Pandering to Persuade[*]

Yeon-Koo Che[†]   Wouter Dessein[‡]   Navin Kartik[§]

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Abstract

A principal chooses one of $n \geq 2$ projects or an outside option. An agent is privately informed about the projects' benefits and shares the principal's preferences except for not internalizing her value from the outside option. We show that strategic communication is characterized by *pandering*: the agent biases his recommendation toward *good-looking* projects—those with appealing observable attributes—even when both parties would be better off with some other project. Projects become more acceptable when *pitched* against a stronger slate of alternatives. We study organizational responses to the pandering distortion, such as delegation and choosing to be less informed.

[†]Department of Economics, Columbia University, and YERI, Yonsei University. Email: yc2271@columbia.edu.

[‡]Graduate School of Business, Columbia University. Email: wd2179@columbia.edu.

[§]Department of Economics, Columbia University. Email: nkartik@columbia.edu.

1 Introduction

A central problem in organizations and markets is that of a decision-maker (DM) who must rely upon advice from a better-informed agent. Starting with Crawford and Sobel (1982), a large literature studies the credibility of "cheap talk" when there are conflicts of interest between the two parties. This paper addresses a novel issue: how do differences in observable or verifiable characteristics of the available alternatives affect cheap talk about non-verifiable private information? In a nutshell, our main insight is that the agent's desire to *persuade* the DM ineluctably leads to recommendations that systematically *pander* toward alternatives that *look better*. We develop the economics of pandering to persuade in strategic communication and study implications for organizational and market responses.

In any number of applications, a DM knows some characteristics of the options she must choose from. For instance, a Dean deciding whether to a hire a new faculty in the economics department can consult candidates' curriculum vitae; a board deciding which capital investment project to fund has some prior experience about which kinds of projects are more or less likely to succeed; and a firm that could hire a consultant to revamp its management processes knows which procedures are being implemented at other firms. This information is not complete, though, since the agent—the economics department, CEO, or consultant respectively—typically has additional "soft" or unverifiable private information. For example, an economics department can evaluate the quality of a candidate's research well beyond the content of a vita. Crucially, the available "hard" information can affect the DM's interpretation of the agent's claims about his soft information. The reason is that any hard information typically creates an asymmetry among the alternative options from the DM's point of view, causing some to look ex-ante more attractive than others. Our interest is in understanding how such asymmetry influences the agent's strategic communication of his soft information.

The incentive issues arise in our model because the DM also has available an outside option, or status quo, in addition to the set of alternatives that the agent is better-informed about. For instance, the outside option for a Dean could be to hire no new faculty member or hire a faculty in a different department, or for a corporate board to not fund any capital investment project. Since the alternatives require resource costs that the agent does not fully internalize, the outside option is typically more desirable to the DM than the agent. In our baseline model, detailed in Section 2, this is the only conflict of interest. More precisely, any alternative project gives the DM and the agent a common benefit; each project's value is randomly drawn and privately observed by the agent. On the other hand, the agent derives no benefit from the outside option, whereas the DM receives some fixed and commonly known

benefit from choosing it.

Consequently, the strategic problem facing the agent is to persuade the DM that some alternative is better than the outside option in a way that maximizes their common benefit amongst the set of alternatives. This captures an essential feature of many applications, including each of the examples mentioned above.

In this setting, we show that cheap-talk communication necessarily takes the form of comparisons.[1] In equilibrium, the agent's message is interpreted as a recommendation about which alternative provides the highest benefit. Our central insight is that any observable differences between alternatives will often cause the agent to systematically distort his true preference ranking over the alternatives: the agent will sometimes recommend an ex-ante attractive alternative that is in fact worse than some other (which was ex-ante less attractive), even though both the agent and the DM would be better off with the latter! In this sense, the agent *panders* toward alternatives with favorable observable information. Although aware of this pandering distortion, in any influential equilibrium, the DM always accepts the agent's recommendation of the most attractive options, while she is more circumspect when the agent recommends an ex-ante unattractive option, in the sense that she may or may not accept such a recommendation.

Despite the common interest the two parties have over alternative projects, the distortion in communication is unavoidable because the agent is also trying to persuade the DM to adopt some alternative over the outside option. If the agent were to always recommend the best alternative, then a recommendation for ex-ante attractive (or "better-looking") alternatives would generate a more favorable assessment from the DM about the benefit of foregoing the outside option. Consequently, for some outside option values, the DM would accept the agent's recommendation of better-looking alternatives but stick with the outside option when a less-attractive (or "weaker-looking") alternative is recommended. This generates the incentive for the agent to distort recommendations toward better-looking alternatives. The incentive to distort becomes more severe when the value of the outside option to the DM is higher.

Building on this basic observation, our analysis proceeds in two ways. First we develop and refine the pandering-to-persuade intuition in a multidimensional cheap-talk model, showing how influential communication can nevertheless take place. The idea is that if the agent recommends an ex-ante unattractive alternative only when it is *sufficiently better*—not just better—than all others, it becomes more acceptable to the DM when recommended. After presenting an illustrative example in Section 3, we turn in Section 4 to a general analysis of when one alternative

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[1]Comparative cheap talk has been studied by Chakraborty and Harbaugh (2007, 2009). As discussed in more detail subsequently, our focus is distinct and complementary to these papers.

"looks better" than another. Formally, this amounts to identifying an appropriate stochastic (partial) order to rank the distributions from which the value of each alternative is drawn. We show that when the stochastic ordering condition holds, pandering toward better-looking alternatives arises in any influential equilibrium of the cheap-talk game once the outside option is sufficiently high for the DM, i.e. when the agent truly needs to persuade the DM. The stochastic ordering of alternatives can be intuitive in some cases, such as when it coincides with the first-order stochastic dominance (FOSD) ranking. But the opposite can also be true: an alternative that is dominated according to FOSD (and even in likelihood ratio) can nevertheless be the one that the agent panders toward. While perhaps counterintuitive, this highlights the economics of communication in the present context: what matters is not the evaluation of an alternative in isolation, but rather *in comparison* to others. Standard stochastic relations pertain to the former, whereas the relation we identify concerns the latter.

Next, we explore several implications of the characterization of pandering. Of note is that weaker-looking alternatives become more credible or acceptable to the DM when they are *pitched* against a stronger slate of alternatives (formally, when the distribution of any alternative improves in the sense of likelihood-ratio dominance). A related point is that weaker-looking alternatives are often better off in a pandering equilibrium (where they are discriminated against) when compared to a truthful ranking. Returning to the hiring application, these two points suggest why candidate A from a lower-ranked department can actually benefit from competing with candidate B from a higher-ranked department than with someone from a similarly-ranked department; moreover, why A is better off when the hiring committee is known to pander toward B rather than just giving a truthful ranking between A and B.

Section 5 turns to studying responses that the DM can take to mitigate the inefficiencies from pandering. Since the communication distortion worsens as the outside option becomes more valuable to the DM, a stronger outside option can hurt the DM. This implies that the DM may benefit from *burning ships*, i.e. reducing the value of the outside option, even at some cost. We also show how a "commitment to buy" or *simple delegation* to the agent (Aghion and Tirole, 1997; Dessein, 2002) is always beneficial to the DM relative to any influential communication if she can can commit to not override the agent's choice ex post, and, moreover, she can make the delegation decision *after* observing some characteristics of the alternatives.

An implication is that observable hard information is valuable in decision-making to the extent that it informs delegation decisions or "no-strings-attached" budget allocations, but no more than that. If, based on the observable information, the DM deems the agent trustworthy enough in the sense that influential communication is possible, then she should instead just give

him full discretion about which alternative to choose. In many circumstances, such a commitment may not be feasible or credible, given that the DM would often have an incentive to override the agent's choice ex post. In these cases, observable information can be harmful to good decision-making because of the pandering distortions it creates in the communication of unverifiable information. Ex-ante, the DM may even prefer *ignorance*—committing never to observe any information about the alternatives—because this mitigates pandering. We also discuss properties of more sophisticated mechanisms when richer commitment possibilities are available, such as stochastic mechanisms, which can be implemented via delegation to a third party.

Before concluding (Section 7), we address in Section 6 some extensions that are potentially important for different applications of the model. Consider the resource allocation problem where a DM decides which projects to provide funding for. The DM may be privately informed about the opportunity cost of resources; she may not only decide which project to fund, but also how much resources to make available; or she may be able to fund more than one project if she wishes to. We show that our baseline model can easily accommodate such extensions, and our main insights regarding pandering and delegation are robust. Another application of the model is to a seller (e.g. consultant) providing advice to a potential buyer (e.g. a firm). It is reasonable that the seller may have a larger profit margin on certain projects, which creates a conflict of interest even between the alternative projects. Again, such conflicts can be introduced in our model and the basic logic of pandering still holds. Unconstrained delegation is only optimal, however, if the conflicts over alternatives are small relative to the observable asymmetries between alternatives.

This paper connects to multiple strands of literature. The logic of pandering is related to Brandenburger and Polak (1996).[2] They elegantly show how a manager who cares about his firm's short-run stock price will distort his investment decision towards an investment that the market believes is ex-ante more likely to succeed. However, their model is not one of strategic communication, but rather has an agent making decisions himself when concerned about external perceptions. As a result, we study a different set of issues, such as organizational and market responses, and we shed light on a broader set of applications, such as buyer-seller relationships and resource allocation processes in firms. Our analysis and findings are also more refined because of a richer framework.[3] Inter alia, we show that an agent may pander towards an alternative with *lower ex-ante expected value*, which does not arise in Brandenburger and Polak (1996).

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[2]See Heidhues and Lagerlof (2003) and Loertscher (2010) for multi-agent versions of a similar theme in the context of electoral competition.

[3]Their model has two states, two noisy signals, and two possible decisions. We have continuous and multi-dimensional state space, perfectly informative signals, an arbitrary finite number of decisions. Moreover, the preferences for the agent in our model are more complex because he also cares about the benefit of the chosen alternative and not just about whether the outside option is foregone.

Crawford and Sobel (1982)'s canonical model of cheap talk has one-dimensional private information and a different preference structure than ours. Within the small but growing literature on multidimensional cheap talk (e.g., Battaglini, 2002; Ambrus and Takahashi, 2008; Chakraborty and Harbaugh, 2009), the most relevant comparison is with Chakraborty and Harbaugh (2007). They show how truthful comparisons can be credible across dimensions even when there is a large conflict of interest within each dimension, so long as there are common interests across dimensions. A key assumption for their result is enough symmetry across dimensions in terms of preferences and the prior. Our analysis is complementary because we study the properties of informative communication when there is enough *asymmetry* across dimensions; this leads to a breakdown of truthful comparisons and instead generates pandering.[4]

Since we compare the outcomes of our cheap-talk model with simple delegation and other mechanisms without transfers, part of this paper is also related to the constrained delegation literature initiated by Holmstrom (1984).[5] Our setting is closest to Armstrong and Vickers (2010). Pandering is not an issue in their paper, however, because they assume that the values of projects are drawn from identical distributions.

Finally, we note that although the notion of pandering may be reminiscent of various kinds of "career concerns" models (e.g. Morris, 2001; Majumdar and Mukand, 2004; Maskin and Tirole, 2004; Prat, 2005; Ottaviani and Sorensen, 2006), the driving forces there are very different from the current paper. In those models, the distortions occur because the agent is attempting to signal either his ability or preferences because of, implicity or explicitly, future considerations. In contrast, our model has no such uncertainty and no dynamic considerations; rather, the distortions occur entirely because the agent wishes to persuade the DM about her current decision. The logic here is also distinct from that of Prendergast (1993), where distortions occur because a worker tries to guess the private information of a supervisor when subjective performance evaluations are used.

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[4] Levy and Razin (2007) identify conditions under which communication can entirely break down in a model of multidimensional cheap talk when the conflict of interest is sufficiently large. While this also occurs in our model for a large enough outside option, their result crucially relies on the state being correlated across dimensions, whereas we assume independence. More importantly, our focus is on the properties of influential communication when the outside option is not too large.

[5] Some recent contributions include Alonso and Matouschek (2008), Goltsman, Hörner, Pavlov and Squintani (2009), Kovác and Mylovanov (2009), and Koessler and Martimort (2009).

