Dynamic Regulation with Stochastic Costs: Signal Dampening, Experimentation and the Ratchet Effect

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Abstract

Regulators and the firms they regulate interact repeatedly. Over the course of these interactions, the regulator collects data that contains information about the firm's invariant private characteristics. This paper studies the case in which the regulator uses information gleaned from past cost observations when designing the current period's contract (that is, contracts are short-term). When costs are stochastic, the regulator's learning process is slowed compared to a deterministic setting. In the absence of additional messages, a separating contract no longer gives her complete information about the firm's type as soon as the first period cost is observed; thus, the second period game is one of asymmetric information.

Second period beliefs depend on the first period contract. In particular, the first period contract determines how much the regulator updates her prior beliefs for any given cost realization. By increasing the distance between first period cost targets ("experimenting"), the regulator increases expected second period welfare, and by decreasing this distance ("signal dampening"), she preserves the good firm's expected second period rent, thus reducing the first period transfer. Given reasonable assumptions on the distribution of noise, the overall effect of the first period contract is to reduce the flow of information; thus, the good agent's efforts are ratcheted up over time.

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JEL Classification: D8, C73, L5

1 Introduction

In regulated industries, firms are usually compensated by directly charging consumers the rates or prices allowed by regulatory boards, while in procurement, the government directly pays the firms they contract with for goods and services provided. Whether elected or appointed, most regulatory board members or procurement officials lack the time and expertise to fully understand a firm's production process and cost structure. Thus, it is natural to assume that firms have better information about expected costs than government officials. Further, firms that have intrinsically low costs, whether due to superior technology, managerial ability, or some other factors, benefit more from this informational advantage than high cost firms.

When designing a contract in a one-off interaction, this informational asymmetry poses a minimal challenge to the regulator. The regulator can design a contract that a firm with low intrinsic costs accepts, but that a firm with high intrinsic costs finds unacceptable. By accepting such a contract, the low cost firm reveals its private information to the regulator. When the regulator and the firm interact only once, this has no bearing on the firm's decision making.

Often, however, government officials and firms interact repeatedly. In repeated interactions, a firm with favorable private information has an increased incentive to keep information about its ability private, while the regulator has an increased incentive to learn about the firm's ability. By keeping information about its intrinsic cost private, the low cost firm enjoys the benefits associated with its informational advantage in the future as well as the present. To learn about the firm's ability, and increase the efficiency of future contracts, the regulator must compensate the low cost firm up-front for ceding its informational advantage. Thus, when the regulator cannot commit to ignore information it gathers about the firm when designing future contracts, the cost of asymmetric information is magnified.

This deadlock between the regulator and the firm is at the heart of the "ratchet effect." Informally, the ratchet effect arises in any multi-period principal-agent interaction in which the principal cannot commit to future incentive schemes and the agent has private information that, once revealed to the principal, results in more demanding incentive schemes in the future. Some of the earliest instances of this phenomenon are documented by Matthewson (1931). Matthewson documents that as skilled factory workers increase their productivity, management either cuts their piece rates or increases output quotas. In response, some workers learn to restrict their output.

More recently, Charness, Kuhn, and Villeval (2011) study the impact of competition on the ratchet effect when workers are compensated by piece rate contracts. Charness et al. use an experimental setting and find a "substantial and significant" ratchet effect in the absence of labor market competition. Once competition is introduced, the ratchet effect is subdued. Macartney (2016) finds evidence of ratchet effects in teachers' value-added incentive schemes.

This dilemma has also been the focus of a large and influential principal agent literature. Weitzman (1980) studies an infinite horizon model in which current output targets depend on both past targets and past output realizations. The ratchet effect arises in Weitzman's model not because the agent has private information about its productivity or talent, but because of the explicit link between present day output targets and past performance. Other principal-agent models in which the ratchet effect arises because of asymmetric information are Dillen and Lundholm (1996), in the case of optimal income taxation, Jeitschko and Mirman (2002) and Jeitschko, Mirman, and Salgueiro (2002), in which a principal hires an agent to produce a good, and Choi and Thum (2003) in the context of a corrupt government official.

In the context of regulation, Laffont and Tirole (1987) and Freixas, Guesnerie, and Tirole (1985) study dynamic interactions between regulator and firm when the firm's intrinsic cost may be either high or low. In these papers, the first period project cost, which the regulator uses to update her beliefs about the firm's type, depends only on the firm's intrinsic cost and its cost reducing effort. Thus, by designing a first period contact that specifies a different cost level for the low and high cost firm, the regulator learns fully about the firm's type after only one period. If the low cost firm accepts such a contract, it permanently loses the benefits associated with its informational advantage.

The regulator must compensate the low cost firm for its foregone rents in order to induce the low cost firm to accept such a contract. Thus, Laffont and Tirole (1987) and Freixas et al. (1985) find that unless the firm places very little weight on future contracts, it is not optimal for the low cost firm to reveal his private information. When the firm's type is drawn from a continuum, the prospect of learning about the firm is even more grim; the best the regulator can hope for is "much pooling" at the beginning of the relationship (see Laffont and Tirole

(1988)).

We examine a dynamic interaction between a regulator and regulated firm when project costs are stochastic. In this environment, the firm no longer has perfect control over project costs; the firm can affect the distribution of costs with its effort, but there remain factors outside of its control that determine the project's realized cost. It is well known from Laffont and Tirole (1986) and Laffont and Tirole (1993) that in a static setting, introducing additive noise to the cost function has no impact on incentives. In a dynamic setting, however, additive noise slows the principal's learning process when contracts depend only on some observable outcome. Therefore, the assumption that costs are affected by noise allows for a more nuanced study of the dynamics of regulation.

Because we study a dynamic environment in which contracts are based on a stochastic outcome, the works most closely related to the current paper are Jeitschko and Mirman (2002) (hereafter, JM) and Jeitschko, Mirman, and Salgueiro (2002) (hereafter, JMS). In both papers, the principal and the agent contract over the production of a good when the agent's output depends on a zero mean noise term. Contracts are based only on output, so the presence of noise implies that, even when the first period contract is separating, the principal is not fully informed about the firm's type at the beginning of the second period. Thus, the good agent enjoys a second period information rent.

Our analysis differs from these papers for two important reasons. First, unlike in JM/JMS, where the agent's type determines his disutility of effort, the functional form of the cost of effort is the same for each firm. However, firms differ in their baseline level of costs. Second, the regulator includes the firm's utility in her objective function, which impacts the rent-extraction versus efficiency tradeoff in each period. These two features of the model combine to have important implications for the dynamic (first period) contract.

The driving force behind the results of this paper is the tradeoff the regulator makes between acquiring better information and reducing the good firm's first period transfer. When costs are stochastic, the regulator's posterior beliefs depend on the first period contract. Designing the contract in a way that increases the accuracy of her information yields a higher expected second period welfare, but increases the low cost firm's temptation to mimic the high cost firm in the first period. The paper shows that the regulator balances these opposing effects when designing the first period contract by manipulating both the low and high cost firm's efforts away from the commitment optimum. Thus, studying dynamic regulation in a stochastic environment yields a more sophisticated understanding of the ratchet effect in which either type of firm may have their effort increased or decreased over the course of the relationship.

The net effect of these competing incentives is in general difficult to ascertain, but under certain natural assumptions, the regulator benefits from having an "arm's-length" relationship with the regulated firm. That is, in certain settings, the regulator finds it optimal to design the first period contract in a way that favors the protection of the low cost firm's second period rent over her desire to learn about the firm's type and improve second period welfare. When this is the case, the low cost firm exerts less than the first best effort in the first period, and has its effort increased over the course of the relationship.

2 Model

Consider a two period interaction between a welfare-maximizing regulator (she) and a regulated firm (he). In each period, the regulator offers the firm a contract to complete a project that has gross-benefit S. In return for completing the project each period, the regulator compensates the firm for the project's costs, c_t, and pays an additional transfer, $t_t(c_t)$, which is a function of project costs, in order to incentivize cost-reducing effort. The project's cost in each period depends on the firm's intrinsic cost parameter, β, its unobservable effort, e_t, and a homoskedastic, zero mean noise term, ε_t:

$$c_t = \beta - e_t + \varepsilon_t, \quad t = 1, 2. \tag{1}$$

The random variable ε_t is assumed to be distributed over the entire real line according to the distribution function $G(\varepsilon)$ with associated density $g(\varepsilon)$. The density satisfies the monotone likelihood ratio property. While the full support assumption is analytically convenient, it raises two issues that bear mention.

The first issue is that, for low enough first period cost realizations, the low cost firm's effort from mimicking the high cost type may be negative in the second period. Static models avoid this situation by assuming that the regulator's prior belief that the firm is the low cost type is small enough that this problem does not arise. However, in this dynamic-stochastic setting, the second period beliefs depend on the first period cost realization, and so the bulk of the analysis allows for negative efforts. Second, full support implies that negative cost realizations are possible. While unrealistic, the possibility of negative costs does not affect the analysis.

It is important to note that ε_t is unobservable both ex-ante and ex-post. Thus, while the regulator is able to observe total cost c_t in each period, she cannot determine the individual impacts of the firm's type, its effort, and noise. This captures the intuition that the firm does not have perfect control over costs; it can certainly affect the distribution of costs by exerting effort, but ultimately costs also depend on factors outside of the firm's control that cannot be perfectly measured. Another interpretation of noise is that of an "accounting error." Given the complexity of accounting rules, and constraints on her time, the regulator may not able to perfectly discern which costs should and shouldn't be reimbursed after observing the firm's income statement or other supporting documents.

The firm's type can be either $\underline{\beta}$ or $\bar{\beta}$, with $0 < \underline{\beta} < \bar{\beta}$, and remains constant over the course of the interaction. Throughout, type $\underline{\beta}$ is referred to as the "low cost type" or "low cost firm," and type $\bar{\beta}$ as the "high cost type" or "high cost firm." The firm's type is its private information; the regulator's prior belief that the firm is the low cost type is given by ρ. The firm experiences a disutility of effort that can be expressed in monetary terms by

$$\psi(e_t) = \begin{cases} \frac{\gamma}{2} e_t^2, & e_t > 0, \\ 0, & e_t \leq 0, \end{cases} \tag{2}$$

where $\gamma > 0$. Thus, the firm's per period utility is given by

$$U_t = t_t - \psi(e_t). \tag{3}$$

Although project costs are stochastic, the firm's effort is not; in each period, the firm chooses his effort before the realization of ε_t.

The regulator's objective in each period is to maximize expected welfare, which is the sum of taxpayer surplus and firm utility. In each period, welfare is given by

$$W_t = S - (1+\lambda)\big(c_t + t_t(c_t)\big) + U_t. \tag{4}$$

Taxpayers enjoy benefit S from the project, compensate the firm for its costs c_t, and pay out the incentive fee $t_t(c_t)$. Since the cost reimbursement and incentive transfer are raised via distortionary taxation, one dollar paid to the firm costs taxpayers \$$(1+\lambda)$, where $\lambda > 0$ denotes the shadow cost of public funds.

The solution concept used is that of a perfect Bayesian equilibrium. In each period, the regulator designs an incentive scheme in order to maximize expected welfare, where the incentive scheme depends on the regulator's beliefs about the firm's type. In the first period, the regulator considers the impacts of the first period contract on expected second period welfare.

At the beginning of the second period, the regulator observes the first period project cost, and updates her beliefs about the firm's type using Bayes' rule. Contracts are short term; thus, when designing the second period contract, the regulator cannot commit to ignore any information she learns about the firm's type from observing the realized first period project cost.

The firm chooses whether to participate or not in each period. If the firm chooses to participate, he chooses his effort to maximize his expected utility given the transfer designed by the regulator. In the first period, he considers the impact that his actions have on the regulator's second period beliefs, and thus his expected second period payoffs.

In practice, it will be convenient to first think about the regulator's problem as choosing an expected cost target for each type of firm in each period. These targets serve two purposes; first, whatever cost the firm decides to target determines the firm's effort. To see this, recall that effort is chosen before the realization of ε_t. Thus, the firm simply chooses its effort such that its expected cost, $E[c_t] = \beta - e_t$, is equal to its chosen cost target.

