Investment banking careers: An equilibrium theory of overpaid jobs

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ABSTRACT

We analyze a general equilibrium labor market model where moral hazard problems are a key concern. We show that variation in moral hazard across types of jobs explains contract terms, work patterns over time, and promotion structures. We explain why high-profile jobs such as investment banking pay more and give higher utility to the employee than other jobs, even if employees have no skill advantage. These jobs also have up-or-out contracts, and inefficiently long hours. Our model also provides a natural theory for promotion to more important tasks after success. We also derive two versions of talent misallocation: High profile employers like investment banks may lure workers whose talent would be more valuable elsewhere, and may reject "over qualified" job applicants – smart workers may be "too hard to manage," because their high outside options make them respond less to firing incentives. Finally, we extend our model to a dynamic economy with demand shocks and show the following results: Workers entering the labor market in recessions suffer life-long disadvantages in the

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labor market, temporary demand shocks have long lasting effects on productivity and the composition of the workforce, and moral hazard problems increase in good times for critical sectors in the economy, leading to both higher pay and higher failure rates.

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Job conditions differ widely. In particular, certain high-profile jobs feature extremely high pay, but also very long hours and low job security. Investment banking is perhaps the quintessential example, but jobs with top law and management consultancy firms fall in this category as well. Despite the onerous work conditions, job applicants seem to view these jobs as especially desirable and prestigious – hence the term "high-profile" in the paper's title.

What explains the high pay in, e.g., investment banking? One possibility is that pay is a compensating differential for the tough work conditions. It is not surprising, for instance, that oil-rig workers or miners are highly compensated, given the intrinsically high-risk nature of their jobs. What makes investment banking different, and more challenging to explain, is that many of the unappealing aspects of the work are not intrinsic but rather chosen by the employer. This is true for both work hours and firing probabilities, and some of these choices seem –at least at first glance– inefficient. For example, it is not uncommon for a newly graduated MBA student who starts with an investment bank to work 100 hour weeks, much of which is spent on rather menial tasks such as gathering data and preparing power point presentations. Arguably, it would be more efficient for the employer to hire one more secretary to do the simpler tasks, have the MBA graduate work less, and lower salaries somewhat.[1]

Another possible explanation is that high pay in investment banking is a skill premium. Maybe the most convincing argument against this can be found in a recent paper by Oyer (2008) on the market for MBA students. Using macroeconomic conditions at the time of graduation as an instrument for the probability of entering investment banking, Oyer shows that an MBA student who enters investment banking has an expected lifetime income that is \$1.5 million to \$5 million higher in present value terms than an equally skilled student who does not. Hence, we do not think either compensating differentials or skill premia alone can explain the high pay and onerous work conditions observed in high-profile jobs.

Our first contribution in this paper is to develop an equilibrium labor-market model in which high pay, onerous work conditions, and high job attractiveness emerge in a very

[1] Randers, Lebitzer, Taylor (96) report evidence that associates of law firms would prefer to work less hours for correspondingly less pay.

natural way from agency problems. Almost all observers agree that agency problems are important in organizations, maybe especially so for the types of tasks performed in high-profile industries. Agency problems naturally generate these features because in jobs with a lot at stake, employers desire a lot of care from their employees. Given agency problems, they must surrender rent to induce this care. Employers then attempt to reduce the rents in any way they can, for example by high firing probabilities and inefficiently long hours.

We next show that agency problems implies up-or-out promotion contracts in high-profile industries. Up-or-out promotion is the subject of a sizeable literature in organizational economics, which largely explains these features in terms of gradual revelation of an individual's skill over time. Because we believe agency problems are certainly important—in part because they provide such a natural explanation for the combination of high-pay and long hours discussed above—we think Occam's razor favors our agency-based explanation. Regardless, our analysis suggests that agency concerns may be a significant factor behind these characteristics, a point not currently appreciated in the existing literature.

Third, having shown that agency concerns naturally and parsimoniously account for the characteristics of high-profile jobs such as investment banking, we next apply our model to study how economic shocks affect firm hiring decisions, firm productivity, and pay levels.

We show that our model naturally accounts for cohort effects in the labor market; for propagation of productivity shocks; and for pro- rather than countercyclical agency problems in high-profile jobs, whereby high-profile employees such as investment bankers are paid more but succeed less in economic good times.

Fourth, and finally, we use our model to analyze misallocations in how employees of observably different talent are matched with firms. When we introduce observable skill differences, we get some surprising results. In particular, our model naturally generates two commonly noted forms of talent misallocation. The first one, which we call "talent lured," is the observation that jobs like investment banking tend to attract talented workers whose skills might be socially more valuable in other jobs, such as engineers and PhDs. In our model, this type of misallocation follows immediately from the fact that the high surplus earned in high moral hazard industries will make it possible for these industries to outbid other employers for workers even if their talent is wasted in investment banking. The second phenomena, which we call "talent scorned," is the opposite – high profile jobs often reject the most talented applicants on the grounds that they are "difficult" or "hard to manage." This can be rational in our model because talented workers, when fired, have higher outside opportunities, which makes it harder to control them with dynamic incentive schemes.

A Paper outline

The paper proceeds as follows. In Section I, we describe the model set-up. In Section II we derive the structure of equilibrium contacts and prove existence of an equilibrium. In Section III we show that workers who are lucky enough to get a job in Sector H when young earn higher lifetime utility than other workers, but are subject to more stressful careers in terms of work and demotion probabilities. In Section IV we derive results on promotion. Section V extends the model to include differences in talent between workers, while Section VI extends the model to a fully dynamic economy where we can study the effects of demand shocks on careers and incentives. Section VII concludes.

I Model

To study the labor market phenomena we are interested in, we need two key elements: Workers of different age, and sectors that vary in their degree of moral hazard problems. There is a continuum of workers of measure 1, and we assume a measure $\frac{1}{2}$ of young workers enter the labor market each period, work for two periods, and then exit. Except for age, workers are identical. They all have the same skill, are risk neutral over both consumption and leisure, start out penniless, and have limited liability. (We will analyze a setup where skills differ across workers in Section V.)

There are two sectors denoted as H (the "high stakes" sector) and L (the "low stakes" sector). A worker in a sector is assigned a project which can either succeed or fail, where the failure cost is what differs across sectors. In Sector H, the failure cost is $k_H > 0$, while in Sector L the failure cost is $k_L = 0$. For each sector $i \in \{H, L\}$, we write the success payoff as $g_i - k_i$, where g_i is determined in equilibrium (see below). One way to think about these payoffs is that k_i is an input cost (e.g., funds provided to a trader) and g_i is the value of output produced when the project succeeds (e.g., gross value after trading). Alternatively, k_i is the value destroyed if a project fails (e.g., a takeover fails), and $g_i - k_i$ is the value created if a project succeeds (e.g., takeover succeeds). Throughout the paper, we write g for the price vector (g_L, g_H).

If a worker spends h hours on the project, it succeeds with probability $p(h)$ and fails with probability $1 - p(h)$. Hence, we can think of g_i as the marginal product of labor. Workers have a per-period time endowment of 1, which they can split between work and leisure, and have linear preferences over leisure. The success probability $p(h)$ is a strictly increasing and strictly concave function with $p'(0) = \infty$ and $p'(1) = 0$.[2] While output (i.e., success or failure) is fully observable, effort is private information to the worker, which leads to a standard moral hazard problem.[3] Analytically, it is slightly easier to express everything in terms of probabilities instead of hours worked: let $\gamma \equiv p^{-1}$, so that the utility cost of a worker achieving success probability p is $\gamma(p)$. The function γ is strictly increasing and strictly convex, with $\gamma'(0) = 0$ and $\gamma'(p(1)) = \infty$.

For the case of financial sectors, the following specific interpretation of the moral hazard problem is worth spelling out. The success payoff g_i is a target (gross) rate of return. A financial sector worker can meet this target either by working hard and discovering genuinely profitable trading opportunities, or by taking "tail" risk. When tail risk is realized all the input funds k_i are lost. By working h hours, the amount of tail risk a worker needs to take to achieve his target return is such that the probability of tail risk being realized is $1 - p(h)$.

For use below, we also make the following fairly innocuous assumption on the shape of the production function:

Assumption 1 *$p\frac{\gamma'''(p)}{\gamma''(p)} > -1$, $\lim_{p\to 0} p\frac{\gamma'''(p)}{\gamma''(p)} < \infty$, and $\lim_{p\to 0} \gamma''(p) < \infty$.*

Economically, the first part of Assumption 1 will ensure that the surplus a worker receives as a result of moral hazard increases at an increasing rate in the effort level p that the firm

[2] The assumption that effort has the same effect on success probabilities in both sectors is less restrictive than it seems. Variation in the amount at stake k across industries has qualitatively the same effect as variation in the effect of effort on success probability, so we choose to normalize by only considering variation in k.

[3] We could have modelled the magnitude of moral hazard problems within a sector in other ways without changing the general message of the paper. For example, instead of varying the money at stake, we could increase the noise between unobservable effort and observable outcome, or we could increase the cost of effort.

wants to induce. The third part ensures that workers exert strictly positive effort in all periods.

A firm in the economy can be active in one or both sectors, where the scope of the firm will be determined endogenously. To close the model, we need to determine the size of the two sectors. For simplicity, we assume there is free entry and perfect competition. The output prices g_H and g_L are determined in equilibrium by the standard market clearing condition that excess demand must equal zero. (Alternatively, if sector i is engaged in trading financial securities, then g_i is inversely related to how many people are following a given trading strategy.)

II Equilibrium contracts

We now describe the contracts in the economy. Taking sector output prices g_H and g_L as given, firms compete to hire young workers by offering them employment contracts. An employment contract consists of a task i that the worker will be assigned to and a pair (v_s, v_f) of continuation utilities, where v_s is the promised utility to the worker in case of success and v_f the promised utility in case of failure. These continuation utilities potentially include cash payments at the end of the first period; however, by standard dynamic contracting results, it is often beneficial to delay wage payments until the end of the second period. We denote the cost to the firm of providing continuation utility v by $w(v; g)$, to be determined below. Moreover, competition among firms, combined with a worker's freedom to quit, means that it is impossible to hold the worker to very low continuation utilities. Formally, only continuation utilities above some cutoff level $v_0(g)$ are possible, where $v_0(g)$ is determined below. (We regularly omit the argument g below.) For each sector that a young worker can start in, firms offer the contract that maximizes worker utility subject to the break even condition; this follows from the free entry assumption. A contract that initially assigns a young worker to sector i therefore solves the following maximization problem:

$$\max_{v_s, v_f \geq v_0(g).} pv_s + (1-p)v_f - \gamma(p) \tag{P1}$$

subject to

$$\gamma'(p) = v_s - v_f, \tag{IC1}$$

$$p\,(g_i - w(v_s; g)) - (1-p)\,w(v_f; g) - k_i \geq 0. \tag{BE1}$$

Condition IC1 is the incentive compatibility condition for the worker who maximizes his choice of effort given the promised continuation utilities. Condition BE1 is the firm's break even condition. It is worth highlighting that both the cost function w and the minimum continuation utility v_0 depend on the *vector* of prices g, since a worker who starts his career in one sector may end his career in another.

We solve for the optimal contracts by backward induction. First, we solve for the cost function $w(v)$, which is given by the subcontract that has to be given to an old worker in order to induce utility v. Then, we characterize properties of the young worker contract, and describe how workers are allocated over firms and sectors over their careers.

A Contracts for old workers

We now solve for the cost function $w(v)$, and partially characterize the minimum feasible continuation utility. We start by deriving the minimal cost $w_i(v)$ for an employer to give a worker exactly utility v via a contract in which the worker works in sector i. We will later determine which sector old workers will be allocated to in equilibrium.

A contract for an old worker is, by necessity, a one-period contract, and consists of a payment w_s for success and w_f for failure. Denote by $\Delta \equiv w_s - w_f$ the bonus for success. The cost $w_i(v)$ is then given by the following maximization problem:

$$w_i(v) = -\max_{\Delta, w_f, p} \{p(g_i - \Delta) - w_f - k_i\}\,. \tag{P2}$$

Note that the cost is just the negative of firm profits. The constraints on the maximization problem are the standard incentive compatibility and limited liability constraints,

$$\gamma'(p) = \Delta \tag{IC2}$$

$$w_f, w_f + \Delta \geq 0, \tag{LL2}$$

along with the promise-keeping constraint that the worker gets utility v,

$$p\Delta + w_f - \gamma(p) = v. \tag{IR2}$$

To solve this problem, first define p_{FBi} as the first best effort which maximizes surplus $pg_i - \gamma(p) - k_i$ from task i, i.e.,

$$\gamma'(p_{FBi}) = g_i.$$

Similarly, denote by v_{FBi} the utility for the agent from setting $w_f = 0$ and $\Delta = g_i$, so that he works at the first best effort:

$$v_{FBi} \equiv p_{FBi} g_i - \gamma(p_{FBi}).$$

Then, it is easy to see that the optimal contract (Δ, w_f) solving program P2 has $w_f = 0$ for $v \leq v_{FBi}$ and $w_f = v - v_{FBi}$ for $v > v_{FBi}$. In other words, paying the agent purely via a bonus for success is optimal as long as the utility promised corresponds to a bonus small enough such that the agent works less than first best. If the utility promised is higher than this, it is better for the firm to live up to the promise partly with a fixed wage $w_f > 0$.

The following result collects some elementary properties of the cost function $w_i(\cdot)$:

Lemma 1 *The cost function $w_i(\cdot)$ is continuous, differentiable, and convex with a unique minimizer. It is equal to k_i at $v = 0$ and at $v = v_{FBi}$. Its derivative is strictly less than* 1 *for $v < v_{FBi}$, and identically equal to* 1 *for $v \geq v_{FBi}$.*

It is important to note that Lemma 1 implies that the cost function w_i has a minimum at some interior $v \in (0, v_{FBi})$. The economic reason is standard: because of moral hazard, the principal finds it optimal to surrender some rent to the agent. Let v_{SBi} denote the utility level at which the minimum cost is acheived, and p_{SBi} the associated effort level, where the subscript refers to "second best." Formally, v_{SBi} and p_{SBi} are defined by

$$\begin{aligned} g_i &= \gamma'(p_{SBi}) + p_{SBi}\gamma''(p_{SBi}) \\ v_{SBi} &= p_{SBi}\gamma'(p_{SBi}) - \gamma(p_{SBi}). \end{aligned}$$

We assume that old workers cannot commit to stay with their initial employer, but can leave the firm and seek work on the outside labor market. Consequently, in equilibrium, it is impossible to give a worker a continuation utility v such that $w_i(v) < 0$ for either $i = L, H$: in such cases, another firm could poach the worker by promising a slightly higher utility level $v + \varepsilon$, and make profits $-w_i(v+\varepsilon) > 0$. Likewise, it is also impossible to give a continuation utility v if there exists $\tilde{v} > v$ such that $w_i(\tilde{v}) < 0$ for either $i = L, H$. So any feasible continuation utility must exeed the threshold $\underline{v}$ defined by

$$\underline{v} \equiv \min\{v : w_i(\tilde{v}) \geq 0 \text{ for } i = L, H \text{ and } \tilde{v} \geq v\}. \tag{2.1}$$

A couple of points are worth stressing. First, and for reasons we elaborate on below, $\underline{v}$ may be too slack a lower bound on what contracts are feasible, in which case $v_0 > \underline{v}$. Second, note that $\underline{v} > 0$ since w_L is strictly negative over $(0, v_{FBi})$. Economically, competition among firms for workers in task L ensures that a worker obtains strictly positive utility from working in task L.

