

Dynamic Lending in the Ghatak/Stiglitz/Weiss adverse selection model, with Comparison to Group Lending

Christian Ahlin

Brian Waters*

[PRELIMINARY AND INCOMPLETE]

October 2011

Abstract

We derive an optimal dynamic lending contract in a simple adverse selection model with limited commitment on the borrower side. An optimal contract involves “penalty” rates after a default, and favorable rates after a success. It also charges higher rates for first-time borrowers than for repeat borrowers, as in “relationship” lending. We compare the efficiency of a group lending contract (of the kind popularized by the micro-credit movement) to the dynamic, individualized contract. We find that the information revelation of both types of unconstrained contracts is similar, but the assumed constraints limit the usefulness of the information revealed in different ways. As a result, group lending can achieve full efficiency under some conditions where dynamic lending fails to. However, when neither type of lending achieves full efficiency, dynamic lending can outperform group lending, by charging high interest rates to borrowers who have defaulted even though this excludes unlucky safe borrowers. We discuss factors push toward one contract form or the other, and also characterize optimal dynamic group contracts.

*We thank Abhijit Banerjee, Philip Bond, Maitreesh Ghatak, Rocco Machiavello, Alberto Motta, Quan Wen, and participants at NEUDC 2009 and ThReD Conferences in Mexico (2008) and Barcelona (2011). All errors are ours.

1 Introduction

Group lending has been proven theoretically capable of improving the functioning of credit markets among poor borrowers lacking collateral. For example, in a variant of the adverse selection context analyzed by Stiglitz and Weiss (1981), Ghatak (1999, 2000) shows that the propensity of borrowers to group homogeneously by risk can allow a lender to include less risky borrowers in the market and revive an otherwise anemic credit market.

While group lending is still widely used by micro-lenders, there is speculation that its popularity is waning in favor of individual loan contracts. Indeed, drawbacks of group lending are apparent even in the models justifying it, for example, its increased risk (Stiglitz, 1990) and contagion properties (Besley and Coate, 1995). Further, it seems clear that in some contexts microfinance is possible without group lending, since a number of apparently leading micro-lenders lend exclusively to individuals (e.g. Bank Rakyat Indonesia); see also the evidence in Gine and Karlan (2008). Theoretically, while group lending models typically compare group contracts to individual contracts in a static context, this may undersell individual lending which is often thought to operate best when it incorporates dynamic features in a repeated relationship.

The first purpose of this paper is to derive an optimal dynamic, individualized lending contract in the Ghatak/Stiglitz/Weiss adverse selection model. We know of no other dynamic analysis of the adverse selection version of Stiglitz and Weiss (1981). This may be partly because dynamic lending is often viewed as a tool to offer borrowers better incentives, a central issue in the moral hazard context but irrelevant in the adverse selection context. (Indeed, the moral hazard version is analyzed dynamically in Stiglitz and Weiss (1983).) However, while dynamic lending will not work by providing dynamic incentives for effort, safe project choice, or repayment in this environment, its effectiveness will come through enabling the lender to better price for risk over time as behavior reveals riskiness.

A second purpose of the paper is to compare the efficiency of group lending contracts, analyzed by Ghatak (1999, 2000), with that of dynamic, individualized lending contracts.

This comparison adds to the literature by using a dynamic contract as the counterfactual benchmark against which to compare group lending. It will also offer insight by comparing across two contracts that reveal the same amount of information and, without constraints, could achieve exactly the same things. Thus, the nature of constraints on the contracts are highlighted as critical for how well they work in practice.

We impose a number of constraints on both types of contracts. A limited liability constraint ensures payments are limited to project returns. In the dynamic lending context, we assume limited commitment on the side of the borrower, that is, that the borrower can opt out of future loans. We also impose monotonicity constraints in both contexts that ensure that the amount due to the bank is not declining in the success level of the borrower (see Innes, 1990, and Gangopadhyay et al., 2005, “GGL”). In other words, contracts cannot give incentives for borrowers to prefer to claim success for themselves or a group member in order to lower their payment to the bank.

The first results concern optimal contracting in a two-type, two-period model of dynamic lending. We find that the optimal contract leads to one of three outcomes, ranked by efficiency, when standard, static individual contracts lead to exclusion of safe borrowers.

1. All safe borrowers are included in both periods. Borrowers pay a high interest rate on their first loan, a low interest rate on their second loan if they have not defaulted, and a moderate interest rate on their second loan if they have defaulted.
2. If 1) is not possible, the interest rate on the second loan of first-period defaulters is raised, leading to the exclusion of unlucky safe borrowers in the second period. Borrowers pay a high interest rate on their first loan and a zero (high) interest rate on their second loan if they have not (have) defaulted.
3. If 2) is not possible, safe borrowers are excluded in both periods.

Thus, under dynamic lending interest rates are front-loaded and discounts are back-loaded – terms get better over time, and this can even be true for defaulters. The model

can thus provide a rationalization of relationship lending. Here, it arises from the ability of borrowers to end the relationship, which pushes the lender to charge more upfront so later rates can be low enough to promote continued borrowing.

In terms of comparison, results show that static group lending (group size of 2) is more able than dynamic lending to achieve perfect efficiency, that is, it includes all borrowers over a larger parameter space than dynamic lending. We pinpoint the source of this difference in efficiency. It is not in the amount of information revelation; both types of contracts reveal the same information. In particular, dynamic lending results in two (time-series) observations of project outcomes based on a borrower's type. Group lending also results in two (cross-section) observations of project outcomes based on a borrower's type, given that groups are matched homogeneously, i.e. are all of the same type. In addition to identical information revelation, one can show that the two types of contracts can achieve identical outcomes if all constraints are ignored.

The difference comes solely from the effect of the constraints on the goal of targeting more repayment to risky borrowers. Group lending targets risky borrowers by relying heavily on joint liability – which hits risky borrowers harder, conditional on success. But it is limited by the monotonicity constraint to impose no more than full liability. Dynamic lending targets risky borrowers by adding an interest rate premium for the second loan if the borrower defaulted on the first loan. But this premium is limited both by the monotonicity constraint – which puts a lower bound of zero on the interest rate after no default – and the limited commitment constraint – which puts an upper bound on the interest rate after default since it must attract safe borrowers. The latter constraints are more binding. In essence, dynamic lending does not work as well at achieving full efficiency because variation in interest rate is limited by the need to keep all borrowers interested in borrowing.

However, dynamic lending can outperform group lending when it gives up on unlucky safe borrowers, that is, by raising the interest premium for default at the expense of pricing defaulting safe borrowers out of the market. In particular, when both dynamic and group

lending fail to achieve full efficiency, dynamic lending can in some cases still achieve nearly-efficient lending with a contract as in case 2) above. In this case, all agents borrow except for safe borrowers after they default. Thus, the overall efficiency comparison is ambiguous.

Interestingly, safe borrowers typically prefer this type of contract, that is, they prefer to be priced out of the market when they fail. Though it generates less total borrower surplus, it more than compensates by increasing the share of this surplus that goes to safe borrowers. It does this by targeting more of the repayment burden to risky borrowers. Thus, there can be an equity-efficiency tradeoff, with equity favored by “punishing” failure more aggressively even though it excludes unlucky safe borrowers.

The results also point to features of the contracting environment that would tilt the scales in favor of group or dynamic lending. One such factor is the nature of correlated risk faced by the borrowers. If risk is heavily serially correlated, then dynamic lending is not effective in revealing information about borrower type; if risk is heavily spatially correlated, then group lending is similarly ineffective. We point to other factors that alter the comparison, some of which can potentially be taken to data.

Of course, group lending and dynamic lending are not mutually exclusive. We characterize the optimal dynamic group contract (2-period, 2-member), and show it can achieve fully efficient lending under weaker conditions than either static group lending or dynamic individual lending. The contract is essentially a hybrid of group and dynamic features: it involves full liability on all loans (as in group lending), and it involves low second period rates after group repayment and moderate rates after group default (as in dynamic lending).

Interestingly, however, dynamic rate adjustments can be tempered by the GGL constraint. The contract could sometimes do better by charging penalty rates after one or two failures in the group, rather than only after two; but the group would then have incentive to hide the failure, since they owe the same on the current loan whether zero or one failed (under full liability), but admitting to one failure would incur penalty rates on the second loan. As a result, the lender chooses to ease up on the dynamic penalty rate after only one

group failure.

The basic model is presented in section 2. Optimal dynamic lending is derived in section 3, and compared with static group lending in section 4. Dynamic group lending is analyzed in section 5. Section 6 concludes, and proofs are in the Appendix.

2 Baseline Model and Results

There is a unit measure continuum of risk-neutral agents. Each is endowed with no capital, one unit of labor, a subsistence option, and a project. The subsistence option requires one unit of labor and gives expected output $\bar{u} \geq 0$. The project requires one unit each of capital and labor. Agents' projects differ in risk, indexed by $p \in \mathcal{P}$. The project of an agent of type p yields gross returns of R_p ("succeeds") with probability p and yields 0 gross returns ("fails") with probability $1 - p$. The key Stiglitz and Weiss (1981) assumption is that projects have the same expected value but differ in risk, in the sense of second-order stochastic dominance:

$$p \cdot R_p = \bar{R}, \quad \forall p \in \mathcal{P}. \quad (\text{A1})$$

The higher is p , the lower is the agent's risk. Agents' types are private information.

Given their lack of capital, agents require outside funding to carry out their projects. We assume limited liability, in particular that agents' exposure in any financial contract is limited to project returns. It follows that an agent who fails owes nothing to an outside lender.

As in Ghatak (1999, 2000), we assume the lender can observe output only coarsely: it can distinguish between $Y = 0$ (fail) and $Y > 0$ (succeed), but not between different levels of $Y > 0$. This assumption along with limited liability makes debt contracts the only feasible financial contracts.¹ We also restrict attention to deterministic contracts.

We consider a non-profit lender with access to capital at gross rate $\rho > 0$. Hence, we focus

¹There are no enforcement issues by assumption: borrowers who can repay, do.

on contracts that maximize total borrower surplus subject to the lender earning expected rate of return ρ on all capital lent. Define

$$\mathcal{N} \equiv \frac{\bar{\mathbf{R}} - \bar{\mathbf{u}}}{\rho} \quad \text{and} \quad \mathcal{G} \equiv \frac{\bar{\mathbf{R}}}{\rho}. \quad (1)$$

These are the net excess return to capital (\mathcal{N}) and the gross excess return to capital (\mathcal{G}) in this market, since the numerator of \mathcal{N} (\mathcal{G}) is the net (gross) return to a unit of capital in this market and the denominator is the return to a unit of capital elsewhere. It is assumed that

$$\bar{\mathbf{R}} > \rho + \bar{\mathbf{u}} \quad \iff \quad \mathcal{N} > 1. \quad (\text{A2})$$

This implies that all projects are expected to return more than the cost of their inputs, capital and labor, so social surplus is monotonically increasing in the number of projects funded. As a result, if the lender exactly breaks even, its objective function is equivalent to lending to as many borrowers as possible, i.e. maximizing outreach.

The analysis will focus on the two-type case: $\mathcal{P} = \{p_r, p_s\}$, with $0 < p_r < p_s < 1$. Let $\theta \in (0, 1)$ be the proportion of risky, and \bar{p} be the mean risk-type: $\bar{p} = \theta p_r + (1 - \theta)p_s$.

Static individual lending. In the static case, standard individual loan contracts involve r paid after success and 0 paid after failure, due to limited liability.² An agent of type $\tau \in \{r, s\}$ will choose to borrow and undertake the project, instead of subsistence, iff

$$\bar{\mathbf{R}} - p_\tau r \geq \bar{\mathbf{u}} \quad \iff \quad r \leq \hat{r}_\tau \equiv \frac{\bar{\mathbf{R}} - \bar{\mathbf{u}}}{p_\tau}, \quad (2)$$

where the left-hand side of the first inequality is the expected revenue of the project less the expected loan payment. We can rearrange to derive the type-specific reservation interest rate, \hat{r}_τ , above which a type- τ agent will opt for subsistence; this is the second inequality.

²Instead of zero, the lender could charge a negative gross interest rate for failure, but one can show that this would be worse for safe borrowers – holding bank profits fixed – and thus not make efficient lending easier to achieve.

It is clear that $\hat{r}_s < \hat{r}_r$: if anyone chooses not to borrow, it will be the safe agents, since they repay the loan with higher probability.

Surplus is maximized when all agents borrow, in which case the bank charges an interest rate that satisfies

$$\bar{p}r = \rho \quad \iff \quad r = \frac{\rho}{\bar{p}},$$

since \bar{p} is the expected repayment rate of the average agent and thus $\bar{p}r$ is the expected return on a unit of capital lent. All are willing to borrow at this interest rate iff it is no greater than \hat{r}_s , i.e. iff

$$\frac{\rho}{\bar{p}} \leq \frac{\bar{R} - \bar{u}}{p_s} \quad \iff \quad \frac{p_s}{\bar{p}} \leq \mathcal{N}.$$

This condition is intuitive – efficient lending can be achieved if the degree of asymmetric information, captured by p_s/\bar{p} , is not greater than the excess return to capital in this market, \mathcal{N} (strictly greater than 1 by assumption A2). In this case, the borrowers get all the surplus from the projects, but risky borrowers earn more than safe due to cross-subsidies.³

If instead

$$\mathcal{N} < \mathcal{N}_{stc,ind} \equiv \frac{p_s}{\bar{p}}, \tag{A3}$$

the lender cannot break even and attract safe agents simultaneously. The next best option is to give up on safe agents and lend only to the risky, charging r to satisfy $p_r r = \rho$. All risky agents borrow at this rate, under assumption A2. Thus, under assumption A3, standard individual lending does not achieve efficiency, as the fraction of efficient projects funded is θ . Inefficiency arises from the lender's inability to price for risk.⁴

³One can also show that successful borrowers can all afford to pay r when $\mathcal{N} \geq p_s/\bar{p}$.