2 The Model

2.1 Setup

There are two players: an agent ("he") and a decision-maker (DM, "she"). The DM must choose a single option, i, from the set $\{0, 1, \ldots, n\}$, where $n \geq 2$.[6] It is convenient to interpret option 0 as a status quo or outside option for the DM, and $N := \{1, \ldots, n\}$ as a set of alternative projects. Both players share a common payoff if one of the alternative projects is chosen, but this value is private information of the agent. Specifically, each project $i \in N$ yields both players a payoff of b_i that is drawn from a prior distribution F_i and privately observed by the agent. (Throughout, payoffs refer to von Neumann-Morgenstern utilities, and the players are expected utility maximizers.) On the other hand, it is common knowledge that if the outside option is chosen, the agent's payoff is zero (a normalization), while the DM's payoff is $b_0 > 0$.

We maintain the following assumptions on $(F_1, \ldots, F_n)$ and b_0:

(A1) For each $i \in N$, $0 \leq \underline{b}_i < b_0 < \bar{b}_i \leq \infty$, where $\underline{b}_i := \inf Support[b_i]$ and $\bar{b}_i := \sup Support[b_i]$.

(A2) For each $i \in N$, F_i is absolutely continuous on $[\underline{b}_i, \bar{b}_i]$, with a differentiable density f_i.

(A3) For each pair $i, j \in N$ with $i \neq j$, $\exists \alpha \in \mathbb{R}_{++}$ such that $\mathbb{E}[b_i | b_i > \alpha b_j] > b_0$.

(A4) For any $i, j \in N$, F_i and F_j are independent distributions, but they need not be identical.

After privately observing $\mathbf{b} := (b_1, \ldots, b_n) \in \mathcal{B} := \prod_{i=1}^{n} [\underline{b}_i, \bar{b}_i]$, which we also refer to as the agent's **type**, the agent sends a cheap-talk or payoff-irrelevant message to the DM, $m \in M$, where M is a large space (e.g. $M = \mathbb{R}_+^n$). The DM then chooses a project $i \in N \cup \{0\}$. Aside from the realization of $\mathbf{b}$, all aspects of the game are common knowledge.

2.2 Discussion of the assumptions

Since both the agent and the DM derive the same payoff, b_i, for any $i \in N$, their interests in choosing from the n projects are completely aligned. Assumption (A1) implies that each project has a positive chance of being better for the DM than the outside option; or else a project would not be a viable choice. More importantly, (A1) also implies that the agent strictly prefers any

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[6]Nothing is lost by excluding $n = 1$, as the analysis is trivial given the rest of the model.

project to the outside option, whereas with positive probability, each project is worse than the outside option for the DM. Thus, the conflict of interest is entirely about the outside option: the agent does not internalize the opportunity cost to the DM of implementing a project. What is essential here is that the DM values the outside option more than the agent relative to the alternative projects; allowing for $\underline{b}_i < 0$ complicates some details of the analysis without adding commensurate insight.[7]

Assumption (A2) is for technical convenience. Assumption (A3) means that the DM's posterior assessment of each project $i \in N$ becomes more favorable than the outside option if project i is known to be sufficiently better than some other project $j \in N$. Note that given (A1), this is automatically satisfied if $\underline{b}_i > 0$ for all i. The precise role of (A3) will be clarified later, but intuitively, it ensures that if the agent only recommends a project when it is sufficiently better than some other, the DM will wish to implement it.

The independence portion of Assumption (A4) is not essential for the main results about pandering, but makes some of the analysis and results more transparent. (A4) also allows for non-identical project distributions. Since this is central to the pandering results, it is worth discussing at some length. A useful way to interpret non-identical distributions is that each project i has some attributes that are publicly observed and some attributes that are privately observed by the agent. For example, if the projects represent academic job candidates, the two components may respectively be a candidate's vita and the department's evaluation of her future trajectory. Both aspects can be viewed as initially stochastic, with the distribution F_i capturing the residual uncertainty about i's value *after* the observable components have been realized (and observed by both DM and agent). Typically, projects will have different realizations of observable information, so that even if projects i and j are initially symmetric, the realizations of their observation information will create an asymmetry in the residual uncertainty about them, so that $F_i \neq F_j$. One can therefore view the distribution of b_i's as parameterized by some observable information v_i; i.e., $F_i(b_i) \equiv F(b_i; v_i)$. The following are two parameterized families of distributions that serve as useful examples:

- **Scale-invariant uniform distributions**: b_i is uniformly distributed on $[v_i, v_i + \overline{u}]$, where $v_1 \geq v_2 \geq \cdots \geq v_n > 0$.

- **Exponential distributions**: b_i is exponentially distributed on $[0, \infty)$ with mean v_i, where

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[7]If $b_i < 0$, then the agent will prefer the outside option over project i. For our purposes, the situation can equivalently be modeled by generating a new distribution for project i, say $\tilde{F}_i$, that is supported on $[0, \overline{b}_i]$ as follows: $\tilde{F}_i(x) = 0$ for all $x < 0$ and $\tilde{F}_i(x) = F_i(x)$ for all $x \geq 0$. Since it is credible for the agent to reveal that $b_i < 0$, the strategic communication problem concerns $\tilde{F}_i$. The resulting atom at zero in $\tilde{F}_i$ can be accommodated in the analysis.

$$v_1 \geq v_2 \geq \cdots \geq v_n > 0.$$

Rather than thinking of some attributes as directly observable to the DM, it may also be plausible that all aspects are privately observed by the agent, but there are two kinds of information: verifiable or "hard" information, and unverifiable or "soft" information. Under a monotone likelihood ratio condition that is satisfied by the above two families but is considerably more general, analogues of standard "unraveling" arguments (Milgrom, 1981; Seidmann and Winter, 1997) support the agent fully revealing the verifiable components. It is then effectively as though the DM directly learns the realizations of these components, and again F_i captures the residual soft information about project i. We formalize this argument in Appendix F.

2.3   Equilibrium simplification

We study a class of perfect Bayesian equilibria. The agent is assumed to use a pure strategy in equilibrium, represented by a function $\mu : \mathcal{B} \to M$.[8] A possibly-mixed strategy for the DM is $\alpha : M \to \Delta(N \cup \{0\})$, where $\Delta(\cdot)$ is the set of probability distributions. We restrict attention to equilibria where the DM does not randomize on the equilibrium path between two or more alternative projects. In other words, in equilibrium, any randomization by the DM must be between the outside option and one project, although which project it is could depend upon the message received. Given that the only conflict between the two players is about the outside option, we view this as a natural class of equilibria to study; indeed, Appendix E proves that this is without loss of generality when $n = 2$, except for knife-edged prior distributions. Hereafter, "equilibrium" refers to a perfect Bayesian equilibrium satisfying these properties.[9]

Since the game is one of cheap talk, the objects of interest are equilibrium mappings from agent types to the DM's (mixtures over) decisions, i.e. $\alpha(\mu(\cdot))$, rather than what messages are used per se. Say that two equilibria are **outcome-equivalent** if they have the same such mapping for almost all types.

Lemma 1. *Any equilibrium is outcome-equivalent to one where no more than n messages are used in equilibrium.*

The proof of this result and all others not in the text are in Appendices A and B. The intuition is straightforward: there are n alternative projects and any message will lead to a

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[8]We conjecture that this is without loss of generality, but have not proved it.

[9]Some readers may prefer to think of the model as a *veto game*: the agent chooses a project from the set N, and the DM can only choose whether to accept the proposal or veto it in favor of the outside option. This would be a natural model for many applications. While the game would no longer be one of cheap talk, our results also hold in this setting with minor qualifiers.

distribution of decisions over the outside option and at most one project. Whenever two or more messages result in a particular project being implemented with positive probability, the agent will only use the message(s) that maximize(s) the acceptance probability of that project. Finally, equilibria in which two or more messages yield the same acceptance probability are outcome-equivalent to an equilibrium in which only one of these messages is ever used.

In light of Lemma 1, we focus hereafter on equilibria where no more than n messages are used, which, without loss of generality, can be taken to be the set N. In other words, *the cheap-talk game is effectively reduced to one in which the agent "recommends" a project $i \in N$*. In turn, the DM's equilibrium strategy can now be viewed as a vector of **acceptance probabilities**, $\mathbf{q} := (q_1, \ldots, q_n) \in [0, 1]^n$, where q_i is the probability with which the DM implements project i *if the agent recommends that project*. Thus, if an agent proposes project i, a DM who adopts strategy $\mathbf{q}$ accepts the recommendation with probability q_i but rejects it in favor of the outside option with probability $1 - q_i$.

We are now in a position to characterize equilibria. The agent's problem is to choose a strategy $\mu : \mathcal{B} \to \Delta(N)$ that maps each profile of project values $\mathbf{b}$ to probabilities $(\mu_1(\mathbf{b}), \ldots, \mu_n(\mathbf{b}))$ of recommending alternative projects in N. Given any $\mathbf{q}$, a strategy μ is optimal for the agent if and only if

$$\mu_i(\mathbf{b}) = 1 \text{ if } q_i b_i > \max_{j \in N \setminus \{i\}} q_j b_j. \tag{1}$$

Accordingly, in characterizing an equilibrium, we can just focus on the DM's acceptance vector, $\mathbf{q}$, with the understanding that the agent best responds according to (1). For any equilibrium $\mathbf{q}$, the optimality of the DM's strategy combined with (1) implies a pair of conditions for each project i such that $\Pr\{\mathbf{b} : q_i b_i = \max_{j \in N} q_j b_j\} > 0$:

$$q_i > 0 \implies \mathbb{E}\left[b_i \mid q_i b_i = \max_{j \in N} q_j b_j\right] \geq \max\left\{b_0, \max_{k \in N \setminus \{i\}} \mathbb{E}\left[b_k \mid q_i b_i = \max_{j \in N} q_j b_j\right]\right\}, \tag{2}$$

$$q_i = 1 \impliedby \mathbb{E}\left[b_i \mid q_i b_i = \max_{j \in N} q_j b_j\right] > \max\left\{b_0, \max_{k \in N \setminus \{i\}} \mathbb{E}\left[b_k \mid q_i b_i = \max_{j \in N} q_j b_j\right]\right\}. \tag{3}$$

Condition (2) says that the DM accepts project i (when it is recommended) only if she finds it weakly better than the outside option as well as the other (unrecommended) projects, given her posterior which takes the agent's strategy (1) into consideration. Similarly, (3) says that if she finds the recommended project to be strictly better than all other options, she must accept that project for sure. These conditions are clearly necessary in any equilibrium; the following result shows that they are also sufficient.

Lemma 2. *If an equilibrium has acceptance vector $\mathbf{q} \in [0, 1]^n$, then (2) and (3) are satisfied for*

all projects i such that $\Pr\{\mathbf{b} : q_i b_i = \max_{j \in N} q_j b_j\} > 0$. Conversely, for any $\mathbf{q} \in [0,1]^n$ satisfying (2) *and* (3) *for all i such that $\Pr\{\mathbf{b} : q_i b_i = \max_{j \in N} q_j b_j\} > 0$, there is an equilibrium where the DM plays $\mathbf{q}$ and the agent's strategy satisfies* (1).

For expositional convenience, we will also focus on equilibria with the property that if a project i has ex-ante probability zero of being implemented on the equilibrium path, then the DM's acceptance vector $\mathbf{q}$ has $q_i = 0$. This is without loss of generality because there is always an outcome-equivalent equilibrium with this property: if $q_i > 0$ but the agent does not recommend i with positive probability, it must be that $q_i \overline{b}_i \leq q_j \underline{b}_j$ for some $j \neq i$, so setting $q_i = 0$ does not change the agent's incentives and remains optimal for the DM with the same beliefs.

2.4 Terminology

We will refer to an equilibrium with $\mathbf{q} = \mathbf{0} := (0, \ldots, 0)$ as a **zero equilibrium**. If $q_i = 1$, we say that the DM **rubber-stamps** project i, since she chooses it with probability one when the agent recommends it. If the principal rubber-stamps all projects, it is optimal for the agent to be **truthful** in the sense that he always recommends the best project. Indeed, in any non-zero equilibrium, it is optimal for the agent to be truthful if and only if the DM rubber-stamps all projects. Accordingly, we will say that a **truthful equilibrium** is one where $\mathbf{q} = \mathbf{1} := (1, \ldots, 1)$.[10] An equilibrium is **influential** if $|\{i \in N : q_i > 0\}| \geq 2$, i.e. there are at least two projects that are implemented on the equilibrium path. We say that the agent **panders toward i over j** if $q_i > q_j > 0$. The reason is that under this condition, the agent will recommend j if it is sufficiently better than the other projects, yet he biases his recommendation toward i over j because he will not recommend j unless $b_j > \frac{q_i}{q_j} b_i$. Note that we do not consider $q_i > 0 = q_j$ as pandering toward i over j because the agent can never get j implemented. An equilibrium is a **pandering equilibrium** if there are some i and j such that the agent panders toward i over j in the equilibrium. Finally, say that an equilibrium $\mathbf{q}$ is **larger** than another equilibrium $\mathbf{q}'$ if $\mathbf{q} > \mathbf{q}'$,[11] and $\mathbf{q}$ **is better than $\mathbf{q}'$** if $\mathbf{q}$ Pareto dominates $\mathbf{q}'$ at the interim stage where the agent has learned his type but the DM has not.