Fix any continuation level $v \geq \underline{v}$. It remains to give the cost $w(v)$ of providing the worker with this utility. If $\min_{i\in\{L,H\}} w_i(\tilde{v}) \geq \min_{i\in\{L,H\}} w_i(v)$ for all $\tilde{v} \geq v$, then clearly the cost is simply $\min_{i\in\{L,H\}} w_i(v)$, and is achieved by assigning the worker to the task $\arg\min_{i\in\{L,H\}} w_i(v)$.

However, because of the underlying moral hazard problem, it is possible that the cost w_H is strictly decreasing over some interval to the right of $\underline{v}$. Formally, this case arises when $v_{SBH} > v_{FBL} = \underline{v}$. In this case, the cheapest way for a firm to deliver continuation utilities between v_{FBL} and v_{SBH} is via a lottery that sometimes assigns the worker to Sector L, and sometimes to Sector H. We restrict the set of lotteries to those that need no commitment from the firm, in that all outcomes in the lottery have the same cost for the firm[4] (though,

[4] Our analysis would be qualitively unaffected if instead we allowed for arbitrary randomizations of this

of course, they give the worker different utility levels). Given this restriction, define v_L by $w_L(v_L) = w_H(v_{SBH})$. Any utility level v between v_L and v_{SBH} is then most cheaply delivered by assigning the worker to receive utility v_L in Sector L with probability $\frac{v_{SBH}-v}{v_{SBH}-v_L}$, and to receive utility v_{SBH} in Sector H with probability $\frac{v-v_L}{v_{SBH}-v_L}$.

Combining the two cases above, for $v \geq \underline{v}$ the minimum cost function is defined by

$$w(v) = \min_{\tilde{v} \geq v, i \in \{L,H\}} w_i(\tilde{v}),$$

and is weakly increasing. Moreover, as v increases, the probability that the worker is assigned to Sector H weakly increases. Figure 1 shows a typical shape of the cost function $w(v)$.

Commitment to the continuation utilities v_s and v_f is straightforward: a firm can offer to give a worker severance/early retirement pay of $w(v_s)$ and $w(v_f)$ if the worker is dismissed after success and failure, respectively. Moreover, the definition of the cost function satisfies renegotiation proofness: given a worker continuation utility v and firm cost $w(v)$, there is no way to both strictly increase the worker's utility and strictly decrease the firm's cost.

Lemma 2 *The second-period effort associated with continuation utility v is weakly increasing in v.*

B Equilibrium contracts and equilibrium existence

We write y_i for total output in sector i. We write ζ_i for the inverse demand curve for the output produced in sector i, i.e., $\zeta_i(y_i)$ is the price such that total demand is y_i.

By itself, the moral hazard problem in Sector L causes no distortion, since when firm profits are zero, there is enough surplus available for the worker to induce him to exert first-best effort. In this sense, Sector H is the more interesting sector, and in order to focus our analysis we make the simplifying assumption that demand for Sector L is perfectly elastic, i.e., $\zeta_L \equiv g_L > 0$. (Our results are qualitively unaffected if this assumption is relaxed; details are available on request from the authors.) For Sector H, we assume that the demand curve slopes strictly down, i.e., ζ_H is strictly decreasing. We also impose the standard Inada condition that $\zeta_H(y_H) \to \infty$ as $y_H \to 0$.

type.

Write $u(v_s, v_f)$ for the expected utility of a young worker under contract (v_s, v_f). (Note that utility depends only on the utility promised, and not on the task to which the worker is assigned.) Write $\pi^i(v_s, v_f; g)$ for the expected profits of a firm employing a young worker using contract (v_s, v_f), and initially assigning the worker to task i.

Given a price vector g, a firm is potentially willing to employ a young worker in sector i—at least initially—using any contract (v_s, v_f) that allows it to at least break even, i.e., $\pi^i(v_s, v_f; g) \geq 0$. Whether the worker remains in sector i—i.e., whether he is retained / promoted / demoted—when old is a topic we discuss in detail below.

Competition between firms drives worker utility up and firm profits down. An important feature of our economy is that, even though workers are ex ante identical, in equilibrium they may end up receiving different expected utilities. To see this, consider the situation in which there is a unique contract v that delivers non-negative profits in the high-stakes Sector H. Reducing compensation relative to this contract causes a reduction in the worker's effort that more than offsets the lower compensation and the firm loses money. In other words, the moral hazard problem forces firms employing workers in Sector H to pay efficiency wages.

Before proceeding further, we establish that an equilibrium exists in our economy.

The details of our proof of equilibrium existence are relegated to Appendix D. The basic approach is standard: we construct a mapping from possible prices g_H into output vectors (y_L, y_H), and use Kakutani's fixed-point theorem to establish that there is a price g_H at which excess demand is zero.

There is one aspect of the proof that we wish to highlight, because it is intimately connected to some of our main results. Consider the case in which the Sector H price g_H is such that firms can exactly break-even using one-period contracts, i.e., $\min w_H = w_H(v_{SBH}) = 0$. Then, on the one hand, if g_H is very slightly lower, any one-period contract leads to strict losses, and it is possible to threaten workers with a continuation utility of v_{FBL} via assignment in Sector L. On the other hand, if g_H is very slightly higher, strictly positive profits are available using a one-period contract in Sector H, and so the worst continuation utility that workers can be threatend with is v_{SBH}.

Consequently, the utility level $\underline{v}$, defined in (2.1), is potentially discontinuous as a function of g_H. This leads to a discontinuity in the incentives that can be given to workers (at a given cost), and hence to a discontinuity in output.

We deal with this complication as follows. The discontinuity occurs at the point at which firms can make exactly zero profits from a one-period Sector H contract, while strictly positive profits are unattainable: formally, $w_H(v_{SBH}) = 0$. If $v_{FBL} < v_{SBH}$ also, this means that firms can make zero profits from a one-period contract either via assigning a worker to Sector L and giving him utility v_{FBL}; or by assigning him to Sector H and giving him more utility; or by any lottery over the two alternatives. Graphically, the cost function w is identically equal to zero for all utility levels between v_{FBL} and v_{SBH}. For such cases, we define the minimum feasible continuation utility as

$$v_0 = (1-\mu)\, v_{FBL} + \mu v_{SBH},$$

where $\mu \in [0,1]$ is a parameter that determines the probability that a worker ends up in Sector H with utility v_{SBH} rather than Sector L with utility v_{FBL}. Importantly, μ is determined in equilibrium, since it must be consistent with total output in the two sectors. By entertaining all possible values $\mu \in [0,1]$ as potential equilibrium values, we obtain a correspondence from prices to output vectors that is upper hemi-continuous, and hence amenable to the application of Kakutani's fixed-point theorem.

Note that in this case the minimum feasible continuation utility v_0 is typically strictly greater than $\underline{v} = v_{FBL}$. Economically, we interpret this as reflecting a secondary labor market for old workers—specifically, there are firms who are willing to hire old workers and assign them to Sector H.

Proposition 1 *An equilibrium exists.*

III Lucky workers and high-profile jobs

A A basic retention result

In Section IV below, we explore how the worker is assigned to different tasks over his lifetime, i.e., promotion and demotion. However, we are already in a position to establish a first basic result: if a young worker begins his career in the high-stakes Sector H, he is always retained in this sector when he succeeds. The intuition for this result can be understood as follows. In Sector L, as noted earlier, the moral hazard problem causes no distortions, which means that the worker is employed in that sector on terms that are independent of where the worker is in his career. It follows that if the worker is always demoted to Sector L, the employment contract is essentially a sequence of one-period contracts, but with assignment to Sector H in the first period and Sector L in the second. This cannot be an equilibrium contract if one of the two one-period assignments strictly dominates the other; the proof also handles the case of indifference.

Lemma 3 *In any equilibrium, if a young worker is employed in Sector H and is successful, he is retained in Sector H.*

Lemma 3 is useful in part because it allows us to work with a constrained version of Problem P1, in which a young worker who starts in Sector H is retained after success. By Lemma 3, this additional constraint never binds in equilibrium. It follows that the constraint (BE1) for young workers starting in Sector H can then be replaced with

$$p\left(g - w_H(v_s; g)\right) - (1-p)\, w(v_f; g) - k \geq 0. \tag{BE1'}$$

B High-profile jobs

As we discussed above, the key feature of our economy is that agency problems may cause ex ante identical workers to be given contracts that deliver different expected utilities. We term a contract that promises strictly higher utility a *high-profile* job, and its recipients as *lucky workers*. As we show below, in addition to offering higher utility, these jobs are associated with many of the other features associated with high-profile jobs: longer hours, higher pay which is more than a compensating differential, and better promotion responsibilities. Note that better promotion possibilities mean that if a young worker is lucky enough to land a high profile job, this has life-long effects on his career.

First, we show that (as one might expect) workers starting in Sector L never receive strictly more utility than workers starting in Sector H:

Proposition 2 *There is no equilibrium in which workers starting in Sector L receive strictly more utility than workers starting in Sector H.*

Next, we show that high-profile jobs in Sector H actually exist. The key condition is that the agency problem in Sector H is sufficiently severe. The strength of the agency problem is in turn directly tied to the stakes k_H in Sector H.

Proposition 3 *Whenever k_H is sufficiently large, high-profile Sector H jobs exists: a strict subset of young workers start in Sector H, and receive strictly more utility than those starting in Sector L.*

It is possible for some workers to receive strictly more utility than other workers in equilibrium only because a reduction in worker utility would actually *reduce* firm profits. In other words, a high-profile worker's endogenous participation constraint—which is determined by what other firms would pay him—is non-binding.

C Properties of high-profile jobs

Any contract for a lucky worker must feature $w(v_f) = 0$: if instead $w(v_f) > 0$, a firm could make strictly positive profits from slightly reducing v_f, and so could profitably lure away workers currently in another type of job, a contradiction. In particular, giving the worker the minimal feasible continuation utility $\underline{v}$ has zero-cost, i.e., $w(\underline{v}) = 0$. Corollary A-1 in Appendix A shows that $\underline{v}$ is in fact the only zero-cost continuation utility when lucky workers exist, and moreover is delivered via assignment to Sector L.

Given that they fare (weakly) worse after failure than any other workers, lucky workers must do better after success, i.e., have better promotion prospects in the sense of a higher continuation utility — otherwise they would not be lucky! So their incentives to work hard when young are also higher than other workers. Finally, by Lemma 2, conditional on succeeding when young lucky workers have higher lifetime compensation than other workers.

Summarizing:

Proposition 4 *High-profile jobs have up-or-out contracts: if they fail, they are assigned to Sector L and receive utility $\underline{v} = v_{FBL}$. Workers in the highest profile jobs work harder when* *young, and, conditional on success when young, have better promotion prospects (i.e., higher v_s) and higher lifetime compensation than other workers.*

IV Promotion

We next use our model to explore, in turn, three distinct aspects of "promotion," namely promotion to more responsibility and higher pay; relief from menial and boring tasks; and promotion to different and more important tasks.

A Promotion to more responsibility

From Proposition 4, lucky young workers who receive more than their reservation utility are motivated partly by the threat of being demoted to Sector L. So far, we have said little about the timing of monetary payments. If a worker is given continuation utility v via assignment to task i, and $v \leq v_{FBi}$, then the worker is compensated only via a bonus for success in the second period. If instead $v > v_{FBi}$, the worker receives an additional payment that is not conditional on the outcome when old. Our model is silent on whether this non-contingent payment is received in period 1 or period 2. We assume it is paid entirely in period 1.

Since lucky young workers receive the minimum feasible continuation utility $\underline{v}$ when they fail, their contracts consist of a bonus Δ_1 for first-period success, and a second bonus Δ_2 paid only in the case of second period success. For exposition, we focus here on the case in which all compensation is postponed, i.e., $\Delta_1 = 0$ (the general case is handled in the appendix). In the second period, the worker is motivated by the bonus Δ_2, and chooses effort p_2 so that $\gamma'(p_2) = \Delta_2$. In the first-period the worker is also motivated by the second-period bonus, but this time he discounts it by the probability it is actually received, by the effort he will have to expend to exert it, and by the fact that even if he fails he still gets a continuation utility of $v_f = \underline{v}$. He chooses effort p_1 so that $\gamma'(p_1) = p_2\Delta_2 - \gamma(p_2) - v_f$. Consequently, if the employee succeeds in the first period, he is "promoted" in the sense that he has more responsibility (i.e., works harder) and is paid more:

Proposition 5 *A lucky worker starting in Sector H works strictly harder in the second period if he succeeds than in the first period, and is also paid more.*

Proposition 5 matches the received wisdom that senior employees in organizations such as investment banks and law firms are both especially productive, and compensated especially well. In our model, this is the case even though senior workers are actually harder to motivate because of their shorter contracting horizons. Rather, promotion is an efficient response to the fact that incentivizing a worker entails surrendering some surplus to him. The surplus that must be promised to a senior worker late in his career to induce hard work serves as a very attractve carrot for workers early in their careers.

It is worth stressing that workers who spend their whole careers in the low-stakes Sector L are *not* promoted in this way, and instead work p_{FBL} throughout their careers. Moreover, their pay is not back-loaded. The reason is straightforward. The moral hazard problem in Sector L is sufficiently small that, in equilibrium, it is non-binding in one-period contracts: as noted previously, since firms receive zero profits, there is enough surplus for workers to induce them to exert the socially efficient amount of effort. Consequently, there is no need to deploy dynamic incentives to ameliorate the moral hazard problem.

Promotion in the sense of Proposition 5 emerges as an efficient way to reduce a worker's surplus. However, Proposition 5 fails to capture a second important aspect of promotion, namely that it is associated with a change in the *type* of work. We next explore two different aspects of changing work.

B Promotion to different and more important tasks

The most cost-efficient way for a firm to deliver a high continuation utility is typically via assignment to Sector H: the high continuation utility is associated with a lot of effort, and this effort is best deployed in the high-stakes sector. This argument delivers the following promotion result: workers who start in the low-stakes sector should progress to the high-stakes sector if they succeed:

Proposition 6 *If k_H is sufficiently small (in particular, $k_H < g_L$), any worker who starts in Sector L is promoted to Sector H with strictly positive probability.*

Proposition 6 is predicated on some workers actually starting in Sector L. Lemma A-6 in Appendix A confirms that whenever the value g_L of Sector L output is sufficiently strong, any equilibrium must indeed entail some workers starting in Sector L.