Second, for a given type of firm, the cost target serves as the mean of the distribution of project costs in each period. Since the incentive transfer is a function of project costs, the expected transfer in each period depends on the cost target. Thus, at the beginning of each period the regulator chooses cost targets that, in expectation, form an incentive feasible menu. Keep in mind that the actual incentive transfer, $t_t(c_t)$, depends on the cost realization. This paper does not discuss the design of the equilibrium reward schedules; however, based on Caillaud, Guesnerie, and Rey (1992) and Picard (1987), an equilibrium reward schedule, based solely on observed cost, exists such that neither type of firm benefits from targeting an expected cost other than the one designed for him by the regulator. For a more thorough discussion of what agency contracts based on noisy signals look like, see Jeitschko and Mirman (2002).

Throughout the paper, the focus is on deriving an equilibrium that is "separating in actions." Because cost observations are noisy, and this uncertainty is not resolved ex-post, the regulator is not able to determine with certainty the firm's type based on observing the cost realization. That is, even when the first period contract is designed in a way that the low cost firm and high cost firm target distinct cost levels, the regulator does not have full information about the firm's type in the second period. Thus, the equilibrium is separating in actions when the regulator designs distinct targets for each type of firm, and each type of firm targets the expected cost designed for for him by the regulator. This means in period $t = 1, 2$, the low cost firm targets $\underline{c}_t$, and the high cost firm targets $\overline{c}_t$.

3 Second period

Sine the model is solved using backward induction, the analysis begins with the second period. Suppose that the first period contract is separating in actions. At the beginning of the second period, the regulator observes the first period cost realization and updates her beliefs about the firm's type using Bayes' rule. Her posterior belief that the firm is the low cost type is given by

$$\rho_2 := \frac{\rho g(c_1 - \underline{c}_1)}{\rho g(c_1 - \underline{c}_1) + (1-\rho) g(c_1 - \overline{c}_1)}, \tag{5}$$

where c_1 is the realized first period cost, $\underline{c}_1$ is the first period cost target for the low cost type, and $\bar{c}_1$ is the first period cost target for the high cost type. In the remainder of the paper, we use the following notation: $\underline{g} = g(c_1 - \underline{c}_1)$, and $\bar{g} = g(c_1 - \bar{c}_1)$.

With beliefs given in (5), the regulator's problem is to choose expected costs $\underline{c}_2$ and $\bar{c}_2$ to maximize expected welfare, subject to incentive and participation constraints (which are derived below):

$$\max_{\underline{c}_2,\bar{c}_2} \quad \rho_2 \int_{\mathbb{R}} \left[S - (1+\lambda)\big(c_2 + t_2(c_2)\big) + t_2(c_2) - \frac{\gamma}{2}(\underline{\beta} - \underline{c}_2)^2\right] g(c_2 - \underline{c}_2) dc_2$$
$$+ (1-\rho_2) \int_{\mathbb{R}} \left[S - (1+\lambda)\big(c_2 + t_2(c_2)\big) + t_2(c_2) - \frac{\gamma}{2}(\bar{\beta} - \bar{c}_2)^2\right] g(c_2 - \bar{c}_2) dc_2 \quad (6)$$

Because the second period game is static, and both the regulator and the firm are risk neutral, zero-mean noise has no impact on incentives. Thus, the binding constraints on the regulator's problem are the low cost type's incentive compatibility constraint and the high cost firm's participation constraint.

First, consider the low cost type's incentive compatibility constraint. The optimal second period cost targets make the low cost firm's expected utility from targeting $\underline{c}_2$ equal to his expected utility from targeting $\bar{c}_2$. When the low cost firm targets $\underline{c}_2$, he chooses his effort in the second period such that $\underline{e}_2 = \underline{\beta} - \underline{c}_2$, and thus his private cost of effort is equal to $\frac{\gamma}{2}\left(\underline{\beta} - \underline{c}_2\right)^2$.

When the low cost firm chooses his effort in this manner, it is easy to see that

$$E[c_2] = E\left[\underline{\beta} - \underline{\beta} + \underline{c}_2 + \varepsilon_2\right] = \underline{c}_2. \quad (7)$$

Therefore, the second period project cost can be written as $c_2 = \underline{c}_2 + \varepsilon_2$, which implies that the density of second period costs is given by $g(c_2 - \underline{c}_2)$. Therefore, the low cost firm's expected second period utility from targeting $\underline{c}_2$ is given by

$$E\left[\underline{U}_2 \mid \underline{c}_2\right] := \int_{\mathbb{R}} \left[t_2(c_2) - \frac{\gamma}{2}(\underline{\beta} - \underline{c}_2)^2\right] g(c_2 - \underline{c}_2) dc_2 = \underline{t}_2 - \frac{\gamma}{2}\left(\underline{\beta} - \underline{c}_2\right)^2, \quad (8)$$

where $\underline{t}_2 := \int_{\mathbb{R}} t_2(c_2) \cdot g(c_2 - \underline{c}_2) dc_2$.

Similarly, when the low cost type targets $\bar{c}_2$, his effort is given by $\bar{e}_2 - \Delta\beta = \underline{\beta} - \bar{c}_2$, and the density of second period costs is given by $g(c_2 - \bar{c}_2)$. Thus, his expected utility from targeting $\bar{c}_2$ is

$$E\left[\underline{U}_2 \mid \bar{c}_2\right] := \int_{\mathbb{R}} \left[t_2(c_2) - \frac{\gamma}{2}(\underline{\beta} - \bar{c}_2)^2\right] g(c_2 - \bar{c}_2) dc_2 = \bar{t}_2 - \frac{\gamma}{2}\left(\underline{\beta} - \bar{c}_2\right)^2, \quad (9)$$

where $\bar{t}_2 := \int_{\mathbb{R}} t_2(c_2) \cdot g(c_2 - \bar{c}_2) dc_2$. The low cost firm's incentive compatibility constraint makes him indifferent, in expectation, between targeting $\underline{c}_2$ and $\bar{c}_2$:

$$E\left[\underline{U}_2 \mid \underline{c}_2\right] = E\left[\underline{U}_2 \mid \bar{c}_2\right] \quad \Longrightarrow \quad \underline{t}_2 - \frac{\gamma}{2}\left(\underline{\beta} - \underline{c}_2\right)^2 = \bar{t}_2 - \frac{\gamma}{2}\left(\underline{\beta} - \bar{c}_2\right)^2. \quad (10)$$

The second period game is designed to extract all expected rent form the high cost type. When the high cost type targets $\bar{c}_2$, his cost of effort is $\bar{e}_2 = \bar{\beta} - \bar{c}_2$, and the density of expected costs is given by $g(c_2 - \bar{c}_2)$. Thus, the high cost type's expected second period rent is given by

$$E\left[\overline{U}_2 \mid \bar{c}_2\right] := \int_{\mathbb{R}} \left[t_2(c_2) - \frac{\gamma}{2}(\bar{\beta} - \bar{c}_2)^2\right] g(c_2 - \bar{c}_2) dc_2 = \bar{t}_2 - \frac{\gamma}{2}\left(\bar{\beta} - \bar{c}_2\right)^2. \tag{11}$$

Thus, the high cost type's participation constraint is given by

$$E\left[\overline{U}_2 \mid \bar{c}_2\right] = 0 \quad \Longrightarrow \quad \bar{t}_2 - \frac{\gamma}{2}\left(\bar{\beta} - \bar{c}_2\right)^2 = 0. \tag{12}$$

Simplifying the objective function in (6) and using (10) and (12) to substitute for the expected transfers leaves the following unconstrained problem:

$$\begin{aligned} \max_{\underline{c}_2, \bar{c}_2} \quad & S - \rho_2 \left[(1+\lambda)\left(\underline{c}_2 + \frac{\gamma}{2}(\underline{\beta} - \underline{c}_2)^2\right) + \lambda\left(\frac{\gamma}{2}(\bar{\beta} - \bar{c}_2)^2 - \frac{\gamma}{2}(\underline{\beta} - \bar{c}_2)^2\right)\right] \\ & - (1-\rho_2)(1+\lambda)\left(\bar{c}_2 + \frac{\gamma}{2}(\bar{\beta} - \bar{c}_2)^2\right), \end{aligned} \tag{13}$$

where $\frac{\gamma}{2}(\bar{\beta} - \bar{c}_2)^2 - \frac{\gamma}{2}(\underline{\beta} - \bar{c}_2)^2$ is the low cost firm's information rent.

The first order conditions of this problem imply the following equilibrium efforts and cost targets:

$$\underline{e}_2 = \underline{\beta} - \underline{c}_2 = \frac{1}{\gamma}, \tag{14}$$

and

$$\bar{e}_2 = \bar{\beta} - \bar{c}_2 = \frac{1}{\gamma} - \frac{\rho_2}{1-\rho_2}\frac{\lambda}{1+\lambda}\Delta\beta. \tag{15}$$

Thus, the low cost type exerts the first best effort in the second period, and the high cost type's effort is distorted away from the first best according to the second period beliefs. Notice that the effort levels given in (14) and (15) correspond to the standard static game in which beliefs are given by ρ_2. This illustrates than in a static setting, additive noise has no impact on incentives when the regulator and firm are risk neutral.

One concern in this model is that the low cost firm's effort from mimicking the high cost firm,

$$\bar{e}_2 - \Delta\beta = \underline{\beta} - \bar{c}_2 = \frac{1}{\gamma} - \frac{1+\lambda-\rho_2}{(1-\rho_2)(1+\lambda)}\Delta\beta, \tag{16}$$

can be less than zero for values of ρ_2 close to one. "Negative effort" captures any measures taken to increase rather than decrease the project cost. To understand why the low cost type might have to increase project costs in order to mimic the high cost type, recall that expected cost for the high cost type are equal to its type minus its cost reducing effort. When the first period cost observation is low, this leads the regulator to believe that she is very likely to be contracting with the low cost type in the second period. In response, she reduces the effort of the high cost type in order to extract rent from the low cost type. When this effort is small enough (i.e. when second period beliefs are close to one), $\bar{c}_2 = \bar{\beta} - \bar{e}_2 > \underline{\beta}$.

This possibility is usually assumed away in static models. However, as ε has full support on the real line, it must be considered in this setting. Since posterior beliefs are monotone decreasing in first period cost realizations, there exists a unique value of ρ_2, defined

$$\rho_2^0 := \rho_2(c_1^0) = \frac{(1+\lambda)(1-\gamma\Delta\beta)}{1+\lambda-\gamma\Delta\beta} < 1, \tag{17}$$

such that for every $c_1 \leq c_1^0$, the low cost type's effort from mimicking the high cost type is negative. Since the firm cannot experience a dis-utility from negative effort, the low cost type's second period incentive compatibility constraint is written

$$\underline{t}_2 - \frac{\gamma}{2}(\underline{\beta} - \underline{c}_2)^2 = \bar{t}_2. \tag{18}$$

The high cost firm's participation constraint remains unchanged. Together, this implies that the regulator's unconstrained problem when $c_1 \leq c_1^0$ is given by

$$\begin{aligned} \max_{\underline{c}_2, \bar{c}_2} \quad & S - \rho_2 \left[(1+\lambda)\left(\underline{c}_2 + \frac{\gamma}{2}(\underline{\beta} - \underline{c}_2)^2 \right) + \lambda \frac{\gamma}{2}\left(\bar{\beta} - \bar{c}_2\right)^2 \right] \\ & - (1-\rho_2)(1+\lambda)\left(\bar{c}_2 + \frac{\gamma}{2}(\bar{\beta} - \bar{c}_2)^2 \right), \end{aligned} \tag{19}$$

where the low cost firm's information rent is now given by $\frac{\gamma}{2}\left(\bar{\beta} - \bar{c}_2\right)^2$. The first order condition for this problem with respect to $\bar{c}_2$ implies the following equilibrium effort for the high cost type (the low cost type still exerts the first best effort):

$$\bar{e}_2^0 = \bar{\beta} - \bar{c}_2 = \frac{1}{\gamma} \frac{(1-\rho_2)(1+\lambda)}{1+\lambda-\rho_2}. \tag{20}$$

The following proposition summarizes the second period game:

Proposition 1. *When $c_1 > c_1^0$, the regulator's problem is given by (13), while for $c_1 \leq c_1^0$, the regulator's problem is given by (19). The first order conditions of (13) and (19) with respect to $\underline{c}_2$ and $\bar{c}_2$ imply that the low cost firm's equilibrium expected rent is given by*

$$\underline{U}_2(\rho_2) = \begin{cases} \frac{\gamma}{2}(\bar{e}_2)^2 - \frac{\gamma}{2}(\bar{e}_2 - \Delta\beta)^2 =: \underline{u}_2, & \textit{if} \quad c_1 > c_1^0, \\ \frac{\gamma}{2}(\bar{e}_2^0)^2 =: \underline{u}_2^0, & \textit{if} \quad c_1 \leq c_1^0, \end{cases} \tag{21}$$

where $\bar{e}_2$ is given in (15), $\bar{e}_2 - \Delta\beta$ in (16), and $\bar{e}_2^0$ in (20). Similarly, equilibrium expected second period welfare is given by

$$W_2(\rho_2) = \begin{cases} S - \rho_2 \left[(1+\lambda)\left(\underline{\beta} - \frac{1}{2\gamma}\right) + \lambda \underline{u}_2\right] - (1-\rho_2)(1+\lambda)\left(\bar{\beta} - \bar{e}_2 + \frac{\gamma}{2}\left(\bar{e}_2\right)^2\right) =: w_2, \\ S - \rho_2 \left[(1+\lambda)\left(\underline{\beta} - \frac{1}{2\gamma}\right) + \lambda \underline{u}_2^0\right] - (1-\rho_2)(1+\lambda)\left(\bar{\beta} - \bar{e}_2^0 + \frac{\gamma}{2}(\bar{e}_2^0)^2\right) =: w_2^0, \end{cases} \tag{22}$$

when c_1 is greater than c_1^0 and less than c_1^0, respectively.