⁴Under perfect information, the bank can perfectly price for risk, charging ρ/p_s to safe borrowers and $\rho/p_r > \rho/p_s$ to risky borrowers. This achieves efficient and equitable lending, with all surplus going to the borrowers.

3 Dynamic, Individual Lending

Assume that agents receive the same endowment (one unit of labor, one project, one subsistence option) in each of two periods. To focus more cleanly on the key issues of risk-pricing and efficiency, discounting and consumption smoothing motives are ignored, so that borrower payoffs are just the sum of their two-period expected payoffs.

We proceed in two steps. First, we restrict attention to simple two-period pooling contracts in which the lender offers one unit of capital in each period to borrowers, payment due after any failure is zero, and no new borrower entry is allowed in period 2. Second, we argue that more general contract forms cannot improve on this simple one (if \mathcal{G} is high enough). In particular, the contract is robust to hidden saving and there is no improvement from charging negative amounts after failure; from screening contracts; from contracts that allow borrowers to enter in period 2; or from contracts that require some form of savings from the first loan and/or self-investment in or collateralization of the second loan.

Simple pooling contracts. A simple pooling contract boils down to three parameters: (r_\emptyset, r_1, r_0) , where r_\emptyset is the interest rate on the first loan and r_1 (respectively, r_0) is the interest rate on the second loan for a borrower who has succeeded (respectively, failed) in his first-period project. The subscripts refer to the agent's borrowing history – agents share the null history in period 1, and their history in period-2 is the number of period-1 successes.

As before, the contract maximizes the total payoffs of the agents subject to the bank earning ρ per unit of capital lent. With the bank's profit constraint binding, and since each project is worthwhile (assumption A2), this is equivalent to maximizing number of loans disbursed.

In addition to limited liability, several constraints are imposed on the contract. First, we assume one-sided commitment to the relationship: the lender can fully commit to the two-period contract, but the borrower cannot commit to taking a second loan. Hence, at date 2 borrowers drop out if the loan terms are such that the outside option is more attractive.

Second, we follow Innes (1990) and Gangopadhyay et al. (2005) by imposing monotonicity

constraints. These constraints ensure that borrowers pay more when they succeed than when they fail. The argument is that feigning success might be relatively easy, e.g. via very short-term loans from relatives or moneylenders, so rewarding success with lower payments is not feasible.⁵ The second-period constraints take the form

$$r_1, r_0 \geq 0 . \tag{3}$$

The right-hand side is the payment due after failure, the left-hand side the amount due after success at each history. If this were not so, the bank could directly reward success on a second-period loan with a negative gross interest rate, and a borrower would want to claim success rather than failure in order to receive the bonus. The first-period monotonicity constraint ensures that claiming success does not result in a better deal over the 2-period relationship:

$$-r_\emptyset + \max\{\bar{R} - p_\tau r_1, \bar{u}\} \leq 0 + \max\{\bar{R} - p_\tau r_0, \bar{u}\}, \tau \in \{r, s\} . \tag{4}$$

The left-hand side gives the payoff from claiming success and paying r_\emptyset , then enjoying the option of a second-period loan at interest rate r_1 (which will be exercised if it gives a better payoff than \bar{u}). The right-hand side is from claiming failure and paying nothing upfront but facing a second-period loan offer at rate r_0 . It is not obvious whether to impose this dynamic monotonicity constraint – even if paying r_\emptyset now is more than compensated by a cheaper future loan, a borrower who just failed may not be able to come up with r_\emptyset .⁶ Our approach will be not to impose it, but to discuss how it can be costlessly satisfied (in terms of borrower surplus) in the subset of the parameter space where it would be violated by the optimal contract derived below.

⁵This constraint can also be motivated as a reduced-form constraint from a costly state verification problem in which the lender only audits when a failure is reported. Since reports of success (with the required payments) go unverified, the constraint ensures there is no incentive to falsely report success.

⁶The bank cannot force a failed borrower to pay r_\emptyset , by assumption. However, borrowers may be able to come up with the money voluntarily, perhaps at some cost, e.g. from relatives or moneylenders.

Define a type- τ agent's two-period payoff from taking the loan in period 1 and, only if optimal, in period 2, as $\Pi_\tau(r_\emptyset, r_1, r_0)$; then

$$\begin{aligned}\Pi_\tau(r_\emptyset, r_1, r_0) &= \bar{R} - p_\tau r_\emptyset + p_\tau \max\{\bar{R} - p_\tau r_1, \bar{u}\} + (1 - p_\tau) \max\{\bar{R} - p_\tau r_0, \bar{u}\} \\ &= 2\bar{R} - p_\tau r_\emptyset - p_\tau^2 \min\{r_1, \hat{r}_\tau\} - p_\tau(1 - p_\tau) \min\{r_0, \hat{r}_\tau\} .\end{aligned}\tag{5}$$

Here, the max-operators reflect the fact that the agent will not borrow in period 2 if the outside option is better; and the second equality uses the fact that \hat{r}_τ (see equation 2) is the agent's reservation rate, implying $\bar{R} - p_\tau \hat{r}_\tau = \bar{u}$.

In the dynamic case also, including safe borrowers is the hard part:

Lemma 1. *If safe agents choose to borrow in period 1, so do risky agents.*

Proof. See Appendix.

Hence, as a first step to maximizing borrower surplus, our strategy is to choose contract parameters to maximize the safe-borrower payoff, subject to lender zero-profit, limited liability, monotonicity, and borrower limited commitment. Several constraints are initially ignored and verified or discussed ex post: limited liability after success, and the dynamic monotonicity constraint. Throughout the section, we assume all agents borrow in period 1, and find conditions under which this assumption holds true.⁷

We next claim that a contract that maximizes the safe borrower's two-period payoff (equation 5) subject to the zero-profit constraint and the other imposed constraints can do no better than to set $r_1 = 0$. Consider first $r_1 \leq \hat{r}_s$. The safe borrower's payoff is

$$\Pi_s(r_\emptyset, r_1, r_0) = 2\bar{R} - p_s r_\emptyset - p_s^2 r_1 - p_s(1 - p_s) \min\{r_0, \hat{r}_s\} .$$

Define Z as the number of agents who fail in the first period and take a loan in the second period, and Z_p as the number of these agents who repay in the second period.⁸ Then the

⁷This is the relevant case since the ultimate question is whether safe borrowers will borrow, and if they do, risky do also, as lemma 1 shows. The proof of Proposition 1 addresses this in more detail.

⁸Then $Z = \theta(1 - p_r)1\{r_0 \leq \hat{r}_r\} + (1 - \theta)(1 - p_s)1\{r_0 \leq \hat{r}_s\}$ and $Z_p = \theta(1 - p_r)p_r 1\{r_0 \leq \hat{r}_r\} + (1 - \theta)(1 -$

bank's zero-profit constraint is

$$\bar{p} \cdot r_\emptyset + \bar{p}^2 \cdot r_1 + Z_p \cdot r_0 = (1 + \bar{p} + Z)\rho .$$

It is straightforward to show that the safe borrower's (linear) indifference curve is less steep (slope: $-1/p_s$) than the bank's isoprofit line in (r_\emptyset, r_1) space (slope: $-\bar{p}/\bar{p}^2$). Thus, to maximize a safe borrower's payoff, the contract can do no better than set r_1 as low as possible, i.e. $r_1 = 0$ (from the monotonicity constraint). In sum, $r_1 = 0$ maximizes the safe borrower payoff for $r_1 \in [0, \hat{r}_s]$.

When $r_1 \in (\hat{r}_s, \infty)$, a safe borrowers' payoffs do not depend directly on r_1 – it is so high they opt out of borrowing in the second period when they succeed. However, r_1 affects them indirectly via the bank's profits, so their preference is for the bank to extract as much revenue as possible from risky borrowers via r_1 , in order to allow for lower r_\emptyset and/or r_0 . The bank clearly does this by charging the risky agents' reservation rate, \hat{r}_r , extracting all the surplus from risky agents' projects in this state of the world. Thus, in this range safe borrowers uniquely prefer $r_1 = \hat{r}_r$.

In sum, to maximize the safe-borrower payoff r_1 should be set either to 0 or to \hat{r}_r . A direct comparison of these two options reveals that $\Pi_s(r_\emptyset, 0, r_0) > \Pi_s(r'_\emptyset, \hat{r}_r, r_0)$ for any value of r_0 , where r_\emptyset (respectively, r'_\emptyset) sets lender profits to zero given $r_1 = 0$ and r_0 (respectively, $r_1 = \hat{r}_r$ and r_0). Thus $r_1 = 0$ is better for safe borrowers. Evidently, the free loan as a reward for success is more valuable to them than the discounted r'_\emptyset that can be offered by extracting successful risky borrowers' period-2 surplus.

Thus a contract tailored to safe borrowers charges the least amount possible at the history where safe borrowers are most prevalent, i.e. after one successful loan. This rate is minimized even at the expense of a higher first-period rate, since this shifts more of the repayment burden onto risky borrowers.

$$p_s)p_s \mathbb{1}\{r_0 \leq \hat{r}_s\}.$$

Now let $r_1 = 0$ and consider r_0 . With $r_0 \leq \hat{r}_s$, the safe borrower's payoff is

$$\Pi_s(r_\emptyset, 0, r_0) = 2\bar{R} - p_s r_\emptyset - p_s(1 - p_s)r_0$$

and, letting $\overline{p(1-p)}$ be the population average of $p(1-p)$, the bank's zero-profit constraint is

$$\bar{p} \cdot r_\emptyset + \overline{p(1-p)} \cdot r_0 = 2\rho . \quad (6)$$

Comparison of the safe borrower's indifference curve and the bank's isoprofit line in (r_\emptyset, r_0) space establishes that a safe borrower prefers r_0 to be as high as possible along any isoprofit line to allow for a lower r_\emptyset . Thus, $r_0 = \hat{r}_s$ is preferred by safe borrowers whenever $r_0 \leq \hat{r}_s$.

For any $r_0 \in (\hat{r}_s, \infty)$, safe borrowers strictly prefer $r_0 = \hat{r}_r$. The argument is as above: if $r_0 > \hat{r}_s$, safe borrowers never face r_0 directly so they prefer the maximal extraction from risky borrowers in order to fund a lower r_\emptyset . That is, in this range they uniquely prefer the risky borrower's reservation rate, $r_0 = \hat{r}_r$. In sum, to maximize safe borrowers' payoff, r_0 should be set either to \hat{r}_s or to \hat{r}_r .

One can show that under assumption A3, safe borrowers prefer $r_0 = \hat{r}_r$ to $r_0 = \hat{r}_s$. Note that either way, their period-2 payoffs are the same: free loan if they succeeded in period 1, reservation payoff if not (whether or not they borrow). The only difference comes in the period-1 rate, r_\emptyset , which in turn depends on whether $r_0 = \hat{r}_r$ or $r_0 = \hat{r}_s$ extracts more profits for the bank, allowing it to lower r_\emptyset . Under assumption A3, it is \hat{r}_r that earns the bank higher profits on borrowers with one failure. Interestingly, safe borrowers thus prefer to be priced out of the market when they fail – either way they get their reservation payoff, but at the higher rate the bank earns more and can charge lower rates on first loans.

Thus the simple pooling contract with the greatest chance of attracting safe borrowers, given that risky also borrow, involves $r_1 = 0$, $r_0 = \hat{r}_r$, and r_\emptyset from the bank's zero-profit constraint:

$$\bar{p} \cdot r_\emptyset + \theta p_r(1 - p_r) \cdot \hat{r}_r = \rho[1 + \bar{p} + \theta(1 - p_r)] ;$$

both left- and right-hand sides reflect the fact that safe borrowers who fail do not borrow in period 2. Solving for r_\emptyset gives

$$r_\emptyset = \frac{\rho[2 - (1 - \theta)(1 - p_s) - \theta(1 - p_r)\mathcal{N}]}{\bar{p}}. \quad (7)$$

When does this contract succeed in attracting safe borrowers in period 1? It does iff $\Pi_s(r_\emptyset, 0, \hat{r}_r) \geq 2\bar{u}$; some algebra shows that this is true iff

$$\mathcal{N} \geq \mathcal{N}_{dyn,ind}^* \equiv \frac{p_s}{\bar{p} + (p_s - \bar{p})\frac{\theta + (1-\theta)p_s}{1 + \theta + (1-\theta)p_s}}. \quad (8)$$

This is clearly a weaker condition than that needed for static individual loans to include safe borrowers, but is not guaranteed to hold: i.e. $1 < \mathcal{N}_{dyn,ind}^* < \mathcal{N}_{stc,ind}$.

Though this contract gives safe borrowers the best payoff and thus provides the weakest conditions for including them in period 1, it does not achieve maximal borrower surplus. This is because it gives up on safe borrowers who fail, and these agents choose the inefficient outside option in period 2. This raises the question: are there contracts that create higher borrower surplus?