Proposition 6 requires k_H to be sufficiently close to $k_L = 0$. The reason is that if instead k_H is very large, assignment to Sector H is efficient only for very high continuation utilities: but very very high continuation utilities are wasted on providing incentives in the low-stakes Sector L. Consequently, firms use promotion to more important tasks as an incentive device only when the two tasks are not too different. Formally:

Proposition 7 *For k_H sufficiently large, a worker who starts in Sector L remains in Sector L.*

C Dog years

A second aspect of promotion and a change in the type of work is that senior employees are exempted from working on straightforward and boring tasks. As we show, this aspect of promotion naturally emerges whenever lucky workers exist, as a way for the firm to further reduce the surplus it surrenders to workers.

To capture this, we introduce what we call *menial* tasks to workers, over and above the regular task. For example, this menial task could involve gathering data, preparing spreadsheets, copying papers, or fetching burgers for more senior employees. The menial task is also easily monitored: the employer can simply stipulate how much of the menial task it wants a worker to do.

Proceeding a little more formally, we take the equilibrium of the economy without menial tasks, and then introduce menial tasks to a null set of firms (this allows us to hold the overall structure of the equilibrium unchanged). To ensure that the menial task is truly menial, we assume that if a worker spends time m on the menial task he produces εm, where ε is very small but positive. A worker can work on both the menial and important tasks: his total hours worked are $\gamma(p) + m$, which must be less than 1, his total time endowment.

We concentrate on the high-profile case where k_H is high enough such that workers starting in Sector L never get promoted to Sector H (Proposition 7), and workers starting in

Sector H earn strictly higher utility than workers starting in Sector L (Proposition 3). We show that the only time the menial task is used is for young workers starting in the high-profile sector; in all other circumstances, firms prefer workers to work on the more efficient tasks:

Proposition 8 *Suppose k_H is high enough such that there are high-profile workers and workers starting in Sector L never get promoted to Sector H. Then, whenever the menial task is sufficiently menial (i.e., ε below some level $\bar{\varepsilon} > 0$), it is assigned* only *to young high-profile workers. Young high-profile workers perform the menial task up to the point that either their time endowment constraint binds, or else their utility is the same as Sector L workers.*

We want to stress two features of this result. First, the menial task is only used in the early stage of the career. If the worker is promoted, he is assigned only to important tasks. The reason is that in the second period, the worker must be promised some surplus to motivate work in the first period, so extracting surplus from the worker in the second period is counterproductive.

Second, since the menial task is used as an inefficient surplus extraction mechanism, its use is concentrated in high-profile industries. This is our "dog years" result: in high-profile industries, such as investment banking or law, there are typically very long hours early on in the career, much of which is spent on less prestigious tasks. This can be a second best solution even when work hours are inefficiently long, and even when the menial task can be performed better or cheaper with less qualified workers.

V Distortions in the allocation of talent

We now add (observable) heterogeneity in talent between workers to study how talent is allocated across industries. In particular, we account for two commonly expressed views about the allocation of talent, namely that (a) talent may be "scorned," in the sense that the most able people do not necessarily get the best jobs, and (b), talent may be "lured," in the sense that, for example, people who should (for social efficiency) be doctors or scientists become investment bankers instead.

We introduce differences in talent by assuming that only a null set of workers have higher skills, while the remaining workers are homogenous as before. This assumption ensures that the basic structure of the equilibrium remains unchanged. Specifically, suppose that a null set of workers have a cost $c_i \gamma(p)$ of achieving success p in sector i, where $\gamma_i < 1$ for both sectors $i = L, H$. One would expect these talented workers to be more generously rewarded than other workers; and social efficiency dictates that they should be given more responsibility (in the sense of working harder) at all stages of their careers. As we show below, however, this does not necessarily happen.

A Talent scorned

The talent scorned effect stems from the fact that more talented people have better outside options. Consequently, it can arise whenever the constraint that continuation utilities exceed v_0 is binding.

For example, consider any equilibrium in which high-profile jobs exist in Sector H. From Proposition 4, any young worker who fails in a high-profile job is moved to Sector L and receives a continuation utility v_{FBL}. However, labor market competition for a more talented workers implies that his continuation utility must be at least

$$\max_{p \geq 0} pg_L - c_L \gamma(p) > v_{FBL}.$$

This better outside option makes the more talented worker harder to incentivize when young. Colloquially, he is "difficult," or "hard-to-manage." It follows that if his talent advantage in Sector H is sufficiently small, i.e., c_H close enough to 1, he will not be hired into a high-profile job since the difficulty of motivating him outweighs his higher productivity.

In this case, the failure of a talented worker to land a high-profile job is disappointing for the worker, but not necessarily socially costly; after all, the fact that his advantage in Sector H is small may mean that he *should* be assigned to Sector L from the perspective of social efficiency. However, we next discuss a revealing case in which talent is scorned in which social inefficiency is clear.

Consider the case in which demand for Sector H output is sufficiently high that the equilibrium takes the following form: *all* workers start their careers in Sector H, and remain there with strictly positive probability even if they fail. In Appendix C, we show, by construction, that there exist parameter values under which an equilibrium of this type arises.

In the equilibrium described, typical—i.e., untalented—old workers who failed when young face a lottery between being assigned to Sector L and receiving utility $\underline{v} = v_{FBL}$, and being assigned to Sector H and receiving strictly higher utility. Firms make zero profits from both of these assignments of old workers. However, the higher value of Sector H output means that talented old workers are *always* assigned to Sector H (unless they are *much* more talented in Sector L, i.e., $c_L << c_H$): if instead they ended up in Sector L, it would be possible to strictly increase both utility and firm profits by moving them to Sector H.

Consequently, an arbitrarily small level of Sector H talent $c_H < 1$ leads to a discrete change in the minimum utility that a firm can credibly promise to a failed worker. This makes talented workers harder to motivate. Whenever the talented workers are not too talented (i.e., c_H not too small), this effect outweighs the direct productivity effect, and talented workers work less hard when young. If firms left the success reward unchanged, they would actually lose money on these talented workers, and so they should respond by lowering the utility offered after success: this means that talented workers do even less work when young; are promoted to positions with less responsibility (i.e., work less hard when old) even when they do succeed; are less highly compensated; and have lower expected utility when they enter the labor force.

B Talent lured

The second example of talent scorned above may also feature a second source of social inefficiency, which we term *talent lured*. To recap slightly more formally, the talent scorned effect arises because whereas, for untalented workers, firms make zero profits both from giving old workers utility v_{FBL} in Sector L and from giving $v_{SBH} > v_{FBL}$ in Sector H, a firm can profitably give a talented old worker utilities $\tilde{v}_L > v_{FBL}$ and $\tilde{v}_H > v_{SBH}$. The disincentive effect follows from the fact that a talented old worker's continuation utility must be strictly more than v_{SBH}.

The talent lured effect arises here whenever the worker ends up in Sector H, i.e., $\tilde{v}_H > \tilde{v}_L$, but his talent is actually socially more valuable in Sector L, $\tilde{v}_L - v_{FBL} > \tilde{v}_H - v_{SBH}$. This is the case in which old workers who are especially talented in Sector L, and should work there from a social efficiency perspective, nonetheless are "lured" into working in Sector H. Indeed, in this case talented workers are scorned when young precisely because they are lured when old.

The key driving force for the talent scorned effect is that the moral hazard problem stops utilities from being equated across sectors in equilibrium: Sector H pays its old workers strictly more utility than Sector L does. Whenever the Sector H stakes are sufficiently large, this same utility differential exists for young workers—there are lucky young workers and high-profile jobs. In this case, talented young workers can be lured into working in Sector H even though their talent would produce a larger increase in output if they were employed in Sector L. This feature of our model is very much in line with popular impressions of investment banks hiring away talented scientists from research careers. The effect arises because firms are prepared to pay for talent only up to the extent to which output is increased. The utility premium enjoyed by Sector H workers means that, unless the difference in talent across sectors (i.e., c_L vs c_H) is very large, Sector H firms can offer more utility to these workers, even if the increase in output is larger in Sector L.

Note that because of the talent scorned effect, it is important that workers have some talent in Sector H: as discussed above, if c_H is close enough to 1, these talented workers will not be hired by Sector H. However, numerical simulations (available upon request) show that, given Sector L talent c_L, there is an interval of Sector H talents c_H such that workers are employed in Sector H even though they would increase output more if employed in Sector L.

VI The effect of demand shocks on career dynamics

Thus far we have assumed that the value of output of both sectors is constant over time, i.e., g_L and ζ_H are constant (recall we assume that demand in Sector L is perfectly elastic). In this section we study the effects of shocks to the value of output. In particular, we consider how shocks affect worker careers, productivity, and prices.

We start with a specification of our basic model in which lucky Sector H workers exist, and workers who start in Sector L remain there (from prior results this is the case whenever k_H is large enough). Write $\bar{\zeta}_H$ and $\bar{g}_L$ for the Sector H demand function and the Sector L price. To keep the analysis as simple as possible, we add shocks by assuming that the economy can be either in a "Good" (G) or "Bad" (B) demand state, with demand higher in the good state, i.e., $\zeta_H^G(\cdot) \geq \zeta_H^B(\cdot)$ and $g_L^G \geq g_L^B$. We assume throughout that ζ_H^G and ζ_H^B are sufficiently close to $\bar{\zeta}_H$, and g_L^G and g_L^B are sufficiently close to $\bar{g}_L$, so that—as we explain below—the stochastic economy continues to have lucky Sector H workers.

A Demand shocks to Sector H

We consider first the case in which the demand shock only affects Sector H, i.e., $g_L^G = g_L^B$ and $\zeta_H^G(\cdot) > \zeta_H^B(\cdot)$. A convenient feature of this specification is that, as we next show, prices and hence contracts remain the same as in the constant-demand version of our model. Instead, all adjustment occurs via hiring decisions into the two sectors. Moreover, in this case, we do not need to make any assumptions on the stochastic process driving the state of the economy.

Let p_1 and p_2 be the first and second period success probabilities for the lucky Sector H workers in the constant-demand version of our economy. Let g_H be the Sector H price, and $\bar{y}_H$ Sector H output.

Turning to the stochastic specification, let λ_t be the number of lucky workers hired at date t. Let y_H^ω be Sector H output level corresponding to the price g_H in state $\omega \in \{G, B\}$, i.e., $\zeta_H^\omega(y_H^\omega) = g_H$. Note that output is higher in state G, i.e., $y_H^G > y_H^B$.

Under the conjecture that prices and contracts remain the same, date t output in Sector H is

$$y_H^t = p_1\lambda_t + p_1\lambda_{t-1}p_2.$$

Consequently, the number of workers hired into Sector H at date t is

$$\lambda_t = \frac{y_H^t}{p_1} - \lambda_{t-1}p_2. \tag{6.2}$$

As one would expect, Sector H hires more workers in good states, and when fewer workers were hired at the previous date.

To verify the conjecture, we need to show that it is possible to vary the number of workers hired by a sufficient amount to fully absorb the demand shock. Formally, this amounts to showing that λ_t remains between 0 (one cannot hire a negative number of new workers), and 1/2 (the total population of young workers).

Define $\underline{\lambda} \equiv \frac{y_H^B - p_2 y_H^G}{p_1\left(1-p_2^2\right)}$ and $\bar{\lambda} \equiv \frac{y_H^G - p_2 y_H^B}{p_1\left(1-p_2^2\right)}$. Then it is straightforward to establish that λ_t remains in the interval $\left[\underline{\lambda}, \bar{\lambda}\right]$.[5] As the shock size approaches 0, i.e., ζ_H^G and ζ_H^B approach $\bar{\zeta}_H$, both $\underline{\lambda}$ and $\bar{\lambda}$ approach $\frac{\bar{y}_H}{p_1(1+p_2)}$, the number of young workers Sector H hires in the constant-demand specification. Hence provided the shocks are sufficiently small, there is indeed enough flexibility to absorb the shocks via hiring decisions, verifying the conjecture.

Straightforward iteration of the hiring equation (6.2) gives

$$\lambda_t = (-p_2)^t \lambda_0 + \frac{1}{p_1} \sum_{s=0}^{t-1} (-p_2)^s y_H^{t-s}, \tag{6.3}$$

which determines date t hiring as a function of the history of shock realizations, summarized by output y_H^{t-s} for $s = 0, \ldots, t-1$. From (6.3), if the economy remains in state $\omega \in \{G, B\}$ for a long time, Sector H hiring converges to

$$\lambda^\omega \equiv \frac{y_H^\omega}{p_1 (1 + p_2)},$$

and the age-profile of Sector H workers converges to p_1 old workers for every young worker.

Now, suppose the economy has been in the high demand state for a long time, and that it is then hit by a bad shock at date 2. From (6.2), the number of new workers hired by

[5] If $\lambda_{t-1} \in \left[\underline{\lambda}, \bar{\lambda}\right]$, then

$$\lambda_t \geq \frac{y_H^B}{p_1} - \bar{\lambda} p_2 = \frac{y_H^B \left(1 - p_2^2\right) - \left(y_H^G - p_2 y_H^B\right) p_2}{p_1 \left(1 - p_2^2\right)} = \frac{y_H^B - p_2 y_H^G}{p_1 \left(1 - p_2^2\right)} = \underline{\lambda}$$

and

$$\lambda_t \leq \frac{y_H^G}{p_1} - \underline{\lambda} p_2 = \frac{y_H^G \left(1 - p_2^2\right) - \left(y_H^B - p_2 y_H^G\right) p_2}{p_1 \left(1 - p_2^2\right)} = \frac{y_H^G - p_2 y_H^B}{p_1 \left(1 - p_2^2\right)} = \bar{\lambda}.$$

Sector H at date 2 falls to

$$\lambda_t = \frac{y_H^B}{p_1} - \lambda^G p_2 < \frac{y_H^B}{p_1} - \lambda^B p_2 = \lambda^B < \lambda^G.$$

In other words, the move from the good to the bad state leads Sector H hiring to fall; in fact, it falls even below its level in the long-run state B case.

The age-profile in Sector H is now skewed towards old workers; and since old workers are more productive than young workers, the initial effect is to *increase* average productivity in Sector H.

However, again from (6.2), the shortfall in date t hiring translates to an increase in date $t+1$ hiring:

$$\lambda_{t+1} = \frac{y_H^\omega}{p_1} - \lambda_t p_2 > \lambda^\omega > \lambda_t.$$

In the case that economy recovers so that the date $t+1$ state is again G, the hiring burst is particularly dramatic, since $\lambda_{t+1} > \lambda^G$. This hiring burst only benefits the date $t+1$ generation of young workers, however; the workers who were young in date 2 and missed out on a high-profile jobs because of the bad shock are not now hired (see Oyer (2008) for evidence). Moreover, Sector H productivity is depressed at date $t+1$, as its firms suffer from the lack of a "missing generation" that was not previously hired: the sector age profile is now unduly tilted towards young workers.