Regardless of the size of c_1, the second period game exhibits the classic rent extraction/efficiency trade-off present in static adverse selection models:

$$\frac{d\underline{U}_2(\rho_2)}{d\rho_2} = \begin{cases} \frac{d\underline{u}_2}{d\bar{e}_2}\frac{d\bar{e}_2}{d\rho_2} = \frac{-1}{(1-\rho_2)^2}\frac{\lambda}{1+\lambda}\gamma\Delta\beta^2 < 0, & \text{if} \quad c_1 > c_1^0, \\ \frac{d\underline{u}_2^0}{d\bar{e}_2^0}\frac{d\bar{e}_2^0}{d\rho_2} = \frac{-\lambda(1+\lambda)^2}{\gamma}\frac{1-\rho_2}{(1+\lambda-\rho_2)^3} < 0, & \text{if} \quad c_1 \leq c_1^0. \end{cases} \tag{23}$$

This is an important consideration for the regulator in the first period, since ρ_2 is a function of $\underline{c}_1$ and $\bar{c}_1$.

To see how second period beliefs, and thus second period welfare, depend on the first period contract, consider $\tilde{c}_1 = \underline{c}_1 + x$. From (5), the closer together are $\underline{c}_1$ and $\bar{c}_1$, the closer together are the values of $\underline{g}(\tilde{c}_1)$ and $\bar{g}(\tilde{c}_1)$. The closer together are $\underline{g}(\tilde{c}_1)$ and $\bar{g}(\tilde{c}_1)$, the closer ρ_2 is to the prior, ρ; indeed, if $\underline{c}_1 = \bar{c}_1$, then $\underline{g}(\tilde{c}_1) = \bar{g}(\tilde{c}_1)$ for all x, and the posterior is equal to the prior. Conversely, the further apart are $\underline{c}_1$ and $\bar{c}_1$, the smaller is $\bar{g}(\tilde{c}_1)$ relative to $\underline{g}(\tilde{c}_1)$, and the closer the posterior is to one (clearly, this argument works in the opposite direction as well).

Thus, the distance between first period cost targets directly influences how much the regulator updates her prior, given a first period cost realization. The further apart are the first period cost targets, the more accurate are the regulator's second period beliefs; the more accurate are the regulator's second period beliefs, the closer second period welfare is to the first-best. However, this information comes at a cost. Since the low cost firm's second period rent is decreasing in ρ_2, spreading the cost targets apart decreases (in expectation) the low cost firm's rent from targeting $\underline{c}_1$, and increases his rent were he to target $\bar{c}_1$ in the first period. This increases the low cost type's first period transfer. Thus, the regulator faces a tradeoff between increasing the expected second period welfare *or* preserving the low cost firm's expected second period rent. Section 4 demonstrates that both incentives exist.

4 First period

The second period beliefs, ρ_2, serve as the link between the first and second period contracts. Therefore, when choosing the first period cost targets, the regulator considers not only the impact that the first period cost targets have on first period welfare, but what impact they have on the second period contract. The regulator's first period problem is to maximize the expectation of first and (discounted) second period welfare, subject to incentive compatibility and participation constraints (which are derived below):

$$\begin{aligned}\max_{\underline{c}_1,\bar{c}_1} \qquad & S-\rho\int_{\mathbb{R}}\left[(1+\lambda)\left(c_1+t_1(c_1)\right)+t_1(c_1)-\frac{\gamma}{2}\left(\underline{\beta}-\underline{c}_1\right)^2\right]g(c_1-\underline{c}_1)dc_1\\ & -(1-\rho)\int_{\mathbb{R}}\left[(1+\lambda)\left(c_1+t_1(c_1)\right)+t_1(c_1)-\frac{\gamma}{2}\left(\bar{\beta}-\bar{c}_1\right)^2\right]g(c_1-\bar{c}_1)\\ & +\delta E[W_2(\rho_2)], \qquad (24)\end{aligned}$$

where $W_2(\rho_2)$ is given in (22), and

$$E[W_2(\rho_2)]=\int_{\mathbb{R}}W_2(\rho_2)\left[\rho g(c_1-\underline{c}_1)+(1-\rho)g(c_1-\bar{c}_1)\right]dc_1. \qquad (25)$$

A well known issue in dynamic games is that the first period payment to the low cost firm may be so large that the high cost type's incentive compatibility constraint binds (the so-called "take the money and run" strategy). For now, consider the low cost firm's incentive compatibility constraint and the high cost firm's participation constraint. Later, it will be shown that noise alleviates the dynamic incentive problem, and that the high cost firm's incentive constraint is slack. The low cost firm's incentive constraint is as follows:

$$\begin{aligned}E[\underline{U}_1|\,\underline{c}_1]&:=\int_{\mathbb{R}}\left[t_1(c_1)-\frac{\gamma}{2}(\underline{\beta}-\underline{c}_1)^2+\delta\underline{U}_2(\rho_2)\right]\underline{g}dc_1\\ &=\int_{\mathbb{R}}\left[t_1(c_1)-\frac{\gamma}{2}(\underline{\beta}-\bar{c}_1)^2+\delta\underline{U}_2(\rho_2)\right]\bar{g}dc_1=:E[\underline{U}_2|\,\bar{c}_2]. \qquad (26)\end{aligned}$$

The left hand side of (26) is the low cost firm's expected utility when he targets $\underline{c}_1$ in the first period. He exerts effort $\underline{e}_1=\underline{\beta}-\underline{c}_1$, and receives an expected first period transfer and expected second period rent, where expectations are taken over the real line according to the density $\underline{g}$. If the low cost firm instead chooses to target $\bar{c}_1$, he experiences a disutility from effort $\bar{e}_1-\Delta\beta=\underline{\beta}-\bar{c}_1$, and receives an expected first period transfer and expected second period rent where expectations are taken according to the density $\bar{g}$.

From the perspective of the high cost firm, the first period game is essentially static since the second period game extracts all the rent from the high cost type. Therefore, the high cost firm's participation constraint requires that his expected first period utility be equal to his outside option of zero:

$$E[\overline{U}_1 \big| \bar{c}_1] := \int_{\mathbb{R}} \left[t_1(c_1) - \frac{\gamma}{2} \left(\bar{\beta} - \bar{c}_1 \right)^2 \right] \bar{g} dc_1 = 0. \tag{27}$$

By defining $\underline{t}_1$ and $\bar{t}_1$ analogously to $\underline{t}_2$ and $\bar{t}_2$, one can simplify (26) and (27) and solve for the low cost firm's expected first period transfer:

$$\underline{t}_1 = \frac{\gamma}{2}(\underline{\beta} - \underline{c}_1)^2 + \frac{\gamma}{2}(\bar{\beta} - \bar{c}_1)^2 - \frac{\gamma}{2}(\underline{\beta} - \bar{c}_1)^2 + \delta \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g}) dc_1. \tag{28}$$

The first three terms on the right hand side of (28) comprise the familiar static transfer: the low cost firm must be compensated for the cost of its effort, and also for the ability to "hide behind" the high cost firm. In dynamic games, there is an additional component of the low cost firm's first period transfer; the distribution induced by targeting $\bar{c}_1$ first order stochastically dominates the distribution induced by targeting $\underline{c}_1$, so that the low cost firm enjoys a higher expected second period rent when he targets $\bar{c}_1$ as opposed to $\underline{c}_1$. Thus, $\int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g}) dc_1 > 0$. The first period transfer must compensate him for this opportunity cost in order to induce him to choose the appropriate cost target.

One of the the chief hurdles in designing a separating contract in a deterministic environment is that in the first period, the low cost firm must be compensated for the second period information rent he forgoes by revealing his private information at the beginning of the relationship. Unless the the firm cares little about the future (i.e., the firm heavily discount future payoffs), this transfer can be prohibitively large. This is because in a deterministic separating equilibrium, information revelation is an "all-or-nothing" proposition; actions perfectly reveal types. Thus, when the the low cost firm follows the equilibrium in the first period, the regulator believes with probability one that she is contracting with the low cost type in the second period, and he is held to his reservation utility.

Further, when the low cost firm takes out-of-equilibrium actions in the first period and mimics the high cost firm, at the beginning of the second period the regulator believes the firm to be the high cost type, and the low cost firm enjoys his highest possible information rent, $\underline{U}_2(0)$. This rationale changes in a stochastic setting. First, simply by following the equilibrium and targeting $\underline{c}_1$ in the first period, the low cost firm enjoys expected second period rent

$$\int_{\mathbb{R}} \underline{U}_2(\rho_2) \underline{g} dc_1 > 0. \tag{29}$$

Now, suppose the low cost firm deviates and targets $\bar{c}_1$ in the second period. The corresponding density of first period costs is $g(c_1 - \bar{c}_1)$, so that the low cost firm's expected second period utility from deviation is

$$\int_{\mathbb{R}} \underline{U}_2(\rho_2)\bar{g}dc_1 < \int_{\mathbb{R}} \underline{U}_2(0)\bar{g}dc_1 = \underline{U}_2(0). \tag{30}$$

Thus, the low cost firm's gains from deviating are diminished in a stochastic environment. Therefore, the low cost type's expected first period transfer in a stochastic setting is less than his transfer in a deterministic environment.

The following assumption guarantees a regular first period problem:

Assumption 1. *The single crossing property holds in the first period. That is,*

$$\begin{aligned}\gamma\left(\bar{\beta} - c\right) &\geq \gamma\left(\underline{\beta} - c\right) + \delta \int_{\mathbb{R}} \frac{d\underline{U}_2}{d\rho_2}\frac{d\rho_2}{dc_1} g(c_1 - c)dc_1 \\ \Longrightarrow \gamma\Delta\beta &\geq \delta \int_{\mathbb{R}} \frac{d\underline{U}_2}{d\rho_2}\frac{d\rho_2}{dc_1} g(c_1 - c)dc_1.\end{aligned} \tag{31}$$

Thus, the high cost type's marginal cost of decreasing the cost target c is higher than the low cost type's marginal cost of decreasing the cost target for every c. From (31), this condition is satisfied when $\frac{d\rho_2}{dc_1}$ is small, i.e. when the posterior beliefs are insensitive to changes in first period cost. Since the magnitude of $\frac{d\rho_2}{dc_1}$ depends on the slope of the density, and the slope of the density goes to zero when the variance is large, this condition is satisfied in sufficiently noisy environments. The single crossing condition is also more likely to be satisfied when the difference between the low and high cost firm's intrinsic cost levels, $\Delta\beta$, is large.