As mentioned before, at zero lender profits, surplus maximization is equivalent to outreach maximization. Thus, any higher-surplus contract must include safe borrowers who fail, since this is the only group excluded in the above contract; that is, it must involve $r_0 \leq \hat{r}_s$. We next add this constraint to the above maximization of safe borrower payoffs. The same logic as above (which was independent of r_0) gives $r_1 = 0$ as optimal for safe borrowers. It was also established above that $r_0 = \hat{r}_s$ is best for safe borrowers when $r_0 \leq \hat{r}_s$. Thus, the best contract for safe borrowers with $r_0 \leq \hat{r}_s$ involves $r_1 = 0$, $r_0 = \hat{r}_s$, and r_\emptyset from the zero-profit constraint 6 given these values:

$$r_\emptyset = \frac{\rho \left[2 - \frac{\overline{p(1-p)}}{p_s} \mathcal{N} \right]}{\bar{p}}. \quad (9)$$

This contract attains full efficiency (everyone always borrows) iff safe borrowers prefer to borrow under this contract, i.e. $\Pi_s(r_\emptyset, 0, \hat{r}_s) \geq 2\bar{u}$. Some algebra shows that this is true iff

$$\mathcal{N} \geq \mathcal{N}_{dyn,ind} \equiv \frac{p_s}{\bar{p} + (p_s - \bar{p})\frac{p_r}{2}}. \quad (10)$$

It is clear that $\mathcal{N}_{dyn,ind} < \mathcal{N}_{stc,ind}$, that is, two-period dynamic lending can achieve full efficiency in some cases where static individual lending cannot. Intuitively, the dynamic contract better prices for risk by varying the second-period interest rate so as to shift the repayment burden toward risky borrowers ($r_1 < r_0$). On the other hand, it is also clear that $\mathcal{N}_{dyn,ind}^* < \mathcal{N}_{dyn,ind}$, i.e. it is sometimes possible to achieve near-efficiency (attracting safe borrowers except after they fail) when full efficiency is not possible.

Summarizing, we have the following proposition:

Proposition 1. *Under assumptions A1-A3 and provided \mathcal{G} is high enough, a simple pooling contract that maximizes borrower surplus subject to limited liability, monotonicity, and limited borrower commitment:*

- *when $\mathcal{N} \in [\mathcal{N}_{dyn,ind}, \mathcal{N}_{stc,ind})$, achieves full efficiency – i.e. includes all borrowers.*
- *when $\mathcal{N} \in [\mathcal{N}_{dyn,ind}^*, \mathcal{N}_{dyn,ind})$, achieves near efficiency – i.e. includes all but failed safe borrowers.*
- *when $\mathcal{N} \in (1, \mathcal{N}_{dyn,ind}^*)$, fails to avert market breakdown – i.e. includes only risky borrowers.*

Proof. See Appendix.

The condition that \mathcal{G} be high enough is needed (under some parameter values) in order to satisfy the ignored limited liability constraint, namely that r_\emptyset be affordable. Thus, it is the net return in this market (\mathcal{N}) that determines maximum efficiency attainable, and the gross return (\mathcal{G}) that ensures borrowers can afford contract stipulations. Note that other than $\mathcal{G} \geq \mathcal{N}$ (since $\bar{u} \geq 0$), \mathcal{G} and \mathcal{N} can vary independently. For example, raising \bar{R} and

\bar{u} in tandem raises \mathcal{G} without changing \mathcal{N} . From another perspective, a condition that \mathcal{G} is high enough for fixed \mathcal{N} can be written as a condition on the outside option \bar{u} being high enough.⁹

What happens when \mathcal{G} is not high enough? In this case, the r_0 derived above is not affordable, so it must be lowered and another rate raised. In the full efficiency case, the rate raised will be r_1 , since r_0 is as high as possible. This limits variation in the second-period interest rate, the key advantage dynamic lending has in risk-pricing, and raises the \mathcal{N} required to achieve efficient lending. In the limit (as $\mathcal{G} \searrow \mathcal{N} \Leftrightarrow \bar{u} \searrow 0$), the minimum \mathcal{N} required for full efficiency goes to $\mathcal{N}_{stc,ind}$, i.e. dynamic lending offers no improvement in achieving full efficiency. A similar story holds in the near-efficiency case; if r_0 is not affordable, it is lowered and r_1 raised, so higher levels of \mathcal{N} are needed to include safe borrowers. However, unlike with the fully efficient contract, at least under some parameter values, even when $\bar{u} = 0$ there is still a range of \mathcal{N} for which nearly-efficient dynamic lending is attainable where static loans would lead to market breakdown.¹⁰

Further analysis is possible here, but the basic point is clear: low enough gross returns limit the amount that can be charged upfront, and thus limit the ability of a dynamic contract to vary interest rates as information is revealed while still keeping customers. On the other hand, high enough gross returns make net returns the exclusive determinant of efficiency attainable.

The dynamic monotonicity constraint 4 is always satisfied by the fully efficient lending

⁹Thus, in these models the assumption of a zero outside option is not always innocuous, at least if the limited liability after success constraint is imposed. Conversely, limited liability after success can typically be ignored and satisfied as needed by scaling up \bar{R} and \bar{u} by the same absolute amount, leaving \mathcal{N} unchanged.

Regarding plausibility, it seems natural in many applications that the outside option be above zero, not least when getting a loan involves a (partial) change in occupation.

¹⁰In some cases, if \mathcal{G} is low enough, then the discount that can be offered in r_1 by charging a high r_0 is so low (since affordability concerns limit the level of r_0) that it instead becomes advantageous to charge $r_1 = \hat{r}_r$, extracting all risky-borrower surplus in period 2 in order to fund a lower period-1 rate. In this case, safe borrow half the time at a relatively low and affordable rate, while risky borrow all the time at both high and low rates. This can be re-written as a menu of contracts – one loan only at a low rate (which safe opt for), or two loans for sure at a higher rate (which risky opt for) – which makes clear the similarity to probabilistic screening in the static case – a chance at a cheap loan (which safe opt for) or a more expensive loan for sure (which risky opt for). Again, though, if \mathcal{G} is high enough, this is dominated both in terms of surplus and safe-borrower payoffs by a high r_0 followed by a cheap r_1 .

contract and typically, but not always, satisfied by the near-efficient contract. When it is not, the problem is that r_\emptyset is so low that borrowers would be willing to pay it in order to get a cheap period-2 loan. This is easy to solve: r_\emptyset can be raised and r_0 lowered at zero profits to eliminate this temptation, maintaining the same number of borrowers and still satisfying all other constraints (see the proof for more discussion). Thus, the dynamic monotonicity constraint may shift surplus from safe to risky borrowers, but does not affect overall efficiency attainable.

The contracts derived above tend to feature back-loaded incentives, with borrowing getting more attractive over time. In particular, with fully efficient lending under assumption A3, even failed borrowers pay a lower rate in period 2 than in period 1: $r_\emptyset > r_0 = \hat{r}_s > r_1 = 0$. Safe agents borrow in period 1 even though they earn less than their outside option, in anticipation of cheaper future loans. The same features sometimes hold with the near-efficient lending contract derived.

The model thus provides a rationale for the feature of relationship lending that borrowers get better terms over time. The story here is that all borrowers face high rates upfront with the promise of more attractive rates – refunds, essentially – in the future. This entices borrowers to keep borrowing while allowing the lender some ability to vary rates (refunds) as information is revealed. By contrast, starting with a neutral rate and hiking it after failure runs the risk of losing unlucky borrowers. Essentially, back-loading overcomes agents' lack of commitment to keep borrowing over time.

Interestingly, under assumption A3 safe borrowers prefer a contract that prices them out of the market in period 2 if they fail in period 1 – even if fully efficient lending is achievable. The implication is that there may be a tradeoff between equity and efficiency, and that, paradoxically, equity is favored by pricing unlucky safe borrowers out of the market. A high interest rate after failure extracts more from risky borrowers and therefore benefits safe borrowers in the first period. However, a low interest rate after failure can raise surplus overall by funding more projects; but it lowers safe borrowers' payoffs (which are always below

risky borrowers’) and raises risky borrowers’ payoffs by more than the increased surplus – mainly due to the cheap period-2 loans when they fail.

More complicated contracts. Given the constraints under which many micro-lenders operate and their apparent preferences for simple products, the simple pooling contracts analyzed thus far may be interesting in their own right. If anything, menus of contracts meant to screen borrowers seem significantly harder to find among micro-lenders than standardized (pooling) contracts. Still, one may wonder if more complicated contracts can raise surplus. Here we argue that the simple pooling contracts cannot be improved upon by a range of more complicated contracts.

First, consider forced savings, collateral, and self-financing.¹¹ In general, each of these instruments could be valuable in this context by raising the amount paid by borrowers after failure, thus shifting more of the repayment burden onto risky borrowers. However, they are only available after a period-1 success, since after failure the borrower continues to have no wealth. More importantly, these instruments offer nothing that cannot be accomplished by a simple contract of the form (r_\emptyset, r_1, r_0) .

In the case of collateral, consider a contract with rates r_\emptyset and r_1 , with required collateral $\kappa > 0$ on the period-2 loan after success.¹² The monotonicity constraint requires $\kappa \leq r_1$; otherwise a borrower who failed would be tempted to claim success to pay the lower amount.¹³ Thus the successful borrower pays r_\emptyset at the end of period 1; pays r_1 if he succeeds in period 2; and pays κ if he fails in period 2. In terms of total two-period payoffs, this is equivalent

¹¹In the following discussion, we implicitly assume $\rho = 1$, that is the bank earns zero net interest on capital. This is done to parallel our simplifying assumption that borrowers do not discount period 2. If borrowers did not discount and banks could earn positive net interest, then forced (or voluntary) savings, e.g., could improve borrower surplus simply by giving them access to a positive rate of return on excess capital, which seems to be a point orthogonal to our focus.

¹²We implicitly assume no deadweight loss associated with collateral; if there were, collateral would seem to be even less useful.

¹³Without the monotonicity constraint here, collateral could indeed improve on the simple contract derived earlier, but only by stipulating more-than-full collateralization of loans. This would make borrowers pay more after failure than success, further shifting repayment burden toward risky borrowers. However, the optimal contract would have the successful borrower (on the period-2 loan after success) paying nothing (getting κ refunded) and the failed borrower paying κ ; thus it would be vulnerable to the failed borrower claiming success in order to have κ refunded.

to the simple contract $r'_0 = r_0 + \kappa$, $r'_1 = r_1 - \kappa \geq 0$, and nothing due after failure in period 2. This modified contract satisfies the same constraints as the previous one, and in fact weakens the limited commitment problem by forcing the borrower to sink κ before taking the second loan.¹⁴ Thus the bank can mimic any collateral policy with a simple contract already analyzed, by raising the period-1 rate and lowering the period-2 rates, essentially collecting the collateral upfront. The same is also true in reverse: one can interpret the optimal simple pooling contract derived earlier, which involves r_0 high and r_1 at zero as instead a more moderate period-1 rate plus a collateral or forced savings stipulation fully guaranteeing the second loan, which comes with a strictly positive interest rate.

The same argument holds for forced savings policy, where the savings is used as collateral. In the strongest case for forced savings, the savings is not refunded if the borrower fails with *or does not take* a second loan; the forced savings is thus used to weaken the limited commitment problem as well as to collateralize the second loan. Monotonicity requires that the amount of savings used to guarantee the loan cannot exceed the gross interest rate on the loan: $\sigma \leq r_1$. So, if σ is the amount of forced savings used as collateral, a simple contract can accomplish the same outcome by adding the forced savings to the period-1 interest rate: $r'_0 = r_0 + \sigma$, $r'_1 = r_1 - \sigma \geq 0$, and nothing due after failure in period 2.

It is also clear that hidden savings poses no problem to the optimal simple pooling contract. When the borrower succeeds, he pays r_0 and then gets a free loan; whether or not he saves some of his returns changes no decisions and does not alter total surplus.

Finally, consider a self-financing requirement on the loan after success. This is similar to collateral, except the payment/investment is made upfront regardless of success or failure. Thus if κ is the self-financing requirement, the successful borrower pays r_0 at the end of period 1; pays κ upfront and then r_1 if he succeeds in period 2; and pays κ upfront and nothing later if he fails in period 2. Monotonicity requires $r_1 \geq 0$. This is equivalent to the simple contract $r'_0 = r_0 + \kappa$, $r'_1 = r_1 \geq 0$, and nothing due after failure in period 2. That

¹⁴It may appear to raise r_0 , potentially making it unaffordable; but note that under the contract with collateral, one successful project must also be able to cover $r_0 + \kappa$.

is, the lender can simply collect the self-financing amount upfront after success in period 1 through the period-1 interest rate.

In sum, then, savings, collateralization, and self-financing are already allowed for by simple contracts. Key questions remain, given some simplifying assumptions made in the previous section's analysis. First, we restricted attention to pooling contracts: can a menu of screening contracts do better? Second, we assumed negative interest rates impossible: can they improve surplus? Finally, we assumed agents with no history could not borrow in period 2: is this restrictive?

Proposition 2. *Under assumptions A1-A3 and provided \mathcal{G} is high enough, the simple pooling contract that maximizes borrower surplus subject to limited liability, monotonicity, and limited borrower commitment cannot be improved upon by any menu of contracts that satisfies the same constraints and can but need not involve negative interest rates and first-time borrowing in period 2.*

Proof. See Appendix.

It is due to the monotonicity constraints that negative interest rates do not help. At times (namely, on the period-2 loan after one success) the lender may want to subsidize success by charging a negative interest rate – but the monotonicity constraint forces it to subsidize failure at least as strongly. Thus there is no increased scope from negative interest rates for a differential subsidy of success. In terms of a refund that is neutral across success and failure, this adds nothing because it can be replicated without negative interest rates by a simple contract that eliminates the refund and lowers earlier payments.