B Demand shocks to both sectors

Next, we expand our analysis to the case in which shocks hit both sectors. As we will see, equilibrium prices and contracts now vary across states. Consequently, we need to impose more structure on the stochastic process driving the state of the economy. We make the standard assumption that the state follows a Markov process, with the transition probability from moving from state $\omega \in \{G, B\}$ at date t to state ψ at date $t+1$ denoted by $\mu^{\omega\psi}$. We assume that the state is at least somewhat persistent, in the sense that if the state is more likely to be good (respectively bad) tomorrow if it is good (respectively, bad) today, $\mu^{GG} > \mu^{BG}$.

We will need a little notation. Let g_i^ω denote the sector i price in state ω. Let $\left(v_s^\omega, v_f^\omega\right)$ be the contract given to young workers starting in Sector H when the state is ω. (Because we are examining the case in which workers who start in Sector L remain there, the Sector L contract is uninteresting.) The continuation utility v_s^ω is itself delivered in a state contingent form, i.e., the continuation utility is $v_s^{\omega G}$ if tomorrow's state is G and is $v_s^{\omega B}$ if tomorrow's state is B, where $v_s^\omega = \mu^{\omega G} v_s^{\omega G} + \mu^{\omega B} v_s^{\omega B}$. Let v_{FBL}^ω be the value of first-best Sector L utility v_{FBL} corresponding to price g_L^ω. Because the cost of delivering a given continuation utility depends on the prices, and hence on the state, and because the distribution of tomorrow's state depends on today's state, the minimal cost of delivering a continuation utility v is $w^\omega(v)$. Finally, the effort chosen by a lucky young worker starting in Sector H in state ω is p_1^ω.

Given this notation, an equilibrium (remember, we are focusing on the lucky worker case) is $\left\{g_H^\omega, p_1^\omega, v_s^\omega, v_f^\omega : \omega = G, B\right\}$ satisfying, for $\omega = G, B$:

Maximal feasible punishment:

$$v_f^\omega = \sum_{\psi=G,B} \mu^{\omega\psi} v_{FBL}^\psi$$

Zero-profits from workers starting in Sector H:

$$p_1^\omega \left(g_H^\omega - w^\omega\left(v_s^\omega\right)\right) - k_H = 0.$$

Incentive compatibility:

$$\gamma'\left(p_1^\omega\right) = v_s^\omega - v_f^\omega.$$

Maximal profits:

$$\frac{1}{\gamma''\left(p_1^\omega\right)}\left(g_H^\omega - w^\omega\left(v_s^\omega\right)\right) - p_1^\omega w^{\omega\prime}\left(v_s^\omega\right) = 0.$$

Our main result is that moral hazard problems are worse in Sector H in good times. Informally, the reason is that in good times, the Sector L price is expected to be relatively high in the future, and consequently v_{FBL} is relatively high. Consequently, the cost of failure for a worker is dampened—he is confident that he will "land on his feet."

Formally, we show that young workers in Sector H work less hard in good times, even at the same that they are paid more:

Proposition 9 *Either lucky young workers work less hard in good times, $p_1^G < p_1^B$; or else all old workers work the socially efficient amount. They are paid strictly more if they succeed, independent of the realized state the following period, i.e., $v_s^{G\psi} \geq v_s^{B\psi}$ for $\psi \in \{G, B\}$.*

VII Conclusion

TO BE WRITTEN

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A Results omitted from main text

Lemma A-1 *If a lucky worker starts in Sector L in equilibrium, his contract* (v_s, v_f) *has* $w(v_s) = g_L$ *and* $w(v_f) = 0$.

Proof of Lemma A-1: If $w(v_s) > g_L$ the firm loses money. If $w(v_s) < g_L$ then $w(v_f) > 0$ by the zero-profit condition, and the firm can then simultaneously raise profits and worker utility by slightly increasing v_s and slightly decreasing v_f. So the only possibility is $w(v_s) = g_L$ and $w(v_f) = 0$. QED

Lemma A-2 *Let* (v_s, v_f) *be a Sector* H *contract such that* $w(v_f) \geq 0$*; firm profits are weakly positive; and the derivative of firm profits with respect to* v_s *is weakly negative. Then profits are strictly decreasing in* v_s *for all higher values of* v_s*.*

Proof of Lemma A-2: Firm profits are $p(g_H - w(v_s)) - (1-p)w(v_f) - k_H$. Since $w(v_f) \geq 0$ and profits are weakly positive, $g_H - w(v_s) + w(v_f) > 0$. The derivative of profits with respect to v_s is $\frac{1}{\gamma''(p)}(g_H - w(v_s) + w(v_f)) - pw'(v_s)$, which has the same sign as $g_H - w(v_s) + w(v_f) - p\gamma''(p)w'(v_s)$. Since the derivative is weakly negative, it follows that $w'(v_s) > 0$. From Lemma 3, $w(v_s) = w_H(v_s)$. We know w_H is convex. So by Assumption 1, the derivative of profits with respect to v_s is strictly negative for all higher values of v_s, completing the proof. QED

Lemma A-3 *If there are lucky workers, any feasible one-period Sector* H *contract produces strict losses,* $\min_{v \geq \underline{v}} w_H(v) > 0$*.*

Proof of Lemma A-3: Suppose to the contrary that there is an equilibrium with lucky workers and $\min_{v \geq \underline{v}} w_H(v) = 0$. There are two cases, depending on which sector the lucky workers are in.

Case: There are lucky workers who start in Sector L*.*

Let $(\check{v}_s, \check{v}_f)$ be the contract received by lucky young workers in Sector L. From the main text, $w(\check{v}_f) = 0$. By the zero-profit condition this implies $w(\check{v}_s) = g_L$. If $w^-(\check{v}_s) > 0$ (where w^- denotes the left-sided derivative), there is a contract perturbation that slightly reduces worker utility while producing strictly positive profits. Since this is inconsistent with lucky workers in Sector L, we must $w^-(\check{v}_s) = 0$. This implies $\min_{v \geq \underline{v}} w_H(v) = g_L > 0$, completing the proof for this case.

Case: There are lucky workers who start in Sector H*.*

Let $(\check{v}_s, \check{v}_f)$ be the contract received by lucky young workers in Sector H. From the main text, $w(\check{v}_f) = 0$. Define $\bar{v} \geq \underline{v}$ by $w_H(\bar{v}) = 0$, and let (Δ, w_f) be the associated contract terms. From the elementary properties of $w_H(\cdot)$, $\bar{v}$ is well-defined, and $w_f = 0$. Consider employing a worker in Sector H using the two-period contract $(\hat{v}_s, \hat{v}_f)$ defined by $\hat{v}_f = \bar{v}$ and $\hat{v}_s = \bar{v} + \Delta$. The firm's profits are $p(g_H - w(\bar{v} + \Delta)) - (1-p)w(\bar{v}) - k_H$, where p is effort induced by the one-period contract (Δ, w_f). We know $w(v) = w_H(v)$ for all $v \geq \bar{v}$, and from the elementary properties of $w_H(\cdot)$ we know $w_H(\bar{v}+\Delta) - w_H(\bar{v}) < \Delta$. So profits under $(\hat{v}_s, \hat{v}_f)$ are strictly greater than $p(g_H - \Delta) - k_H = 0$.

Since $(\check{v}_s, \check{v}_f)$ is used in equilibrium, $\hat{v}_s < \check{v}_s$: if instead $\hat{v}_s \geq \check{v}_s$, one can easily construct a perturbation of the contract $(\hat{v}_s, \hat{v}_f)$ that gives the firm strictly positive profits and the worker strictly higher utility than $(\check{v}_s, \check{v}_f)$.

Note that the contract $(\hat{v}_s, \check{v}_f)$ would also give strictly positive profits. It follows from Lemma A-2 that for every $v_s \in [\hat{v}_s, \check{v}_s)$, the contract $(v_s, \check{v}_f)$ gives strictly positive profits. (To see this, suppose to the contrary that some such contract $(v_s, \check{v}_f)$ gives weakly negative profits. By continuity, there exists a contract $(v_s, \check{v}_f)$ giving exactly zero profits, and such that the derivative with respect to v_s is weakly negative. Lemma A-2 then implies that profits are negative at the contract $(\check{v}_s, \check{v}_f)$, a contradiction.) But then $(\check{v}_s, \check{v}_f)$ cannot be an equilibrium contract, since a firm could make strictly positive profits by offering a contract of the form $(\check{v}_s - \varepsilon, \check{v}_f)$ to unlucky workers. The contradiction completes the proof. QED

Corollary A-1 *If there are lucky workers, $\underline{v}$ is the only continuation utility v such that $w(v) = 0$, and moreover, $\underline{v}$ can be provided only via assignement to Sector L.*

Proof of Corollary A-1: Immediate from Lemma A-3 and the definition of $\underline{v}$. QED

Lemma A-4 *If there are lucky workers starting in Sector H, there is no Sector H contract that gives strictly positive profits (regardless of the utility given to workers).*

Proof of Lemma A-4: Suppose to the contrary that there is an equilibrium with lucky young workers in Sector H, receiving contract $(\check{v}_s, \check{v}_f)$ say, and such that there exists a contract $(\hat{v}_s, \hat{v}_f)$ that gives strictly positive profits. From Corollary A-1, the lucky worker contract has $\check{v}_f = \underline{v}$. So $v_f \geq \check{v}_f$, and hence $v_s < \check{v}_s$ since otherwise $(\check{v}_s, \check{v}_f)$ cannot be an equilibrium contract. The contract $(\hat{v}_s, \underline{v})$ would also give strictly positive profits. One then obtains a contradiction exactly as in the final step of Lemma A-3. QED

Lemma A-5 *If there are some lucky workers starting in Sector H, all workers who start in Sector H are lucky and receive the same contract.*

Proof of Lemma A-5: Fix an equilibrium with some lucky workers starting in Sector H. Observe first that any workers who start in Sector H must receive a contract with $v_f = \underline{v}$: otherwise, if $v_f > \underline{v}$, then from Corollary A-1 one can construct a Sector H contract giving strictly positive profits, contradicting Lemma A-4.

Second, note that any equilibrium contract must have zero profits, and a derivative of profits with respect to v_s that is weakly negative. From Lemma A-2 there exists at most one such contract with $v_f = \underline{v}$. Combined with the first observation, this completes the proof. QED

Lemma A-6 *Whenver g_L is sufficiently high, some workers must start in Sector L.*

Proof of Lemma A-6: We show that no equilibrium exists in which all workers start in Sector H. Suppose to the contrary that such an equilibrium exists.

We first show that for any price g_H, workers who start in Sector H work at least p_{SBH}, where $\gamma'(p_{SBH}) + p\gamma''(p_{SBH}) = g_H$. To see this, we show that if workers work less than p_{SBH}, it is possible to strictly increase both worker utility and firm profits by increasing v_s. Evaluating, an increase in v_s strictly increases worker utility, and changes profits by $\frac{1}{\gamma''(p)}(g_H - (w(v_s) - w(v_f))) - pw'(v_s)$, where $w(v_s) - w(v_f) \leq v_s - v_f = \gamma'(p)$ and $w'(v_s) \leq 1$. So the change in profits is at least $\frac{1}{\gamma''(p)}(g_H - \gamma'(p)) - p$, which (by Assumption 1) is strictly positive if $p < p_{SBH}$.

Hence the price of good H must be less than some upper bound, $\bar{g}_H$ say: this follows since prices above $\bar{g}_H$ would require very little output, but by the above argument output is bounded away from zero.

Let demand g_L be high enough that the w_L function is everywhere strictly below the w_H function (for $g_H = \bar{g}_H$). But then all old workers must work in Sector L. But by supposition all workers start in Sector H. This contradicts Lemma 3, completing the proof. QED

B Proofs of results stated in main text

Proof of Lemma 2: Much of the proof is very straightforward, and omitted. Here, we show that w_i is strictly convex for $v < v_{FB_i}$ (above v_{FBi} it is linear with slope 1). Straightforward substitution implies $w_i(v) = -p(v)(g_i - \gamma'(p(v))) - k_i$ where $p(v)\gamma'(p(v)) - \gamma'(p(v)) = v$. So $p'(v) = (p(v)\gamma''(p(v)))^{-1}$, which by Assumption 1 implies that $p''(v) < 0$. Substituting into $w'(v)$ this implies $w_i'(v) = -p'(v)(g_i - \gamma'(p(v))) + 1$ and so $w_i''(v) = -p''(v)(g_i - \gamma'(p(v))) + p'(v)^2\gamma''(p(v))$. This is strictly positive since for $v < v_{FBi}$ we know $p(v) < p_{FBi}$ and so $\gamma'(p(v)) < \gamma'(p_{FBi}) = g_i$. QED

Proof of Lemma 2: This is immediate over ranges in which the continuation level is provided via assignment to the same sector, i.e., $[\underline{v}, v_L]$ and $[v_{SBH}, \infty)$, where v_L is as defined in the main text. To complete the proof, note that the worker exerts more effort under the contract delivering utility v_{SBH} in Sector H than under the contract delivering v_L in Sector L, since $v_{SBH} \geq v_L$ and he receives no fixed wage in the Sector H contract. QED

Proof of Lemma 3: Suppose instead that a young worker employed in Sector H is sometimes demoted to Sector L after success. Let (v_s, v_f) be the equilibrium contract, and p_y the associated effort choice for the young worker.

Note first that we can assume without loss that the worker is *always* demoted, as follows. After success, the worker is given a (possibly degenerate) lottery over different continuation utilities and task assignments, all of which have the same cost for the firm. All continuation levels in this lottery must be the same, since otherwise the firm could strictly raise profits by assigning the highest continuation utility with probability one. By hypothesis, one element of this lottery must be associated with demotion to task L. So there is no loss in assuming that the lottery places probability one on demotion after success. After failure, we must have $v_f < v_s$ (since otherwise $p_y = 0$ and the firm loses money), and so the worker is also always demoted after failure.

We first argue that the equilibrium contract for the young worker has $v_f = \underline{v}$. To see this, suppose instead that $v_f > \underline{v}$, and consider the perturbation in which v_s is raised by dv_s and v_f is lowered by $\frac{p_y}{1-p_y}dv_s$. By the envelope theorem, the worker's utility is unchanged; and his effort changes by $dp = \frac{1}{\gamma''(p_y)}\left(1 + \frac{p_y}{1-p_y}\right)dv_s$. Denoting the left- and right-sided derivatives of w by w^- and w^+, the firm's profits are changed by

$$\left(\frac{g_H - w(v_s) + w(v_f)}{\gamma''(p_y)(1-p_y)} - p_y w^+(v_s) + p_y w^-(v_f)\right)dv_s.$$

Note than $w^{-}(v_f) = 1$ since by supposition the worker is in sector 1 after failure; and $w^{+}(v_s) \leq 1$. So the above expression is strictly positive, since $k_H > 0$ implies that $g_H - w(v_s) > 0$ in order for the firm to make non-negative profits. But then the contract perturbation strictly raises firm profits while leaving worker utility unchanged.