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[10]There can be a zero equilibrium where the agent always recommends the best project; this exists if and only if for all $i \in N$, $\mathbb{E}[b_i | b_i = \max_{j \in N} b_j] \leq b_0$. We choose not to call this a truthful equilibrium.

[11]Throughout, we use standard vector notation: $\mathbf{q} > \mathbf{q}'$ if $q_i \geq q_i'$ for all i with strict inequality for some i; $\mathbf{q} \gg \mathbf{q}'$ if $q_i > q_i'$ for all i.

3 Illustrative Example

As a prelude to the general results, we begin with a simple numerical example to illustrate the key idea of pandering to persuade. Suppose there are two projects whose values b_1 and b_2 are distributed uniformly on $[\frac{1}{3}, \frac{4}{3}]$ and $[0, 1]$ respectively. In any usual sense, the DM's prior favors project one, or project one "looks better" than project two. A direct computation shows that

$$\mathbb{E}[b_1|b_1 > b_2] = 0.91 > 0.78 = \mathbb{E}[b_2|b_2 > b_1].^{12} \tag{4}$$

Naturally, what kind of equilibrium will arise and how effective the agent's communication will be depends on b_0, the value of outside option to the DM. If $b_0 \leq 0.78$, then (4) implies that the DM will rubber-stamp the agent's recommendation as long as he is truthful. In particular, the DM will have no incentive to pick either the outside option or a project that has not been recommended.[13] When the DM rubber-stamps both projects, the agent's optimal strategy is to recommend truthfully. Hence, a truthful equilibrium exists, as shown in Panel A of Figure 1.

If $b_0 > 0.78$, however, the truthful equilibrium cannot be supported. To see why, suppose the agent recommends truthfully. When he recommends project two, the DM will not rubber-stamp it as (4) implies that she would rather choose the outside option. Hence, a truthful equilibrium does not exist. In fact, one can show that if $b_0 \geq 0.86$, the only equilibrium is the zero equilibrium where the DM always opts for the outside option.

What happens when $b_0 \in (0.78, 0.86)$? There is an influential equilibrium $\mathbf{q}^* = (1, q_2^*)$ for some $q_2^* \in (0, 1)$. In this equilibrium, the DM rubber-stamps project one whenever it is proposed but rejects project two with positive probability when it is proposed. This causes the agent to *pander toward the better-looking project*: he biases his recommendation toward project one as he proposes project two if and only if $b_2 > \frac{b_1}{q_2^*} > b_1$. To understand the logic of this pandering equilibrium, recall that the DM strictly prefers to reject project two if the agent were to employ the truthful strategy. Suppose the DM rejects the recommendation of project two with some small probability. Intuitively, the DM gets "tougher" on project two but not to a degree that she will always reject it. Faced with this strategy, the agent finds it in his best interest to recommend project two more selectively: he recommends it only when it is so much better than project one that it is worth the risk of rejection. (This can be seen in Panels B-D of Figure 1: the agent recommends project two only if (b_1, b_2) lies in the dark shaded region, well above the 45 degree line.) Naturally, when the agent is selective in this manner, the DM's posterior belief about

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[12] All numbers in the example are rounded to two decimal places.

[13] The latter observation also uses the fact that $\mathbb{E}[b_1|b_2 > b_1] = 0.56$ and $\mathbb{E}[b_2|b_1 > b_2] = 0.42$.

Panel A: Truthful Equilibrium
$b_0 \leq 0.78$

$b_2 > b_1$

$E(b_2) = 0.78$
$[E(b_1) = 0.56]$

$b_1 > b_2$

$E(b_1) = 0.91$
$[E(b_2) = 0.42]$

Panel B: Pandering Equilibrium
$b_0 = 0.8$, $q_2^* = 0.83$

$(0.83)b_2 > b_1$

$E(b_2) = 0.8$
$[E(b_1) = 0.5]$

Pandering

$b_1 > (0.83)b_2$

$E(b_1) = 0.89$
$[E(b_2) = 0.45]$

Panel C: Pandering Equilibrium
$b_0 = 0.85$, $q_2^* = 0.61$

$(0.61)b_2 > b_1$

$E(b_2) = 0.85$
$[E(b_1) = 0.42]$

Pandering

$b_1 > (0.61)b_2$

$E(b_1) = 0.86$
$[E(b_2) = 0.48]$

Panel D: No Influential Equilibrium
$b_0 = 0.86$

$(0.57)b_2 > b_1$

$E(b_2) = 0.86$
$[E(b_1) = 0.41]$

Pandering

$b_1 > (0.57)b_2$

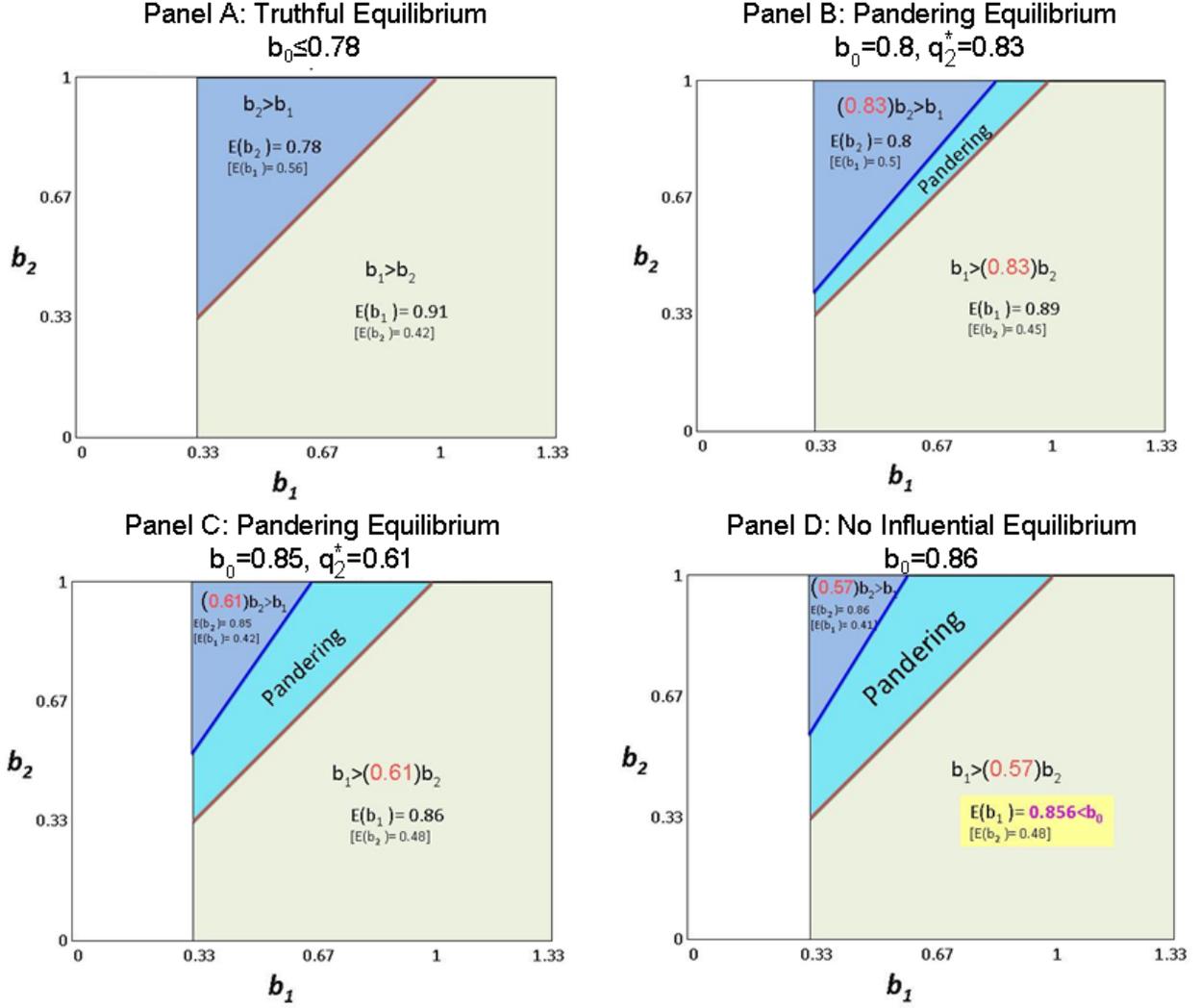$E(b_1) = 0.856 < b_0$
$[E(b_2) = 0.48]$

Figure 1 – Example with $b_1 \sim U[\frac{1}{3}, \frac{4}{3}]$ and $b_2 \sim U[0, 1]$.

project two when it is recommended improves compared to when the agent is truthful. In fact, the posterior can improve to such a degree that project two becomes acceptable. It can be shown that one can always find $q_2^* \in (0, 1]$ such that

$$b_0 = \mathbb{E}[b_2 | q_2^* b_2 > b_1], \tag{5}$$

which makes the DM indifferent between accepting and rejecting the recommendation of project two, which then rationalizes the DM's randomization when project two is proposed.[14]

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[14]While the equilibrium is in mixed strategies, the underlying intuition and logic of pandering equilibria does not hinge on mixed strategies. The mixed strategy equilibrium can be purified in the sense of Harsanyi (1973). Suppose the agent has some uncertainty about the value of the outside option, b_0, while the value is privately known to

Notice that even if $b_0 \geq 0.86$, there will be a solution q_2^* to (5). So why does the pandering equilibrium only exist when $b_0 \in (0.78, 0.86)$? The reason is that the DM must also find project one acceptable when it is recommended, since she is rubber-stamping it. When the agent panders more toward project one, the DM's posterior about project one when recommended worsens, as the recommendation is not as informative. The pandering equilibrium requires

$$\mathbb{E}[b_1 | b_1 > q_2^* b_2] \geq b_0. \tag{6}$$

It can be verified that when $b_0 \geq 0.86$, the q_2^* that solves (5) does not satisfy (6). This is shown in Panel D of Figure 1 for $b = 0.86$.

A related observation is that the degree of pandering changes as b_0 rises within $(0.78, 0.86)$. As b_0 gets larger, the agent must be more selective against recommending project two for the DM to find it acceptable, which in turn requires the DM to reject project two with higher probability when it is recommended. Formally, as b_0 rises, q_2^* must fall to keep (5) satisfied. This is also seen in Figure 1, where Panels B and C show how the acceptance rate of the worse looking project falls as b_0 rises.

The pandering equilibrium is not the only equilibrium when $b_0 \in (0.78, 0.86)$. If $b_0 \leq \frac{5}{6} = \mathbb{E}[b_1]$, there is a non-influential equilibrium $\mathbf{q} = (1, 0)$. Here the agent always proposes project one and the DM rubber-stamps it, despite the fact that both players would be better off by implementing project two whenever $b_2 > b_1$. If $b_0 > \frac{5}{6}$, there is a zero equilibrium, which can be supported by the agent always recommending project one.[15] The agent is clearly strictly better off with a larger acceptance vector and hence prefers the influential pandering equilibrium to either of these non-influential equilibria. More interestingly, the DM also prefers the pandering equilibrium. To see this, consider any (b_1, b_2) such that the agent would recommend b_2 in the pandering equilibrium; since $b_2 \geq b_1/q_2^* > b_1$ in such a case, the DM will be strictly better off from choosing project two than project one, which shows that she strictly prefers the pandering equilibrium to the $(1, 0)$ equilibrium. In addition, conditions (5) and (6) imply that the DM prefers the pandering equilibrium to the zero equilibrium (strictly, unless both conditions hold with equality).

We end the example's analysis by emphasizing the importance of the projects being non-

the DM. For example, let b_0 be uniformly distributed on $[v_0 - \varepsilon, v_0]$, with $\varepsilon > 0$ small. Then for $v_0 \in (0.78, 0.86)$ a pandering equilibrium $\tilde{\mathbf{q}} = (1, \tilde{q}_2)$ exists, where $\tilde{q}_2$ is the solution to $\mathbb{E}[b_2 | \tilde{q}_2 b_2 > b_1] = v_0 - (1 - \tilde{q}_2)\varepsilon$. In this equilibrium, the DM plays a pure strategy where she accepts project one whenever it is proposed but she accepts project two when it is proposed if and only if $b_0 \leq \mathbb{E}[b_2 | \tilde{q}_2 b_2 > b_1]$, which from the agent's perspective occurs with probability $\tilde{q}_2$. As $\varepsilon \to 0$, $\tilde{q}_2 \to q_2^*$. See also Section 6.1.