More interestingly, screening contracts offer no improvement. This is due to the fact that borrower and lender payoffs are diametrically opposed (with exceptions having to do with agents opting out of borrowing; however, these are readily dealt with). That is, a risky borrower's indifference curve coincides exactly with an isoprofit curve of the lender for risky borrowers, and any adjustment of rates that a risky borrower pays that changes

risky borrowers' total payoffs by X changes the lender's profits by $-X$.¹⁵ It is also true that the binding incentive constraint is the risky borrower's: at the optimum he is indifferent between his contract and the safe borrower's. Applying these two facts to a pair of optimal screening contracts, note that the risky borrower does equally well if instead he is given the safe borrower's contract; and since his payoff and the lender's payoff move one for one, the lender also does equally well. Thus, giving both risky and safe the safe borrower's contract – i.e. a simple pooling contract – is as good as can be done.

Finally, consider contracts that induce first-time period-2 borrowing. It turns out all of these can be replicated without first-time period-2 borrowing by giving the first-time period-2 borrowers the same deal in period 1 and an unattractive deal in period 2. This can be done without changing payoffs in equilibrium or in deviation. Thus, contracts that induce first-time period-2 borrowing offer nothing that cannot be accomplished without allowing it.

4 Comparing Group Lending and Dynamic Lending

Static group lending. Next, we review results from the model of Ghatak (1999, 2000) and Gangopadhyay et al. (2005), in which it is shown that joint liability lending can improve efficiency when individual lending breaks down. The environment is exactly the same as the one studied above, except that the setting is static and agents are now assumed to know each other's risk; risk is still unobservable to the outside lender.

The theory considers contracts written with pairs of borrowers, now with two parameters: r being the amount paid by an agent who succeeds, and c being the additional amount paid by an agent who succeeds and whose partner fails. Without loss of generality, we restrict attention to pooling contracts and ignore negative payments after failure.¹⁶ The expected

¹⁵This would be different if borrowers were risk-averse and the lender risk-neutral, e.g., since then payoffs of the two parties would not move one for one in opposite directions, but could even both move in the same direction due to the gains from selling insurance.

¹⁶Arguments similar to those of Proposition 2 demonstrate that these assumptions are without loss of generality for surplus maximization. However, our focus on pooling contracts causes the derivation here to differ slightly from the earlier papers.

payoff of a borrower of type i paired with a borrower of type j , $(i, j) \in \{r, s\}^2$, is then

$$\overline{R} - p_i r - p_i(1 - p_j)c = \overline{R} - p_i[r + (1 - p_j)c] .$$

Ghatak (1999, 2000) shows that the unique stable match involves groups that are homogeneous in risk-type when $c > 0$. Thus in equilibrium the payoff above involves $p_j = p_i$.

As in Gangopadhyay et al. (2005), we impose the constraint that liability for one's partner cannot be more than his debt obligation, i.e. the group-level monotonicity constraint

$$r + c \leq 2r \quad \iff \quad c \leq r .$$

If this were not true, a successful borrower would have the incentive to claim his partner succeeded even when he failed, in order to pay $2r$ rather than $r + c$.

In the group lending case also, including safe borrowers is the hard part.¹⁷ Hence, our strategy is to choose contract parameters to maximize the safe-borrower payoff, subject to lender zero-profit, limited liability, and monotonicity. Limited liability after success is initially ignored and verified ex post. We also assume all agents borrow in period 1, and find conditions under which this assumption holds true.¹⁸

If all agents borrow, the bank must set r and c to satisfy

$$\overline{p} \cdot r + \overline{p(1 - p)} \cdot c = \rho , \tag{11}$$

where $\overline{p(1 - p)}$ is again the population average of $p(1 - p)$ and the left-hand side is the

¹⁷Consider any (r, c) with $c \leq r$. It suffices to establish the first inequality in the following:

$$\overline{R} - p_s[r + (1 - p_s)c] \leq \overline{R} - p_r[r + (1 - p_r)c] \iff (p_s - p_r)r \geq c(p_s - p_r)(p_s + p_r - 1) \iff r \geq c(p_s + p_r - 1).$$

The last inequality is implied by $c \leq r$.

¹⁸This is the relevant case since the ultimate question is whether safe borrowers will borrow, and if they do, risky do also.

expected return on a unit of capital lent. Consider a safe borrower's payoff:

$$\bar{R} - p_s r - p_s(1 - p_s)c .$$

It is straightforward to show that the safe borrower's (linear) indifference curve is steeper than the bank's isoprofit line 11, in (r, c) space. The corner solution of $c = r$ (since monotonicity requires $c \leq r$) thus maximizes the safe borrower's utility for any profit level of the bank.

With $c = r$, the isoprofit condition becomes

$$\overline{p(2-p)} \cdot r = \rho \quad \iff \quad r = \frac{\rho}{\overline{p(2-p)}} ,$$

where $\overline{p(2-p)}$ is the population average of $p(2-p)$. All agents are willing to borrow at this interest rate iff safe borrowers are, i.e.

$$\bar{R} - \frac{\rho}{\overline{p(2-p)}} p_s(2 - p_s) \geq \bar{u} \quad \iff \quad \mathcal{N} \geq \mathcal{N}_{stc,grp} \equiv \frac{p_s(2 - p_s)}{\overline{p(2-p)}} = \frac{p_s}{\bar{p} + (p_s - \bar{p})\frac{p_r}{2-p_s}} . \quad (12)$$

It is clear that $\mathcal{N}_{stc,grp} < \mathcal{N}_{stc,ind}$, i.e. group lending can achieve efficiency while individual lending cannot if $\mathcal{N} \in [\mathcal{N}_{stc,grp}, \mathcal{N}_{stc,ind})$. It is also clear that $\mathcal{N}_{stc,grp} > 1$, so that for $\mathcal{N} \in (1, \mathcal{N}_{stc,grp})$, group lending cannot achieve full efficiency, since the best possible contract for safe borrowers fails to attract them. In this case, the only alternative is to give up on safe borrowers and include only risky.

The previous discussion ignored the constraint of limited liability after success – that is, whether $r + c$ is affordable. As in the previous section, this constraint is satisfied, for any \mathcal{N} , if \mathcal{G} is high enough. Combining, we have the result that under assumptions A1-A3 and provided \mathcal{G} is high enough, the simple pooling contract that maximizes borrower surplus subject to limited liability and monotonicity:

- when $\mathcal{N} \in [\mathcal{N}_{stc,grp}, \mathcal{N}_{stc,ind})$, achieves full efficiency – i.e. includes all borrowers.

- when $\mathcal{N} \in (1, \mathcal{N}_{stc,grp})$, fails to avert market breakdown – i.e. includes only risky borrowers.

Comparing static group lending with dynamic individual lending. We next compare maximal surplus achievable by the two types of contracts; for now, we assume \mathcal{G} high enough to support the best contracts derived. The goal is to derive insight into the strengths and weaknesses of two seemingly popular contract forms in improving lending markets.

Of course, the environments needed for the two types of contracts are not identical. The dynamic lending contract assumes a two-period project endowment and ability for the lender to commit dynamically, while the group lending contract assumes a tight-knit community where borrowers know each others' types and can frictionlessly match. Hence, one way to interpret the following comparisons is in terms of surplus attainable across different environments, one in which dynamic lending is feasible but group lending is not, and vice versa for the other. Another informal way to interpret the comparisons is to imagine both types of lending are feasible, but for some reason or another (e.g. lack of information) lenders are not necessarily choosing the optimal kind of contract. In this scenario, the comparison would give guidance as to which form of lending is preferable.¹⁹

One can compare the relative effectiveness of group lending and dynamic lending at achieving full efficiency (for \mathcal{G} high enough) by comparing conditions 12 and 10. By inspection, $\mathcal{N}_{stc,grp} < \mathcal{N}_{dyn,ind}$, so that for $\mathcal{N} \in [\mathcal{N}_{stc,grp}, \mathcal{N}_{dyn,ind})$, static group (paired) lending can achieve efficient lending while dynamic individual (2-period) lending cannot.

In terms of achieving full efficiency, group lending thus dominates dynamic lending. Why? Note that both types of lending reveal the exact same information, namely two draws from the borrower's distribution. The two draws are time series observations in the dynamic lending case and cross section observations in the group lending case. The cross-sectional observations are just as informative of the borrower's type, given homogeneously matched

¹⁹However, if both types of lending are feasible, a combination of dynamic and group lending may do better – we explore this in the next section.

groups.²⁰

Note also that absent the monotonicity and limited commitment constraints, the two types of contracts are equally effective at fully efficient lending. The group lending payoff is

$$\bar{R} - p_\tau r - p_\tau(1 - p_\tau)c = \bar{R} - p_\tau[(r + c) - p_\tau \mathbf{c}],$$

while the average per-period dynamic lending payoff is

$$\frac{2\bar{R} - p_\tau r_\emptyset - p_\tau^2 r_1 - p_\tau(1 - p_\tau)r_0}{2} = \bar{R} - p_\tau \left[\frac{(r_\emptyset + r_0)}{2} - p_\tau \frac{(\mathbf{r}_0 - \mathbf{r}_1)}{2} \right].$$

All average per-period payoffs (including the bank's) resulting from the dynamic contract (r_\emptyset, r_1, r_0) can be replicated by group contract $r = (r_\emptyset + r_1)/2$ and $c = (r_0 - r_1)/2$. Conversely, any group contract (r, c) can be replicated by dynamic contract $r_\emptyset = 2r$, $r_1 = 0$, and $r_0 = 2c$. Technically, both contracts create payoffs that are quadratic in risk-type, and with two types, two contract instruments are sufficient to position the quadratic optimally so as to eliminate any cross-subsidy and attract all borrowers.

Thus, the contracts do not differ in information revelation and, unconstrained, in ability to tailor payoffs by risk-type. All differences come from constraints. As it turns out group lending is less constrained in its ability to target discounts to safe borrowers. To see this, note that the bracketed terms in the above payoffs can be considered the “effective interest rate” under each type of contract.²¹ Further, embedded in each of the effective interest rates is a “discount” term (in bold), multiplying risk-type and thus differentially enjoyed

²⁰Thus the lender's Bayesian posterior assessment that a borrower is safe, e.g., would be the same after an individual succeeded in both projects under a dynamic contract as after both group members succeeded under a group contract $(\frac{(1-\theta)p_s^2}{\theta p_r^2 + (1-\theta)p_s^2})$; after an individual succeeded in one project out of two under a dynamic contract as after exactly one group member succeeded under a group contract $(\frac{(1-\theta)p_s(1-p_s)}{\theta p_r(1-p_r) + (1-\theta)p_s(1-p_s)})$; and after an individual failed in both projects under a dynamic contract as after both group members failed under a group contract $(\frac{(1-\theta)(1-p_s)^2}{\theta(1-p_r)^2 + (1-\theta)(1-p_s)^2})$.

²¹The effective interest rate is the analog to the interest rate from a static individual loan contract, where the payoff is $\bar{R} - p_\tau r$.

by safe borrowers: c for group lending, $(r_0 - r_1)/2$ for dynamic lending.²² Both contracts are constrained in how high this discount can be. With group lending, the monotonicity constraint forces $c \leq r$. With dynamic lending, the monotonicity constraint forces $r_1 \geq 0$ and the limited commitment constraint keeps $r_0 \leq \hat{r}_s$; the overall discount is then bounded by $\hat{r}_s/2$. It turns out that the dynamic lending bound is more restrictive, and thus the dynamic contract has a harder time targeting discounts to safe borrowers.

In sum, the need to keep safe borrowers in the market over time limits the variation in interest rate that the lender can use to target discounts to safe borrowers via dynamic lending. Put differently, when “penalty” rates need to attract unlucky safe borrowers, they cannot be too severe.

The discussion above compares the two types of contracts for achieving full efficiency, and group lending dominates. However, when dynamic lending falls short of full efficiency, it can sometimes achieve near-efficiency; this is not true of group lending. This raises the question: when group lending fails to include safe borrowers, can dynamic lending do better by including safe borrowers at least most of the time?

Comparison of condition 8 for nearly-efficient dynamic lending with condition 12 for efficient group lending gives that the former is strictly weaker iff

$$p_r < \frac{(2 - p_s)[\theta + (1 - \theta)p_s]}{1 + \theta + (1 - \theta)p_s}.$$

When this inequality holds,²³ then $\mathcal{N}_{dyn,ind}^* < \mathcal{N}_{stc,grp}$, so for $\mathcal{N} \in [\mathcal{N}_{dyn,ind}^*, \mathcal{N}_{stc,grp})$ dynamic lending is more efficient than group lending. Here, group lending fails to include safe agents at all, while dynamic lending includes safe agents except in period 2 after a failure.

Dynamic lending can thus dominate group lending in terms of total surplus, but only

²²Thus, under the group contract the safe borrower’s effective interest is lower than the risky’s by $(p_s - p_r)c$; under the dynamic contract, the comparable figure is $(p_s - p_r)(r_0 - r_1)/2$. The reason c acts as a discount in the group lending case is that a higher c shifts repayment toward states of the world where risky borrowers are relatively more prevalent, namely one success and one failure (where $r + c$ is due), as opposed to two successes (where $2r$ is due). Similarly, in the dynamic lending case a higher $r_0 - r_1$ shifts repayment toward states of the world where risky borrowers are relatively more prevalent.

²³A sufficient condition for this inequality is $p_r \leq 1/2$.

when it prices unlucky safe borrowers out of the market in order to lower the period-1 rate. In general, though, the efficiency comparison between group and dynamic lending is not unambiguous – one style of lending does not dominate in every circumstance, and context can be important in determining optimal contract.