Since by supposition the old worker is in Sector L, $\underline{v} = v_{FBL}$ and $w(v_s) = v_s - v_{FBL} = \gamma'(p_y)$. So p_y is the largest solution to $p_y(g_H - \gamma'(p_y)) - k_H = 0$; note that $p_y < p_{FBH}$. The young worker's associated utility is $p_y\gamma'(p_y) - \gamma(p_y) + \underline{v}$. This argument also implies that a one-period contract in Sector H with a cash payment of $v_s - v_{FBL}$ after success, and nothing after failure, gives the firm zero profits and the worker $p_y\gamma'(p_y) - \gamma(p_y)$ in utility. So $w_H(p_y\gamma'(p_y) - \gamma(p_y)) \leq 0$, and since $p_y < p_{FBH}$, $w'_H(p_y\gamma'(p_y) - \gamma(p_y)) < 1$.

One possible contract that could be offered to a young worker in Sector L is a continuation utility of $\underline{v} + g_L$ after success and $\underline{v}$ after failure. The contract gives the firm non-negative profits, and since the worker chooses effort p_{FBL}, his utility is $v_{FBL} + \underline{v}$. We claim that $p_y\gamma'(p_y) - \gamma(p_y) \geq v_{FBL}$. If instead $p_y\gamma'(p_y) - \gamma(p_y) < v_{FBL}$, this contract strictly raises the worker's utility while giving the firm zero-profits. Then the further perturbation in which the young worker is employed in Sector L with continuation utilities $v_s = \underline{v} + g_L - \varepsilon$ and $v_f = \underline{v}$ utility strictly raises the worker's utility while giving the firm strictly positive profits, contradicting the equilibrium assumption and establishing the claim.

The above arguments imply that either $w_H(\tilde{v}) < w_L(\tilde{v})$ for all $\tilde{v} > \underline{v}$, or that $p_y\gamma'(p_y) - \gamma(p_y) > \underline{v}$ and $w_H(p_y\gamma'(p_y) - \gamma(p_y)) = 0$. In either case, it follows that any continuation utility strictly above $\underline{v}$ is delivered using a strictly positive probability of assignment to task H. Since $v_s > \underline{v}$, this provides a contradiction and completes the proof. QED

Proof of Proposition 2: Suppose such an equilibrium exists. Let $(\hat{v}_s, \hat{v}_f)$ be the contract given to the worker in Sector L. From Lemma A-1, $w(\hat{v}_f) = 0$ and $w(\hat{v}_s) = g_L$. Moreover, from the proof of Lemma A-3, we know $w^{-}(\hat{v}_s) = 0$, where w^{-} denotes the left-hand derivative. So $\hat{v}_s \leq \arg\min_{\tilde{v}} w_H(\tilde{v})$, and $\hat{v}_f = \underline{v}$. Let (v_s, v_f) be the contract given to a worker starting in Sector H. From Lemma 3, $v_s \geq \arg\min_{\tilde{v}} w_H(\tilde{v})$. Since certainly $v_f \geq \underline{v}$, his utility is weakly higher than the worker starting in Sector L, a contradiction. QED

Proof of Proposition 3:

We establish the proof via a series of Lemmas:

Lemma A-7 *As $k_H \to \infty$, we have that the price $g_H \to \infty$, that the bonus Δ given after success to an old worker employed in Sector H goes to infinity, that the effort p exerted by the worker goes to $\bar{p}$, and that the utility of the worker goes to infinity.*

Proof of Lemma A-7: We claim first that $g_H > k_H$, and so the price $g_H \to \infty$ as $k_H \to \infty$. This is immediate from the zero-profit condition if young workers are employed in Sector H. If instead only old workers are employed in Sector H, note that $w_H(v)$ can be written as $w_H(v) = -(pg_H - (v + \gamma(p)) - k_H)$. For old workers to be employed in Sector H we must have $w_H(v) \leq w_L(v)$, and certainly $w_L(v) \leq v$. Hence $k_H + \gamma(p) \leq pg_H$, implying $g_H > k_H$.

To prove the rest of the Lemma, note that if an old worker is employed in Sector H, his utility is at least $\arg\min_{\tilde{v}} w_H(\tilde{v}; g)$. Evaluating, this equals $p_{SBH}\gamma'(p_{SBH}) - \gamma(p_{SBH})$, where $\gamma'(p_{SBH}) + p_{SBH}\gamma''(p_{SBH}) = g_H$. By Assumption 1, $p_{SBH} \to \bar{p}$ as $g_H \to \infty$. We also have $\Delta \to \infty$ as $g_H \to \infty$, where $\Delta = \gamma'(p_{SBH})$ is the bonus used to induce effort p_{SBH}. Finally, we have $\arg\min_{\tilde{v}} w_H(\tilde{v}; g) \to \infty$ as $g_H \to \infty$. This can be seen as follows: Because $\gamma'(0) = 0$, and $\lim_{p \to 0} \gamma''(p) < \infty$ by assumption 1, there is a p such that $\bar{p} > p > 0$ and such that $\gamma(p) < \infty$. The utility from exerting effort p for the worker is $p\Delta - \gamma(p)$, which goes to infinity as Δ goes to infinity. Since this utility is a lower bound for the utility of the worker if he exerts optimal effort, we must have $\arg\min_{\tilde{v}} w_H(\tilde{v}; g) \to \infty$ as $g_H \to \infty$. **End Proof.**

Lemma A-8 *As $k_H \to \infty$, if a worker starts in Sector L, he must remain there.*

Proof of Lemma A-8: Since $g_H \to \infty$ as $k_H \to \infty$ from Lemma A-7, we must have that output in Sector H approaches 0, or else demand cannot equal supply. We next claim that this in turn implies that the fraction of workers in Sector H approaches 0, and hence the fraction of workers in Sector L approaches 1. To see this, note that as $g_H \to \infty$, the effort of any old worker employed in Sector H approaches $\bar{p}$. So the only way in which the number of workers in Sector H can remain bounded away from 0 is consequently if some young workers are employed in Sector H, and their probability of remaining in Sector H approaches 0. By Lemma 3, successful workers remain in Sector H, and so the success rate in the first period must approach 0. A small increase in v_s affects profits by $\frac{dp}{dv_s}\left(g_H - w\left(v_s\right) + w\left(v_f\right)\right) - pw'\left(v_s\right)$, which equals $\frac{dp}{dv_s}\left(\frac{k_H + w\left(v_f\right)}{p}\right) - pw'\left(v_s\right)$ from the zero-profit condition. But if p approaches 0 as k_H grows large this is strictly positive for k_H large, since $w'\left(v_H\right)$ is bounded above by 1. But then the firm could change its contract to strictly increase both profits and worker utility, giving a contradiction. Hence, the fraction of workers in Sector H must approach 0.

Next, suppose contrary to the claim in the Lemma that some young workers employed in Sector L get promoted to Sector H. Note that no firm pays a young worker in Sector L strictly more than g_L, since otherwise the firm would lose money. Since $g_L < \zeta_L(0)$, and since $\zeta_L(0)$ is bounded, g_L is bounded.[6]. So as $k_H \to \infty$, for promotion to be a possibility there must exist a corresponding sequence of continuation utilities v such that $w_H(v)$ remains bounded above by $\zeta_L(0)$, and such that $v \geq \arg\min_{\tilde{v}} w_H(\tilde{v})$. Since $w_H(v) = -\left(pg_H - p\Delta - k_H\right)$, (where Δ is the bonus payment used to deliver utility v), and since we know from Lemma A-7 that $p \to \bar{p}$ and $\Delta \to \infty$ as $k_H \to \infty$, we must have that $pg_H - k_H \to \infty$ as $k_H \to \infty$, since otherwise $w_H(v)$ cannot remain bounded from above. Finally, note that for k_H large enough $\underline{v} = v_{FBL}$, since otherwise all old workers would be employed in Sector H. Hence $\underline{v}$ remains bounded as $k_H \to \infty$.

There are two possible cases to consider. First, suppose the young worker's utility in Sector L remains bounded as $k_H \to \infty$. Consider employing this worker in Sector H instead using the contract $v_s = v$, $v_f = \underline{v}$. As $k_H \to \infty$, v grows without bound from Lemma A-7. Since $v - \underline{v}$ goes to infinity, p in the first period goes to $\bar{p} > 0$. Profits are given by $p\left(g_H - w_H(v)\right) - k_H$, which goes to infinity as $k_H \to \infty$ since $w_H(v)$ is bounded. Hence, utility for the worker is increased and the firm earns positive profits, which is a contradiction to the equilibrium assumption.

Second, suppose the young worker's utility in Sector L grows unboundedly. Write

[6] It is worth noting that even if $\zeta_L(y_L) \to \infty$ as $y_L \to 0$, it is still possible to establish an upper bound for g_L as $k_H \to \infty$. Suppose to the contrary that there is no such bound. Then output in sector L must approach 0. This is only possible if success probabilities approach 0. But this is inconsistent with g_L growing arbitrarily large, since the success probability of any old worker employed in sector L is p_{FBL}, and this approaches $\bar{p}$ as $g_H \to \infty$; and moreover, as $k_H \to \infty$ almost all old workers must be employed in sector 1.

$v_{SBH} = \arg\min_{\tilde{v}} w_H(\tilde{v}; g)$. We know $w_H(v_{SBH}) \leq g_L$, since otherwise a young worker who starts in Sector L can never end up in Sector H. If $w_H(v_{SBH}) = g_L$, then the young worker starting in Sector L is employed on a contract with $w(v_s) = g_L$ and $v_f = \underline{v}$. Then employing a young worker in Sector H using the contract $v_s = v_{SBH}$ and $v_f = \underline{v}$ weakly increases the worker's utility, and since it induces effort $p \to \bar{p}$ as $k_H \to \infty$, it produces strictly positive profits for all k_H large enough. This contradicts the equilibrium assumption.

If instead $w_H(v_{SBH}) < g_L$, then the young worker must always be promoted to Sector H after success. Since as $k_H \to \infty$ almost all workers must be in Sector L, this implies both that the young worker's success probability converges to 0, and that the worker's probability of remaining in Sector L after failure converges to 1. But the latter implication in turn implies that $v_s - v_f$ explodes as $k_H \to \infty$, implying the success probability converges to $\bar{p}$, so this cannot be the case.

Hence, we must have that $w_H(v) > \zeta_L(0)$ for all v as k_H large, which in turn implies that for k_H large enough, any young worker who starts in Sector L remains in Sector L. **End Proof.**

From Lemma A-8, some young workers must start in Sector H, since otherwise there is no Sector H output. From Lemma 3, these workers remain in Sector H if they succeed. From Lemma A-7, as $k_H \to \infty$, utility after success grows arbitrarily large, so the utility of a young worker starting in Sector H grows arbitrarily large. On the other hand, the utility of a young worker starting in Sector L is bounded above (since he remains in Sector L, and the price g_L is bounded above). So there are young workers who start in Sector H with utility strictly higher than those starting in Sector L. QED

Proof of Proposition 5: From Proposition 3, we know that a young worker starting in Sector H remains in Sector H after success. There are two cases to consider. The first case, in which $v_s < v_{FBH}$, is handled in the main text. Here, we deal with the second case in which $v_s \geq v_{FBH}$. In this case, if the worker succeeds when young, he then exerts effort p_{FBH} when old.

Let p denote his effort when young. For any effort level $\tilde{p}$, let $S(\tilde{p}) = \tilde{p}g_H - \gamma(\tilde{p}) - k_H$ be total one-period surplus (i.e., the sum of firm profits and worker utility) associated with effort

$\tilde{p}$. Note that $v_s - w(v_s) = S(p_{FBH})$. Since the worker is lucky, $w(v_f) = 0$ by Proposition 4. Hence firm profits are

$$S(p) + \gamma(p) + p(S(p_{FBH}) - v_s).$$

Denote by $U(p)$ the one-period utility for a worker from being induced to work p by receiving a bonus $\gamma'(p)$ after success:

$$U(p) \equiv p\gamma'(p) - \gamma(p).$$

Substituting in for $U(\cdot)$ and $\gamma'(p) = v_s - v_f$, firm profits equal

$$S(p) - U(p) + p(S(p_{FBH}) - v_f).$$

The derivative of profits with respect to p (and hence v_s) must be zero: if it were strictly positive, it would be possible to both strictly increase worker utility and firm profits, while if it were strictly negative, lucky workers could not exist in equilibrium. So

$$0 = S'(p) - U'(p) + S(p_{FBH}) - v_f.$$

To complete the proof, suppose that, contrary to the claimed result, $p \geq p_{FBH}$. We know both $S'(p) \leq 0$ and $U'(p) \geq U'(p_{FBH})$ (since u is convex in p by Assumption 1), implying

$$0 \leq -U'(p_{FBH}) + S(p_{FBH}) - v_f. \tag{A-1}$$

Finally, note that $S(p_{FBH}) = p_{FBH}g_H - \gamma(p_{FBH}) - k_H \leq u(p_{FBH})$; and $U(0) = 0$ together with the convexity of U in p (Assumption 1) implies $U(p_{FBH}) \leq p_{FBH}U'(p_{FBH}) < U'(p_{FBH})$. Hence the righthand side of (A-1) is strictly negative, giving a contradiction and completing the proof that the worker exerts more effort.

Finally, he must be paid more, as follows: in the case under consideration, $v_s = v_{FBH} + \Delta_1$ and $v_f = \underline{v}$. Since $v_{FBH} \geq \underline{v}$, and the worker's first period effort is less than the first-best level p_{FBH}, it follows that the date 1 bonus Δ_1 is strictly less than $\gamma'(p_{FBH})$, which is the date 2 bonus. QED

Proof of Proposition 6: Fix any $k_H < g_L$. Suppose that, contrary to the claimed result, there exists an equilibrium in which workers who start in Sector L remain in Sector L. Let (v_s, v_f) be the contract given to these workers.

First, we argue that $w(v_s) = g_L$ and $w(v_f) = 0$. Since $w(v_s) > g_L$ implies the firm loses money, to show $w(v_s) = g_L$ it is sufficient to rule out the case $w(v_s) < g_L$. In this case, $w(v_f) > 0$, since otherwise the firm is making strictly positive profits. Consider slightly increasing v_s by dv_s and slightly decreasing v_f by dv_f while holding worker utility unchanged, i.e., $pdv_s + (1-p) dv_f = 0$, where p is the worker's effort choice under the original contract (this is the envelope theorem). The change in firm profits is

$$(g_L - (w(v_s) - w(v_f))) dp - pw'(v_s) dv_s - (1-p) w'(v_f) dv_f.$$

Since by supposition the worker remains in Sector L, $w(v_s) = w_L(v_s)$. From the construction of w, it follows that $w(v) = w_L(v_s)$ for all $v \in [\underline{v}, v_s]$, and hence $w'(v) = 1$ over this interval. Hence $pw'(v_s) dv_s + (1-p) w'(v_f) dv_f = 0$. Since the contract change induces more effort, i.e., $dp > 0$, it follows that the change in profits is strictly positive. Hence this case cannot arise. Finally, note that $w(v_s) = g_L$ implies $w(v_f) = 0$ by the zero-profit equilibrium condition.