[15]Both these non-influential equilibria can be supported with passive beliefs for the DM in the off-path event that project two is recommended.

identically distributed for pandering. If, instead, $F_1 = F_2$, then there would be some cutoff value of the outside option such that for lower outside options there would be a truthful equilibrium where the DM rubber-stamps whichever project is recommended, while for higher outside options there would only be a zero equilibrium.

4   General Analysis

This section generalizes the ideas illustrated above, focussing initially on the two projects case since it permits the clearest development of our main themes. At the end of this section, we discuss how most of the results can be extended to more than two projects.

The fundamental logic of pandering to persuade is very general: so long as the two projects are not identically distributed, the DM's beliefs when the agent is truthful will typically favor one project, say project one, over the other. Our goal is to identify when there is a *systematic pattern* of pandering, namely to understand what attributes of the projects—in terms of their value distributions—cause one project to be pandered toward regardless of the selection of equilibrium and the value of the outside option. Moreover, we would like systematic comparative statics, for instance how the outside option affects the degree of pandering. Such analysis requires an appropriate stochastic ordering of the project value distributions.

Definition 1. For $n = 2$, projects are **strongly ordered** if

$$\mathbb{E}[b_1|b_1 > b_2] > \mathbb{E}[b_2|b_2 > b_1], \tag{R1}$$

and, for any $i, j \in \{1, 2\}$ with $i \neq j$,

$$\mathbb{E}[b_i|b_i > \alpha b_j] \text{ is nondecreasing in } \alpha \in \mathbb{R}_+ \tag{R2}$$

so long as the expectation is well-defined.

The first part of the ordering condition is mild since when $F_1 \neq F_2$, generally $\mathbb{E}[b_1|b_1 > b_2] \neq \mathbb{E}[b_2|b_2 > b_1]$; in this sense, (R1) can be viewed as a labeling convention. The important part of the definition is (R2). Consider $\mathbb{E}[b_1|b_1 > \alpha b_2]$: there are two effects on this expectation when α increases. On the one hand, for any given realization of b_2, the conditional expectation of b_1 increases; call this a *conditioning* effect. However, there is a countering *selection* effect: as α rises, lower realizations of b_2 become increasingly likely. Perhaps counter-intuitively, the selection effect can dominate the conditioning effect, so that in general, $\mathbb{E}[b_1|b_1 > \alpha b_2]$ can decrease when

α increases.[16] (R2) requires the conditioning effect to at least offset the selection effect. This is satisfied, for example, by the two parameterized families of distributions introduced earlier (scale-invariant uniform and exponential distribution families).

Theorem 1. *Assume $n = 2$ and the projects are strongly ordered.*

1. *If $\mathbf{q}$ is an equilibrium with $q_1 > 0$, then $q_1 \geq q_2$; if in addition $q_2 < 1$, then $q_1 > q_2$.*

2. *There is a largest equilibrium, $\mathbf{q}^*$, in the sense that for any other equilibrium $\mathbf{q} \neq \mathbf{q}^*$, $\mathbf{q}^* > \mathbf{q}$. Moreover, $\mathbf{q}^*$ is the best equilibrium. There exist $b_0^* := \mathbb{E}[b_2 | b_2 \geq b_1]$ and some $b_0^{**} \geq b_0^*$ such that:[17]*

   (a) *If $b_0 \leq b_0^*$, then the best equilibrium is the truthful equilibrium, $\mathbf{q}^* = (1, 1)$.*

   (b) *If $b_0 \in (b_0^*, b_0^{**})$, the best equilibrium is a pandering equilibrium, $\mathbf{q}^* = (1, q_2^*)$ for some $q_2^* \in (0, 1)$. Moreover, in this region of b_0, an increase in b_0 strictly increases pandering in the best equilibrium (i.e. q_2^* strictly decreases) and strictly decreases the interim expected payoffs of both players in the best equilibrium.[18]*

   (c) *If $b_0 > b_0^{**}$, only the zero equilibrium exists, $\mathbf{q}^* = (0, 0)$.*

Part 1 of the Theorem implies that in any equilibrium where project one is proposed on path, either the equilibrium is truthful or there is pandering toward project one. Part 2 characterizes the *largest* equilibrium, which is appealing to focus on for a number of reasons, not the least of which is that it is the *best* equilibrium. The possible values of the outside option can be partitioned into three distinct regions: when b_0 is low, the best equilibrium is truthful; when b_0 is intermediate, it is a pandering equilibrium; and when b_0 is large enough, only the zero equilibrium exists. Part 2(a) of the Theorem implies that a truthful equilibrium can exist even if $\mathbb{E}[b_i] < b_0$ for $i = 1, 2$. This is intuitive, since what matters for the DM's decision when the agent is truthful is her posterior conditional on one project being better than the other, which can be substantially higher than the unconditional expectation. Note that $b_0^* \geq \max\{\mathbb{E}[b_1], \mathbb{E}[b_2]\}$, hence a sufficient condition for a truthful equilibrium is that one of the projects has higher ex-ante expectation than the outside option.

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[16]This is easily seen in a a discrete example: suppose b_1 and b_2 are both uniformly distributed on $\{1, 3\}$ and $\{0, 2\}$ respectively. Then $\mathbb{E}[b_1 | b_1 > b_2] = \frac{1}{3}(1) + \frac{2}{3}(3) = \frac{7}{3}$, while $\mathbb{E}[b_1 | b_1 > 2b_2] = \frac{1}{2}(1) + \frac{1}{2}(3) = 2$.

[17]Typically, $b^{**} > b^*$. A sufficient condition that guarantees the strict inequality is that $\mathbb{E}[b_2 | \alpha b_2 > b_1]$ is strictly decreasing in α at $\alpha = 1$. This is satisfied, for example, by both our leading parametric distribution families: scale-invariant uniform and exponential.

[18]For the agent, this means that his interim expected payoff is weakly smaller for all $\mathbf{b}$ and strictly so for some $\mathbf{b}$.

Part 2(b) of the Theorem contains two comparative statics as the outside option increases in the region where the best equilibrium has pandering. First, as one would expect, there is strictly more pandering, because the agent must distort more for the DM to be willing to accept project two when recommended. Surprisingly, the DM's welfare strictly decreases with a higher outside option. To see the logic, note that in a pandering equilibrium, the DM is indifferent between accepting project two when recommended and choosing the outside option. This implies that holding fixed the agent's recommendation strategy, the DM's utility is the same whether she plays $\mathbf{q}^* = (1, q_2^*)$ or just rubber-stamps both projects, $\mathbf{q} = (1, 1)$. Since, in the relevant region, a higher b_0 induces more pandering, a DM who plays $\mathbf{q} = (1, 1)$ would be choosing the better project less often when b_0 is higher, which implies the welfare result.

When $b_0 < b_0^*$, the value of the outside option is irrelevant for welfare since the best equilibrium is truthful. Once $b_0 > b_0^{**}$, the DM's welfare is strictly increasing in b_0 since the outside option is always chosen. Altogether then, the outside option has a *non-monotonic* effect on the DM's expected payoff. Naturally, the agent's welfare is weakly decreasing in b_0. It is constant and identical to the DM's when $b_0 \leq b_0^*$, then strictly declines in b_0 in the pandering interval (b_0^*, b_0^{**}), and finally drops to zero once $b_0 > b_0^{**}$.

The characterization of Theorem 1 provides another interesting insight: when pandering arises, the agent does not benefit from a commitment to truthfully recommend the best alternative. To see this, observe that if the agent were constrained to rank the projects truthfully, the DM would play $\mathbf{q} = (1, 0)$ when $b_0 \in (b_0^*, b_0^{**})$. The agent interim—hence, ex-ante—prefers the pandering equilibrium vector $(1, q_2^*)$, since he can still get project one whenever he wants but also chooses to propose project two if $b_2 q_2^* > b_1$. Hence, unlike in the leading cases of Crawford and Sobel (1982), cheap-talk is not self-defeating in the current model: for intermediate conflicts of interest (captured by b_0), the agent prefers the equilibrium pandering to ex-ante "tieing his hands" to a truthful ranking.[19] Indeed, for the relevant outside options, if the DM were to think naively that the agent is telling the truth (e.g. because she is not aware of the conflict of interest), the agent would want to change the DM's beliefs and behavior by convincing the DM that he is in fact pandering (e.g. by making her aware of the conflict of interest).

A related insight is that the *alternatives* themselves can also benefit from pandering. This is again because when $b_0 \in (b_0^*, b_0^{**})$, project two would never be implemented if the agent ranks projects truthfully while it is implemented with positive probability in the pandering equilibrium. The idea can be illustrated via a faculty hiring application: without pandering, a candidate from a lesser-ranked school would be recommended whenever a committee finds him to be the best,

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[19] Optimal commitment by the agent is not our focus in this paper; see Kamenica and Gentzkow (2009) for some work in this direction.

but such a recommendation may never be accepted by the Dean. On the other hand, with pandering, the candidate is only proposed when he sufficiently dominates a candidate from a better-ranked school; this happens less often, but the candidate benefits because he is at least approved sometimes when recommended. Moreover, a candidate from a better-ranked school also benefits from pandering because he is recommended more often (even when moderately worse that the other candidate) and is approved when recommended.

The mechanics of Theorem 1 can be illustrated with our two leading parametric distribution families.

Example 1 (Scale-invariant uniform distributions). *Assume that b_2 is uniformly distributed on $[0, 1]$, while b_1 is uniformly distributed on $[v, 1 + v]$ with $v > 0$.[20] Strong ordering is satisfied, so Theorem 1 applies. As shown in Appendix D, $b_0^* = \frac{2+v}{3}$, $q_2^* = \frac{v}{3b_0 - 2}$, and b_0^{**} is the (unique) solution to $b_0^{**} = \mathbb{E}\left[b_1 \middle| b_1 > \left(\frac{v}{3b_0^{**} - 2}\right) b_2\right]$, which is indeed larger than b_0^*. Pandering is increasing in b_0, i.e. q_2^* is decreasing in b_0. Moreover, b_0^* and q_2^* are increasing in v; in this sense, project two becomes more acceptable when project one is stronger.*

Example 2 (Exponential distributions). *Assume that b_1 and b_2 are exponentially distributed with means v_1 and v_2, where $v_1 > v_2 > 0$.[21] Strong ordering is satisfied, so Theorem 1 applies. Appendix D computes that $b_0^* = v_2 + \frac{v_1 v_2}{v_1 + v_2}$, $q_2^* = \frac{v_1}{v_2}\left(\frac{2v_2 - b_0}{b_0 - v_2}\right)$, and $b_0^{**} = \frac{3v_1 v_2}{v_1 + v_2} > b_0^*$. Pandering is increasing (q_2^* is decreasing) in b_0. Again, b_0^* and q_2^* are increasing in v; in this sense, project two becomes more acceptable when project one is stronger. Appendix D also provides a closed-form solution for the DM's expected payoff, which may be useful for applications.*

What makes one project look better than the other? To better understand the direction of pandering, it is instructive to consider the strong ordering condition in more detail. Intuition suggests that the agent will pander toward a project that is ex-ante attractive. Within our leading families of distributions (scale-invariant uniform distributions and exponential distributions), the strong ordering condition agrees with all usual stochastic ordering notions, including likelihood-ratio ordering. Specifically, if project one dominates project two in the sense of $v_1 > v_2$ in either of these families, then b_1 likelihood-ratio dominates b_2, and hence if there is pandering, the agent panders toward the project that would be ranked higher in any usual sense. In particular, the agent panders toward the project with higher ex-ante expected value.

In general, however, our ordering condition does not correspond to standard notions of stochastic ordering. The reason for this divergence is important in understanding the mechanics

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[20] Assumptions (A1) and (A3) require that $b_0 < 1$.