4.1 Factors that alter the comparison

Beyond the basic point that no single contract form always dominates, the model can give some guidance about conditions that argue for one or the other type of contract. One obvious point, mentioned above, is that different assumptions are required for each type of contract. Group lending assumes good local information and low-friction, purposeful matching. Realistically, these are only available in degrees, and may be especially lacking in more anonymous urban contexts.²⁴ Dynamic lending assumes commitment by the lender to adjust rates in a pre-announced way based on past history. This may be easier said than done, especially when outreach is the main goal – lenders may not credibly price many borrowers out of the market, and they may not follow through with large discounts. Also, borrowers’ need for capital may not extend over several periods, so that future loans cannot be used to price for risk after information has been revealed.

The above discussion also brings out an interesting parallel between group lending and dynamic lending: the analog between size of group in group lending and number of periods in a dynamic lending relationship, both of which determine the amount of information revelation. Specifically, a k -person group contract reveals k draws from the distribution of a borrower’s type (under homogeneous matching), as does a k -period dynamic contract. One might conjecture that larger groups and longer contracts reveal more information and are better able to price for efficiency. While a formal analysis is outside the scope of this paper,²⁵

²⁴In favor of this assumption in one context, Ahlin (2009) rejects random risk-matching among micro-credit groups in Thailand in the direction of positive assortative matching by risk.

²⁵Ahlin (2011) analyzes optimal group size in the static group lending model of Ghatak and finds efficiency increasing with group size. Generalizing the dynamic lending contract to T periods appears to be harder. We have analyzed the $T = 3$ and $T = 4$ cases; the results in these cases confirm that more time extends

this suggests that contextual limits on either group size or relationship length would tilt the scale against one or the other type of contracts.

The perspective of information revelation also suggests ways in which different types of correlated risk might affect each contract. Consider serial correlation. In the extreme, if the correlation is perfect, then a 2-period dynamic contract gives the lender not two draws from the borrower's distribution, but one. The dynamic contract is no better than a static one. Consider spatial correlation. In the extreme, if borrowers' projects in a village are perfectly correlated, a static 2-person group contract gives the lender only one draw from the borrower's distribution, and the group contract is no better than an individual contract. While a more formal analysis is beyond the scope of the paper, it appears that serial correlation works against dynamic lending while spatial correlation works against group lending, so that the nature of correlated risk can tilt the scale toward one or the other type of contract. It seems plausible that spatial correlation would be often prevalent in agricultural endeavors; serial correlation could be more endemic to small enterprise start-ups, if some ideas/managers are simply more viable long-term than others.

Another potential determinant of the relative attractiveness of the two types of contracts is the level of payoffs, \mathcal{G} . The above comparisons assume \mathcal{G} is large enough so that the best-case contracts can be offered, conditional on \mathcal{N} . If \mathcal{G} is small, however, the contracts must be modified: liability must be reduced in the group contract case, and period-2 discounts for success must be reduced in the dynamic case. While (again) a detailed analysis is beyond the scope of the paper, several points can be made. First, under assumptions A1-A3, it is straightforward to show that limited liability is more binding in the efficient dynamic contract than in the group contract: i.e. the first-period interest rate in the efficient dynamic contract, r_θ , is higher than the highest possible amount due under the group contract, $r + c$. Thus, in addition to maximum efficiency attainable, affordability also seems to argue in

the parameter space for which efficient lending is achievable. However, we have not been able to prove our conjecture that fixing any parameters satisfying assumptions A1-A3, efficiency is always attainable if T is high enough.

favor of group lending when we restrict attention to fully efficient lending. Second, the affordability comparison is ambiguous when we compare efficient group lending with *near*-efficient dynamic lending. One can show that when $\mathcal{N}_{dyn,ind}^* < \mathcal{N}_{stc,grp}$, such that for $\mathcal{N} \in [\mathcal{N}_{dyn,ind}^*, \mathcal{N}_{stc,grp})$ near-efficient dynamic lending is achievable while group lending fails, then affordability can swing in favor of dynamic lending. There can exist values of \mathcal{G} such that for \mathcal{N} in a neighborhood above $\mathcal{N}_{stc,grp}$, dynamic lending still dominates group lending because full liability is not affordable under group lending. In sum, affordability concerns can tilt the scale further toward (nearly-efficient) dynamic lending.²⁶

5 Dynamic Group Lending

Of course, if conditions are met for both group lending and dynamic lending to be feasible – dynamic endowments, local knowledge, frictionless matching, lender commitment – there need not be a dichotomy the two types of contracts. Indeed, group lending programs often seem to embed a dynamic element as they adjust terms (or deny loans) over time.

We next make the assumptions of both environments and consider dynamic group lending, with groups of two borrowers over two periods. For brevity, we only consider simple pooling contracts with non-negative period-2 rates²⁷ that achieve full efficiency. Thus, at each history loan offers come with a direct interest rate and a joint liability payment. The first loan comes with interest rate r_\emptyset , and joint liability c_\emptyset ; the second loan comes with interest rate r_σ and joint liability c_σ , where history σ is 11 if both succeeded, 10 if the borrower succeeded and his partner failed, 01 if the borrower failed while his partner succeeded, and 00 if neither succeeded. The contract consists of ten parameters: $(r_\emptyset, c_\emptyset, r_{00}, c_{00}, r_{01}, c_{01}, r_{10}, c_{10}, r_{11}, c_{11})$.

Limited commitment continues to be imposed, so that period-2 contracts at each history

²⁶The reverse may not be true, even if the group contract is more affordable. This is because, if both the group contract and the nearly-efficient dynamic contract are feasible ignoring affordability, the group contract will be chosen since it raises higher surplus. Thus, adding affordability concerns can only tilt the scale toward dynamic lending, if they change anything.

²⁷Similar arguments to those of Proposition 2 could be made to show this is without loss of generality with respect to more complicated contracts.

must be good enough to attract both types of borrowers; we impose only the more binding safe-borrower constraints and verify the risky-borrower constraints ex post. We also focus only on contracts that induce homogeneous matching. Thus, we will assume homogeneous matching when deriving the optimal contract, and verify that it obtains ex post. Limited liability after success will also be ignored ex ante and verified ex post; limited liability after failure will simply be reflected in that nothing is due from a borrower after failure.

Group-level monotonicity constraints are imposed, as before. The period-2 constraints (MC-2) are

$$2r_\sigma \geq r_\sigma + c_\sigma \geq 0, \quad \sigma \in \{00, 01, 10, 11\}.$$

These inequalities guarantee that the group payment is monotonically increasing in the number of successes in the group. The period-1 monotonicity constraints (MC-1) are

$$\begin{aligned} 2r_\emptyset + 2p_\tau[r_{11} + (1 - p_\tau)c_{11}] &\geq r_\emptyset + c_\emptyset + p_\tau[r_{01} + (1 - p_\tau)c_{01}] + p_\tau[r_{10} + (1 - p_\tau)c_{10}] \\ &\geq 2p_\tau[r_{00} + (1 - p_\tau)c_{00}], \quad \tau \in \{r, s\}. \end{aligned}$$

These guarantee that the total group payments – current, and expected future – are monotonically increasing in number of successes in the group. As before, it is not obvious whether to impose these constraints – the ability to feign current success may be limited by low current output. Our approach will be to impose it but to discuss how the contract would differ without it. In deriving the optimal contract, we ignore all dynamic monotonicity constraints except the following:

$$2r_\emptyset + 2p_s[r_{11} + (1 - p_s)c_{11}] \geq r_\emptyset + c_\emptyset + p_s[r_{01} + (1 - p_s)c_{01}] + p_s[r_{10} + (1 - p_s)c_{10}] ; \quad (13)$$

the others are verified ex post.

We proceed by maximizing the safe-borrower payoff (assuming homogeneous matching):

$$2\bar{R} - p_s r_\emptyset - p_s(1 - p_s)c_\emptyset - p_s^3 r_{11} - p_s^3(1 - p_s)c_{11} - p_s^2(1 - p_s)r_{01} - p_s^2(1 - p_s)^2 c_{01} \\ - p_s^2(1 - p_s)r_{10} - p_s^2(1 - p_s)^2 c_{10} - p_s(1 - p_s)^2 r_{00} - p_s(1 - p_s)^3 c_{00},$$

subject to the safe-borrower limited commitment constraints, the period-2 monotonicity constraints, dynamic monotonicity constraint 13, and the bank's zero-profit constraint:

$$2\rho \leq \overline{p}r_\emptyset + \overline{p(1-p)}c_\emptyset + \overline{p^3}r_{11} + \overline{p^3(1-p)}c_{11} + \overline{p^2(1-p)}r_{01} + \overline{p^2(1-p)^2}c_{01} \\ + \overline{p^2(1-p)}r_{10} + \overline{p^2(1-p)^2}c_{10} + \overline{p(1-p)^2}r_{00} + \overline{p(1-p)^3}c_{00},$$

where the bars indicate population means of various functions of risk-type p . The remaining constraints are verified ex post.

A first claim is that the safe-payoff maximizing contract can do no better than treat borrowers symmetrically, regardless of their histories. That is, it can do no better than to set $r_{01} = r_{10}$ and $c_{01} = c_{10}$, charging a borrower the same whether it was he or his partner who failed. This is a consequence of homogeneous matching, which implies that observation of either borrower's outcome is equally informative about both borrowers' type.

Lemma 2. *To maximize the safe borrower's payoffs subject to the above-mentioned constraints, a contract can do no better than equal treatment after one failure in the group: $r_{01} = r_{10}$ and $c_{01} = c_{10}$.*

Proof. See Appendix.

This lemma simplifies notation a bit, allowing the interest rate and degree of liability in period 2 to depend only on total number of successes in the group without keeping track of who failed. The contract now can be written $(r_\emptyset, c_\emptyset, r_2, c_2, r_1, c_1, r_0, c_0)$; the subscripts denote number of successes in the group in the previous period. We next show that full liability in period 2 is optimal for safe borrowers.

Lemma 3. *Full liability in period 2 maximizes the safe-borrower payoff subject to the above-mentioned constraints: $c_2 = r_2$, $c_1 = r_1$, and $c_0 = r_0$.*

Proof. See Appendix.

The contract now boils down to $(r_\emptyset, c_\emptyset, r_2, r_1, r_0)$, where second period loans are understood to carry full group liability. The rest of the contract can be solved for using straightforward Lagrangean methods.

Lemma 4. *The contract that maximizes the safe-borrower payoff subject to the imposed constraints sets $r_2 = r_1 = 0$; sets r_0 as high as possible subject to the limited commitment constraint, i.e. $r_0 = \hat{r}_s/(2 - p_s)$; and imposes full liability in period 1, i.e. $c = r$.*

Proof. See Appendix.

In sum, the contract best for safe borrowers subject to the imposed constraints has the following features: full liability on all loans; low-interest loans after group repayment (whether one or both succeeded) in the first-period; and high-interest loans after group failure. It is thus a hybrid of the optimal static group contract – with its full liability – and the optimal dynamic individual contract – with free loans after success and more expensive ones after failure. And, as with the dynamic individual contract, terms often get better over time. In particular, $r_\emptyset > r_0$ when static group lending breaks down and, typically, even when static group lending can achieve efficiency but static individual lending breaks down. Thus, the first loan is the most expensive, but terms get better over time, much as in relationship lending.

Using the contract terms of Lemma 4, r_\emptyset can be solved for from the zero-profit constraint:

$$\overline{p(2-p)r_\emptyset} + \overline{p(1-p)^2(2-p)} \frac{\hat{r}_s}{2-p_s} = 2\rho \quad \iff \quad r_\emptyset = \frac{\rho \left[2 - \frac{p(1-p)^2(2-p)}{p_s(2-p_s)} \mathcal{N} \right]}{p(2-p)}.$$

The safe-borrower payoff under this contract exceeds $2\bar{u}$ iff

$$\mathcal{N} \geq \mathcal{N}_{dyn,grp} \equiv \frac{p_s(2-p_s)}{p(2-p) + \frac{p(1-p)^2(2-p) - p(2-p)(1-p_s)^2}{2}} = \frac{p_s}{\bar{p} + (p_s - \bar{p}) \frac{p_r [3 - p_s - p_r (2 - \frac{p_s + p_r}{2})]}{2 - p_s}}.$$

By inspection, this condition is weaker than the one for static group lending ($\mathcal{N}_{dyn,grp} < \mathcal{N}_{stc,grp}$) and hence for dynamic individual lending (since $\mathcal{N}_{stc,grp} < \mathcal{N}_{dyn,ind}$); thus dynamic group lending can achieve full efficiency when neither static group lending nor dynamic individual lending can. This is not surprising, since the lender combines both methods of improved risk-pricing in this dynamic group contract. However, $\mathcal{N}_{dyn,grp} > 1$, so that even 2-period dynamic group lending does not achieve efficient for $\mathcal{N} \in (1, \mathcal{N}_{dyn,grp})$. Summarizing, we have the following proposition:

Proposition 3. *Assuming A1-A3 and that both group and dynamic lending are feasible, and provided \mathcal{G} is high enough, a homogeneous-matching simple pooling contract subject to limited liability, monotonicity, and limited borrower commitment achieves maximal borrower surplus iff $\mathcal{N} \geq \mathcal{N}_{dyn,grp} (> 1)$.*

Proof. See Appendix.

The omitted constraints are verified in the proof. The contract is shown to induce homogeneous matching under assumption A3. Matching is not as straightforward as in static (or repeated-static) group lending, since the dynamic interest rate penalty works against homogeneous matching;²⁸ however, this effect is not enough to overcome the incentives for homogeneous group formation provided by joint liability.

One dynamic monotonicity constraint affects the contract interestingly, in particular the imposed constraint that the group owes at least as much after two successes as it does after one success. Consider substituting a more static period-1 monotonicity constraint that simply guarantees current group payments increase in number of successes: $0 \leq r_\emptyset + c_\emptyset \leq 2r_\emptyset$.

