1, \widehat{b}_2)$ is implemented in Stages 3 and 4. To see that the sharing rule is budget balanced, add (26) and (27) to get

$$S_1(m_1, m_2; \alpha_1, \alpha_2) + S_2(m_1, m_2; \alpha_1, \alpha_2) = \Gamma + (\gamma_1 - c_1) m_1 + (\gamma_2 - c_2) m_2.$$

Finally, we set

$$\Upsilon = -(\alpha_1 - \underline{\alpha}_1) r_1 \widehat{b}_1 m_1 + \alpha_2 r_2 \widehat{b}_2 m_2 + \overline{\pi}_2.$$

Expert 2's profit in the continuation equilibrium is

$$\begin{aligned}
S_2(m_1, m_2; \alpha_1, \alpha_2) &= \Upsilon - \alpha_2 r_2 \widehat{b}_2 m_2 + \alpha_1 r_1 \widehat{b}_1 m_1 + (\gamma_1 - c_1) m_1 \\
&= -(\alpha_1 - \underline{\alpha}_1) r_1 \widehat{b}_1 m_1 + \alpha_2 r_2 \widehat{b}_2 m_2 + \overline{\pi}_2 - \alpha_2 r_2 \widehat{b}_2 m_2 + (\alpha_1 - \underline{\alpha}_1) r_1 \widehat{b}_1 m_1 \\
&= \overline{\pi}_2.
\end{aligned}$$

Because the Partnership's profit is $\Gamma + (\gamma_1 - c_1) m_1 + (\gamma_2 - c_2) m_2 = \overline{\pi}_1 + \overline{\pi}_2$, Senior Partner Expert 1's profit in the continuation equilibrium is $\overline{\pi}_1$. Given the Principal's contract in (23) to (25), Expert 1 obtains the same payoff and profit as in the auxiliary game. It is an equilibrium for Expert 1 to accept the Principal's contract and to offer the sharing rule in (26) and (27) in Stage 2.■

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