[21] As shown in the Appendix D, Assumption (A3) requires that $b_0 < 2v_2$.

of strategic persuasion in our setting. When the agent recommends a project to the DM, he is essentially making a *comparative statement* about alternative projects by conveying that the project he recommends is better than the others. Crucially, a project that looks best "in isolation" need not be the one that looks best when "pitched comparatively," because the posterior on the recommended project can depend very much on the project it is compared against. To illustrate, suppose b_1 is uniformly distributed on $[1, 3]$, while b_2 is uniformly distributed on $[2, 3]$. Think of these as two job candidates from Ph.D. programs: candidate 2 is from a top-5 ranked department and candidate 1 is from a top-30 ranked department. Candidate 2 clearly dominates candidate 1 when one views each candidate in isolation; for instance, $\mathbb{E}[b_2] = 2.5 > 2 = \mathbb{E}[b_1]$, and b_2 in fact likelihood-ratio dominates b_1. This does not mean, however, that candidate 2 looks better than candidate 1 when "pitched comparatively." Pitching candidate 2 favorably against candidate 1 is not a very strong endorsement of candidate 2, since he is expected to be better than candidate 1. This is likely to convey only that candidate 1 is expectedly weak; specifically, the DM's posterior belief puts more weight on b_1 being in the lower half interval, $[1, 2]$, rather than b_2 being in the upper tail of of $[2, 3]$. On the other hand, pitching candidate 1 favorably against candidate 2 is a huge endorsement of candidate 1, for it suggests that candidate 1's value is likely in the upper tail of $[2, 3]$. Indeed,

$$\mathbb{E}[b_1 | b_1 > b_2] = 2.66 > \mathbb{E}[b_2 | b_2 > b_1] = 2.55.$$

From the perspective of comparative pitching, the candidate who is weaker in isolation looks better than the candidate who is stronger in isolation! It can be shown that candidate 1 dominates candidate 2 in our strong ordering, and any pandering is therefore toward candidate 1. Note, however, that this does not mean that the agent will recommend candidate 1 *more often* than candidate 2. Since he is weak in isolation, candidate 1 will often have realized values significantly smaller than candidate 2; at the margin, however, the agent is biased toward the former.

The next result further develops the economics of comparative pitching. We say that distribution F **likelihood-ratio dominates** distribution $\tilde{F}$ if their respective densities $\tilde{f}$ and f satisfy $\frac{f(b')}{f(b)} \geq \frac{\tilde{f}(b')}{\tilde{f}(b)}$ for any $b' > b$ such that both ratios have either a non-zero numerator or denominator. The likelihood-ratio domination is **strict** if the inequality holds strictly for a set of positive measure of (b', b) satisfying $b' > b$.

Theorem 2. *Fix b_0 and an environment $\mathbf{F} = (F_1, F_2)$ that satisfies strong ordering. Let $\tilde{\mathbf{F}} = (\tilde{F}_1, \tilde{F}_2)$ be an environment with a weaker slate of alternatives: $\tilde{F}_j = F_j$ for some j, and for $i \neq j$, either (a) F_i strict likelihood-ratio dominates $\tilde{F}_i$ and $\tilde{\mathbf{F}}$ satisfies strong ordering, or (b) $\tilde{F}_i$ is a degenerate distribution at zero. Letting $\mathbf{q}^*$ and $\tilde{\mathbf{q}}^*$ denote the best equilibria in each of the*

respective environments, we have $\mathbf{q}^* \geq \tilde{\mathbf{q}}^*$. Moreover, $\mathbf{q}^* > \tilde{\mathbf{q}}^*$ if $\mathbf{q}^* > \mathbf{0}$ and $\tilde{\mathbf{q}} < \mathbf{1}$.

Theorem 2 considers two senses in which the slate of alternatives becomes stronger when switching from environment $\tilde{\mathbf{F}}$ to $\mathbf{F}$: in case (a), the number of projects is held constant, but the distribution of one project improves in the sense of strict likelihood-ratio dominance; in case (b), the environment $\tilde{\mathbf{F}}$ consists of only one project while the environment $\mathbf{F}$ is obtained by adding a new project to $\tilde{\mathbf{F}}$. In either case, the best equilibrium in the stronger environment is at least as large as the original environment, and strictly larger if the original environment did not have a truthful equilibrium and the stronger environment has a non-zero equilibrium. (These caveats are necessary or else both environments would have the same best equilibrium, either truthful or zero respectively.)

An important implication of Theorem 2 is that the best equilibrium in the stronger environment can be strictly larger if the value distribution that improves is that of project one, *even though project one is already accepted with probability one when proposed.* In this sense, project two can become more acceptable to the DM when project one becomes stronger, even though project two's distribution is unchanged. This is entirely due to the property of comparative rankings: an improvement in F_1 improves the conditional expectation of project two when it is proposed, holding fixed the equilibrium acceptance vector. Strong ordering then implies the existence of a larger equilibrium if the original equilibrium was not truthful. Examples 1 and 2 illustrate this point: there, a likelihood-ratio improvement of project one corresponds to increasing v or v_1 respectively, and as noted in the examples, this causes b_0^* and q_2^* to both increase.

Case (b) of Theorem 2 implies that the agent never benefits from "hiding a project." To fix ideas, suppose the availability of project one is common knowledge between the DM and the agent, but the availability of project two is not. Project two is only available with some probability, and its availability is privately information of the agent. The Theorem implies that if the agent can credibly prove the availability of project two, it is always optimal for the agent to do so when feasible. It is also possible, for instance, that $\mathbb{E}[b_1] < b_0$ but $\mathbb{E}[b_i|b_i > b_j] > b_0$, $i, j = 1, 2, i \neq j$, in which case the agent can get the better project accepted if two is available but neither project would be accepted if only project one were available.

What happens if the projects are not strongly ordered? While strong ordering is essential for delivering the full force of Theorems 1 and 2, a weaker stochastic ordering suffices to identify a systematic direction of pandering.

Definition 2. For $n = 2$, projects are **weakly ordered** if

$$\forall \alpha \geq 1, \mathbb{E}[b_1 | b_1 > \alpha b_2] > \mathbb{E}[b_2 | \alpha b_2 > b_1].$$

It is straightforward that strong ordering implies weak ordering. The latter is weaker because it does not require (R2). Rather, weak ordering allows $\mathbb{E}[b_i | b_i > \alpha b_j]$ (for any $i \neq j$) to decrease in α, but requires that the ranking assumed in (R1), i.e. that $\mathbb{E}[b_1 | b_1 > \alpha b_2] > \mathbb{E}[b_2 | \alpha b_2 > b_1]$ when $\alpha = 1$, must be preserved for all larger α.[22]

Theorem 3. *Assume $n = 2$ and the two projects are weakly ordered. Then, any influential but non-truthful equilibrium has pandering toward project one.*

Proof. Under weak ordering, first note that there cannot be an equilibrium with $1 > q_2 = q_1 > 0$ because then the agent will be truthful, in which case $\mathbb{E}[b_1 | b_1 > b_2] > \mathbb{E}[b_2 | b_2 > b_1]$ implies that it cannot be optimal for the DM to play (q_1, q_2). So any non-truthful but influential equilibrium must have either $q_1 > q_2 > 0$ or $q_2 > q_1 > 0$. But the latter configuration cannot be an equilibrium because for $\alpha = \frac{q_2}{q_1} > 1$, $\mathbb{E}[b_1 | b_1 > \alpha b_2] > \mathbb{E}[b_2 | b_1 < \alpha b_2] \geq b_0$, hence the DM's optimality requires $q_1 = 1$, a contradiction. *Q.E.D.*

This result is tight in the sense that the weak ordering condition is *not only sufficient but also almost necessary* for pandering to systematically go in the direction of one project. Suppose $\mathbb{E}[b_1 | b_1 > b_2] > \mathbb{E}[b_2 | b_2 > b_1]$ but for some $\alpha' > 1$, $\mathbb{E}[b_1 | b_1 > \alpha' b_2] < \mathbb{E}[b_2 | \alpha' b_2 > b_1]$. Then, pandering is not systematic, because its direction varies with the value of b_0. Specifically, for some $b_0 \in (\mathbb{E}[b_2 | b_2 > b_1], \mathbb{E}[b_2 | b_2 > b_1] + \varepsilon)$, for small $\varepsilon > 0$, there exists a pandering equilibrium $\mathbf{q} = (1, q_2)$ with $q_2 \in (0, 1)$, i.e. the agent panders toward project one. Yet, for some $b_0 \in (\mathbb{E}[b_1 | b_1 > \alpha' b_2], \mathbb{E}[b_1 | b_1 > \alpha' b_2] + \varepsilon)$, for small $\varepsilon > 0$, there is an equilibrium $\mathbf{q}' = (q_1', 1)$ with $q_1' \approx 1/\alpha' \in (0, 1)$; namely, the agent now panders toward project two.

More than two projects. Theorems 1 and 2 can be extended to more than two projects by strengthening the stochastic order.

Definition 3. For $n > 2$, projects are **strongly ordered** if

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[22] Truncated Normal distributions typically satisfy weak ordering but not strong ordering. For an example, let G_1 be a Normal distribution with mean 5 and variance 1, while G_2 is Normal with mean 4.5 and variance 1. The corresponding densities are denoted g_1 and g_2 respectively. For $i = 1, 2$, each b_i is distributed on $[0, \infty)$ with density $f_i(x) = \frac{g_i(x)}{1 - G_i(0)}$. One can verify that for $\alpha \geq 1$, $\mathbb{E}[b_1 | b_1 > \alpha b_2]$ initially rises in α but then starts to fall, hence strong ordering fails. However, it can also be verified that weak ordering is satisfied.

1. For any $i < j$, and any $k \in \mathbb{R}_+$,

$$\mathbb{E}[b_i | b_i > b_j, b_i > k] > \mathbb{E}[b_j | b_j > b_i, b_j > k]. \tag{R1$'$}$$

whenever both expectations are well-defined.

2. For any i and j, and any $k \in \mathbb{R}_+$,

$$\mathbb{E}[b_i | b_i > \alpha b_j, b_i > k] \text{ is nondecreasing in } \alpha \in \mathbb{R}_+ \tag{R2$'$}$$

so long as the expectation is well-defined.

The only difference between (R1$'$) and (R1), or (R2$'$) and (R2), is the extra conditioning on the relevant random variable being above the non-negative constant k. Obviously, when $k = 0$, (R1$'$) and (R2$'$) are respectively identical to (R1) and (R2), because of our maintained assumption (A1). Since Definition 3 requires (R1$'$) and (R2$'$) to hold for all $k \in \mathbb{R}_+$, this notion of strong ordering is more demanding than that of Definition 1, even if there are only two projects. Intuitively, the roles of (R1$'$) and (R2$'$) are analogous to that of (R1) and (R2), but modified to account for the fact that when $n > 2$, a recommendation for a project i is a comparative statement not only against project j, but also the other $n - 2$ projects. In other words, the DM's posterior about i when the agent recommends project i rather than project j must also account for the fact that i is sufficiently better than all the other non-j projects as well, for each realization of their values.[23]

Given this extension of the strong ordering notion, our main conclusions generalize to any $n \geq 2$; see Theorems 8 and 9 in Appendix C. We show there that there are threshold values, b_0^* and b_0^{**}, such that (i) for $b_0 < b_0^*$, there is a truthful equilibrium; (ii) for $b_0 \in (b_0^*, b_0^{**})$, the largest equilibrium has pandering towards better-looking projects (those with a lower index, by Definition 3); and (iii) for $b_0 > b_0^{**}$, the only equilibrium is the zero equilibrium.[24] In particular, these results apply to the scale-invariant and exponential families of distributions because both these families satisfy (R1$'$) and (R2$'$).

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[23]In this light, some readers may find it helpful to consider the following alternative to part one of the definition: For any $i < j$ and any $(\alpha_k)_{k \neq i,j} \in \mathbb{R}_{++}^{n-2}$, $\mathbb{E}[b_i | b_i > b_j, b_i > \max_{k \neq i,j} \alpha_k b_k] > \mathbb{E}[b_j | b_j > b_i, b_j > \max_{k \neq i,j} \alpha_k b_k]$ whenever these expectations are well-defined. A similar modification can also be used for the second part of the definition. While these requirements are slightly weaker and would suffice, we chose the earlier formulation for greater clarity.

[24]One caveat is that the largest equilibrium need not be the best equilibrium when $n > 2$. Nevertheless, we argue in Appendix C that the largest equilibrium is still compelling to focus on.

5 Delegation, Commitment, and Other Responses

We now turn to studying some responses that the DM may take to mitigate the pandering distortions in communication. Throughout this section, we assume there are two projects that are strongly ordered, and refer to the largest equilibrium $\mathbf{q}^*$ defined in Theorem 1.