²⁸To see this, consider a similar contract without joint liability at any date or history: $r, r_0 > 0$ and $r_2 = r_1 = c_\sigma = 0$. Then, one can show that heterogeneous matching (i.e. negative assortative matching, safe with risky as much as possible) is the only stable match if $p_s + p_r > 1$. See also Guttman (2008).

With this new constraint and if $p_s + p_r > 1$, the contract best for safe borrowers would charge a penalty rate on the loan after one group success ($r_1 = r_0$, high), in addition to imposing full liability on the first loan. This would be a way to shift repayment from fresh borrowers onto those with one failure on the first loan, which is more prevalent for risky than safe borrowers if $p_s + p_r > 1$.²⁹ However, this would leave groups with one failure owing the full group obligation on the first loan, $2r_\emptyset$, and facing penalty rates on the second loan – they would be tempted simply to claim two successes, pay the same current amount $2r_\emptyset$, but get much better future rates. Thus, the dynamic monotonicity constraint forces the lender to moderate the penalty for one group failure, and in particular to choose between joint liability or dynamic interest rate adjustments (or some moderate combination of each). Joint liability wins out as the more effective approach, so that whether one or two borrowers succeeded, the group owes the same current amount and gets the same preferential interest rate on the second loan.

6 Conclusion

[TO BE COMPLETED]

References

- [1] Christian Ahlin and Neville Jiang. Can micro-credit bring development? *Journal of Development Economics*, forthcoming.
- [2] Christian Ahlin and Robert M. Townsend. Using repayment data to test across theories of joint liability lending. *Economic Journal*, 117(517):F11–F51, February 2007a.
- [3] Christian Ahlin and Robert M. Townsend. Selection into and across contracts: Theory and field research. *Journal of Econometrics*, 136(2):665–698, February 2007b.

²⁹In fact, this contract allow full efficiency to be attained for any value of $\mathcal{N} > 1$, at many values of (p_r, p_s) .

- [4] Christian Ahlin and Brian Waters. Dynamic versus group lending with adverse selection. April 2007.
- [5] Beatriz Armendariz de Aghion and Jonathan J. Morduch. *The Economics of Microfinance*. MIT Press, Cambridge MA, 2005.
- [6] Abhijit V. Banerjee, Timothy Besley, and Timothy W. Guinnane. Thy neighbor's keeper: The design of a credit cooperative with theory and a test. *Quarterly Journal of Economics*, 109(2):491–515, May 1994.
- [7] Eric Bellman. Invisible hand: Entrepreneur gets big banks to back very small loans. *Wall Street Journal*, page A1, May 15, 2006.
- [8] Timothy Besley and Stephen Coate. Group lending, repayment incentives and social collateral. *Journal of Development Economics*, 46(1):1–18, February 1995.
- [9] Morris H. DeGroot. *Probability and Statistics*. Addison Wesley, 2nd edition, 1986.
- [10] Jeremy T. Fox. Estimating matching games with transfers. Working Paper, January 9, 2006.
- [11] Shubhashis Gangopadhyay, Maitreesh Ghatak, and Robert Lensink. On joint liability lending and the peer selection effect. *Economic Journal*, 115(506):1005–1015, October 2005.
- [12] Maitreesh Ghatak. Group lending, local information and peer selection. *Journal of Development Economics*, 60(1):27–50, October 1999.
- [13] Maitreesh Ghatak. Screening by the company you keep: Joint liability lending and the peer selection effect. *Economic Journal*, 110(465):601–631, July 2000.
- [14] Maitreesh Ghatak and Timothy W. Guinnane. The economics of lending with joint liability: Theory and practice. *Journal of Development Economics*, 60(1):195–228, October 1999.

- [15] Jean Dickinson Gibbons and Subhabrata Chakraborti. *Nonparametric Statistical Inference*. Marcel Dekker, New York, 4th edition, 2003.
- [16] Joseph Kaboski and Robert M. Townsend. Borrowing and lending in semi-urban and rural Thailand. Mimeo, 1998.
- [17] Patrick Legros and Andrew F. Newman. Monotone matching in perfect and imperfect worlds. *Review of Economic Studies*, 69(4):925–942, October 2002.
- [18] Paolo Mauro. Corruption and growth. *Quarterly Journal of Economics*, 110(3):681–712, August 1995.
- [19] Jonathan Morduch. The microfinance promise. *Journal of Economic Literature*, 37(4):1569–1614, December 1999.
- [20] Loic Sadoulet. Equilibrium risk-matching in group lending. Working paper, June 1999.
- [21] Joseph E. Stiglitz. Peer monitoring and credit markets. *World Bank Economic Review*, 4(3):351–366, September 1990.
- [22] Joseph E. Stiglitz and Andrew Weiss. Credit rationing in markets with imperfect information. *American Economic Review*, 71(3):393–410, June 1981.

Proof of Lemma 1. One can derive the reservation interest rate in period 1 given r_1 and r_0 , call it \hat{r}_τ for a type- τ agent, from the condition $\Pi_\tau(\hat{r}_\tau, r_1, r_0) = 2\bar{u}$. Using payoff 5 and the definition of \hat{r}_τ (see equation 2), \hat{r}_τ can be written

$$\hat{r}_\tau = 2\hat{r}_\tau - p_\tau \min\{r_1, \hat{r}_\tau\} - (1 - p_\tau) \min\{r_0, \hat{r}_\tau\} .$$

We then have

$$\begin{aligned} \hat{r}_r - \hat{r}_s &= 2(\hat{r}_r - \hat{r}_s) + [p_s \min\{r_1, \hat{r}_s\} - p_r \min\{r_1, \hat{r}_r\}] \\ &\quad - [(1 - p_r) \min\{r_0, \hat{r}_r\} - (1 - p_s) \min\{r_0, \hat{r}_s\}] . \end{aligned}$$

It can be verified that the bracketed term on the first line is positive, and that the bracketed term on the second line is positive and maximized at $r_0 = \hat{r}_r$; thus,

$$\hat{r}_r - \hat{r}_s \geq 2(\hat{r}_r - \hat{r}_s) - (1 - p_r)\hat{r}_r + (1 - p_s)\hat{r}_s = \hat{r}_r - \hat{r}_s + p_r\hat{r}_r - p_s\hat{r}_s = \hat{r}_r - \hat{r}_s > 0 ,$$

where the last equality and the inequality use the definition of \hat{r}_τ . Thus, $\hat{r}_s < \hat{r}_r$, so if at some contract (r_\emptyset, r_1, r_0) safe borrowers choose to borrow in period 1 ($r_\emptyset \leq \hat{r}_s$), so do risky ($r_\emptyset \leq \hat{r}_r$). ■

Proof of Proposition 1. Consider the first claim, for $\mathcal{N} \in [\mathcal{N}_{dyn,ind}, \mathcal{N}_{stc,ind})$. The argument in the text finds a contract $(r_1 = 0, r_0 = \hat{r}_s, r_\emptyset)$ from the zero-profit constraint that satisfies most constraints and includes all safe borrowers (and all risky borrowers) iff $\mathcal{N} \geq \mathcal{N}_{dyn,ind}$, where $\mathcal{N}_{dyn,ind} \in (1, \mathcal{N}_{stc,ind})$. Thus, if the remaining constraints are satisfied and for $\mathcal{N} \in [\mathcal{N}_{dyn,ind}, \mathcal{N}_{stc,ind})$, full efficiency is attainable, and since the objective is surplus maximization, it will be achieved. The ignored constraints were limited liability after success and the dynamic monotonicity constraint 4. The dynamic monotonicity constraint boils down to $r_\emptyset \geq p_s\hat{r}_s = \overline{R} - \bar{u}$, which can be shown to hold under assumption A3. Limited liability after success clearly does not bind in period 2, where the maximum interest rate is \hat{r}_s , which can be shown affordable for safe (and hence risky) borrowers. The potentially binding limited liability constraint is the affordability of r_\emptyset , which by assumption A1 is more binding for safe borrowers:

$$R_{p_s} \geq r_\emptyset \quad \iff \quad \mathcal{G} \geq \frac{2p_s - p(1-p)\mathcal{N}}{\bar{p}} .$$

The second inequality multiplies the first by p_s/ρ and uses equation 9 for r_\emptyset and the definitions of \mathcal{N} and \mathcal{G} (equation 1). This constraint is tighter the lower is \mathcal{N} , so if

$$\mathcal{G} \geq \mathcal{G}_{dyn,ind} \equiv \frac{2p_s - \overline{p(1-p)}\mathcal{N}_{dyn,ind}}{\bar{p}},$$

it is satisfied for all $\mathcal{N} \in [\mathcal{N}_{dyn,ind}, \mathcal{N}_{stc,ind}]$.³⁰ Summing up, if $\mathcal{G} \geq \mathcal{G}_{dyn,ind}$ and \mathcal{N} is in this range, fully efficient lending is achieved.

Consider next the last claim, for $\mathcal{N} \in (1, \mathcal{N}_{dyn,ind}^*)$. The argument in the text finds the best contract for safe borrowers – assuming everyone borrows in period 1 – subject to most constraints. It then shows that safe borrowers would indeed choose to borrow in period 1 under this contract iff $\mathcal{N} \geq \mathcal{N}_{dyn,ind}^*$, where $\mathcal{N}_{dyn,ind}^* \in (1, \mathcal{N}_{dyn,ind})$. This proves that for $\mathcal{N} \in (1, \mathcal{N}_{dyn,ind}^*)$, there is no way to include safe borrowers in period 1 (and hence in period 2, given the assumption of no loans to new entrants in period 2) – assuming risky borrow in period 1. (Adding the omitted monotonicity and limited liability constraints would not help; it would weakly lower the maximum achievable safe borrower payoff.) The remaining option left is to check is a contract that includes safe borrowers in period 1 but not risky; but Lemma 1 shows this is impossible. We have thus proved that safe borrowers cannot be included for \mathcal{N} in this range.

Finally, consider the second claim, for $\mathcal{N} \in [\mathcal{N}_{dyn,ind}^*, \mathcal{N}_{dyn,ind})$. The argument in the text finds a contract ($r_1 = 0$, $r_0 = \hat{r}_r$, r_\emptyset from the zero-profit constraint) that satisfies most constraints and includes all safe borrowers except after failure (and all risky borrowers) if $\mathcal{N} \geq \mathcal{N}_{dyn,ind}^*$, where $\mathcal{N}_{dyn,ind}^* \in (1, \mathcal{N}_{dyn,ind})$. Thus, if the remaining constraints are satisfied and if no higher surplus is achievable for $\mathcal{N} \in [\mathcal{N}_{dyn,ind}^*, \mathcal{N}_{dyn,ind})$, then surplus is maximized by this contract and involves everyone borrowing except safe borrowers after failure.

First, is higher surplus achievable? Any contract that achieved higher surplus would have to include failed safe borrowers, since this is the only group left out in the candidate contract. But, the best contract for safe borrowers – assuming all borrow in period 1 – subject to most constraints and including failed safe borrowers ($r_0 \leq \hat{r}_s$) is solved for in the text and is exactly the full efficiency contract discussed in a previous paragraph. This contract fails to include safe borrowers when $\mathcal{N} < \mathcal{N}_{dyn,ind}$. (By Lemma 1, we do not have to worry about contracts that exclude risky borrowers in period 1, since such contracts will also exclude safe borrowers in all states.) Thus, there is no way to include failed safe borrowers for $\mathcal{N} \in [\mathcal{N}_{dyn,ind}^*, \mathcal{N}_{dyn,ind})$, and thus no way to improve surplus.

³⁰One can show that when $\mathcal{N} \in [\mathcal{N}_{dyn,ind}, \mathcal{N}_{stc,ind})$, $[2p_s - \overline{p(1-p)}\mathcal{N}]/\bar{p} > \mathcal{N}_{stc,ind}(> \mathcal{N})$, so the constraint is not automatically satisfied, i.e. requires $\mathcal{G} > \mathcal{N} \Leftrightarrow \bar{u} > 0$.