Proposition 2. *The regulator's full first period problem is given by*

$$\begin{aligned}\max_{\underline{c}_1,\bar{c}_1} \; & S - \rho\left[(1+\lambda)\left(\underline{c}_1 + \frac{\gamma}{2}(\underline{\beta} - \underline{c}_1)^2\right) + \lambda\left(\frac{\gamma}{2}(\bar{\beta} - \bar{c}_1)^2 - \frac{\gamma}{2}(\underline{\beta} - \bar{c}_1)^2 + \delta\int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g})dc_1\right)\right] \\ & - (1-\rho)(1+\lambda)\left(\bar{c}_1 + \frac{\gamma}{2}(\bar{\beta} - \bar{c}_1)^2\right) + \delta E[W_2(\rho_2)],\end{aligned} \tag{32}$$

where $E[W_2(\rho_2)]$ is given in (25). The first order conditions of (32) with respect to $\underline{c}_1$ and $\bar{c}_1$ imply the following equilibrium first period efforts (and cost targets):

$$\underline{e}_1 = \underline{\beta} - \underline{c}_1 = \frac{1}{\gamma} + \frac{\delta}{\gamma\rho(1+\lambda)}\frac{d}{d\underline{c}_1}\left[\rho\lambda\int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g})dc_1 - E[W_2]\right], \tag{33}$$

and

$$\begin{aligned}\bar{e}_1 = \bar{\beta} - \bar{c}_1 =& \frac{1}{\gamma} - \frac{\rho\lambda}{(1-\rho)(1+\lambda)}\Delta\beta \\ &+ \frac{\delta}{\gamma(1-\rho)(1+\lambda)}\frac{d}{d\bar{c}_1}\left[\rho\lambda\int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g})dc_1 - E[W_2]\right].\end{aligned} \tag{34}$$

Notice that for each type of firm, the equilibrium effort is distorted relative to the commitment optimum by the term in brackets. Given the expression for the low and high cost firm's equilibrium efforts, one can verify that the high cost firm's incentive constraint is satisfied in sufficiently noisy environments. The proof is left for the appendix, but the intuition is that as the variance increases, the effort distortions in (33) and (34) go to zero along with the dynamic portion of the low cost type's first period transfer, $\int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g})dc_1$. Thus, for a high enough variance, the high cost type's dynamic incentive constraint resembles his static incentive constraint.

4.1 Signal dampening

At the beginning of the first period, the low cost type's decision to follow a first period separating equilibrium, or deviate and mimic the high cost type, depends on how much he stands to gain from this deviation. Deviation is less profitable in a stochastic setting than in a deterministic one, but he is still able to increase his expected second period rent by choosing to target $\bar{c}_1$ instead of $\underline{c}_1$ in the first period. The equilibrium first period transfer, which ensures that neither type of agent benefits from targeting $c_1 \notin \{\underline{c}_1, \bar{c}_1\}$, compensates him for this opportunity cost.

Exactly how much the low cost type stands to gain from mimicking the high cost type depends on the regulator's second period beliefs. Since $c_1 = \underline{\beta} - e_1 + \varepsilon_1$ is a random variable, and second period beliefs are a function of first period project costs, ρ_2 is also a random variable. This means the regulator cannot directly choose second period beliefs at the beginning of the game. The regulator can, however, affect the distribution of this random variable by her choice of the first period cost targets, $\underline{c}_1$ and $\bar{c}_1$.

To see this, refer to the discussion immediately following Proposition 1. The closer together are the first period cost targets, the closer is ρ_2 to the prior for any given cost realization. The less the regulator learns from the first period cost realization, the closer is the low cost firm's expected second period rent from targeting $\underline{c}_1$ compared to when he deviates and targets $\bar{c}_2$. This decreases the low cost type's incentives to mimic the high cost type in the first period, which reduces the low cost type's first period transfer, and thus alleviates the first period incentive problem.

The following proposition and subsequent proof solidifies this logic by, for the time being, ignoring the impacts of the first period contract on expected second period welfare. The proof makes use of the connection between effort and cost targets; an increased cost target implies a decrease in effort, and vice-versa. The proof formalizes the intuition that the regulator can decrease the low cost firm's first period transfer by decreasing the distance between $\bar{c}_1$ and $\underline{c}_1$. To do this, the proof shows that the first period transfer is decreasing in $\underline{c}_1$ and increasing in $\bar{c}_1$. This equilibrium transfer effect decreases (increases) the low cost (high cost) type's equilibrium first period effort.

Tying cost targets to efforts also allows a discussion of how the ratchet effect behaves in a stochastic setting versus a deterministic one. In a deterministic separating equilibrium, the high cost type has his effort ratcheted up over time, while the low cost type always exerts the first best effort (see Laffont and Tirole (1993)). As the following proposition shows, and as the above intuition argues, in a stochastic setting the regulator distorts the efforts of both types of firm in the first period, as opposed to just the high cost firm. In particular, the low cost type exerts less than the first best effort, and the high cost type exerts more effort than is optimal in a deterministic separating equilibrium.

Proposition 3. *The effect of the dynamic portion of the low cost firm's first period transfer is to decrease (increase) the low cost (high cost) firm's first period effort. That is,*

$$\frac{d}{d\underline{c}_1}\left[\rho\lambda\int_{\mathbb{R}}\underline{U}_2(\rho_2)(\bar{g}-\underline{g})dc_1\right]<0, \tag{35}$$

and

$$\frac{d}{d\bar{c}_1}\left[\rho\lambda\int_{\mathbb{R}}\underline{U}_2(\rho_2)(\bar{g}-\underline{g})dc_1\right]>0. \tag{36}$$

Proof. Consider the expression for the low cost type's first period effort given by (33). Ignoring the effect of the first period contract on expected second period welfare, the low cost type's equilibrium first period effort is less than in a deterministic separating equilibrium (that is, less than $\frac{1}{\gamma}$, the first best) when (35) is true. To show that (35) holds, consider

$$\begin{aligned}\frac{d}{d\underline{c}_1}\int_{\mathbb{R}}\underline{U}_2(\rho_2)(\bar{g}-\underline{g})dc_1 &= \frac{d}{d\underline{c}_1}\left[\int_{-\infty}^{c_1^0}\underline{u}_2^0(\bar{g}-\underline{g})dc_1+\int_{c_1^0}^{\infty}\underline{u}_2(\bar{g}-\underline{g})dc_1\right]\\ &= \int_{-\infty}^{c_1^0}\frac{d\underline{u}_2^0}{d\rho_2}\frac{d\rho_2}{d\underline{c}_1}(\bar{g}-\underline{g})+\underline{u}_2^0\underline{g}'dc_1+\int_{c_1^0}^{\infty}\frac{d\underline{u}_2}{d\rho_2}\frac{d\rho_2}{d\underline{c}_1}(\bar{g}-\underline{g})+\underline{u}_2\underline{g}'dc_1. \end{aligned} \tag{37}$$

Integrate the second term under each integral on the right hand side of (37) by parts. Doing so yields

$$\int_{-\infty}^{c_1^0} \frac{d\underline{u}_2^0}{d\rho_2}\left[\frac{d\rho_2}{d\underline{c}_1}\bar{g} - \left(\frac{d\rho_2}{d\underline{c}_1} + \frac{d\rho_2}{dc_1}\right)\underline{g}\right] dc_1 + \underline{u}_2^0 \underline{g}\Big|_{-\infty}^{c_1^0} + \int_{c_1^0}^{\infty} \frac{d\underline{u}_2}{d\rho_2}\left[\frac{d\rho_2}{d\underline{c}_1}\bar{g} - \left(\frac{d\rho_2}{d\underline{c}_1} + \frac{d\rho_2}{dc_1}\right)\underline{g}\right] dc_1 + \underline{u}_2 \underline{g}\Big|_{c_1^0}^{\infty}. \quad (38)$$

Now,

$$\frac{d\rho_2}{d\underline{c}_1} = \frac{-\rho(1-\rho)\underline{g}'\bar{g}}{D^2}, \quad (39)$$

and

$$\frac{d\rho_2}{dc_1} = \frac{\rho(1-\rho)[\underline{g}'\bar{g} - \underline{g}\bar{g}']}{D^2}, \quad (40)$$

where $D = \rho\underline{g} + (1-\rho)\bar{g}$. Thus,

$$\frac{d\rho_2}{d\underline{c}_1} + \frac{d\rho_2}{dc_1} = \frac{-\rho(1-\rho)\underline{g}\bar{g}'}{D^2}. \quad (41)$$

Further,

$$\underline{u}_2^0 \underline{g}\Big|_{-\infty}^{c_1^0} + \underline{u}_2 \underline{g}\Big|_{c_1^0}^{\infty} = \underline{g}(c_1^0)\left[\underline{u}_2^0(\rho_2^0) - \underline{u}_2(\rho_2^0)\right]. \quad (42)$$

When $\rho_2 = \rho_2^0$, it is easily verified that

$$\underline{u}_2^0(\rho_2^0) = \frac{\gamma}{2}\Delta\beta^2 = \underline{u}_2(\rho_2^0). \quad (43)$$

After substituting for the relevant terms and simplifying, (38) becomes

$$\int_{-\infty}^{c_1^0} \frac{d\underline{u}_2^0}{d\rho_2} k\left[\underline{g}'\bar{g}^2 - \underline{g}^2\bar{g}'\right] dc_1 + \int_{c_1^0}^{\infty} \frac{d\underline{u}_2}{d\rho_2} k\left[\underline{g}'\bar{g}^2 - \underline{g}^2\bar{g}'\right] dc_1, \quad (44)$$

where $k = \frac{-\rho(1-\rho)}{D^2}$.

Because $\frac{d\underline{u}_2}{d\rho_2} < 0$ and $\frac{d\underline{u}_2^0}{d\rho_2} < 0$, to show that

$$\underline{g}'\bar{g}^2 - \underline{g}^2\bar{g}' < 0 \quad (45)$$

for all c_1 is sufficient to show that the above integrals are negative over their respective limits of integration. This task is a simple extension of the proof of Theorem 2 in Jeitschko and Mirman (2002) and is omitted. Thus,

$$\frac{d}{d\underline{c}_1}\rho\lambda\int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g})dc_1 = \rho\lambda\left[\int_{-\infty}^{c_1^0} \frac{d\underline{u}_2^0}{d\rho_2} k\left[\underline{g}'\bar{g}^2 - \underline{g}^2\bar{g}'\right] dc_1 + \int_{c_1^0}^{\infty} \frac{d\underline{u}_2}{d\rho_2} k\left[\underline{g}'\bar{g}^2 - \underline{g}^2\bar{g}'\right] dc_1\right] < 0, \quad (46)$$

and the low cost firm's first period effort is decreased. A similar proof shows that

$$\frac{d}{d\bar{c}_1}\left[\int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g}-\underline{g})dc_1\right] = -\frac{d}{d\underline{c}_1}\left[\int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g}-\underline{g})dc_1\right] > 0. \tag{47}$$

Thus, the effect of the dynamic portion of the low cost firm's first period transfer is to decrease the distance between cost targets, and reduce how much the regulator updates her prior for any given cost realization. □

This proof establishes that even though the regulator cannot commit to ignore information she learns about the firm when designing the second period contract, in a stochastic environment the regulator can commit to learn less via her choice of first period cost targets. Doing so preserves the low cost firm's equilibrium expected second period rent and decreases his gains from deviation, which in turn decreases his first period transfer, alleviating the dynamic incentive problem.

4.2 Experimentation

Proposition 3 establishes that the regulator has an incentive to reduce the flow of information. This subsection shows that the opposite incentives exist as well. Intuitively, the more accurate the regulator's second period beliefs are, the better off she is; after all, the second period game is static, so that any distortions away from the first best arise solely because of asymmetric information, and not because of any dynamic considerations. The following lemma establishes, in the sense of Blackwell (1951) that, indeed, information is valuable to the regulator.