5.1 Delegation

One mechanism that has received attention in the literature is that of **full delegation**, where the DM simply transfers decision-making authority to the agent. In a setting with incomplete contracts (Grossman and Hart, 1986; Hart and Moore, 1990), the simplicity of this mechanism is appealing. At first glance, delegation involves a tradeoff for the DM in our context. On the one hand, delegation eliminates pandering (since the agent will always recommend the best project), but on the other, it sometimes leads to a project being implemented even when the DM prefers the outside option. Nevertheless, we find that delegation is unambiguously optimal for the DM whenever influential communication is possible:

Theorem 4. *If the largest equilibrium $\mathbf{q}^*$ of the communication game is nonzero, then the DM is ex-ante weakly better off by delegating authority to the agent no matter which equilibrium of the communication game would be played, and strictly so if $\mathbf{q}^* < \mathbf{1}$.*

To see the intuition, suppose the largest communication equilibrium $\mathbf{q}^*$ is non-zero. By Theorem 1, $\mathbf{q}^* \gg \mathbf{0}$. If $\mathbf{q}^* \neq \mathbf{1}$, then the DM is randomizing between accepting project two and rejecting it when it is recommended, so she must be indifferent between project 2 and the outside option in that case. Therefore, holding the agent's strategy fixed, the DM's expected utility is the same whether she plays $\mathbf{q}^*$ or $\mathbf{1}$. Delegation effectively commits the DM to playing the latter strategy and also has the additional benefit of eliminating any pandering since the agent will choose the best project truthfully. So the DM is weakly better off by delegating, and strictly so unless $\mathbf{q}^* = \mathbf{1}$. As is clear from this argument, Theorem 4 does not actually require the projects to be ordered.

A few remarks are useful in interpreting Theorem 4. First, Dessein (2002) has established a similar result for the uniform-quadratic specification of Crawford and Sobel (1982). However, he also shows that more generally, delegation in Crawford and Sobel (1982) is only optimal if the conflict of interest is sufficiently small, rather than whenever communication is influential. In contrast, Theorem 4 proves that delegation is preferred by the DM in the current model whenever communication can be influential. Second, the Theorem only gives sufficient conditions

for delegation to be optimal. In fact, even if b_0 is large so that $\mathbf{q}^* = \mathbf{0}$, delegation will be strictly preferred by the DM so long as b_0 is not too large. Obviously, if b_0 is sufficiently large, the DM would prefer to retain authority and just choose the outside option. Third, the Theorem in fact holds for any $n \geq 2$ (if $n > 2$, $\mathbf{q}^*$ is defined by Theorem 8 in Appendix C). Fourth, whenever the DM prefers delegation to communication, **constrained delegation**—where the DM allows the agent to choose from a particular subset of all possible options—is of no additional benefit to the DM, since the agent will never choose the outside option and there is perfect alignment of interests among the alternative projects.

Finally, when $\mathbf{q}^* < \mathbf{1}$, it is crucial that the DM be able to commit to delegation, because ex-post, the DM will be tempted to override the agent and choose the outside option whenever the agent chooses a project that is ex-ante poor-looking, namely project two. Of course, if the agent expects the DM to override his choice ex-post, his incentives change dramatically, and the outcome would be some equilibrium $\mathbf{q}$ of the communication game.

5.2 Stochastic mechanisms

Full or constrained delegation can be viewed as commitments by the DM to particular vectors of acceptance probabilities, where each $q_i \in \{0, 1\}$. From this perspective, it is natural to ask what acceptance strategies the DM would wish to use if she can commit to a stochastic acceptance rule. Stochastic mechanisms are of more than purely theoretical interest. As will be seen, the optimal stochastic mechanism can be implemented via delegating authority to a third party who internalizes a different value than b_0 from the outside option.

Formally, the optimal stochastic mechanism is a solution to the following problem:

$$\max_{\mathbf{q} \in [0,1]^2} \mathbb{E}\left[\sum_{i \in \{1,2\}} q_i(b_i - b_0) \cdot \mathbb{1}_{\{q_i b_i > q_j b_j, \forall j\}} \right] + b_0. \tag{7}$$

In other words, the DM chooses an acceptance profile $\mathbf{q}$ knowing that the agent will respond optimally to it in terms of which project to propose. In particular, the DM is allowed to choose the profile $\mathbf{q}$ under which she may accept a proposed project with positive probability even though its posterior value may be strictly less than b_0 (which of course requires credible commitment). We have already seen that for moderate outside options, full delegation strictly dominates any equilibrium with communication. Notice that full delegation is subsumed as a feasible solution in the above problem, where $\mathbf{q} = \mathbf{1}$.

Let $\mathbf{q}^c$ denote the solution to (7). It is of interest whether full delegation coincides with the optimal commitment rule. The answer is that it generally does not.

Theorem 5. *If the largest communication equilibrium has $\mathbf{q}^* < \mathbf{1}$, then the optimal commitment rule has $\mathbf{q}^c < \mathbf{1}$. If $\mathbf{0} < \mathbf{q}^* < \mathbf{1}$, then the optimal commitment rule is $\mathbf{q}^c = (1, q_2^c)$ where $q_2^c \in (q_2^*, 1)$.*

The Theorem says that, whenever the DM does not rubber-stamp the adviser's recommendation in the largest communication equilibrium $\mathbf{q}^*$, she should also refuse to rubber-stamp it in an optimal stochastic mechanism; that is, she should not fully delegate. Interpreted differently, whenever the (largest) communication equilibrium involves pandering, the optimal commitment rule also induces the agent to pander. The intuition is as follows. Assume project two is ex-ante undesirable in the sense that $\mathbb{E}[b_2|b_2 \geq b_1] < b_0$. By Theorem 1, this is necessary and sufficient for the DM to not rubber-stamp project two when it is recommended by the agent. Starting from $\mathbf{q} = \mathbf{1}$, suppose the DM lowers q_2 slightly below 1. The benefit is that when project two is proposed, the outside option b_0 will be sometimes realized instead of b_2. The cost is that this induces some pandering. However, the benefit is of first-order importance since $\mathbb{E}[b_2|b_2 > b_1] < b_0$ while the cost is of second-order importance, since there is no pandering at $\mathbf{q} = \mathbf{1}$. On balance, reducing q_2 slightly below 1 is beneficial.

The second statement of Theorem 5 is that the optimal stochastic mechanism involves reduced pandering compared to any influential communication equilibrium whenever the largest communication equilibrium is nonzero. The logic is that starting from $\mathbf{q}^* < \mathbf{1}$, if the DM slightly increases q_2, this has a first-order benefit of reducing pandering and only a second-order cost because $\mathbb{E}[b_2|q_2^* b_2 > b_1] = b_0$. Extending this logic shows that we must have $\mathbf{q}^c > \mathbf{q}^*$. There exists a $b_0' \in (0, b_0)$ such that if the DM delegates decision-making to a third party who values the outside option at b_0' instead of b_0, then communication between the agent and the third party will end up implementing $\mathbf{q}^c$. The presence of such a third party is plausible in a hierarchical organization. For instance, in such a setting, often the intermediate boss, or a supervisor, typically internalizes the value of the outside option more than the agent but not as much as the principal.

5.3 Endogenous choice of outside option

So far we have taken the outside option, b_0, to be exogenously given. In many applications, it is reasonable to think that the DM can endogenously choose b_0, perhaps improving it at a cost. Formally, suppose that prior to communication, the DM can endogenously choose the

outside option at a cost $c(b_0)$, where $c(\cdot)$ is strictly increasing, and assume there exists $\hat{b}_0 \in$ arg$\max_{b_0 \in \mathbb{R}_+}[b_0 - c(b_0)]$. Suppose further that the DM's choice of b_0 is publicly observed prior to the communication game.

Let Π_D denote the expected utility the DM receives from full delegation. More precisely, this is the payoff that the DM enjoys if the agent is truthful *and* the DM rubber-stamps the agent's recommendations. The following observations can be readily drawn.

Theorem 6. *If $\Pi_D \geq \hat{b}_0 - c(\hat{b}_0)$, then it is optimal for the DM to set $b_0 = 0$ and rubber-stamp the agent's recommendation ($\mathbf{q}^* = \mathbf{1}$). If $\Pi_D < \hat{b}_0 - c(\hat{b}_0)$, however, it is optimal for the DM to set $b_0 = \hat{b}_0$ and never accept the agent's recommendation ($\mathbf{q}^* = \mathbf{0}$).*

The rationale for this result is simple: Since full delegation is better for the DM than communicating with any pandering, it does not pay the DM to invest in the outside option unless it is so attractive that it will always be chosen. By choosing a minimal outside option, the DM effectively delegates the project choice to the agent. So the issue for the DM boils down to whether this is better or worse than investing in an outside option so large that it is always implemented.

The logic of Theorem 6 applies even when destroying outside option is costly, implying that "burning ships" can sometimes be optimal. In particular, as was shown in Theorem 1, the DM may strictly benefit from decreasing her outside option, precisely because it reduces pandering. This may be desirable for the DM even if it is costly do so, so long as the cost is not too great.

5.4 Ignorance can be bliss

The pandering distortion results from the DM's partial knowledge of the projects' attributes. Such knowledge may be obtained by the DM investigating the projects herself or from the agent's communication of verifiable information to the DM. The results obtained so far suggest that the DM may actually benefit by committing herself to remain ignorant, either by not investigating or by not engaging in any verifiable communication with the agent. In particular, any information that can only improve the DM's understanding of *how alternative projects compare with one another but not how they compare relative to the outside option* is likely to exacerbate the pandering problem without any countervailing benefit. We now formalize this idea by identifying a type of information that the DM will never wish to learn.

Suppose there are two projects A and B whose values b_A and b_B are ex-ante identically distributed. Suppose the DM, either through her own investigation or verifiable communication

with the agent, can *costlessly* obtain a signal $s \in S$ prior to the agent's communication of soft information. Assume for convenience that S is finite. We consider two regimes: (1) No information: The DM does not observe s; and (2) Information: the DM observes the realized value of s. We say that the signal is **value-neutral** if $\mathbb{E}[\max\{b_A, b_B\}|s]$ is the same for all $s \in S$, and it is **non-trivial** if $\mathbb{E}[b_A|b_A > b_B, s] \neq \mathbb{E}[b_A|b_A > b_B, s']$ for some $s, s' \in S$.[25] Value-neutrality captures the notion of the signal being valuable only insofar as it informs the DM about which of the projects is better, but not about how the best project compares against the outside option.[26]

Theorem 7. *Focus on the best equilibrium under each information regime. If the signal is value-neutral, then the DM prefers (at least weakly) not observing the signal to observing the signal. If the signal is also non-trivial, then there exists a non-empty interval $[\hat{b}_0, \bar{b}_0]$ such that the preference for ignorance is strict for $b_0 \in (\hat{b}_0, \bar{b}_0)$.*

Theorem 7 shows that observable information can be harmful, and the DM would benefit from ignorance in the sense of not observing such information.[27] While the result assumes that the projects are ex-ante identical, it is robust to relaxing this assumption because the DM's payoffs from no information and information vary continuously (upon selecting the best equilibrium) when the assumption is slightly relaxed.

6 Discussion and Extensions

The model we have studied is quite stylized, but we believe it yields broadly relevant insights about strategic communication when alternatives differ in observable characteristics. Our analysis generalizes readily in many respects, producing additional insights for applications. In lieu of an exhaustive analysis, we sketch several ideas for extending our baseline model, restricting attention to two projects and assuming strong ordering between them. As will be seen, these extensions preserve the themes that given strong ordering, (i) ranking equilibria exist, featuring pandering toward better-looking projects for appropriate values of the outside option, and (ii)

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[25]In keeping with strong ordering, one may assume that $\mathbb{E}[b_A|b_A > b_B, s] - \mathbb{E}[b_B|b_B > b_A, s]$ is increasing in s. This additional structure is not needed, however.

[26]While value-neutrality is generally a strong assumption, it holds for example with the widely-used binary signal structure: $S = \{s_A, s_B\}$ such that for any real valued function $h(b_A, b_B)$, $\mathbb{E}[h(b_A, b_B)|s_A] = \mathbb{E}[h(b_B, b_A)|s_B]$. Given symmetric binary signals, $\mathbb{E}[\max\{b_A, b_B\}|s_A] = \mathbb{E}[\max\{b_B, b_A\}|s_B] = \mathbb{E}[\max\{b_A, b_B\}|s_B]$.