Since $g_L > k_H$, the basic properties of w_L and w_H (see Lemma 1) imply that $\underline{v} = v_{FBL}$ and $w(v) = w_L(v) = v - v_{FBL}$ for all $v \geq \underline{v}$. In words, any contract simply offers an old worker the right to work in Sector L under the first-best contract terms (payment g_L in the case of success), along with a possible cash payment (in the case of continution utility strictly above v_{FBL}). Moreover, by strict convexity of w_H for values v such that $w_H(v) \leq k_H$, note that there is no Sector H one-period contract that delivers weakly positive profits and worker utility weakly above $\underline{v} = v_{FBL}$.

By the Inada condition for good H demand, some workers start in Sector H. From above, we can write the continuation utilities for their contract, after success and failure respectively, as $v_{FBL} + w + \Delta$ and $v_{FBL} + w$, with corresponding firm costs $w + \Delta$ and w. This is effectively just a one-period contract. But we have just argued that the firm loses money on any one-period Sector H contract with utility above v_{FBL}. So these workers have a liketime utility strictly below $2v_{FBL}$. But the workers who start in Sector L receive the contract $v_s = v_{FBL} + g_L$, $v_f = v_{FBL}$, with a correpsonding lifetime utility of exactly $2v_{FBL}$. This contradicts Proposition 2, completing the proof. QED

Proof of Proposition 7: This is proved in Lemma A-8 as part of the proof of the existence of lucky young workers (Proposition 3). QED

Proof of Proposition 8: We study the relaxed problem in which the worker's period 2 time constraint is disregarded. We will show that the menial task is never used in period 2 in the solution to the relaxed problem. Consequently, the solution to the relaxed problem satisfies the worker's period 2 time constraint, and coincides with the solution to the full problem.

Under the conditions stated, $\underline{v} = v_{FBL}$ and

$$w(v) = \begin{cases} w_L(v) = v - v_{FBL} & \text{if } v \in [v_{FBL}, v_{FBL} + w_H(v_{SBH})] \\ w_H(v_{SBH}) & \text{if } v \in [v_{FBL} + w_H(v_{SBH}), v_{SBH}] \\ w_H(v) & \text{if } v \geq v_{SBH} \end{cases}.$$

Moreover, $g_L < w_H(v_{SBH})$; this is why Sector L workers are never promoted.

When the menial task is introduced, the new cost function $w^*(v)$ is now given by

$$w^*(v) = \min_{m \geq 0} w(v+m) - m\varepsilon. \tag{A-2}$$

This follows since if we want to give the worker utility v while he is assigned m of the menial task, he has to earn a utility $v+m$ from his work on the non-menial task. The cheapest way to deliver this utility is via the original w function.

From (A-2) and the shape of $w(v)$, it follows that $w^*(v)$ takes the following form. Define $\hat{v} > v_{SBH}$ by $w'(\hat{v}) = \varepsilon$, and $\check{v}$ by $\check{v} - v_{FBL} = w^*(\hat{v}) - \varepsilon(\hat{v} - \check{v})$. Then

$$w^*(v) = \begin{cases} w(v) & \text{if } v \in [v_{FBL}, \check{v}] \\ w(\check{v}) + \varepsilon(v - \check{v}) & \text{if } v \in (\check{v}, \hat{v}) \\ w(v) & \text{if } v \geq \hat{v} \end{cases}.$$

Note that the menial task is only used for $v \in (\check{v}, \hat{v})$. Note also that as $\varepsilon \to 0$, $\hat{v} \to v_{SBH}$ and $\check{v} \to v_{FBL} + w_H(v_{SBH})$.

Case: Workers starting in sector H

From Lemma A-5, for the non-menial task case there is a unique contract, (v_s, v_f) say, that gives zero-profits. Any other contract $(\tilde{v}_s, \tilde{v}_f)$ gives strictly negative profits. Recall that $v_f = v_{FBL}$. It is straightforward to show that $v_s > v_{SBH}$.

Consequently, for all $\alpha > 0$ sufficiently small, there exists some $\delta(\alpha) > 0$ such that losses of at least α are produced by any contract $(\tilde{v}_s, \tilde{v}_f)$ with $\tilde{v}_s \notin (v_s - \delta(\alpha), v_s + \delta(\alpha))$ and/or $\tilde{v}_f \notin (v_f - \delta(\alpha), v_f + \delta(\alpha))$. Moreover, $\delta(\alpha) \to 0$ as $\alpha \to 0$.

Fix α sufficiently small such that $v_s > v_{SBH} + 2\delta(\alpha)$, $2\delta(\alpha) < w_H(v_{SBH})$, and such that $\delta(\alpha)$ is less than the utility difference between the young Sector H and Sector L workers.

Next, consider how the contract changes when menial tasks are possible. Given α, choose $\varepsilon \in (0, \alpha)$ small enough that $\check{v} > v_{FBL} + w_H(v_{SBH}) - \delta(\alpha)$ and $\hat{v} < v_{SBH} + \delta(\alpha)$.

Since the direct profits from menial tasks in period 1 are bounded above by ε, and $\varepsilon < \alpha$, it follows that any equilibrium contract $\left(v_s^*, v_f^*\right)$ with menial tasks must have $v_s^* \in (v_s - \delta(\alpha), v_s + \delta(\alpha))$ and $v_f^* \in (v_f - \delta(\alpha), v_f + \delta(\alpha))$. Hence $v_s^* > \hat{v}$ and $v_f^* < \hat{v}$, implying that the menial task is never used in period 2.

Finally, in period 1 it is optimal to have the worker do the menial task until either his time constraint binds, or his utility is reduced to the utility of Sector L workers.

Case: Workers starting in sector L

For the non-menial task case, the equilibrium Sector L contract is $(v_s, v_f) = (v_{FBL} + g_L, v_{FBL})$, as follows. Recall we are examining the case in which Sector L workers remain in Sector L in any equilibrium contract. Under the constraint that a worker remains in Sector L, the contract stated maximizes social surplus (i.e., firm profits plus worker utility) while giving the firm zero-profits. Moreover, raising v_s would give the firm strictly negative profits; while any other change to the contract would move the agent's effort decisions away from the first-best, strictly reducing total social surplus — and hence either worker utility or firm profits. Worker utility under the contract is $2v_{FBL}$.

Next, consider how the contract changes when menial tasks are possible. The contract must still deliver utility of at least $2v_{FBL}$ to the worker. By an exactly parallel argument to the Sector H case, it follows that for all $\varepsilon > 0$ sufficiently small, an equilibrium menial task contract is close to the equilibrium contract without menial tasks, and that no menial task is assigned in period 2. In particular, an equilibrium menial task contract has $v_s, v_f < \check{v}$, and the worker remains in Sector L.

Finally, since an equilibrium menial task contract must deliver utility at least $2v_{FBL}$, and the worker remains in Sector L, and the menial task is socially inefficient, it follows that the equilibrium menial task contract must be $(v_{FBL} + g_L, v_{FBL})$, and no menial task is assigned in period 1. QED

Proof of Proposition 9:

We start with a couple of preliminary lemmas. Notationally, write $p_2^{\omega\psi}$ for a lucky worker's effort when old if he succeed when young: notice that this depends on the state the worker started in, ω, and also the current state, ψ.

Lemma A-9 *Suppose* $g_H^G > g_H^B$. *For* $\omega = G, B$, $p_2^{\omega G} > p_2^{\omega B}$ *and* $g_H^G - \gamma'\left(p_2^{\omega G}\right) > g_H^B - \gamma'\left(p_2^{\omega B}\right)$.

Proof of Lemma A-9: Because the continuation utilities v_s^{ω} are delivered in a cost-minimizing way,

$$w_H^{\omega\prime}\left(v_s^G\right) = w_H'\left(v_s^{\omega G}\right) = w_H'\left(v_s^{\omega B}\right).$$

Consequently, either $p_2^{\omega G} = p_{FBH}^G$ and $p_2^{\omega B} = p_{FBH}^B$ and the result is immediate; or $p_2^{\omega G} < p_{FBH}^G$ and $p_2^{\omega B} < p_{FBH}^B$. In the latter case,

$$\begin{aligned} w_H'\left(v_s^{\omega G}\right) &= 1 - \frac{g_H^G - \gamma'\left(p_2^{\omega G}\right)}{p_2^{\omega G}\gamma''\left(p_2^{\omega G}\right)} \\ w_H'\left(v_s^{\omega B}\right) &= 1 - \frac{g_H^B - \gamma'\left(p_2^{\omega B}\right)}{p_2^{\omega B}\gamma''\left(p_2^{\omega B}\right)}. \end{aligned}$$

Consequently, $p_2^{\omega G} \leq p_2^{\omega B}$ would contradict $w_H'\left(v_s^{\omega G}\right) = w_H'\left(v_s^{\omega B}\right)$, and so $p_2^{\omega G} > p_2^{\omega B}$. Given $p_2^{\omega G} > p_2^{\omega B}$, it follows that $g_H^G - \gamma'\left(p_2^{\omega G}\right) > g_H^B - \gamma'\left(p_2^{\omega B}\right)$. QED

Lemma A-10 *Suppose* $g_H^G > g_H^B$ *and* $w^{B\prime}\left(v_s^B\right) < 1$. *Then* $w^G\left(v_s^G\right) \geq w^B\left(v_s^B\right)$ *implies* $w^{G\prime}\left(v_s^G\right) > w^{B\prime}\left(v_s^B\right)$.

Proof of Lemma A-10: If $w^{G\prime}\left(v_s^G\right)=1$ the result is immediate. For the remainder of the proof, assume $w^{G\prime}\left(v_s^G\right)<1$. The inequality $w^G\left(v_s^G\right)\geq w^B\left(v_s^B\right)$ is equivalent to

$$\begin{aligned} & \mu^{GG}p_2^{GG}\left(g_H^G-\gamma'\left(p_2^{GG}\right)\right)+\mu^{GB}p_2^{GB}\left(g_H^B-\gamma'\left(p_2^{GB}\right)\right) \\ \leq\; & \mu^{BG}p_2^{BG}\left(g_H^G-\gamma'\left(p_2^{BG}\right)\right)+\mu^{BB}p_2^{BB}\left(g_H^B-\gamma'\left(p_2^{BB}\right)\right). \end{aligned}$$

We show that at least one of $p_2^{GG}>p_2^{BG}$ and $p_2^{GB}>p_2^{BB}$ must hold. Suppose to the contrary that $p_2^{GG}\leq p_2^{BG}$ and $p_2^{GB}\leq p_2^{BB}$. Then since profits are decreasing in effort (if they were increasing, both the firm and worker could be made better off),

$$\begin{aligned} p_2^{GG}\left(g_H^G-\gamma'\left(p_2^{GG}\right)\right) &\geq p_2^{BG}\left(g_H^G-\gamma'\left(p_2^{BG}\right)\right) \\ p_2^{GB}\left(g_H^B-\gamma'\left(p_2^{GB}\right)\right) &\geq p_2^{BB}\left(g_H^B-\gamma'\left(p_2^{BB}\right)\right). \end{aligned}$$

From Lemma A-9, $p_2^{GG}\left(g_H^G-\gamma'\left(p_2^{GG}\right)\right)>p_2^{GB}\left(g_H^B-\gamma'\left(p_2^{GB}\right)\right)$. But together these last three inequalities contradict $w^G\left(v_s^G\right)\geq w^B\left(v_s^B\right)$, implying that at least one of $p_2^{GG}>p_2^{BG}$ and $p_2^{GB}>p_2^{BB}$.

To complete the proof of the claim, note that because the continuation utilities v_s^ω are delivered in a cost-minimizing way, we know

$$\begin{aligned} w^{G\prime}\left(v_s^G\right) &= w'\left(v_s^{GG}\right)=w'\left(v_s^{GB}\right) \\ w^{B\prime}\left(v_s^B\right) &= w'\left(v_s^{BG}\right)=w'\left(v_s^{BB}\right). \end{aligned}$$

Because

$$\begin{aligned} w_H'\left(v_s^{\omega G}\right) &= 1-\frac{g_H^G-\gamma'\left(p_2^{\omega G}\right)}{p_2^{\omega G}\gamma''\left(p_2^{\omega G}\right)} \\ w_H'\left(v_s^{\omega B}\right) &= 1-\frac{g_H^B-\gamma'\left(p_2^{\omega B}\right)}{p_2^{\omega B}\gamma''\left(p_2^{\omega B}\right)}, \end{aligned}$$

then $p_2^{GG}>p_2^{BG}$ implies $w_H'\left(v_s^{GG}\right)>w_H'\left(v_s^{BG}\right)$ and $p_2^{GB}>p_2^{BB}$ implies $w_H'\left(v_s^{GB}\right)>w_H'\left(v_s^{BB}\right)$. This completes the proof. QED

Turning to the main proof or Proposition 9, it is immediate from $v_{FBL}^G>v_{FBL}^B$ that

$g_H^G > g_H^B$.

Suppose that, contrary to the claomed result, there is an equilibrium with $p_1^G \geq p_1^B$ in which old workers sometimes depart from the socially efficient amount of effort.

Note first that if $w^{B\prime}\left(v_s^B\right) = 1$, then since by supposition $p_1^G \geq p_1^B$, and also $v_f^G > v_f^B$, we must have $v_s^G > v_f^B$, implying $w^{G\prime}\left(v_s^G\right) = 1$. But then old workers always work the socially efficient amount. Hence we must have $w^{B\prime}\left(v_s^B\right) < 1$.

To obtain a contradiction, we show that if all equations other than the state G zero-profit equation hold, then state G profits are in fact strictly positive, contradicting the equilibrium assumption.

Suppose that all equations other than the state G zero-profit equation hold, and that state G profits are weakly negative. By supposition $p_1^G \geq p_1^B$, so $g_H^G - w^G\left(v_s^G\right) \leq g_H^B - w^B\left(v_s^B\right)$, implying $w^G\left(v_s^G\right) > w^B\left(v_s^B\right)$. Again since by supposition $p_1^G \geq p_1^B$, it also implies $w^{G\prime}\left(v_s^G\right) \leq w^{B\prime}\left(v_s^B\right)$. But this contradicts Lemma A-10, completing the proof.

Finally, we establish that in state G the continuation utilites are higher state-by-state.

In the case in which old workers always work the first-best, and $p_1^G = p_1^B$, we know the average continuation utility is higher, and the continuation utility can be delivered in either state: so there is certainly an equilibrium in which the continuation utility is higher state-by-state.