Lemma 1. *Information is valuable. That is,*

$$\frac{d^2W_2(\rho_2)}{d\rho_2^2} > 0. \tag{48}$$

Proof. From the perspective of the second period, expected second period welfare is given by (22). When $c_1 > c_1^0$, welfare can be expressed as

$$\begin{aligned} w_2 = \underset{\underline{e}_2,\bar{e}_2}{\operatorname{argmax}}\ & S - \rho_2\left((1+\lambda)\left(\underline{\beta} - \underline{e}_2 + \frac{\gamma}{2}(\underline{e}_2)^2\right) + \lambda\underline{u}_2\right) \\ & - (1-\rho_2)(1+\lambda)\big(\bar{\beta} - \bar{e}_2 + \frac{\gamma}{2}(\bar{e}_2)^2\big). \end{aligned} \tag{49}$$

By the envelope theorem,

$$\begin{aligned} \frac{dw_2}{d\rho_2} &= -(1+\lambda)\big(\underline{\beta} - \underline{e}_2(\rho_2) + \frac{\gamma}{2}(\underline{e}_2(\rho_2))^2\big) - \lambda\underline{u}_2(\bar{e}_2(\rho_2)) + (1+\lambda)\big(\bar{\beta} - \bar{e}_2(\rho_2) + \frac{\gamma}{2}(\bar{e}_2(\rho_2))^2\big) \\ &= (1+\lambda)\big(\Delta\beta + \frac{1}{2\gamma}\big) - \lambda\underline{u}_2(\bar{e}_2(\rho_2)) - (1+\lambda)\big(\bar{e}_2(\rho_2) - \frac{\gamma}{2}(\bar{e}_2(\rho_2))^2\big). \end{aligned} \tag{50}$$

Thus,

$$\frac{d^2 w_2}{d\rho_2^2} = -\lambda \frac{d\underline{u}_2}{d\bar{e}_2}\frac{d\bar{e}_2}{d\rho_2} - (1+\lambda)(1-\gamma\bar{e}_2(\rho_2))\frac{d\bar{e}_2}{d\rho_2} > 0, \tag{51}$$

since $\frac{d\underline{u}_2}{d\bar{e}_2} > 0$ and $\frac{d\bar{e}_2}{d\rho_2} < 0$, and the high cost type's effort is less than the first best, which implies $(1-\gamma\bar{e}_2(\rho_2)) > 0$. Because $(1-\gamma\bar{e}_2^0) > 0$ and $\frac{d\underline{u}_2^0}{d\bar{e}_2^0} > 0$ and $\frac{d\bar{e}_2^0}{d\rho_2} < 0$ as well, the proof is identical for w_2^0. Thus, information is valuable. □

Given that information is valuable, the next step is to show that the regulator has incentives to increase the distance between first period cost targets, which increases how much she updates her prior for any given first period cost realization. Certainly, the low cost firm could have a high cost realization that leads the regulator to update her prior in the wrong direction, but this is unlikely when the low cost firm targets $\underline{c}_1$. Since welfare distortions in the second period arise because of asymmetric information, and since the information asymmetry diminishes as the distance between first period cost targets grows, an increase in the spread between $\underline{c}_1$ and $\bar{c}_1$ increases expected second period welfare.

Similar to the previous subsection, this incentive to manipulate first period cost targets in order to increase the flow of information can be interpreted in terms of equilibrium first period efforts. Naturally, the incentives that arise from the desire to acquire information run counter to the incentives that exist to decrease the low cost firm's first period transfer; as the following proposition shows, the effect of expected second period welfare on the the firms' first period efforts opposes the effect of the low cost type's first period transfer.

Proposition 4. *The effect of expected second period welfare is to increase (decrease) the low cost (high cost) firm's first period effort. That is,*

$$\frac{dE[W_2(\rho_2)]}{d\underline{c}_1} < 0, \tag{52}$$

and

$$\frac{dE[W_2(\rho_2)]}{d\bar{c}_1} > 0 \tag{53}$$

Proof. From the perspective of the first period,

$$E[W_2(\rho_2)] = \int_{-\infty}^{c_1^0} w_2^0 \left[\rho \underline{g} + (1-\rho)\bar{g}\right] dc_1 + \int_{c_1^0}^{\infty} w_2 \left[\rho \underline{g} + (1-\rho)\bar{g}\right] dc_1. \tag{54}$$

First, consider

$$\frac{dE[W_2(\rho_2)]}{d\underline{c}_1} = \int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2}\frac{d\rho_2}{d\underline{c}_1}\left[\rho\underline{g} + (1-\rho)\bar{g}\right] - w_2^0\rho\underline{g}'dc_1 + \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2}\frac{d\rho_2}{d\underline{c}_1}\left[\rho\underline{g} + (1-\rho)\bar{g}\right] - w_2\rho\underline{g}'dc_1. \quad (55)$$

Integrate the second term under each integral by parts. Doing so yields

$$\int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2}\left[\left(\frac{d\rho_2}{d\underline{c}_1} + \frac{d\rho_2}{dc_1}\right)\rho\underline{g} + \frac{d\rho_2}{d\underline{c}_1}(1-\rho)\bar{g}\right]dc_1 - w_2^0\rho\underline{g}\Big|_{-\infty}^{c_1^0} + \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2}\left[\left(\frac{d\rho_2}{d\underline{c}_1} + \frac{d\rho_2}{dc_1}\right)\rho\underline{g} + \frac{d\rho_2}{d\underline{c}_1}(1-\rho)\bar{g}\right]dc_1 - w_2\rho\underline{g}\Big|_{c_1^0}^{\infty}. \quad (56)$$

Now,

$$-\,w_2^0\rho\underline{g}\Big|_{-\infty}^{c_1^0} - w_2\rho\underline{g}\Big|_{c_1^0}^{\infty} = -\,w_2^0\rho\underline{g}\Big|_{c_1^0} + w_2\rho\underline{g}\Big|_{c_1^0} = 0. \quad (57)$$

From the proof of Proposition 3, we have

$$\frac{d\rho_2}{d\underline{c}_1} = \frac{-\rho(1-\rho)\underline{g}'\bar{g}}{D^2}, \quad (58)$$

$$\frac{d\rho_2}{dc_1} = \frac{\rho(1-\rho)[\underline{g}'\bar{g} - \underline{g}\bar{g}']}{D^2}, \quad (59)$$

and

$$\frac{d\rho_2}{d\underline{c}_1} + \frac{d\rho_2}{dc_1} = \frac{-\rho(1-\rho)\underline{g}\bar{g}'}{D^2}. \quad (60)$$

Substituting the above into (56) yields

$$\int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2}\left[\frac{-\rho(1-\rho)\underline{g}\bar{g}'}{D^2}\rho\underline{g} - \frac{\rho(1-\rho)\underline{g}'\bar{g}}{D^2}(1-\rho)\bar{g}\right]dc_1 + \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2}\left[\frac{-\rho(1-\rho)\underline{g}\bar{g}'}{D^2}\rho\underline{g} - \frac{\rho(1-\rho)\underline{g}'\bar{g}}{D^2}(1-\rho)\bar{g}\right]dc_1 \quad (61)$$

$$= -\left[\int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2}\rho_2^2(1-\rho)\bar{g}'dc_1 + \int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2}(1-\rho_2)^2\rho\underline{g}'dc_1\right] - \left[\int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2}\rho_2^2(1-\rho)\bar{g}'dc_1 + \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2}(1-\rho_2)^2\rho\underline{g}'dc_1\right]. \quad (62)$$

Using the fact that $(1-\rho_2)^2 = 1 - \rho_2 - \rho_2(1-\rho_2)$, re-write (62) as

$$-\int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2}\rho_2\left[\rho_2(1-\rho)\bar{g}' - \rho(1-\rho_2)\underline{g}'\right]dc_1 - \int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2}(1-\rho_2)\rho\underline{g}'dc_1 - \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2}\rho_2\left[\rho_2(1-\rho)\bar{g}' - \rho(1-\rho_2)\underline{g}'\right]dc_1 - \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2}(1-\rho_2)\rho\underline{g}'dc_1. \quad (63)$$

Since $\rho_2 = \frac{\rho \underline{g}}{D}$ and $1 - \rho_2 = \frac{(1-\rho)\bar{g}}{D}$,

$$\rho_2(1-\rho)\bar{g} - \rho(1-\rho_2)\underline{g}' = \frac{\rho(1-\rho)}{D}[\bar{g}'\underline{g} - \bar{g}\underline{g}'] = -\frac{d\rho_2}{dc_1}D. \tag{64}$$

Thus, (63) becomes

$$\int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2}\frac{d\rho_2}{dc_1}\rho_2 D dc_1 - \int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2}(1-\rho_2)\rho\underline{g}' dc_1 + \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2}\frac{d\rho_2}{dc_1}\rho_2 D dc_1 - \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2}(1-\rho_2)\rho\underline{g}' dc_1. \tag{65}$$

Once again, use the fact that $D\rho_2 = \rho\underline{g}$, and (65) becomes

$$\int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2}\frac{d\rho_2}{dc_1}\rho\underline{g} dc_1 - \int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2}(1-\rho_2)\rho\underline{g}' dc_1 + \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2}\frac{d\rho_2}{dc_1}\rho\underline{g} dc_1 - \int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2}(1-\rho_2)\rho\underline{g}' dc_1. \tag{66}$$

Integrate the second and fourth integrals in (66) by parts:

$$\int_{-\infty}^{c_1^0} \frac{dw_2^0}{d\rho_2}(1-\rho_2)\rho\underline{g}' dc_1 = \frac{dw_2^0}{d\rho_2}(1-\rho_2)\rho\underline{g}\Big|_{-\infty}^{c_1^0} - \int_{-\infty}^{c_1^0} \left(\frac{d^2w_2^0}{d\rho_2^2}\frac{d\rho_2}{dc_1}(1-\rho_2) - \frac{dw_2^0}{d\rho_2}\frac{d\rho_2}{dc_1}\right)\rho\underline{g} dc_1, \tag{67}$$

and

$$\int_{c_1^0}^{\infty} \frac{dw_2}{d\rho_2}(1-\rho_2)\rho\underline{g}' dc_1 = \frac{dw_2}{d\rho_2}(1-\rho_2)\rho\underline{g}\Big|_{c_1^0}^{\infty} - \int_{c_1^0}^{\infty} \left(\frac{d^2w_2}{d\rho_2^2}\frac{d\rho_2}{dc_1}(1-\rho_2) - \frac{dw_2}{d\rho_2}\frac{d\rho_2}{dc_1}\right)\rho\underline{g} dc_1. \tag{68}$$

Substituting back in to (66) yields

$$\frac{dE[W_2(\rho_2)]}{d\underline{c}_1} = \int_{-\infty}^{c_1^0} \frac{d^2w_2^0}{d\rho_2^2}(1-\rho_2)\frac{d\rho_2}{dc_1}\rho\underline{g} dc_1 + \int_{c_1^0}^{\infty} \frac{d^2w_2}{d\rho_2^2}(1-\rho_2)\frac{d\rho_2}{dc_1}\rho\underline{g} dc_1 + (1-\rho_2)\rho\underline{g}\Big(\frac{dw_2}{d\rho_2} - \frac{dw_2^0}{d\rho_2}\Big)\Big|_{c_1^0}. \tag{69}$$

Since $\frac{d\rho_2}{dc_1} < 0$ by the monotone likelihood ratio property, by Lemma 1 the integrals are negative for all c_1. It is left to show that, when evaluated at c_1^0,

$$\frac{dw_2}{d\rho_2} - \frac{dw_2^0}{d\rho_2} = 0. \tag{70}$$

Lemma 1 gives the expression for $\frac{dw_2}{d\rho_2}$, and a similar argument yields

$$\frac{dw_2^0}{d\rho_2} = (1+\lambda)\big(\Delta\beta + \frac{1}{2\gamma}\big) - \lambda\underline{u}_2^0(\rho_2) - (1+\lambda)\big(\bar{e}_2^0(\rho_2) - \frac{\gamma}{2}(\bar{e}_2^0(\rho_2))^2\big). \tag{71}$$

Thus, when evaluated at c_1^0,

$$\frac{dw_2}{d\rho_2} - \frac{dw_2^0}{d\rho_2} = \lambda\left[\underline{u}_2^0(\rho_2^0) - \underline{u}_2(\rho_2^0)\right] + (1+\lambda)\left[\bar{e}_2^0(\rho_2^0) - \bar{e}_2(\rho_2^0) + \frac{\gamma}{2}(\bar{e}_2(\rho_2^0))^2 - \frac{\gamma}{2}(\bar{e}_2^0(\rho_2^0))^2\right]. \tag{72}$$

From Proposition 1, $\underline{u}_2^0(\rho_2^0) - \underline{u}_2(\rho_2^0) = 0$. Further,

$$\bar{e}_2^0(\rho_2^0) = \Delta\beta = \bar{e}_2(\rho_2^0). \tag{73}$$

Thus,

$$\frac{dw_2}{d\rho_2} - \frac{dw_2^0}{d\rho_2} = 0, \tag{74}$$

and

$$\frac{dE[W_2(\rho_2)]}{d\underline{c}_1} = \int_{-\infty}^{c_1^0} \frac{d^2w_2^0}{d\rho_2^2}(1-\rho_2)\frac{d\rho_2}{dc_1}\rho\underline{g}dc_1 + \int_{c_1^0}^{\infty} \frac{d^2w_2}{d\rho_2^2}(1-\rho_2)\frac{d\rho_2}{dc_1}\rho\underline{g}dc_1 < 0. \tag{75}$$

A similar proof shows that

$$\frac{dE[W_2(\rho_2)]}{d\bar{c}_1} = -\left[\int_{-\infty}^{c_1^0} \frac{d^2w_2^0}{d\rho_2^2}(1-\rho_2)\frac{d\rho_2}{dc_1}\rho\underline{g}dc_1 + \int_{c_1^0}^{\infty} \frac{d^2w_2}{d\rho_2^2}(1-\rho_2)\frac{d\rho_2}{dc_1}\rho\underline{g}dc_1\right] > 0. \tag{76}$$

Thus, the effect of expected second period welfare is to increase the distance between the first period cost targets. □

5 Equilibrium ratchet effect

The analysis has shown that the optimal first period contract is determined by two competing forces. The effect of the low cost type's first period transfer is to decrease the distance between first period cost targets, while expected second period welfare has the opposite impact. This section determines the net effect that these opposing incentives have on the first period contract by considering the transfer and welfare effects together.