[27]Evidently, the nature of information matters for this conclusion. Just as we have characterized the nature of information that can only make the DM worse off, certain kinds of information benefit the DM. It can be shown that the DM will always benefit from learning information with the dual characteristics, i.e. observing a signal that is ranking-neutral in the sense that $\mathbb{E}[b_A|b_A > b_B, s]$ and $\mathbb{E}[b_B|b_A < b_B, s]$ are constant across $s \in S$ and also value-non-neutral in that $\mathbb{E}[\max\{b_A, b_B\}|s]$ varies with s.

the organizational implications of such pandering, particularly the benefit of delegation, carry over with some interesting caveats.

6.1 Private information about outside option

In many applications, the value of the outside option may not be known when communication takes place, or it may be known only privately to the DM. For example, when a department in a university makes its hiring recommendation to its Dean, the Dean may have private information about the hiring opportunities and needs for other departments, or may still be waiting to hear from them. Similarly, a seller may not be privy to a buyer's reservation value of her product, or a CEO may know more than a manager about the cost of capital.

We can easily accommodate this by assuming that the value of the outside option, b_0, is observed privately by the DM prior to the agent's communication about **b**. Suppose that b_0 is drawn from a distribution $G(\cdot)$ with strictly positive density on $[0, \infty)$. In this setting, a ranking equilibrium is described not by a vector of acceptance probabilities, but rather by a threshold vector (b_0^1, b_0^2) such that the DM follows the agent's recommendation of project $i \in \{1, 2\}$ if and only if $b_0 \leq b_0^i$, choosing her outside option otherwise. Since the agent's best response is to recommend project 1 if $b_1 > \frac{G(b_0^2)}{G(b_0^1)} b_2$ and recommend project 2 otherwise, the threshold vector (b_0^1, b_0^2) is an equilibrium if and only if

$$\mathbb{E}\left[b_1 \middle| b_1 > \frac{G(b_0^2)}{G(b_0^1)} b_2\right] = b_0^1 \text{ and } \mathbb{E}\left[b_2 \middle| b_1 < \frac{G(b_0^2)}{G(b_0^1)} b_2\right] = b_0^2. \tag{8}$$

As long as the expected value of b_i given $b_i \geq \alpha b_j$ ($i, j \in \{1, 2\}$ and $i \neq j$) is bounded for any $\alpha > 0$, there exists a solution to (8). Moreover, strong ordering implies that any solution has $b_0^1 > b_0^2$. In other words, *there is pandering in any ranking equilibrium.* The intuition is that since the agent is uncertain about the outside option, on the margin, when b_1 is only slightly below b_2, he prefers to recommend project one so as to increase the probability of acceptance, which yields a benefit of higher-order magnitude compared with the associated utility loss of getting project one rather than project two.

Given the equilibrium pandering, the DM has an incentive to convince the agent that her outside option is low, for the agent will pander less if he believes the outside option to have a lower value. Such communication from the DM is not credible, however, so long as the DM's information is soft. By contrast, if the information is hard, then there will be unraveling, resulting in full revelation of the outside option value.

What about the DM's decision to delegate? Since the DM's private information affects the decision she makes following the agent's recommendation, it may conceivably affect the DM's incentive to delegate project choice to the agent. For instance, when $b_0 \in (b_0^1, b_0^2)$, the DM only approves project one but rejects project two, when the respective projects are recommended. One may think that for such values of b_0, the DM does not want to delegate and instead just accepts project one when it is recommended. This logic is incomplete, however, because the DM's decision not to delegate would reveal that her outside option is high and thereby exacerbate the agent's pandering. Strikingly, it can be shown that the DM delegates project choice in equilibrium if and only if $b_0 \leq \mathbb{E}[\max\{b_1, b_2\}]$ — just as in the baseline model where b_0 was common knowledge.

6.2 Preference conflicts over alternative projects

Another natural extension is to allow the DM and the agent to have non-congruent preferences over the set of alternative projects. For instance, a seller may obtain a larger profit margin on a particular product, or a Dean may have a gender bias or favor economists that do research in a certain area. A simple way to introduce such conflicts is to assume that the agent derives a benefit $a_i b_i$ from project i, where $a_i > 0$ is common knowledge, while the DM continues to obtain b_i from project i.[28] The parameter $a := a_1/a_2$ is a sufficient statistic for the preference conflict between projects and indicates whether the agent is biased toward the better-looking project ($a > 1$) or the worse-looking project ($a \in (0, 1)$). A ranking equilibrium is characterized by the DM's vector of acceptance probabilities $q = (q_1, q_2)$, but now the agent recommends project 1 if $aq_1 b_1 > q_2 b_2$ and project 2 otherwise. We will say that the agent panders toward project i if $q_i > q_j > 0$ ($i \neq j$), i.e. he biases his recommendation in favor of i from the perspective of his preferences, not from the DM's. Consequently, the DM may benefit from pandering, as we discuss below. To avoid uninteresting cases, assume that the truthful equilibrium does not exist, i.e. either $\mathbb{E}[b_1|ab_1 > b_2] < b_0$ or $\mathbb{E}[b_2|ab_1 < b_2] < b_0$, and a non-zero equilibrium does exist.

Under strong ordering, one can show that there exists a critical threshold of conflict, $\bar{a} \in (1, \infty]$, such that *if $a \in (0, \bar{a})$, the largest equilibrium has pandering toward the better-looking project ($\mathbf{q}^* = (1, q_2^*)$ with $q_2^* \in (0, 1)$) while for $a > \bar{a}$, the largest equilibrium has pandering toward the worse-looking project ($q^* = (q_1^*, 1)$ with $q_1^* \in (0, 1)$).*[29] Notice that if $a < 1$, the agent has a preference bias for project two but nevertheless panders toward project one in order to persuade the DM. If $a \in (1, \bar{a})$, then pandering reinforces the agent's bias to over-recommend

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[28]While such a multiplicative form of bias is especially convenient to study, it is also straightforward to incorporate an additive or other forms of bias.

[29]Formally, $\bar{a}$ is the solution to $\mathbb{E}[b_2|b_2 > \bar{a}b_1] = \mathbb{E}[b_1|b_2 < \bar{a}b_1]$ if a solution exists (uniqueness is guaranteed by strong ordering), and $\bar{a} = \infty$ otherwise.

project one from the DM's perspective. Interestingly, in this case, the acceptance probability of project two in the largest equilibrium is increasing in the preference conflict a; the reason is that the agent's preference bias toward the better-looking project makes his recommendation of a worse-looking project more credible than the same recommendation made by an unbiased agent.[30] Finally, when $a > \bar{a}$, the agent's preferences are so biased toward the good-looking project that a recommendation of project one is less credible than that of project 2; hence the persuasion motive leads him to pander toward project two. It is not hard to check that even though the agent is pandering toward project two relative to his true preferences, he still over-recommends project one from the DM's perspective, i.e. $aq_1 \geq q_2$ in any equilibrium.

An important difference from the baseline model is that if $a < 1$ or $a > \bar{a}$, the DM benefits from some pandering, for it counteracts the agent's preference bias. This affects the DM's gains from delegation. If $a < 1$, a small degree of pandering reduces the agent's bias toward project two; in fact the DM's utility is maximized at $q_2^* = a$. Delegation then dominates communication only when the largest equilibrium has sufficiently severe pandering. If $a > \bar{a}$ so that the agent is strongly biased in favor of project one, the agent's pandering toward project two (recall, this is relative to the agent's preferred alternative) is always beneficial to the DM, so delegation is never optimal. Only when $a \in [1, \bar{a})$ is delegation is optimal for any level of pandering.

The above observations highlight a fundamental difference between pandering due to conflicts of interests over alternatives and pandering due to observable differences between alternatives. In particular, if projects are identical ($F_1 = F_2$) but $a \neq 1$, then pandering is always beneficial to the DM: the agent knows that a proposal of a pet project is less credible, so he restrains himself from proposing—i.e., panders against—such a project. Delegation is suboptimal in such a case. By contrast, in our baseline model where $F_1 \neq F_2$ and $a = 1$, we have seen that pandering is always detrimental to the DM, and delegation is strictly preferred whenever pandering occurs in the largest equilibrium.

6.3   Variable project size

The decisions a DM is called upon to make are often not binary. For example, a board of a corporation may not only decide which project to fund but also how much resources to make available for a chosen project. Or, a buyer may purchase variable units of a good from a seller.

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[30]This does not imply that the DM prefers a more biased agent when $a \in (0,1)$. The reason is that $q_2^* \in (0,1)$ implies $\mathbb{E}[b_2 | q_2^* b_2 > ab_1] = b_0$; hence, in equilibrium, an increase in a triggers a change in q_2^* but keeps q_2^*/a constant and thus does not affect the agent's recommendation strategy. Since the DM is indifferent across all $q_2 \in [0,1]$, holding fixed the agent's strategy, the DM's welfare does not change.

In these cases, it is reasonable that projects with higher expected values receive more resources from the DM.

Assume that the DM must decide how much to invest in one of two projects. Project $i \in \{1, 2\}$ yields profits of $b_i q_i - \frac{\gamma}{2} q_i^2$, when the DM invests q_i for the project and the project has quality b_i, which is again private information of the agent. The agent derives the benefit $b_i q_i$ but does not internalize the investment cost $\frac{\gamma}{2} q_i^2$. In this setting, a ranking equilibrium is characterized by an investment vector $\mathbf{q} = (q_1, q_2)$, where $q_i \in \mathbb{R}_+$ is the investment size chosen by the DM when project i is recommended. The agent recommends project one whenever $q_1 b_1 > q_2 b_2$, which implies that in any equilibrium, $\gamma q_1 = \mathbb{E}[b_1 | q_1 b_1 > q_2 b_2]$ and $\gamma q_2 = \mathbb{E}[b_2 | q_1 b_1 < q_2 b_2]$. Strong ordering implies that $q_1 > q_2$ in any equilibrium, so the agent always panders toward the better-looking project no matter the value of the outside option. The intuition is most clearly seen when b_1 is only slightly smaller than b_2: in this case, since the agent cares about the amount of resources he receives, he will recommend project one if the DM believes this to be truthful.

The resulting distortion can be mitigated by delegation, provided the DM can put a cap on the maximum investment the agent can make (knowing that the agent will always invest the maximum allowed). In particular, relative to the communication game, it is always beneficial for the DM to delegate project choice to the agent with a cap equal to the quantity the DM would invest herself in the better-looking project when it is recommended in the communication game.

This extension permits a comparison with Blanes i Vidal and Moller (2007), who show that a principal may select a project that she privately knows is inferior but is perceived to be of higher quality by an agent who must exert costly effort to implement the project. Intuitively, the DM in our model is the agent in theirs whose implementation effort is increasing in his posterior on project quality.

6.4 Non-exclusive projects

There are situations in which the DM may choose multiple projects. For example, a corporate board may approve several capital investment projects if the expected profits of each exceed their cost of capital, or a Dean may want to hire two economists if both candidates appear to be outstanding.

To fix ideas, assume that the DM may choose to implement neither, either, or both projects. If both projects are chosen, both the DM and the agent obtain a payoff of $b_1 + b_2$; if only project $i \in \{1, 2\}$ is chosen, the DM gets $b_i + b_0$ while the agent gets b_i; and if neither is chosen, the DM gets $2b_0$ while the agent gets 0. In this setting, one may wonder if the intuition

of pandering toward better-looking projects in order to persuade would still apply. In particular, is it possible that the agent, in equilibrium, panders toward the worse-looking project in order to increase the chances that *both* projects are selected? Such a possibility is particularly relevant if b_0 is such that

$$\mathbb{E}[b_1|b_1 > b_2] > \mathbb{E}[b_2|b_2 > b_1] > \mathbb{E}[b_1|b_2 > b_1] > b_0 > \mathbb{E}[b_2|b_1 > b_2]. \tag{9}$$

Restricting attention to ranking equilibria where the agent simply reports whether $\alpha b_1 > b_2$ or $b_2 > \alpha b_1$ for some $\alpha > 0$, one can show that any influential equilibrium still has pandering toward the better-looking project (i.e, $\alpha > 1$). In such an equilibrium, when the agent ranks project one ahead of project two, the DM accepts it and also accepts project two with some probability. If the agent ranks project two ahead of project one, the DM accepts it but rejects project one. Hence, even if the DM can implement both projects, the communication is still biased toward the better-looking project. Existence of an influential ranking equilibrium requires $\mathbb{E}[b_2] \geq b_0$. If $\mathbb{E}[b_2] < b_0$, the only equilibrium is the one where the DM always chooses project one.