Next, consider the case in which $p_1^G < p_1^B$. The zero profit conditions imply

$$g_H^G - w^G\left(v_s^G\right) > g_H^B - w^B\left(v_s^B\right).$$

The maximal profit condition implies

$$\frac{\left(g_H^\omega - w^\omega\left(v_s^\omega\right)\right)}{\gamma''\left(p_1^\omega\right)p_1^\omega} = w^{\omega\prime}\left(v_s^\omega\right),$$

so since $\gamma''\left(p_1^G\right)p_1^G < \gamma''\left(p_1^B\right)p_1^B$ and $g_H^G - w^G\left(v_s^G\right) > g_H^B - w^B\left(v_s^B\right)$, we have $w^{G\prime}\left(v_s^G\right) > w^{B\prime}\left(v_s^B\right)$. This in turn implies $w'\left(v_s^{G\psi}\right) > w'\left(v_s^{B\psi}\right)$, or

$$\frac{g_H^\psi - \gamma'\left(p_2^{G\psi}\right)}{p_2^{G\psi}\gamma''\left(p_2^{G\psi}\right)} > \frac{g_H^\psi - \gamma'\left(p_2^{B\psi}\right)}{p_2^{B\psi}\gamma''\left(p_2^{B\psi}\right)},$$

which implies

$$p_2^{G\psi} > p_2^{B\psi}.$$

QED

C Constructive proof of existence of the talent scorned equilibrium

In the main text, we observe that if the equilibrium is such that all young workers start in Sector H, and remain in Sector H with strictly positive probability after failure, then the equilibrium is one in which "talent is scorned." Here, we demonstrate, by construction, that equilibria of this sort actually exist.

In detail, we show that an equilibrium with the following properties exists. All workers start in Sector H, and receive the same contract. They receive a contract that promises utility v_s if they succeed, v_f if they fail, and work p. If they succeed they work p_s in Sector H, while if they fail, with probability $1-\mu$ they work p_{FBL} in Sector L and receive utility v_{FBL}, and with probability μ they work p_{SBH} in Sector H and receive utility v_{SBH}. The price of output in Sector H is g_H.

The values v_s, p_s, v_f, p, μ, g_H must satisfy the following six conditions.

The first-period incentive constraint,

$$\gamma'(p) = v_s - v_f.$$

The "promise-keeping" constraint,

$$v_f = (1-\mu)\, v_{FBL} + \mu v_{SBH}.$$

The consistency of p_s with v_s,

$$v_s = p_s \gamma'(p_s) - \gamma(p_s) \text{ if } v_s < v_{FBH}, \text{ and } p_s = p_{FBH} \text{ if } v_s \geq v_{FBH}.$$

The market-clearing condition for Sector H output,

$$\zeta_H\left(\frac{1}{2}p + \frac{1}{2}\left(pp_s + (1-p)\,\mu p_{SBH}\right)\right) = g_H.$$

The randomization condition,

$$w_H\left(v_{SBH}\right) = 0.$$

The zero-profit condition,

$$p\left(g_H - w\left(v_S\right)\right) - (1-p)\,w\left(v_f\right) - k_H = 0.$$

In addition to satisfying these six conditions, we then need to check that the contract for young workers is indeed optimal, i.e., that there is no way for a firm to strictly increase the worker's utility while making zero profits.

Here is a specific numerical example. The exogenous parameters of the model are the cost of effort function γ, the demands g_L and $\zeta_H(\cdot)$, and the Sector H fixed cost k_H. We use $\gamma(p) = (1-p)\ln(1-p) + p$, implying $\gamma'(p) = -\ln(1-p)$ and $\gamma''(p) = \frac{1}{1-p}$; $g_L = 0.96$; $\zeta_H(y_H) = \frac{2.408}{y_H}$; and $k_H = 1.2$.

One can then check that the following values satisfy the six conditions stated above:

$v_s = 2.2322$

$p_s = 0.9447$

$v_f = 0.3511$

$p = 0.8476$

$\mu = 0.1491$

$g_H = 2.8958$

In performing this check, the following intermediate values are useful:

$v_{FBL} = 0.3429$

$v_{SBH} = 0.3979$

$v_{FBH} = 1.9510$

$p_{FBL} = 0.6171$

$p_{SBH} = 0.6490$

$p_{FBH} = 0.9447$

Figures 2 and 3 then show the worker utility available from different contracts in each of the two sectors. In both figures, the horizontal line is drawn at the utility level in the conjectured equilibrium. The other lines are then constructed by selecting an alternative value of $\tilde{v}_f$ that is at least as high as the minimum utility that can be threatened in the conjectured equilibrium, namely $(1-\mu)\,v_{FBL} + \mu v_{SBH}$. For each alternative $\tilde{v}_f$, $\tilde{v}_s$ is then chosen as high as possible while consistent with non-negative profits. The two figures correspond to the cases $(1-\mu)\,v_{FBL} + \mu v_{SBH} \leq \tilde{v}_f \leq v_{SBH}$ and $\tilde{v}_f \geq v_{SBH}$. As one can see, the worker's utility from any alternative contract is indeed lower than in the conjectured equilibrium, establishing that the conjectured equilibrium is indeed an equilibrium.

D Proof of Proposition 1 (equilibrium existence)

Recall that contracts with either v_f or v_s strictly below $\underline{v}\,(g)$ are infeasible, since at these utility levels a firm could make strictly positive profits while strictly raising an old worker's utility above the promised amount. In addition to $\underline{v}\,(g)$, define

$$\bar{v}\,(g) \equiv \max\left\{v : v \geq \underline{v}\,(g) \text{ such that } w\,(v) \leq 0\right\}.$$

In other words, $w\,(\cdot; g)$ is identically equal to zero between $\underline{v}\,(g)$ and $\bar{v}\,(g)$. Note that in many cases, $\underline{v}\,(g) = \bar{v}\,(g)$.

When $\underline{v}\,(g) < \bar{v}\,(g)$, it may be infeasible for firms to hold continuation utilities below $\bar{v}\,(g)$. The reason is that the continuation utility $\underline{v}\,(g)$ is the zero-profit continuation utility for Sector L, and the continuation utility $\bar{v}\,(g)$ is the zero-profit continuation utility for Sector H. So if a secondary labor market exists for old workers, an old worker's continuation utility must be at least $(1-\mu)\,\underline{v}\,(g) + \mu\bar{v}\,(g)$, where $\mu \in [0,1]$ indexes the relative preponderance of Sector H and Sector L firms in the secondary market for old workers.

Contract-task pairs (v, i) such that $\pi^i\,(v; g) < 0$ will never be observed in equilibrium. Moreover, for a given μ, if a contract-task pair (v, i) is such that there exists $(\tilde{v}, \tilde{\imath})$ such that $\pi^{\tilde{\imath}}\,(\tilde{v}; g) > 0$, $u\,(\tilde{v}) > u\,(v)$ and $\tilde{v}_s, \tilde{v}_f \geq (1-\mu)\,\underline{v}\,(g) + \mu\bar{v}\,(g)$, then (v, i) cannot be observed in equilibrium, since in this case another firm can make strictly positive profits by offering to strictly raise a worker's utility.

For any $\mu \in [0,1]$, define

$$\mathcal{E}(g;\mu) = \left\{ \begin{array}{l} (v,i) : v_f, v_s \geq (1-\mu)\underline{v}(g) + \mu\bar{v}(g), \pi^i(v;g) \geq 0, \\ \text{and } \nexists (\tilde{v},\tilde{\imath}) \text{ such that } \pi^{\tilde{\imath}}(\tilde{v};g) > 0, u(\tilde{v}) > u(v), \tilde{v}_f, \tilde{v}_s \geq (1-\mu)\underline{v}(g) + \mu\bar{v}(g) \end{array} \right\}.$$

Then let

$$\mathcal{E}(g) = \bigcup_{\mu\in[0,1]} \mathcal{E}(g;\mu).$$

The set $\mathcal{E}(g)$ is the set of possible equilibrium contracts. The basic outline of the proof of equilibrium existence is then as follows. First, we conjecture an output vector (y_L, y_H). For the goods market to clear, the output vector implies a price vector (g_L, g_H). The prices in turn imply a set of possible equilibrium contracts, $\mathcal{E}(g)$. The equilibrium contracts determine output. If output coincides with our initial conjecture, we have found an equilibrium. Formally, we define a correspondence mapping output vectors to output vectors, and use Kakutani's fixed point theorem to prove a fixed-point exists. The key step is Lemma A-13, which establishes upper hemi-continuity.

We give the details of this argument below. First, however, we explain why the fixed point $y = (y_L, y_H)$ is indeed an equilibrium. Write g for the corresponding prices.

The equilibrium potentially entails randomization of several different initial contracts. That is, when young workers initially enter the labor force, they are randomly assigned to one of M different contracts. For concreteness, note that since y is two-dimensional vector, Carathéodory's theorem implies that the randomization is over at most 3 different contract-task pairs. Formally, the randomization is over $\{(v^m, i^m) : m = 1, \ldots, M\}$, with probabilities $q^1, \ldots, q^M$. For each contract-task pair (v^m, i^m), there exists μ^m such that $(v^m, i^m) \in \mathcal{E}(g;\mu^m)$.

Fix m, and consider the workers who start with contract-task (v^m, i^m). If $\min\left\{v_f^m, v_s^m\right\} \geq \bar{v}(g)$ then continuation levels are delivered by retention within the firm: note that because $w(\tilde{v}) > 0$ for all $\tilde{v} > \bar{v}(g)$, workers with these continuation levels cannot be profitably hired away by competing employers. Next, consider the case of $v_f^m < \bar{v}(g)$ and/or $v_s^m < \bar{v}(g)$.

These continuation utilities are delivered via a lottery over assignment to task L with a low continuation utility, and assignment to task H with a high continuation utility. Because the cost of delivering continuation levels below $\bar{v}(g)$ is zero, we assume these continuation levels are delivered outside the original firm; in other words, there is a secondary labor market for old workers who started under contract-task (v^m, i^m), and are owed a continuation utility that has no cost to deliver it. Any worker initially assigned contract (v^m, i^m) is free to enter this secondary labor market; hence by construction, there is no contract-task $(\tilde{v}, \tilde{\imath})$ that can simultaneously deliver strictly positive profits and strictly improve a worker's utility over (v^m, i^m). Finally, note that the secondary labor markets for $m, \tilde{m}$ are separate.

The proof of the existence of a fixed point follows:

Lemma A-11 *Let $\{g^n\}$ be a sequence of prices such that $g^n \to g$ and $\underline{v}(g^n)$ converges. Then $\lim \underline{v}(g^n) \in [\underline{v}(g), \bar{v}(g)]$.*

Proof of Lemma A-11:

First, we show $\lim \underline{v}(g^n) \geq \underline{v}(g)$. Suppose to the contrary that $\lim \underline{v}(g^n) < \underline{v}(g)$. Then there exists $v \in (\lim \underline{v}(g^n), \underline{v}(g))$ such that either $w_L(v; g) < 0$ or $w_H(v; g) < 0$. By continuity of $w_i(\cdot; \tilde{g})$ in $\tilde{g}$, for all n sufficiently large, either $w_L(v; g^n) < 0$ or $w_H(v; g^n) < 0$. But this contradicts the definition of $\underline{v}(g^n)$.

Second, we show $\lim \underline{v}(g^n) \leq \bar{v}(g)$. Suppose to the contrary that $\lim \underline{v}(g^n) > \bar{v}(g)$. Then there exists $v \in (\bar{v}(g), \lim \underline{v}(g^n))$ such that $w(v; g) > 0$. So by the definition of w, for all $\tilde{v} \geq v$, $w_i(\tilde{v}; g) \geq w(v; g) > 0$ for $i = L, H$. Let $\check{v}$ be such that for all $\tilde{g}$ in the neighborhood of g, $w_i(\tilde{v}; \tilde{g}) > 0$ for $i = L, H$ and all $\tilde{v} \geq \check{v}$. So in particular, for all $\tilde{v} \in [v, \check{v}]$, $w_i(\tilde{v}; g) \geq w(v; g) > 0$ for $i = L, H$. By continuity of $w(\cdot; \tilde{g})$ in $\tilde{g}$, for all n sufficiently large, $w_i(\tilde{v}; g^n) > 0$ for for all $\tilde{v} \in [v, \check{v}]$. Moreover, for all n sufficiently large, $w_i(\tilde{v}; g^n) > 0$ for all $\tilde{v} \geq \check{v}$. Hence for all n sufficiently large, $w_i(\tilde{v}; g^n) > 0$ for all $\tilde{v} \geq v$, for $i = L, H$. But since $v < \lim \underline{v}(g^n)$, this contradicts the definition of $\underline{v}(g^n)$ for n sufficiently large. QED

Lemma A-12 *Let $\{g^n\}$ be a sequence of prices such that $g^n \to g$, $\underline{v}(g^n)$ and $\bar{v}(g^n)$ converge, and $\underline{v}(g^n) < \bar{v}(g^n)$. Then $\lim \underline{v}(g^n) = \underline{v}(g)$ and $\lim \bar{v}(g^n) = \bar{v}(g)$.*

Proof of Lemma A-12:

When $\underline{v}(g^n) < \bar{v}(g^n)$, it must be the case that $\underline{v}(g^n) = v_{FBL}$, and since v_{FBL} is continuous in g, it follows that $\lim \underline{v}(g^n) = \underline{v}(g)$. Also, we must have g_H such that $w_H(v)$ reaches a minimum at zero at $\underline{v}(g^n)$. Note that this uniquely defines g_H, and hence $\underline{v}(g^n)$ is fixed for all n at $\bar{v}(g)$, so it follows trivially that $\lim \bar{v}(g^n) = \bar{v}(g)$. QED

Lemma A-13 *The correspondence $\mathcal{E}$ is non-empty, compact valued, and upper hemi-continuous.*

Proof of Lemma A-13:

For any g, the set $\mathcal{E}(g)$ is non-empty since there always exists a contract that delivers non-negative profits in Sector L, and because $\mathcal{E}(g)$ certainly contains the contract-task pair that maximizes worker utility subject to non-negative firm profits.

For any given g, the set $\mathcal{E}(g)$ is bounded. It is closed, as follows. Let $\{(v^n, i^n)\}$ be a sequence of contract-task pairs lying in $\mathcal{E}(g)$, with limit (v, i). Suppose to the contrary that $(v, i) \notin \mathcal{E}(g)$. So in particular, $(v, i) \notin \mathcal{E}(g; \mu = 1)$, and there exists $(\tilde{v}, \tilde{\imath})$ such that $\pi^{\tilde{\imath}}(\tilde{v}; g) > 0$, $u(\tilde{v}) > u(v)$, and $\tilde{v}_f, \tilde{v}_s \geq \bar{v}(g)$. By continuity of u in the contract terms, it follows that there exists (v^n, i^n) such that $u(\tilde{v}) > u(v^n)$. But this contradicts $(v^n, i^n) \in \mathcal{E}(g; \mu)$ for all $\mu \in [0, 1]$.

We next show that $\mathcal{E}(\cdot)$ is an upper hemi-continuous correspondence. To do so, consider a sequence $g^n \to g$ with $(v^n, i^n) \in \mathcal{E}(g^n)$. Given Proposition 11.11 in Border, it suffices to show that there is a convergent subsequence of $\{(v^n, i^n)\}$ with limit in $\mathcal{E}(g)$.

By the Bolzano-Weierstrass theorem, there exists a subsequence of $\{(v^n, i^n)\}$ such that $\{(v^n, i^n)\}$, $\{\underline{v}(g^n)\}$ and $\{\bar{v}(g^n)\}$ are all convergent. Let (v, i) be the limit of $\{(v^n, i^n)\}$. Note that $\pi^i(v; g) \geq 0$. Suppose that $(v, i) \notin \mathcal{E}(g)$.