Second, are the remaining constraints satisfied? Limited liability after success again does not bind in period 2, where safe borrowers face only a zero rate (the others drop out), while some risky borrowers face \hat{r}_r , which can be shown affordable for them. Again, the potentially binding limited liability constraint is the affordability of r_\emptyset , which by assumption A1 is more binding for safe borrowers:

$$R_{p_s} \geq r_\emptyset \quad \Longleftrightarrow \quad \mathcal{G} \geq \frac{p_s[2 - (1 - \theta)(1 - p_s) - \theta(1 - p_r)\mathcal{N}]}{\bar{p}}.$$

The second inequality multiplies the first by p_s/ρ and uses equation 7 for r_\emptyset and the definitions of \mathcal{N} and \mathcal{G} (equation 1). This constraint is tighter the lower is \mathcal{N} , so if

$$\mathcal{G} \geq \mathcal{G}_{dyn,ind}^* \equiv \frac{p_s[2 - (1 - \theta)(1 - p_s) - \theta(1 - p_r)\mathcal{N}_{dyn,ind}^*]}{\bar{p}},$$

it is satisfied for all $\mathcal{N} \in [\mathcal{N}_{dyn,ind}^*, \mathcal{N}_{dyn,ind}]$.³¹ Finally, consider the dynamic monotonicity constraint 3, which given the contract boils down to

$$r_\emptyset \geq p_r \hat{r}_r = p_s \hat{r}_s = \bar{R} - \bar{u} \quad \Longleftrightarrow \quad \mathcal{N} \leq \frac{2 - (1 - \theta)(1 - p_s)}{1 - (1 - \theta)(1 - p_s)},$$

where the second inequality divides the first by ρ , uses equation 7 for r_\emptyset , and solves for \mathcal{N} . It turns out this constraint is not always satisfied in this range of \mathcal{N} . The contract would thus have to be altered but, it turns out, not in any consequential way, because it fails exactly when \mathcal{N} is relatively high. Consider the following modification to the contract: raise r_\emptyset and lower r_0 along the zero-profit constraint until the dynamic monotonicity constraint binds, i.e. until $r_\emptyset = \bar{R} - \bar{u}$. The new contract involves $r_\emptyset = \bar{R} - \bar{u}$, $r_1 = 0$, and r_0 from the zero-profit constraint. It is straightforward to verify that both safe and risky borrowers choose to borrow in this case, and that this modified contract satisfies all constraints, including the limited liability constraint since $R_{p_s} > \bar{R} \geq \bar{R} - \bar{u} = r_\emptyset$. It is also clear that r_0 will still be higher than \hat{r}_s ,³² so the contract continues to include all borrowers except failed safe ones. With the bank earning zero profits, then, borrower surplus is unchanged since the same agents borrow; the only difference is a shift in surplus from safe to risky borrowers. In summary, imposing the dynamic monotonicity constraint may alter the contract but does not change the Proposition's conclusions; and the omitted limited liability constraint is satisfied for \mathcal{G} high enough. ■

³¹Under some parameter values and for \mathcal{N} high enough in this range, this constraint is not binding, i.e. is satisfied for any $\bar{u} \geq 0$ (i.e. $\mathcal{G}_{dyn,ind}^* \leq \mathcal{N}$). But for \mathcal{N} near $\mathcal{N}_{dyn,ind}^*$, it requires $\mathcal{G} > \mathcal{N} \Leftrightarrow \bar{u} > 0$.

³²This is clear since if not, this contract would achieve full efficiency, which we have shown is impossible for \mathcal{N} in this range; it can also be verified algebraically.

Proof of Proposition 2. A menu of contracts now involves several parameters, r_σ^τ and x_σ^τ , where σ represents the state/history and $\tau \in \{r, s\}$ represents the type. There are four histories, $\sigma \in \{\emptyset_1, 0, 1, \emptyset_2\}$: the null history in each period (\emptyset_2 representing new borrowers in period 2) and repeat borrowers in period 2 having 0 or 1 success. For each state and type, r_σ^τ represents the amount due after success from a loan taken by that type at that history, and x_σ^τ the amount due after failure. Both must be kept track of to allow for negative interest rates. Thus, we are considering menus of two contracts described by parameters $(r_{\emptyset_1}^s, r_{\emptyset_1}^r, x_{\emptyset_1}^s, x_{\emptyset_1}^r, r_1^s, r_1^r, x_1^s, x_1^r, r_0^s, r_0^r, x_0^s, x_0^r, r_{\emptyset_2}^s, r_{\emptyset_2}^r, x_{\emptyset_2}^s, x_{\emptyset_2}^r)$.

The proof is in three steps. In part A, we show there is no loss of generality in setting (most) $x_\sigma^\tau = 0$. In part B, we show no loss of generality in offering a single contract rather than a menu of contracts, assuming no period-2 first-time borrowing. In part C, we show that contracts that do induce period-2 first-time borrowing offer no improvement in surplus over those that do not.

A Define $\kappa_\sigma^{\tau'|\tau} \equiv p_\tau r_\sigma^{\tau'} + (1 - p_\tau)x_\sigma^{\tau'}$ as the expected payment of a type- τ agent for a loan intended for type τ' at history σ . The two-period payoff of a type- τ agent who borrows – if and when optimal – under the contract intended for type τ' can be written $2\bar{R} - Z^{\tau'|\tau}$, where $Z^{\tau'|\tau} = \min\{M^{\tau'|\tau}, N^{\tau'|\tau}\}$,

$$\begin{aligned} M^{\tau'|\tau} &= \kappa_{\emptyset_1}^{\tau'|\tau} + p_\tau \min\{p_\tau \hat{r}_\tau, \kappa_1^{\tau'|\tau}\} + (1 - p_\tau) \min\{p_\tau \hat{r}_\tau, \kappa_0^{\tau'|\tau}\}, \quad \text{and} \\ N^{\tau'|\tau} &= p_\tau \hat{r}_\tau + \min\{p_\tau \hat{r}_\tau, \kappa_{\emptyset_2}^{\tau'|\tau}\}. \end{aligned}$$

Here $M^{\tau'|\tau}$ is the two-period expected payment when the agent takes a loan in period 1: the expected payment for a loan at state \emptyset_1 ; if successful the choice between a loan at state 1 or dropping out, which is equivalent to a loan at rate \hat{r}_τ ; and if not, the choice between a loan at state 0 or dropping out. Similarly, $N^{\tau'|\tau}$ is the two-period expected payment when the agent does not take a loan in period 1: this is equivalent to a loan at rate \hat{r}_τ in period 1 and then a choice between a loan at state \emptyset_2 or staying out.

For parts A and B of the proof, we consider the problem of maximizing the safe-borrower payoff, i.e. minimizing Z^{sls} , subject to most of the constraints: limited liability after failure, $x_\sigma^\tau \leq 0$, $\forall \sigma \in \{\emptyset_1, 0, 1, \emptyset_2\}, \tau \in \{r, s\}$; period-2 monotonicity, $x_\sigma^\tau \leq r_\sigma^\tau$, $\forall \sigma \in \{0, 1, \emptyset_2\}, \tau \in \{r, s\}$; the risky incentive compatibility constraint, $Z^{r|r} \leq Z^{s|r}$; and the zero-profit constraint,

which can be written, defining μ_τ as the measure of type- τ agents ($\mu_r = 1 - \theta, \mu_s = \theta$)

$$0 \leq \sum_{\tau \in \{r, s\}} \mu_\tau \left[\mathbf{1}\{N^{\tau|\tau} < M^{\tau|\tau}\} \mathbf{1}\{\kappa_{\emptyset_2}^{\tau|\tau} \leq p_\tau \hat{r}_\tau\} (\kappa_{\emptyset_2}^{\tau|\tau} - \rho) + \mathbf{1}\{M^{\tau|\tau} \leq N^{\tau|\tau}\} \left[\kappa_{\emptyset_1}^{\tau|\tau} - \rho + p_\tau \mathbf{1}\{\kappa_1^{\tau|\tau} \leq p_\tau \hat{r}_\tau\} (\kappa_1^{\tau|\tau} - \rho) + (1 - p_\tau) \mathbf{1}\{\kappa_0^{\tau|\tau} \leq p_\tau \hat{r}_\tau\} (\kappa_0^{\tau|\tau} - \rho) \right] \right]$$

Note that adjusting x_σ^τ up and r_σ^τ down keeping $\kappa_\sigma^{\tau|\tau} = p_\tau r_\sigma^\tau + (1 - p_\tau) x_\sigma^\tau$ fixed does not alter $Z^{s|s}$, $Z^{r|r}$, or the zero-profit constraint. This adjustment may affect $Z^{s|r}$, but if so it raises it, since

$$\frac{dx_\sigma^s}{dr_\sigma^s} \Big|_{\kappa_\sigma^{s|s}} = -p_s / (1 - p_s) < -p_r / (1 - p_r) = \frac{dx_\sigma^s}{dr_\sigma^s} \Big|_{\kappa_\sigma^{s|r}}.$$

Thus, without loss x_σ^τ can be set at their upper bounds defined by limited liability or monotonicity. The intuition is that minimizing the subsidy for failure minimizes the temptation for the risky borrower to pick the safe-borrower contract.

Next, consider x_0^τ at some optimum. If $r_0^\tau \geq 0$, then $x_0^\tau = 0$ (the limited liability upper bound); otherwise, $x_0^\tau = r_0^\tau < 0$ (the monotonicity upper bound). In the latter case, consider lowering x_0^τ by $|x_0^\tau|$ and raising x_0^τ and r_0^τ to zero. This leaves $Z^{s|s}$, $Z^{r|r}$, $Z^{s|r}$, and the zero-profit constraint unchanged, and continues to satisfy the imposed limited liability and monotonicity constraints. Thus, without loss $x_0^\tau = 0$. A similar argument establishes that $x_1^\tau = 0$ without loss – if it were negative, then $x_1^\tau = r_1^\tau < 0$, but these could be raised to zero and $r_{\emptyset_1}^\tau$ lowered by the same amount without changing any key terms or violating any constraints. Finally, consider $x_{\emptyset_1}^\tau$. The argument of the previous paragraph established that it will be at its upper bound, which is zero from limited liability. In sum, we can safely assume $x_0^\tau = x_1^\tau = x_{\emptyset_1}^\tau = 0$, and that either $x_{\emptyset_2}^\tau = 0$ or $x_{\emptyset_2}^\tau = r_{\emptyset_2}^\tau < 0$. For purposes of maximizing the safe-borrower payoff, then, the menu of contracts thus simplifies to $(r_{\emptyset_1}^s, r_{\emptyset_1}^r, r_1^s, r_1^r, r_0^s, r_0^r, r_{\emptyset_2}^s, r_{\emptyset_2}^r, x_{\emptyset_2}^s, x_{\emptyset_2}^r)$.

B Next, we ignore the possibility of first-time borrowing in period 2; we return to this later. That is, we now restrict attention in the constrained minimization of $Z^{s|s}$ to contracts that do not offer first-time borrowing in period 2. (This can easily be accomplished by setting the interest rates in state \emptyset_2 high enough.) Our goal here is to show that within this set of contract menus, requiring identical contracts for safe and risky borrowers is not a binding constraint with respect to the constrained minimization of $Z^{s|s}$.

Consider two subcases, 1) in which at the optimum safe borrowers do not borrow in period 1 (and hence not in period 2), and 2) in which at the optimum safe borrowers borrow in period 1. In case 1), we know that at the optimum $Z^{s|s} = 2p_s \hat{r}_s$; this is the cost corresponding to not borrowing. One way to accomplish this is to set $r_\sigma^\tau = \hat{r}_r$ for $\tau \in \{r, s\}$ and $\sigma \in \{\emptyset_1, 0, 1\}$.

Thus, imposing that risky and safe have identical contracts is without loss in this case.

In case 2), we argue that the risky borrower IC constraint must bind. If the opposite were true, then $Z^{r|r} < Z^{s|r} \leq 2p_r \hat{r}_r$, that is risky borrowers also borrow in period 1. But then the lender could raise $r_{\theta_1}^r$ and lower $r_{\theta_1}^s$ along the zero-profit constraint, lowering $Z^{s|s}$ without violating any constraint. Thus, in this case we must have $Z^{r|r} = Z^{s|r}$. We also argue that without loss, $r_\sigma^\tau \leq \hat{r}_r$ for $\tau \in \{r, s\}$ and $\sigma \in \{0, 1\}$. If not, i.e. if $r_\sigma^\tau > \hat{r}_r$ for some $\tau \in \{r, s\}$ and $\sigma \in \{0, 1\}$, consider lowering it to \hat{r}_r . One can show this does not affect $Z^{s|s}$, $Z^{r|r}$, $Z^{s|r}$, and the imposed limited liability and monotonicity constraints. Iff $\tau = r$, it can affect the zero-profit constraint, but then only positively, raising more revenue because the drop in interest rate attracts risky borrowers in period 2 and extracts all their surplus, which can be used to lower $r_{\theta_1}^s$ and thus $Z^{s|s}$. Combining this period-2 interest rate cap of \hat{r}_r with the binding IC constraint, $Z^{r|r} = Z^{s|r}$, we have

$$\min\{2\hat{r}_r, r_{\theta_1}^r + p_r r_1^r + (1 - p_r)r_0^r\} = \min\{2\hat{r}_r, r_{\theta_1}^s + p_r r_1^s + (1 - p_r)r_0^s\};$$

this uses the definition of $Z^{\tau|\tau}$ above and divides by p_r . Define $Q^{r|r}$ as the second term in brackets on the left-hand side, and $Q^{s|r}$ as the analogous term on the right-hand side; thus $\min\{2\hat{r}_r, Q^{r|r}\} = \min\{2\hat{r}_r, Q^{s|r}\}$. The zero-profit constraint simplifies to

$$\theta \cdot \mathbf{1}\{Q^{r|r} \leq 2\hat{r}_r\}(p_r Q^{r|r} - 2\rho) + (1 - \theta) \cdot [\dots] \geq 0;$$

the first term represents risky borrowers' expected total payment minus capital costs, if they borrow ($Q^{r|r} \leq 2\hat{r}_r$), while the second term represents safe borrowers and depends solely on the contract through r_σ^s terms, which are omitted for brevity.