We are able to show that when the distribution of noise puts little weight on cost observations less than c_1^0, the regulator favors rent preservation to learning. That is, the overall effect is to decrease the distance between first period cost targets; the low cost firm exerts less effort effort than in the commitment optimum, while the high cost firm exerts more.

There are two reasons for considering this restriction. First, while the full support assumption is analytically convenient, it implies that negative (or unrealistically low) cost observations are possible. Ignoring this possibility paints a more accurate picture of the regulator's trade-offs when designing the first period contract.

Second, consider the regulator's problem when $\bar{e}_2 - \Delta\beta < 0$. This mimicking effort can only be negative when the regulator's belief that the firm is the low cost type is close to one. Under such beliefs, the regulator sets the high cost firm's cost-reducing effort close to zero in order extract rent from the high cost type. Therefore, it becomes costly to offer a contract for both types of firm; on the off chance that the firm is the high cost type, he exerts almost no cost reducing effort.

If the value of the project, S, is not so large that the project is worth implementing "at all costs," the regulator finds it beneficial to offer only the first best contract to the low cost type when beliefs are close to one.[1] The possibility that the regulator shuts down the high cost firm is akin to ignoring low cost observations. This is because the first-best contract does not depend on beliefs, and leaves no rent to the low cost firm. When the regulator shuts down the high cost firm, the first period contract does not have any impact on the regulator's second period problem when the first period cost is low.

In light of this discussion, consider that the regulator's full first period problem can be written

$$\begin{aligned}\max_{\underline{c}_1, \bar{c}_1} \quad & S - \rho\left[(1+\lambda)\left(\underline{c}_1 + \frac{\gamma}{2}(\underline{\beta} - \underline{c}_1)^2\right) + \lambda\left(\frac{\gamma}{2}(\bar{\beta} - \bar{c}_1)^2 - \frac{\gamma}{2}(\underline{\beta} - \bar{c}_1)^2\right)\right] \\ & - (1-\rho)(1+\lambda)\Big(\bar{c}_1 + \frac{\gamma}{2}(\bar{\beta} - \bar{c}_1)^2\Big) + \delta\left[\rho\underline{w}^{FB} + (1-\rho)\left(\overline{w}^{FB} - \frac{1+\lambda}{2\gamma}\right)\right] \\ & + \delta\int_{-\infty}^{c_1^0}\left\{(1-\rho)(1+\lambda)\bar{e}_2^0 - (1+\lambda-\rho)\frac{\gamma}{2}(\bar{e}_2^0)^2\right\}\bar{g}dc_1 \\ & + \delta\int_{c_1^0}^{\infty}\left\{(1-\rho)(1+\lambda)\bar{e}_2 - (1+\lambda-\rho)\frac{\gamma}{2}\bar{e}_2^2 + \rho\lambda\frac{\gamma}{2}(\bar{e}_2 - \Delta\beta)^2\right\}\bar{g}dc_1, \end{aligned} \tag{77}$$

where $\underline{w}^{FB} = S - (1+\lambda)\left(\underline{\beta} - \frac{1}{2\gamma}\right)$ and $\overline{w}^{FB} = S - (1+\lambda)\left(\bar{\beta} - \frac{1}{2\gamma}\right)$ are the first best welfare

[1] A necessary condition for this to be optimal is $S - (1+\lambda)\bar{\beta} < 0$.

for the low and high cost firm, respectively. Now, define

$$A := (1-\rho)(1+\lambda)\bar{e}_2^0 - (1+\lambda-\rho)\frac{\gamma}{2}(\bar{e}_2^0)^2, \tag{78}$$

and

$$B := (1-\rho)(1+\lambda)\bar{e}_2 - (1+\lambda-\rho)\frac{\gamma}{2}\bar{e}_2^2 \ + \rho\lambda\frac{\gamma}{2}(\bar{e}_2 - \Delta\beta)^2. \tag{79}$$

The first order conditions of this problem imply the following effort levels for the low and high cost firm:

$$\underline{\beta} - \underline{c}_1 = \frac{1}{\gamma} - \frac{\delta}{\rho(1+\lambda)\gamma}\frac{d}{d\underline{c}_1}\left[\int_{-\infty}^{c_1^0} A\,\bar{g}\,dc_1 + \int_{c_1^0}^{\infty} B\,\bar{g}\,dc_1\right], \tag{80}$$

$$\begin{aligned}\bar{\beta} - \bar{c}_1 =& \frac{1}{\gamma} - \frac{\rho\lambda}{(1-\rho)(1+\lambda)}\Delta\beta \\ &- \frac{\delta}{(1-\rho)(1+\lambda)\gamma}\frac{d}{d\bar{c}_1}\left[\int_{-\infty}^{c_1^0} A\,\bar{g}\,dc_1 + \int_{c_1^0}^{\infty} B\,\bar{g}\,dc_1\right].\end{aligned} \tag{81}$$

The following proposition establishes the result that when the distribution of noise places little weight on cost realizations less than c_1^0, the effort in (80) is less than $\frac{1}{\gamma}$, while the effort in (81) is greater than $\frac{1}{\gamma} - \frac{\rho\lambda}{(1-\rho)(1+\lambda)}\Delta\beta$.

Proposition 5. ***The Ratchet Effect:*** *When the distribution of first period costs places little weight on cost observations less than c_1^0, the regulator's desire to protect information rents is stronger than the desire to learn. That is,*

$$\frac{d}{d\underline{c}_1}\left[\int_{-\infty}^{c_1^0} A\,\bar{g}\,dc_1 + \int_{c_1^0}^{\infty} B\,\bar{g}\,dc_1\right] > 0, \tag{82}$$

$$\frac{d}{d\bar{c}_1}\left[\int_{-\infty}^{c_1^0} A\,\bar{g}\,dc_1 + \int_{c_1^0}^{\infty} B\,\bar{g}\,dc_1\right] < 0. \tag{83}$$

Proof. First, consider

$$\frac{d}{d\underline{c}_1}\left[\int_{-\infty}^{c_1^0} A\,\bar{g}\,dc_1 + \int_{c_1^0}^{\infty} B\,\bar{g}\,dc_1\right] = \int_{-\infty}^{c_1^0} A'\frac{d\bar{e}_2^0}{d\rho_2}\frac{d\rho_2}{d\underline{c}_1}\bar{g}dc_1 + \int_{c_1^0}^{\infty} B'\frac{d\bar{e}_2}{d\rho_2}\frac{d\rho_2}{d\underline{c}_1}\bar{g}dc_1, \tag{84}$$

where

$$A' = (1-\rho)(1+\lambda) - (1+\lambda-\rho)\gamma\bar{e}_2^0 \tag{85}$$

and

$$B' = (1-\rho)(1+\lambda) - (1+\lambda-\rho)\gamma\bar{e}_2 + \rho\lambda\gamma(\bar{e}_2 - \Delta\beta). \tag{86}$$

Using the definition of $\frac{d\rho_2}{d\underline{c}_1}$, (84) can be re-written

$$\frac{-\rho}{1-\rho}\left[\int_{-\infty}^{c_1^0} A'\frac{d\bar{e}_2^0}{d\rho_2}(1-\rho_2)^2\underline{g}'dc_1 + \int_{c_1^0}^{\infty} B'\frac{d\bar{e}_2}{d\rho_2}(1-\rho_2)^2\underline{g}'dc_1\right] \tag{87}$$

Integration by parts yields

$$\begin{aligned} \frac{-\rho}{1-\rho}\Bigg[& A'\frac{d\bar{e}_2^0}{d\rho_2}(1-\rho_2)^2\underline{g}\Big|_{-\infty}^{c_1^0} \\ & - \int_{-\infty}^{c_1^0}\left\{-(1+\lambda-\rho)\gamma\left(\frac{d\bar{e}_2^0}{d\rho_2}\right)^2(1-\rho_2) + A'\frac{d^2\bar{e}_2^0}{d\rho_2^2}(1-\rho_2) - 2A'\frac{d\bar{e}_2^0}{d\rho_2}\right\}\frac{d\rho_2}{dc_1}(1-\rho_2)\underline{g}dc_1 \\ & + B'\frac{d\bar{e}_2}{d\rho_2}(1-\rho_2)^2\underline{g}\Big|_{c_1^0}^{\infty} + \int_{c_1^0}^{\infty}(1-\rho)(1+\lambda)\gamma\left(\frac{d\bar{e}_2}{d\rho_2}\right)^2(1-\rho_2)^2\frac{d\rho_2}{dc_1}\underline{g}dc_1\Bigg]. \end{aligned} \tag{88}$$

This can be re-written

$$\begin{aligned} \frac{-\rho}{1-\rho}\Bigg[& \left(A'\frac{d\bar{e}_2^0}{d\rho_2} - B'\frac{d\bar{e}_2}{d\rho_2}\right)(1-\rho_2)^2\underline{g}\Big|_{c_1^0} \\ & - \int_{-\infty}^{c_1^0}\left\{-(1+\lambda-\rho)\gamma\left(\frac{d\bar{e}_2^0}{d\rho_2}\right)^2(1-\rho_2) + A'\frac{d^2\bar{e}_2^0}{d\rho_2^2}(1-\rho_2) - 2A'\frac{d\bar{e}_2^0}{d\rho_2}\right\}\frac{d\rho_2}{dc_1}(1-\rho_2)\underline{g}dc_1 \\ & + \int_{c_1^0}^{\infty}(1-\rho)(1+\lambda)\gamma\left(\frac{d\bar{e}_2}{d\rho_2}\right)^2(1-\rho_2)^2\frac{d\rho_2}{dc_1}\underline{g}dc_1\Bigg]. \end{aligned} \tag{89}$$

Now, consider the first term in (89). When evaluated at $c_1 = c_1^0$, $\bar{e}_2 = \bar{e}_2^0 = \Delta\beta$. Thus, $A' = B' = (1-\rho)(1+\lambda) - (1+\lambda-\rho)\gamma\Delta\beta$. Further, when $c_1 = c_1^0$, posterior beliefs are given by (17). Thus,

$$\frac{d\bar{e}_2^0}{d\rho_2}\Big|_{c_1^0} = \frac{-\lambda(1+\lambda)}{\gamma}\frac{1}{(1+\lambda-\rho_2^0)^2} = \frac{-(1+\lambda-\gamma\Delta\beta)^2}{\lambda(1+\lambda)\gamma}, \tag{90}$$

and

$$\frac{d\bar{e}_2}{d\rho_2}\Big|_{c_1^0} = \frac{-1}{(1-\rho_2^0)^2}\frac{\lambda}{1+\lambda}\Delta\beta = \frac{-(1+\lambda-\gamma\Delta\beta)^2}{\lambda(1+\lambda)\gamma^2\Delta\beta}. \tag{91}$$

Therefore, the first term in (89) can be expressed

$$[(1-\rho)(1+\lambda) - (1+\lambda-\rho)\gamma\Delta\beta]\left[\frac{(1-\gamma\Delta\beta)(1+\lambda-\gamma\Delta\beta)^2}{\lambda(1+\lambda)\gamma^2\Delta\beta}\right]\frac{\lambda^2\gamma^2\Delta\beta^2}{(1+\lambda-\gamma\Delta\beta)^2}\underline{g}(c_1^0) \tag{92}$$

$$= (1-\rho)\left[1 - \frac{1+\lambda-\rho}{(1-\rho)(1+\lambda)}\gamma\Delta\beta\right]\lambda\Delta\beta(1-\gamma\Delta\beta)\underline{g}(c_1^0) \tag{93}$$

Consider each term in (93). Clearly, $1-\rho < 1$. The term in brackets is equal to $\gamma(\bar{e}_2(\rho)-\Delta\beta)$, which is the derivative of $\frac{\gamma}{2}\left(\bar{e}_2(\rho)-\Delta\beta\right)^2$ with respect to $\bar{e}_2(\rho)$ in a static game in which beliefs are given by the prior, ρ. This can be assumed to be positive, and is less than one since this mimicking effort is less than the first best. Further, $0 < 1-\gamma\Delta\beta < 1$, so that as long as $\underline{g}(c_1^0) \approx 0$, this term can be ignored when signing the equilibrium effect on the low cost firm's effort.