First, we consider the case in which for all n large enough, $\underline{v}(g^n) = \bar{v}(g^n)$. From Lemma A-11, let $\mu \in [0, 1]$ be such that $\lim \underline{v}(g^n) = (1 - \mu)\underline{v}(g) + \mu\bar{v}(g)$. Since in particular $(v, i) \notin \mathcal{E}(g; \mu)$, and $v_s, v_f \geq \lim \underline{v}(g^n)$, it follows that there exists $(\tilde{v}, \tilde{\imath})$ such that $\tilde{v}_s, \tilde{v}_f \geq \lim \underline{v}(g^n)$, $u(\tilde{v}) > u(v)$ and $\pi^{\tilde{\imath}}(\tilde{v}; g) > 0$. Hence there also exists $(\tilde{v}, \tilde{\imath})$ such that $\tilde{v}_s, \tilde{v}_f > \lim \underline{v}(g^n)$, $u(\tilde{v}) > u(v)$ and $\pi^{\tilde{\imath}}(\tilde{v}; g) > 0$. So for all n large enough, $\tilde{v}_s, \tilde{v}_f > \underline{v}(g^n)$, $u(\tilde{v}) > u(v)$ and $\pi^{\tilde{\imath}}(\tilde{v}; g^n) > 0$. But since $\underline{v}(g^n) = \bar{v}(g^n)$ for all n large enough, this implies that, for all $\tilde{\mu} \in [0, 1]$, $(v^n, i^n) \notin \mathcal{E}(g^n; \tilde{\mu})$, and hence $(v^n, i^n) \notin \mathcal{E}(g^n)$, a contradiction.

Second, we consider the alternate case in which there is no subsequence such that for all n large enough, $\underline{v}(g^n) = \bar{v}(g^n)$. This implies that there is a subsequence such that $\underline{v}(g^n) < \bar{v}(g^n)$. From Lemma A-12, $\lim \underline{v}(g^n) = \underline{v}(g)$ and $\lim \bar{v}(g^n) = \bar{v}(g)$.

There are two subcases.

In the first and easier subcase, $\min\{v_f, v_s\} \geq \bar{v}(g)$. Since $(v,i) \notin \mathcal{E}(g; \mu = 1)$, there exists $(\tilde{v}, \tilde{\imath})$ such that $\tilde{v}_s, \tilde{v}_f > \bar{v}(g)$, $u(\tilde{v}) > u(v)$ and $\pi^{\tilde{\imath}}(\tilde{v}; g) > 0$. So for all n large enough, $\tilde{v}_s, \tilde{v}_f > \bar{v}(g^n)$, $u(\tilde{v}) > u(v^n)$ and $\pi^{\tilde{\imath}}(\tilde{v}; g^n) > 0$. But then for all n large enough, for all $\tilde{\mu} \in [0,1]$, $(v^n, i^n) \notin \mathcal{E}(g^n; \tilde{\mu})$, and hence $(v^n, i^n) \notin \mathcal{E}(g^n)$, a contradiction.

In the second subcase, $\min\{v_f, v_s\} \in [\underline{v}(g), \bar{v}(g))$. Let μ be such that $\min\{v_f, v_s\} = (1-\mu)\underline{v}(g) + \mu\bar{v}(g)$. Since $(v,i) \notin \mathcal{E}(g;\mu)$, there exists $(\tilde{v}, \tilde{\imath})$ such that $\tilde{v}_s, \tilde{v}_f > (1-\mu)\underline{v}(g) + \mu\bar{v}(g)$, $u(\tilde{v}) > u(v)$ and $\pi^{\tilde{\imath}}(\tilde{v}; g) > 0$. So there exists $\varepsilon > 0$ such that $\tilde{v}_s, \tilde{v}_f > (1-\mu-\varepsilon)\underline{v}(g) + (\mu+\varepsilon)\bar{v}(g)$. For all n large enough, for all $\tilde{\mu} \in [0, \mu+\varepsilon]$, $\tilde{v}_s, \tilde{v}_f > (1-\tilde{\mu})\underline{v}(g) + \tilde{\mu}\bar{v}(g)$. So for all n large enough, for all $\tilde{\mu} \in [0, \mu+\varepsilon]$, $(v^n, i^n) \notin \mathcal{E}(g^n; \tilde{\mu})$. Moreover, for all n large enough, $\min\{v_f^n, v_s^n\} < \min_{\tilde{\mu} \in [\mu+\varepsilon, 1]} (1-\tilde{\mu})\underline{v}(g^n) + \tilde{\mu}\bar{v}(g^n)$, and so for all $\tilde{\mu} \in [\mu+\varepsilon, 1]$, $(v^n, i^n) \notin \mathcal{E}(g^n; \tilde{\mu})$. But then for all n large enough, for all $\tilde{\mu} \in [0,1]$, $(v^n, i^n) \notin \mathcal{E}(g^n; \tilde{\mu})$, and hence $(v^n, i^n) \notin \mathcal{E}(g^n)$, a contradiction.

QED

Lemma A-14 *Let $\alpha : E \to\to F, \beta : F \to\to G$ be upper hemi-continuous, α closed-valued, and $\beta(y)$ bounded for all $y \in F$. Then $\beta \circ \alpha : E \to\to G$ is upper hemi-continuous and compact-valued.*

Proof of Lemma A-14: Upper hemi-continuity is standard (see Proposition 11.23 of Border). We show that $\beta \circ \alpha$ is compact-valued. Given that $\beta(y)$ is bounded for all $y \in F$, it suffices to show that $\beta \circ \alpha$ is closed-valued. Fix $x \in E$, and consider any convergent sequence $\{z^n\} \subset \beta \circ \alpha(x)$, with limit z. For each n, there exists $y^n \in \alpha(x)$ such that $z^n \in \beta(y^n)$. By Bolzano-Weierstrass, y^n has a convergent subsequence. By upper hemi-continuity of β, $z \in \beta(\lim y^n)$. By closed-valuedness of $\alpha(x)$, $\lim y^n \in \alpha(x)$. Hence $z \in \beta \circ \alpha(x)$, completing the proof. QED

Lemma A-15 *For any continuation utility v, let $\mathcal{Y}^c(v)$ be the set of expected output pairs (i.e., output in Sector L and output in Sector H) that are associated with the cost-minimizing way of delivering v. Then $\mathcal{Y}^c$ is compact-valued and upper hemi-continuous.*

Proof of Lemma A-15: TO BE WRITTEN. STANDARD.

We write y_i for total output in sector i. As stated in the main text, we assume that the value of output g_i is decreasing in output y_i. Formally, $g_i = \zeta_i(y_i)$, where ζ_i is a strictly decreasing and continuous function.

Even if all workers work in sector i, and always succeed, total output is still just 1, and so we know $(y_L, y_H) \in [0,1]^2$. We assume that $\zeta_i(1) > 0$ for $i = L, H$. Finally, we assume $\zeta_L(0) < \infty$, while $\zeta_H(y_H) \to \infty$ as $y_H \to 0$. Note that the only reason we make the latter assumption is that we will focus below on economies in which the stakes k_H in task H are high, and we need to make sure this does not lead to zero production in Sector H.

Proof of Proposition 1:

To establish existence, we construct a correspondence that maps the set of possible output levels, $[0,1]^2$, into itself, and then apply Kakutani's fixed point theorem. We first define a correspondence on $[0,1] \times (0,1]$, and then extend it to cover $[0,1] \times [0,1]$.

For any $(y_L, y_H) \in [0,1] \times (0,1]$, the associated prices are $g = (g_L(y_L), g_H(y_H))$.

Given g, define $\mathcal{Y}(g)$ as the set of per-period expected output pairs produced by a worker given a contract-task pair $(v,i) \in \mathcal{E}(g)$ when young, i.e.,

$$\mathcal{Y}(g) = \bigcup_{(v_f, v_s, i) \in \mathcal{E}(g)} \left\{ \begin{array}{c} \frac{1}{2}\left((p,0) * \mathbf{1}_{(i=L)} + (0,p) * \mathbf{1}_{(i=H)} + p y_s + (1-p) y_f\right) \text{ such that} \\ \gamma'(p) = v_s - v_f,\ y_s \in \mathcal{Y}^c(v_s) \text{ and } y_f \in \mathcal{Y}^c(v_f) \end{array} \right\}.$$

In this expression, y_s and y_f are the vectors of output in the two sectors associated with delivering utilities v_s and v_f, respectively. It follows straightforwardly from Lemma A-14 that $\mathcal{Y}$ is upper hemi-continuous and compact-valued. It is also non-empty because $\mathcal{E}$ is. Define $\bar{\mathcal{Y}}(g)$ as the convex hull of $\mathcal{Y}$. The correspondence $\bar{\mathcal{Y}}$ is compact and convex valued, and by Proposition 11.29 of Border, it is upper hemi-continuous.

Consequently, $\bar{\mathcal{Y}}(g_L(y_L), g_H(y_H))$ defines a correspondence from $[0,1] \times (0,1]$ into $[0,1] \times$

$[0,1]$. For any y_L, as $y_H \to 0$ the price $g_H(y_H) \to \infty$, while $g_L(y_L)$ remains fixed, and so the set $\mathcal{Y}(g)$ converges to $\{(0, \bar{p})\}$, where $\bar{p}$ is the maximal attainable success probability. So defining $\bar{\mathcal{Y}}(g_L(y_L), g_H(0))$ as $\{(0, \bar{p})\}$ ensures upper hemi-continuity or the correspondence $\bar{\mathcal{Y}}$.

By Kakutani's fixed point theorem, $\bar{\mathcal{Y}}$ has a fixed point, y say. Let $g = (g_L(y_L), g_H(y_H))$ be the associated price. QED

E Figures

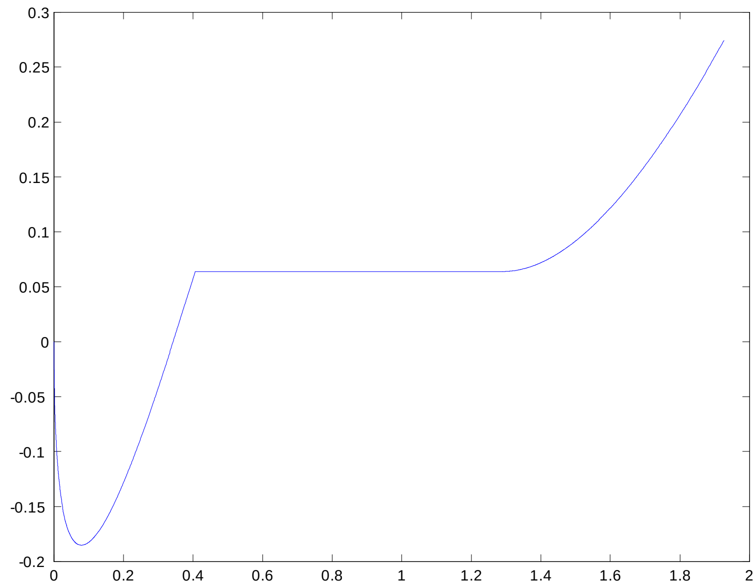

Figure 1: A representative w (cost) function. The horizontal axis determines the continuation utility, while the vertical axis determines the cost to the firm. The first convex portion corresponds to w_L, and the second convex portion corresponds to w_H. The horizontal line linking the two corresponds to delivering utility via a lottery between the two sectors L and H, with the firm indifferent between lottery outcomes.

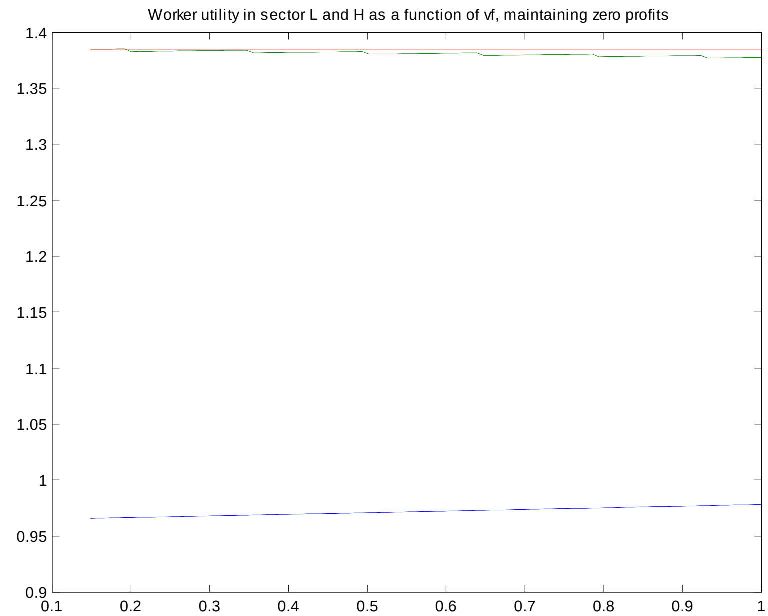

Figure 2: Confirmation that any alternative zero-profit contract gives the worker a lower utility than the conjectured equilibrium contract of Appendix C: see text for a fuller description.

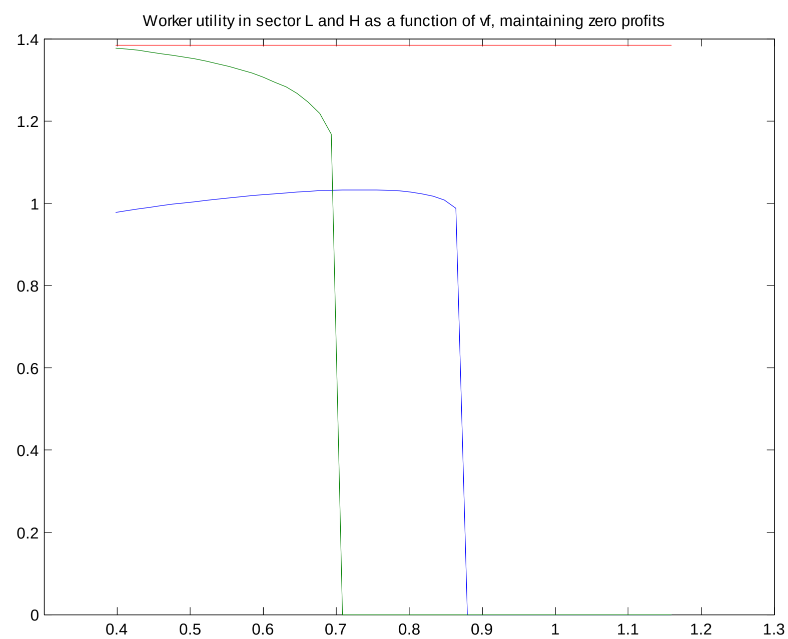

Figure 3: Confirmation that any alternative zero-profit contract gives the worker a lower utility than the conjectured equilibrium contract of Appendix C: see text for a fuller description.