Continuing case 2), we next show that $Q^{s|r} < 2\hat{r}_r$; this will guarantee that $Q^{s|r} = Q^{r|r}$. Note that since safe agents borrow in period 1, by assumption, then $Z^{s|s} = p_s Q^{s|s}$, where

$$Q^{s|s} \equiv r_{\theta_1}^s + p_s \min\{\hat{r}_s, r_1^s\} + (1 - p_s) \min\{\hat{r}_s, r_0^s\};$$

also, $Z^{s|s} \leq 2p_s \hat{r}_s$. Thus, $Q^{s|s} \leq 2\hat{r}_s$. Note that

$$\begin{aligned} Q^{s|r} - Q^{s|s} &= p_r r_1^s - p_s \min\{\hat{r}_s, r_1^s\} + (1 - p_r)r_0^s - (1 - p_s) \min\{\hat{r}_s, r_0^s\} \\ &\leq 0 + (1 - p_r)\hat{r}_r - (1 - p_s)\hat{r}_s = \hat{r}_r - \hat{r}_s. \end{aligned}$$

The inequality comes from noting that the first difference in the sum is maximized (over $[0, \hat{r}_r]$) at $r_1^s = 0$ or $r_1^s = \hat{r}_r$, while the second difference in the sum is maximized (over $[0, \hat{r}_r]$)

at $r_0^s = \hat{r}_r$. Thus,

$$Q^{s|r} \leq Q^{s|s} + \hat{r}_r - \hat{r}_s \leq \hat{r}_r + \hat{r}_s < 2\hat{r}_r .$$

Combining this with the earlier analysis, we have that $Q^{r|r} = Q^{s|r}$. One can then modify any contract menu to give the risky borrower the safe borrower's contract, i.e. to offer only a single pooling contract at the safe borrower's contract. Note that this would not change the risky borrower's payoff, since $Q^{s|r} = Q^{r|r}$, and it would not change the zero-profit constraint above for the same reason (note that it only depends on $Q^{r|r}$).³³ All other constraints continue to be satisfied; thus, any optimal contract of this kind can be replicated with a simple pooling contract.

We have shown that ignoring contracts that induce new entry in period 2 and subject to most constraints, no contract can provide a higher payoff for safe borrowers than a simple pooling contract, the best among which is the near efficient-contract derived in the text. This implies that for $\mathcal{N} < \mathcal{N}_{dyn,ind}^*$, safe agents will not borrow, since no menu of contracts can provide a higher payoff than a simple pooling contract, which fails to attract safe borrowers for \mathcal{N} in this range. We also know that for $\mathcal{N} \in [\mathcal{N}_{dyn,ind}, \mathcal{N}_{stc,ind})$, full efficiency is achieved with a simple pooling contract, and thus a menu of contracts cannot raise surplus.

Finally, we argue that for $\mathcal{N} \in [\mathcal{N}_{dyn,ind}^*, \mathcal{N}_{dyn,ind})$, no menu of contracts can improve surplus relative to the near-efficient simple pooling contract. We have shown no menu of contracts can raise safe payoffs, but the possibility remains that a menu might lower safe payoffs but raise overall surplus. Note that to raise surplus, the menu of contracts would need to include failed safe borrowers, since this is the only group excluded under the best simple pooling contract. But one can then repeat the entire argument with the additional constraint that $r_0^s \leq \hat{r}_s$ and show that no menu of contracts can give safe borrowers higher payoffs than the simple pooling contract that maximizes safe-borrower payoffs subject to this constraint, the efficient contract derived in the text. Thus, for \mathcal{N} in this range no menu of contracts that has a chance at delivering higher efficiency than the near-efficient contract can provide high enough payoffs for safe borrowers to include them, since the efficiency simple pooling contract cannot.

Thus, if parameters are such that the simple pooling contract satisfies all constraints, which we have shown true if \mathcal{G} is high enough, no other contract (disallowing entry in period 2) can raise surplus relative to the optimal simple pooling contract.

C In the remainder of the proof, we argue that a contract that induces first-time entry in period 2 cannot raise surplus relative to the optimal pooling contract that does not

³³This is a key statement of the proof, and it relies on the lender's and borrower's payoffs being diametrically opposed. This implies there is no loss in putting the risky borrower on his IC constraint exactly where the safe borrower is, since anywhere on the IC constraint is equally good for both risky borrower and lender.

allow period-2 entry.

Consider a menu of contracts that induces first-time risky borrowing in period 1 and first-time safe borrowing in period 2, involving $r_{\emptyset_2}^s$ and $x_{\emptyset_2}^s$. Consider substituting a different contract for safe borrowers that charges $\widetilde{r}_{\emptyset_1}^s = r_{\emptyset_2}^s$ and $\widetilde{x}_{\emptyset_1}^s = x_{\emptyset_2}^s$, and puts all other interest rates above \hat{r}_r (and other $x_{\sigma}^s = 0$). This gives the exact same deal to safe borrowers – one loan at $r_{\emptyset_2}^s$ and $x_{\emptyset_2}^s$, one dropout period – so safe will choose the safe-borrower contract and borrow in period 1 only. Further, it leaves unchanged or reduces the risky borrowers' temptation to take the safe contract – the temptation is now to get one loan at $r_{\emptyset_2}^s$ and $x_{\emptyset_2}^s$ and then drop out, while before it was also a loan at $r_{\emptyset_2}^s$ and $x_{\emptyset_2}^s$ plus one period doing no worse than dropping out. In sum, under the alternate contract, safe agents borrow in period 1 then drop out, while risky borrow in period 1 under their original contract; all payoffs and total surplus remains unchanged. Thus, the original menu of contracts can be replicated by one that does not allow first-time borrowing in period 2.

The same basic argument applies to a menu of contracts that induces first-time safe borrowing in period 1 and first-time risky borrowing in period 2. The risky contract's period-2 rates can be switched to period 1, with period-2 rates set above \hat{r}_r . This new menu continues to satisfy incentive compatibility and results in the same payoffs and total surplus. Similarly, a menu of contracts that induces both types to first-time borrow in period 2 can be replicated with the same rates switched to period 1 and period-2 rates set above \hat{r}_r .

In sum, any menu of contracts that induces first-time period-2 entry can be replicated with a contract that does not allow it. This brings all such contracts into the same category as those ruled out in rest of the proof as offering no improvement relative to the simple pooling contract, under the stated conditions. ■

Proof of Lemma 2. Fix the sums $r_{01} + r_{10}$ and $c_{01} + c_{10}$ in a contract that maximizes the safe-borrower payoff subject to the imposed constraints. It is straightforward to verify that setting $r'_{01} = r'_{10} = (r_{01} + r_{10})/2$ and $c'_{01} = c'_{10} = (c_{01} + c_{10})/2$ does not change the maximand or any constraints, since these depend on the sums only and not the terms separately – with the exception of the (safe-borrower) limited commitment and period-2 monotonicity constraints. Concerning the limited commitment constraints, it is straightforward to show (by summing the two relevant constraints) that these alternative parameter values also satisfy them; the same argument applies to the period-2 monotonicity constraints. ■

Proof of Lemma 3. One can show that raising c_k and lowering r_k along the safe borrower's indifference curve, for any $k \in \{0, 1, 2\}$, does not affect the safe-borrower payoff, limited commitment constraints, or dynamic monotonicity constraint 13. It does, however,

slacken the zero-profit constraint; the excess profits can be refunded in a way that does not violate constraints (e.g. lowering c_\emptyset), raising safe-borrower payoffs. This proves that c_k must be at its upper bound, defined by period-2 monotonicity: $c_k = r_k$, for all $k \in \{0, 1, 2\}$. ■

Proof of Lemma 4. Consider maximizing the safe-borrower payoff

$$2\bar{R} - p_s r_\emptyset - p_s(1 - p_s)c_\emptyset - p_s^3(2 - p_s)r_2 - 2p_s^2(1 - p_s)(2 - p_s)r_1 - p_s(1 - p_s)^2(2 - p_s)r_0$$

subject to the bank's zero-profit constraint (multiplier μ)

$$\bar{p}r_\emptyset + \overline{p(1-p)}c_\emptyset + \overline{p^3(2-p)}r_2 + \overline{2p^2(1-p)(2-p)}r_1 + \overline{p(1-p)^2(2-p)}r_0 \geq 2\rho ;$$

dynamic monotonicity constraint 13 (multiplier γ)

$$r_\emptyset + 2p_s(2 - p_s)r_2 \geq c_\emptyset + 2p_s(2 - p_s)r_1 ;$$

limited commitment constraint (multiplier λ_0)

$$p_s r_0 + p_s(1 - p_s)r_0 \leq \hat{r}_s \quad \iff \quad \frac{\hat{r}_s}{p_s(2 - p_s)} - r_0 \geq 0 ;$$

and monotonicity constraints $r_2 \geq 0$, $r_1 \geq 0$ (multipliers λ_2 and λ_1 , respectively). The first-order conditions are

$$\begin{array}{llll} [r_\emptyset] & - p_s & +\bar{p}\mu + \gamma & = 0 \\ [c_\emptyset] & - p_s(1 - p_s) & +\overline{p(1-p)}\mu - \gamma & = 0 \\ [r_2] & - p_s^3(2 - p_s) & +\overline{p^3(2-p)}\mu + \gamma 2p_s(2 - p_s) & +\lambda_2 = 0 \\ [r_1] & - 2p_s^2(1 - p_s)(2 - p_s) & +\overline{2p^2(1-p)(2-p)}\mu - \gamma 2p_s(2 - p_s) & +\lambda_1 = 0 \\ [r_0] & - p_s(1 - p_s)^2(2 - p_s) & +\overline{p(1-p)^2(2-p)}\mu & -\lambda_0 = 0 \end{array}$$

Solving this linear system of five equations reveals that all five multipliers are strictly positive. Thus, all constraints bind. This pins down r_0 , r_1 , and r_2 directly, at the values stated in the lemma; given $r_2 = r_1$, the dynamic monotonicity constraint gives $c_\emptyset = r_\emptyset$; and finally, the preceding facts in the zero-profit constraint pins down r_\emptyset . We ignored several imposed constraints, period-2 monotonicity after two failures and limited commitment after one or two successes; these are clearly satisfied. ■

Proof of Proposition 3. The text establishes that if $\mathcal{N} < \mathcal{N}_{dyn,grp}$, the simple pooling contract best for safe borrowers – subject to most constraints and assuming all agents borrow – does not attract them. Clearly adding omitted constraints would not attract them. Thus, there is no way to induce everyone to borrow with a simple pooling contract for \mathcal{N} in this range.

We also showed that for $\mathcal{N} \geq \mathcal{N}_{dyn,grp}$, the same contract attracts safe borrowers in both periods. If it also attracts risky borrowers in both periods and satisfies the omitted constraints, then maximal borrower surplus is attained.

Risky borrowers do indeed borrow under these conditions. One can show that at any period-2 loan where $c_\sigma \leq r_\sigma$, which is implied by monotonicity, risky borrowers find it profitable to borrow if safe borrowers do; i.e. the risky borrower’s limited-commitment constraint is implied by the safe borrower’s. One can also show that given $r_2 = r_1 = 0$ and $r_0 = c_0 = \hat{r}_s/(2 - p_s)$, and $c_\emptyset = r_\emptyset$, the risky borrower’s reservation rate for the first loan is higher than the safe borrower’s, so safe choosing to borrow imply that risky also do.

Regarding matching, one can show that under assumption A3, $\Pi_{ss} - \Pi_{sr} > \Pi_{rs} - \Pi_{rr}$, where Π_{ij} is the two-period payoff of a borrower of type i matched with a borrower of type j under the contract derived in the text. This guarantees that homogeneous matching by risk-type is indeed the unique stable match. It remains stable in period 2 due to the positive joint liability.

Limited liability after failure is automatically satisfied by the contract in that nothing is ever due from borrowers who fail. Limited liability after success is satisfied, as before, if \mathcal{G} is high enough. To see this, note that $r_\emptyset + c_\emptyset = 2r_\emptyset$ and $r_0 + c_0 = 2r_0$ must be affordable to any borrower, the successful safe borrower being the poorer one. Thus we need

$$R_{p_s} \geq 2 \max\{r_\emptyset, r_0\} \quad \iff \quad \mathcal{G} \geq 2p_s \max \left\{ \frac{2 - \frac{p(1-p)^2(2-p)}{p_s(2-p_s)} \mathcal{N}}{p(2-p)}, \frac{\mathcal{N}}{p_s(2-p_s)} \right\} ;$$

the second inequality multiplies the first by p_s/ρ and uses the derived expressions for r_\emptyset and r_0 . Since \mathcal{N} is in a bounded set $(1, \mathcal{N}_{stc,ind})$, the right-hand side is bounded and a high enough \mathcal{G} guarantees this inequality is satisfied.

Finally, consider the dynamic monotonicity constraints. One of the four was imposed in the maximization, guaranteeing that safe groups are not tempted to claim two successes instead of one; the contract clearly satisfies this for risky groups too, since $r_1 = r_2 = 0$ and $c_\emptyset = r_\emptyset$. The other two guarantee disclosure of two failures rather than just one:

$$r_\emptyset + c_\emptyset + 2p_\tau(2 - p_\tau)r_1 \geq 2p_\tau(2 - p_\tau)r_0, \quad \tau \in \{r, s\},$$

the more binding of which boils down to

$$r_\emptyset \geq p_s(2 - p_s)r_0 = \bar{R} - \bar{u} .$$

This typically, but not always, holds under assumption A3. When it does not, the period-1 interest rate and joint liability are so low relative to the period-2 penalty rate that they are worth paying in order to qualify for a free loan in period 2. This only happens, it turns out, when \mathcal{N} is high, in which case safe borrowers are not on the margin and the contract is easily adjusted to satisfy the constraint. In particular, consider modifying the best-for-safe contract by raising r_\emptyset (and $c_\emptyset = r_\emptyset$) and lowering r_0 (and $c_0 = r_0$) in tandem along the zero-profit constraint, until r_\emptyset reaches $\bar{R} - \bar{u}$ (one can show under assumption A3 that r_0 does not reach zero). This clearly satisfies the above dynamic monotonicity constraint. It also continues to satisfy all other monotonicity and limited commitment constraints, homogeneous matching, and limited liability if \mathcal{G} is high enough. Finally, one can show that it attracts safe borrowers (and hence risky). Since lender profits are zero, borrower surplus is still maximal. In sum, in the cases where the best-for-safe contract violates the above dynamic monotonicity constraint, the contract can be modified to satisfy all constraints and still achieve maximal surplus. ■