Return attention to

$$\frac{-\rho}{1-\rho}\left[-\int_{-\infty}^{c_1^0}\left\{-(1+\lambda-\rho)\gamma\left(\frac{d\bar{e}_2^0}{d\rho_2}\right)^2(1-\rho_2)+A'\frac{d^2\bar{e}_2^0}{d\rho_2^2}(1-\rho_2)-2A'\frac{d\bar{e}_2^0}{d\rho_2}\right\}\frac{d\rho_2}{dc_1}(1-\rho_2)\underline{g}dc_1 \right.$$
$$\left. + \int_{c_1^0}^{\infty}(1-\rho)(1+\lambda)\gamma\left(\frac{d\bar{e}_2}{d\rho_2}\right)^2(1-\rho_2)^2\frac{d\rho_2}{dc_1}\underline{g}dc_1\right]. \tag{94}$$

For the low cost firm to exert less than the first best in the first period, the sign of (94) should be positive. Consider the second integral; since $\frac{d\rho_2}{dc_1} < 0$ by the monotone likelihood ratio property, and every other term under the integral is positive, the integral is negative for every $c_1 \in (c_1^0, \infty)$. The first integral is more difficult to sign. To show that the term in brackets is negative would be sufficient to achieve the desired result. However, for

$$\rho_2 \in \left[\frac{1+\lambda-\rho+2\lambda\rho}{1+3\lambda-\rho}, 1\right], \tag{95}$$

the term in brackets is positive. Thus, the first integral in (94) must be outweighed by the second. This is the case when the distribution of first period costs puts little weight on cost realizations less than c_1^0. In this case,

$$\int_{-\infty}^{c_1^0}\left\{-(1+\lambda-\rho)\gamma\left(\frac{d\bar{e}_2^0}{d\rho_2}\right)^2(1-\rho_2)+A'\frac{d^2\bar{e}_2^0}{d\rho_2^2}(1-\rho_2)-2A'\frac{d\bar{e}_2^0}{d\rho_2}\right\}\frac{d\rho_2}{dc_1}(1-\rho_2)\underline{g}dc_1 \approx 0, \tag{96}$$

and

$$\frac{d}{d\underline{c}_1}\left[\int_{-\infty}^{c_1^0} A\,\bar{g}\,dc_1 + \int_{c_1^0}^{\infty} B\,\bar{g}\,dc_1\right]$$
$$\approx \frac{-\rho}{1-\rho}\int_{c_1^0}^{\infty}(1-\rho)(1+\lambda)\gamma\left(\frac{d\bar{e}_2}{d\rho_2}\right)^2(1-\rho_2)^2\frac{d\rho_2}{dc_1}\underline{g}dc_1 > 0. \tag{97}$$

A similar proof shows that

$$\frac{d}{d\bar{c}_1}\left[\int_{-\infty}^{c_1^0} A\,\bar{g}\,dc_1 + \int_{c_1^0}^{\infty} B\,\bar{g}\,dc_1\right] = -\frac{d}{d\underline{c}_1}\left[\int_{-\infty}^{c_1^0} A\,\bar{g}\,dc_1 + \int_{c_1^0}^{\infty} B\,\bar{g}\,dc_1\right]. \tag{98}$$

Thus, when one restricts the distribution of first period costs to put little weight on cost observations less than c_1^0, the low cost firm's effort in the first period is below the first best, and his effort is increased over the course of the interaction with the regulator. □

Under the conditions outlined in Proposition 5, the value of information is decreased in a repeated relationship; not only is the regulator content to have imperfect information in the second period, but she chooses to learn less than she could by implementing the optimal static cost targets. This is because the benefit of better information in the second period does not outweigh the concomitant increase in the low cost type's expected first period transfer.

6 Conclusion

In this two-period model of regulation, the regulator and the firm contract over the completion of a socially valuable project. The firm has private information about its intrinsic cost level, which can be high or low, and has imperfect control over the project's final cost (costs are stochastic). In this setting, the regulator determines how much information she gathers about the firm's type via her choice of first period cost targets.

The regulator can gather more information about the firm by increasing the distance between first period cost targets. The better the regulator's information is about the firm's type in the second period, the higher is expected second period welfare. Conversely, the regulator can commit to gather less information about the firm by decreasing the distance between first period cost targets. The worse the regulator's information is about the firm's type in the second period, the higher is the low cost firm's equilibrium second period welfare, and the lower is its benefit from mimicking the high cost firm. Thus, by decreasing the distance between first period cost targets, the regulator decreases the low cost firm's first period transfer.

If the distribution of noise places little weight on low cost realizations, or if the project is not worth implementing when the high cost firm exerts little cost-reducing effort, the net effect of the first period contract is to decrease the distance between the first period cost targets.

Thus, the regulator's desire to reduce the first period transfer is stronger than her desire to improve expected second period welfare.

In these settings, the low cost type exerts less than the first-best effort in the first period, and has his effort ratcheted up over the course of his interaction with the regulator. Anecdotal evidence of the ratchet effect, as in Matthewson (1931), suggests that agents with favorable private information preserve their future rents by taking actions to keep this information private. Thus, the prediction that the low cost firm increases his effort over time aligns closely with observed repeated principal-agent interactions.

7 Appendix

7.1 High cost type's first period incentive constraint

Since the high cost type's participation constraint binds in expectation, it is sufficient to check that

$$\underline{t}_1 - \frac{\gamma}{2}\left(\bar{\beta} - \underline{c}_1\right)^2 \leq 0. \tag{99}$$

Substituting for $\underline{t}_1$ from (28) and simplifying, this requires

$$\frac{\delta}{\gamma \Delta\beta} \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g}) dc_1 \leq \bar{c}_1 - \underline{c}_1. \tag{100}$$

Now, from (33) and (34),[2]

$$\begin{aligned} \bar{c}_1 - \underline{c}_1 = & \frac{1+\lambda-\rho}{(1-\rho)(1+\lambda)}\Delta\beta \\ & + \frac{\delta}{\gamma\rho(1-\rho)(1+\lambda)} \frac{d}{d\underline{c}_1}\left[\rho\lambda \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g}) dc_1 - E[W_2]\right]. \end{aligned} \tag{102}$$

[2]And using the fact that

$$\frac{d}{d\bar{c}_1}\left[\rho\lambda \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g}) dc_1 - E[W_2]\right] = -\frac{d}{d\underline{c}_1}\left[\rho\lambda \int_{\mathbb{R}} \underline{U}_2(\rho_2)(\bar{g} - \underline{g}) dc_1 - E[W_2]\right] \tag{101}$$

Thus, the high cost firm's incentive constraint is satisfied when

$$\begin{aligned}\frac{1+\lambda-\rho}{(1-\rho)(1+\lambda)}\Delta\beta \geq & \frac{\delta}{\gamma\rho(1-\rho)(1+\lambda)}\frac{d}{d\underline{c}_1}E[W_2] \\ & -\frac{\delta\lambda}{\gamma(1-\rho)(1+\lambda)}\frac{d}{d\underline{c}_1}\int_{\mathbb{R}}\underline{U}_2(\rho_2)(\bar{g}-\underline{g})dc_1 \\ & +\delta\int_{\mathbb{R}}\underline{U}_2(\rho_2)(\bar{g}-\underline{g})dc_1. \end{aligned} \tag{103}$$

From Proposition 4, $\frac{d}{d\underline{c}_1}E[W_2] < 0$. Therefore, it must be checked that when the variance is sufficiently large,

$$-\frac{\delta\lambda}{\gamma(1-\rho)(1+\lambda)}\frac{d}{d\underline{c}_1}\int_{\mathbb{R}}\underline{U}_2(\rho_2)(\bar{g}-\underline{g})dc_1 + \delta\int_{\mathbb{R}}\underline{U}_2(\rho_2)(\bar{g}-\underline{g})dc_1 \approx 0. \tag{104}$$

From Proposition 3,

$$\frac{d}{d\underline{c}_1}\int_{\mathbb{R}}\underline{U}_2(\rho_2)(\bar{g}-\underline{g})dc_1 = \int_{c_1^0}^{\infty}\frac{d\underline{u}_2}{d\rho_2}k\left[\underline{g}'\bar{g}^2-\underline{g}^2\bar{g}'\right]dc_1 + \int_{-\infty}^{c_1^0}\frac{d\underline{u}_2^0}{d\rho_2}k\left[\underline{g}'\bar{g}^2-\underline{g}^2\bar{g}'\right]dc_1. \tag{105}$$

As the variance of first period cost increases, the slope of the density goes to zero. As the slope of the density goes to zero, so too does $\frac{d}{d\underline{c}_1}\int_{\mathbb{R}}\underline{U}_2(\rho_2)(\bar{g}-\underline{g})dc_1$.

Turning attention to $\int_{\mathbb{R}}\underline{U}_2(\rho_2)(\bar{g}-\underline{g})dc_1$, integration by parts yields

$$\int_{\mathbb{R}}\underline{U}_2(\rho_2)(\bar{g}-\underline{g})dc_1 = -\left[\int_{-\infty}^{c_1^0}\frac{d\underline{u}_2^0}{d\rho_2}\frac{d\rho_2}{dc_1}\left[\bar{G}-\underline{G}\right]dc_1 + \int_{c_1^0}^{\infty}\frac{d\underline{u}_2}{d\rho_2}\frac{d\rho_2}{dc_1}\left[\bar{G}-\underline{G}\right]dc_1\right]. \tag{106}$$

Since $\frac{d\rho_2}{dc_1} = \frac{\rho(1-\rho)[\underline{g}'\bar{g}-\underline{g}\bar{g}']}{D^2}$ goes to zero as the slope of the density goes to zero, this term is close to zero when the variance is large. Thus, the high cost type's incentive constraint is satisfied in noisy enough environments.

7.2 Robustness to specification of cost of effort

This section demonstrates that the competing effects of the low cost firm's first period transfer and expected second period welfare on the first period contract do not depend on the specification of the firm's cost of effort. Suppose instead of the quadratic cost of effort case considered above, the firm's cost of effort was given by $\psi(e)$, where ψ is increasing and convex. In the second period, there still exists a c_1^0 such that for $c_1 < c_1^0$, the low cost firm's cost of effort from mimicking the high cost firm is negative. Thus, for $c_1 > c_1^0$, the second period incentive compatibility constraint for the low cost type is given by

$$\underline{t}_2 - \psi(\underline{\beta}-\underline{c}_2) = \bar{t}_2 - \psi(\underline{\beta}-\bar{c}_2), \tag{107}$$

and the high cost type's participation constraint by

$$\bar{t}_2 - \psi(\bar{\beta} - \bar{c}_2) = 0. \tag{108}$$

Again, $\underline{t}_2$ and $\bar{t}_2$ are the expectation of $t_2(c_2)$ over the distributions $\underline{g}$ and $\bar{g}$, respectively. The regulator's second period problem is

$$\max_{\underline{c}_2, \bar{c}_2} \quad S - \rho_2 \Big[(1+\lambda) \Big(\underline{c}_2 + \psi(\underline{\beta} - \underline{c}_2) \Big) + \lambda \Phi(\bar{\beta} - \bar{c}_2) \Big] - (1-\rho_2)(1+\lambda) \Big(\bar{c}_2 + \psi(\bar{\beta} - \bar{c}_2) \Big) \tag{109}$$

The first order conditions imply the following equilibrium effort conditions for the low and high cost firm:

$$\psi'(\underline{e}_2) = 1 \tag{110}$$

and

$$\psi'(\bar{e}_2) = 1 - \frac{\rho_2}{1-\rho_2} \frac{\lambda}{1+\lambda} \Phi'(\bar{e}_2). \tag{111}$$

The low cost firm's rent is given by

$$\Phi(\bar{e}_2) = \psi(\bar{e}_2) - \psi(\bar{e}_2 - \Delta\beta). \tag{112}$$

A standard result is that the low cost type's rent is increasing in the high cost type's effort,

$$\frac{\partial \Phi}{\partial \bar{e}_2} = \psi'(\bar{e}_2) - \psi'(\bar{e}_2 - \Delta\beta) > 0, \tag{113}$$

since $\Delta\beta > 0$ and $\psi'' > 0$.

From (111), we can see that the high cost type's second period effort depends on the regulator's second period beliefs, ρ_2. By implicitly differentiating (111), we see that

$$\frac{\partial \bar{e}_2}{\partial \rho_2} = - \frac{\frac{1}{(1-\rho_2)^2} \frac{\lambda}{1+\lambda} \Phi'(\bar{e}_2)}{\psi''(\bar{e}_2) + \frac{\rho_2}{1-\rho_2} \frac{\lambda}{1+\lambda} \Phi''(\bar{e}_2)} < 0. \tag{114}$$

Together, (113) and (114) imply

$$\frac{\partial \Phi(\bar{e}_2)}{\partial \rho_2} = \frac{\partial \Phi(\bar{e}_2)}{\partial \bar{e}_2} \cdot \frac{d\bar{e}_2}{d\rho_2} < 0. \tag{115}$$

When $c_1 \leq c_1^0$, $\bar{e}_2 - \Delta\beta \leq 0$. Thus, the low cost type's incentive compatibility constraint is given by

$$\underline{t}_2 - \psi(\underline{\beta} - \underline{c}_2) = \bar{t}_2. \tag{116}$$

The high cost type's participation constraint remains unchanged, therefore the regulator's second period problem is given by

$$\max_{\underline{c}_2,\bar{c}_2} \quad S-\rho_2\Big[(1+\lambda)\Big(\underline{c}_2+\psi(\underline{\beta}-\underline{c}_2)\Big)+\lambda\psi(\bar{\beta}-\bar{c}_2)\Big]-(1-\rho_2)(1+\lambda)\Big(\bar{c}_2+\psi(\bar{\beta}-\bar{c}_2)\Big) \tag{117}$$

The first order condition for the low cost type's cost target implies the first best effort, (110). The first order condition for the high cost type, however, implies that

$$\psi'(\bar{e}_2)=\frac{(1-\rho_2)(1+\lambda)}{1+\lambda-\rho_2}. \tag{118}$$

When $c_1 > c_1^0$, second period welfare is given by

$$W_2(\rho_2)=S-\rho_2\Big[(1+\lambda)\Big(\underline{\beta}-\underline{e}^*+\psi(\underline{e}^*)\Big)+\lambda\Phi(\bar{e}_2)\Big]-(1-\rho_2)(1+\lambda)\Big(\bar{\beta}-\bar{e}_2+\psi(\bar{e}_2)\Big), \tag{119}$$

and when $c_1 \leq c_1^0$,

$$W_2^0(\rho_2)=S-\rho_2\Big[(1+\lambda)\Big(\underline{\beta}-\underline{e}^*+\psi(\underline{e}^*)\Big)+\lambda\psi(\bar{e}_2)\Big]-(1-\rho_2)(1+\lambda)\Big(\bar{\beta}-\bar{e}_2+\psi(\bar{e}_2)\Big). \tag{120}$$

From the perspective of the first period, the low cost type's incentive compatibility constraint

$$\begin{aligned}\underline{t}_1-\frac{\gamma}{2}(\underline{\beta}-\underline{c}_1)^2+\delta\Big[\int_{c_1^0}^{\infty}\Phi(\bar{e}_2)\underline{g}dc_1+\int_{-\infty}^{c_1^0}\psi(\bar{e}_2)\underline{g}dc_1\Big]&\\ =\bar{t}_1-\frac{\gamma}{2}(\underline{\beta}-\bar{c}_1)^2+\delta\Big[\int_{c_1^0}^{\infty}\Phi(\bar{e}_2)\bar{g}dc_1+\int_{-\infty}^{c_1^0}\psi(\bar{e}_2)\bar{g}dc_1\Big].&\end{aligned} \tag{121}$$

Expected second period welfare is

$$E[W_2]=\int_{c_1^0}^{\infty}W_2(\rho_2)[\rho\underline{g}+(1-\rho)\bar{g}]dc_1+\int_{-\infty}^{c_1^0}W_2^0(\rho_2)[\rho\underline{g}+(1-\rho)\bar{g}]dc_1 \tag{122}$$

. The regulator's full first period problem is

$$\begin{aligned}\max_{\underline{c}_1,\bar{c}_1} \quad S-\rho\Big[(1+\lambda)\Big(\underline{c}_1+\frac{\gamma}{2}(\underline{\beta}-\underline{c}_1)^2\Big)+\lambda\Big(\frac{\gamma}{2}(\bar{\beta}-\bar{c}_1)^2-\frac{\gamma}{2}(\underline{\beta}-\bar{c}_1)^2+\delta\int_{c_1^0}^{\infty}\Phi(\bar{e}_2)(\bar{g}-\underline{g})dc_1\\ +\delta\int_{-\infty}^{c_1^0}\psi(\bar{e}_2)(\bar{g}-\underline{g})dc_1\Big)\Big]-(1-\rho)(1+\lambda)\Big(\bar{c}_1+\frac{\gamma}{2}(\bar{\beta}-\bar{c}_1)^2\Big)\Big]+\delta E[W_2].\end{aligned} \tag{123}$$

7.2.1 Signal dampening

The first order condition of the regulator's full first period problem, ignoring the first period contract's effects on expected second period welfare, is given by

$$\underline{\beta}-\underline{c}_1=\frac{1}{\gamma}+\frac{\delta\lambda}{\gamma(1+\lambda)}\frac{d}{d\underline{c}_1}\Big[\int_{c_1^0}^{\infty}\Phi(\bar{e}_2)(\bar{g}-\underline{g})dc_1+\int_{-\infty}^{c_1^0}\psi(\bar{e}_2)(\bar{g}-\underline{g})dc_1\Big] \tag{124}$$

When the cost of effort has a general functional form, the regulator signal dampens.

$$\frac{d}{d\underline{c}_1}\Big[\int_{c_1^0}^{\infty}\Phi(\bar{e}_2)(\bar{g}-\underline{g})dc_1+\int_{-\infty}^{c_1^0}\psi(\bar{e}_2)(\bar{g}-\underline{g})dc_1\Big]$$
$$=\int_{c_1^0}^{\infty}\frac{d\Phi(\bar{e}_2)}{d\rho_2}\frac{d\rho_2}{d\underline{c}_1}(\bar{g}-\underline{g})+\Phi(\bar{e}_2)\underline{g}'dc_1+\int_{-\infty}^{c_1^0}\frac{d\psi(\bar{e}_2)}{d\rho_2}\frac{d\rho_2}{d\underline{c}_1}(\bar{g}-\underline{g})+\psi(\bar{e}_2)\underline{g}'dc_1. \quad (125)$$

Integrate the second term under each integral by parts. This yields

$$\int_{c_1^0}^{\infty}\frac{d\Phi(\bar{e}_2)}{d\rho_2}\Big[\frac{d\rho_2}{d\underline{c}_1}\bar{g}-\big(\frac{d\rho_2}{d\underline{c}_1}+\frac{d\rho_2}{dc_1}\big)\underline{g}\Big]dc_1+\Phi(\bar{e}_2)\underline{g}\Big|_{c_1^0}^{\infty}$$
$$+\int_{-\infty}^{c_1^0}\frac{d\psi(\bar{e}_2}{d\rho_2}\Big[\frac{d\rho_2}{d\underline{c}_1}\bar{g}-\big(\frac{d\rho_2}{d\underline{c}_1}+\frac{d\rho_2}{dc_1}\big)\underline{g}\Big]dc_1+\psi(\bar{e}_2)\underline{g}\Big|_{-\infty}^{c_1^0} \quad (126)$$

Now,

$$\psi(\bar{e}_2)\underline{g}\Big|_{c_1^0}-\Phi(\bar{e}_2)\underline{g}\Big|_{c_1^0}=\underline{g}(c_1^0)\big[\psi(\bar{e}_2^0)-\psi(\bar{e}_2^0)+\psi(0)\big]=0. \quad (127)$$

Thus, (126) becomes

$$\int_{c_1^0}^{\infty}\frac{d\Phi(\bar{e}_2)}{d\rho_2}k\Big[\underline{g}'\bar{g}^2-\underline{g}^2\bar{g}'\Big]dc_1+\int_{-\infty}^{c_1^0}\frac{d\psi(\bar{e}_2)}{d\rho_2}k\Big[\underline{g}'\bar{g}^2-\underline{g}^2\bar{g}'\Big]dc_1. \quad (128)$$

where again $k=\frac{-\rho(1-\rho)}{D^2}$. Now, both $\frac{d\Phi(\bar{e}_2)}{d\rho_2}$ and $\frac{d\psi(\bar{e}_2)}{d\rho_2}$ are negative, so their respective products with k are positive. By a proof identical to the proof of Proposition 1, the term in brackets in each integral is negative. Thus, the regulator signal dampens.

7.2.2 Experimentation

Lemma 2. *Information is valuable. That is,*

$$\frac{d^2W_2(\rho_2)}{d\rho_2^2}>0. \quad (129)$$

Proof. Notice

$$W_2(\rho_2)=\underset{\underline{e}_2,\bar{e}_2}{\operatorname{argmax}}\, S-\rho_2\Big[(1+\lambda)\big(\underline{\beta}-\underline{e}_2+\psi(\underline{e}_2)\big)+\lambda\Phi(\bar{e}_2)\Big]$$
$$-(1-\rho_2)(1+\lambda)\big(\bar{\beta}-\bar{e}_2+\psi(\bar{e}_2)\big). \quad (130)$$

By the envelope theorem,

$$\frac{dW_2(\rho_2)}{d\rho_2}=(1+\lambda)\big(\Delta\beta+\underline{e}^*-\psi(\underline{e}^*)\big)-\lambda\Phi(\bar{e}_2(\rho_2))-(1+\lambda)\big(\bar{e}_2(\rho_2)-\psi(\bar{e}_2(\rho_2))\big) \quad (131)$$

$$\Rightarrow\frac{d^2W_2(\rho_2)}{d\rho_2^2}=-\lambda\frac{d\Phi}{d\bar{e}_2}\frac{d\bar{e}_2}{d\rho_2}-(1+\lambda)\big(1-\psi'(\bar{e}_2(\rho_2))\big)\frac{d\bar{e}_2}{d\rho_2}>0. \quad (132)$$

Thus, information is valuable. □

The proof that the regulator experiments is identical in the case of a general cost of effort, except for the final step. Recall that

$$\frac{dE[W_2]}{d\underline{c}_1} = \int_{c_1^0}^{\infty} \frac{d^2W_2}{d\rho_2^2}(1-\rho_2)\frac{d\rho_2}{dc_1}\rho\underline{g}dc_1 + \int_{-\infty}^{c_1^0} \frac{d^2W_2^0}{d\rho_2^2}(1-\rho_2)\frac{d\rho_2}{dc_1}\rho\underline{g}dc_1 + (1-\rho_2)\rho\underline{g}\Big(\frac{dW_2}{d\rho_2} - \frac{dW_2^0}{d\rho_2}\Big)\Big|_{c_1^0}. \quad (133)$$

It is left to show that, when evaluated at c_1^0,

$$\frac{dW_2}{d\rho_2} - \frac{dW_2^0}{d\rho_2} = 0. \quad (134)$$

From Lemma 1, when evaluated at c_1^0,

$$\frac{dW_2}{d\rho_2} = (1+\lambda)\big(\Delta\beta + \underline{e}^* - \psi(\underline{e}^*)\big) - \lambda\Phi(\bar{e}_2(\rho_2^0)) - (1+\lambda)\big(\bar{e}_2(\rho_2^0) - \psi(\bar{e}_2(\rho_2^0))\big) \quad (135)$$

and

$$\frac{dW_2^0}{d\rho_2} = (1+\lambda)\big(\Delta\beta + \underline{e}^* - \psi(\underline{e}^*)\big) - \lambda\psi(\bar{e}_2^0(\rho_2^0)) - (1+\lambda)\big(\bar{e}_2^0(\rho_2^0) - \psi(\bar{e}_2^0(\rho_2^0))\big) \quad (136)$$

One can easily verify that $\bar{e}_2(\rho_2^0) = \bar{e}_2^0(\rho_2^0)$. Thus, the effect of expected second period welfare on the first period contract is to decrease the distance between cost targets.

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