Interestingly, if $\mathbb{E}[b_2] < b_0$, the DM would benefit from committing herself to implement at most one project. Indeed, the inequalities in (9) imply that the agent will then truthfully reveal the better project, and the DM will follow this recommendation. By contrast, if the DM does not make such a commitment, the desire to get both projects adopted destroys the credibility of the agent's communication.

7   Conclusion

This paper has studied strategic communication by an agent who has some non-verifiable private information about the benefit of various alternatives and wants a decision maker (DM) to select the best one, but does not fully internalize the DM's benefit from an outside option. In the baseline model, this is the only source of conflict: conditional on not choosing the outside option, the two parties' preferences are aligned over the alternatives.

This agency problem is salient in many settings of *resource allocation*. Examples include a seller who vies for a consumer's purchase, a supplier competing for a firm's contract, a venture capitalist raising funds from wealthy individuals or institutions, a philanthropist choosing between charities, or a firm allocating its resources between its divisions. In each of these applications, the agent typically does not fully internalize the opportunity cost of resources: suppliers, money managers, venture capitalists, charities, and division managers all derive private bene-

fits when they sell more, are allocated more resources, manage more money, or are given larger budgets.

Our main finding is that there is a systematic distortion in the agent's recommendations, and hence the DM's decisions, toward alternatives that "look ex-ante better" to the DM. An alternative can look better than another based on characteristics that are either directly observable or revealed via some verifiable information by the agent himself. The reason for the distortion is that recommending an option that looks ex-ante less attractive makes the DM pessimistic about all the options, even when the DM believes the agent is truthfully recommending the best available option. Since the agent wants to persuade the DM against the outside option, the agent ineluctably panders toward "better-looking" alternatives.

Our analysis shows that which alternative the agent panders toward can be subtle. Perhaps surprisingly, recommendations can be distorted toward alternatives with lower expected values, because in a comparative ranking, these can sometimes be "better looking" than alternatives with higher expected values. We also show that equilibrium pandering has a number of interesting implications. For example, worse-looking options benefit from pandering as it makes them acceptable to the DM while they would not be under truthful recommendations. At the same time, better-looking projects benefit because they are recommended more often.

The second part of our analysis concerned organizational or market responses to pandering. A simple process that improves on pure communication is for the DM to first request all verifiable information from the agent and then decide whether to either entirely delegate decision-making to the agent (with a commitment never to override the agent's choice) or just choose the outside option. In this organization structure, hard information on the options is all that matters, as it determines whether delegation to the agent is warranted, and cheap-talk or soft recommendations do not add any value. We therefore provide a rationale for "no-strings attached" budget allocations, delegation of hiring decisions to subgroups, commitments to buy in buyer-seller relationships, and requirements from venture capitalists or investment funds that investors commit their money for some period of time. We also show that if the DM cannot commit to such delegation, she may be better off by not observing any hard information at all.

Given the ubiquity of resource allocation problems, and more generally the importance of strategic advice when options "look different" based on partial information, we believe the framework presented here can be fruitfully used for various applications. Our model is amenable to a number of extensions and tailoring, as illustrated in Section 6. There is also much left to understand about the economics of pandering to persuade at a broader level. For example, several agents may compete for resources, effectively endogenizing the DM's value of the outside

option as far as any single agent is concerned. The DM may also have private information about some aspects of her preferences over alternatives, making an agent cautious to express a strong preference for one of the alternatives. These are just two of the directions we are pursuing in ongoing work.

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A Proofs

Proof of Lemma 1. See supplementary appendix B. *Q.E.D.*

Proof of Lemma 2. See supplementary appendix B. *Q.E.D.*

Proof of Theorem 1. For any $y > 0$, let $\Lambda(y) := \mathbb{E}[b_1|b_1 > yb_2] - \mathbb{E}[b_2|yb_2 > b_1]$, whenever this is well-defined. Strong ordering implies that $\Lambda(y) > 0$ for any $y \geq 1$ at which it is well-defined.

STEP 1: We begin with Part 1 of the Theorem. Pick any equilibrium $\mathbf{q}$ with $q_1 > 0$. Assume, to contradiction, that $q_2 > q_1$. Then $\mathbb{E}[b_1|q_1b_1 \geq q_2b_2] = \mathbb{E}\left[b_1|b_1 \geq \frac{q_2}{q_1}b_2\right] > \mathbb{E}\left[b_2|\frac{q_2}{q_1}b_2 \geq b_1\right] = \mathbb{E}[b_2|q_2b_2 \geq q_1b_1] \geq b_0$, where the strict inequality is by strong ordering and that $q_2/q_1 \geq 1$, while the weak inequality is because $q_2 > 0$. But equilibrium now requires that $q_1 = 1$ (recall condition (3)), a contradiction with $q_2 > q_1$. Similarly, if $0 < q_1 = q_2 < 1$, the same argument applies, except that the contradiction is not with $q_2 > q_1$ but rather with $q_1 < 1$.

Part 2 of the Theorem is proved in a number of steps. Steps 2–4 concern the essential properties of the largest equilibrium, $\mathbf{q}^*$, and the outside option thresholds, b_0^* and b_0^{**}.

STEP 2: Plainly, there is a truthful equilibrium $\mathbf{q}^* = (1, 1)$ when $b_0 < b_0^* := \mathbb{E}[b_2|b_2 > b_1]$, and this is the largest equilibrium for such b_0. Note that for any $b_0 > b_0^*$, there is no equilibrium $\mathbf{q}$ with $q_1 = 0 < q_2$, because in that case the agent always recommends project two, but then $\mathbb{E}[b_2] \leq \mathbb{E}[b_2|b_2 > b_1] = b_0^* < b_0$, contradicting equilibrium condition (2). By the first part of the Theorem, we conclude that for $b_0 > b_0^*$, any non-zero equilibrium $\mathbf{q}$ has $q_1 > q_2$.

STEP 3: Suppose that for all $y > 0$, $\Lambda(y) > 0$. Set $b_0^{**} := \sup_{y \in (0,1]} \mathbb{E}[b_2|b_1 < yb_2]$. Since $b_0^* = \mathbb{E}[b_2|b_2 > b_1]$, it follows that $b_0^{**} \geq b_0^*$, with equality if and only if $\mathbb{E}[b_2|b_1 < yb_2] = \mathbb{E}[b_2|b_1 < b_2]$ for all $y \in (0, 1)$. Since $\mathbb{E}[b_2|b_1 < yb_2]$ is continuous in y for all $y > 0$, it follows that for any $b_0 \in (b_0^*, b_0^{**})$, there is some $q_2^* \in (0, 1)$ that solves $b_0 = \mathbb{E}[b_2|b_1 < q_2^*b_2]$; if there are multiple solutions, pick the largest one. Suppose the agent recommends project two if and only if $q_2^*b_2 > b_1$. Since $\mathbb{E}[b_1|b_1 \leq q_2^*b_2] < \mathbb{E}[b_2|b_1 < q_2^*b_2] = b_0$, it is optimal for the DM to accept project two with probability q_2^* when it is proposed. Moreover, $\mathbb{E}[b_2|b_1 > q_2^*b_2] \leq \mathbb{E}[b_2] \leq \mathbb{E}[b_2|q_2^*b_2 > b_1] = b_0 < \mathbb{E}[b_1|b_1 > q_2^*b_2]$, where the last inequality is by the hypothesis that $\Lambda(y) > 0$ for all $y > 0$; hence it is also optimal for the DM to accept project one when proposed. Therefore, $\mathbf{q}^* = (1, q_2^*)$ is a pandering equilibrium, which by construction is larger than any other equilibrium $\mathbf{q}$ with $q_1 = 1$. Moreover, any non-zero equilibrium $\tilde{\mathbf{q}}$ with $\tilde{q}_1 < 1$ has $\tilde{q}_1 > \tilde{q}_2$ (see Step 2), hence there would be a larger equilibrium $\mathbf{q} = (1, \tilde{q}_2/\tilde{q}_1)$, which in turn is weakly smaller than $\mathbf{q}^*$. Finally, we don't need to consider $b_0 \geq b_0^{**}$ because this violates Assumption (A3).

STEP 4: Suppose now that $\Lambda(y) = 0$ for some $y > 0$. Let $\hat{y} := \max\{y : \Lambda(y) = 0\}$. Since $\Lambda(1) > 0$, it follows that $\hat{y} < 1$ and $\Lambda(y) > 0$ for all $y > \hat{y}$. Set $b_0^{**} = \mathbb{E}[b_2|b_1 < \hat{y}b_2]$. Plainly, $b_0^{**} \geq b_0^*$, with strict inequality if $\mathbb{E}[b_2|b_1 < yb_2]$ is strictly decreasing at $y = 1$. It follows from the continuity of $\mathbb{E}[b_2|b_1 \leq yb_2]$ in y for $y \in [\hat{y}, 1]$ that for all $b_0 \in (b_0^*, b_0^{**}]$, there is a solution

$q_2^* \in (\hat{y}, 1)$ to $b_0 = \mathbb{E}[b_2 | b_1 < q_2^* b_2] < \mathbb{E}[b_1 | b_1 > q_2^* b_2]$; if there are multiple solutions, pick the largest one. By arguments similar to those used in Step 1, $q^* = (1, q_2^*)$ is a pandering equilibrium that is also the largest among all equilibria. Finally, we must argue that for $b_0 > b_0^{**}$, the only equilibrium is $\mathbf{q}^* = \mathbf{0}$. Note there is no influential equilibrium by the construction of b_0^{**}, and by Step 2, any non-influential equilibrium $\mathbf{q}$ must have $q_2 = 0$. But $\mathbb{E}[b_1] \leq \mathbb{E}[b_1 | b_1 > \hat{y} b_2] = \mathbb{E}[b_2 | b_1 < \hat{y} b_2] = b_0^{**} < b_0$, so there is no equilibrium $\mathbf{q}$ with $q_1 > 0 = q_2$.

STEP 5: This step shows that $\mathbf{q}^*$ is the best equilibrium. Since $\mathbf{q}^* = \mathbf{0}$ is the only equilibrium when $b_0 > b_0^{**}$, assume $b_0 < b_0^{**}$. Clearly, the agent prefers a larger equilibrium (in the sense that his expected payoff is weakly larger for all $\mathbf{b}$ and strictly larger for some $\mathbf{b}$), so we need only show that the DM's welfare is highest at $\mathbf{q}^*$. As shown earlier, $b_0 < b_0^{**}$ implies that the largest equilibrium $\mathbf{q}^*$ has $q_1^* = 1$. Suppose there exists another equilibrium $\mathbf{q} < \mathbf{q}^*$.

Consider first $q_1 = 0$ and $q_2 = 0$. The DM weakly prefers $\mathbf{q}^*$ since she always chooses a project that gives her on expectation at least b_0. Consider next $q_1 = 0$ and $q_2 > 0$. Then $\mathbb{E}[b_2] \geq b_0$, which implies by strong ordering that $\mathbb{E}[b_1 | b_1 \geq b_2] > \mathbb{E}[b_2 | b_2 \geq b_1] \geq \mathbb{E}[b_2]$, hence $\mathbf{q}^* = \mathbf{1}$. Clearly, the DM strictly prefers $\mathbf{q}^*$ over $\mathbf{q}$. Finally, suppose $q_1 > 0$. Then, by the first part of the Theorem, $q_1 \geq q_2$. We can assume $q_1 = 1$, for otherwise there exists another equilibrium $\mathbf{q}' = \frac{1}{q_1}\mathbf{q}$ which the DM prefers at least weakly to $\mathbf{q}$. Since $\mathbf{q} < \mathbf{q}^*$, it now follows that $q_2 < q_2^*$. Let $\Pi(\tilde{\mathbf{q}})$ denote the DM's expected payoff in an arbitrary equilibrium $\tilde{\mathbf{q}}$. Notice that in computing $\Pi(\mathbf{q}^*)$ or $\Pi(\mathbf{q})$, we can keep the agent's strategy fixed and assume the DM instead adopts both projects with probability one when recommended (even though she may not in equilibrium), because of the DM's indifference when she adopts project two with strictly interior probability. Thus,

$$
\begin{aligned}
\Pi(\mathbf{q}^*) &= \mathbb{E}\left[b_1 \cdot \mathbb{1}_{\{b_1 > q_2^* b_2\}} + b_2 \cdot \mathbb{1}_{\{q_2 b_2 < b_1 < q_2^* b_2\}} + b_2 \cdot \mathbb{1}_{\{b_1 < q_2 b_2\}} \right] \\
&> \mathbb{E}\left[b_1 \cdot \mathbb{1}_{\{b_1 > q_2^* b_2\}} + b_1 \cdot \mathbb{1}_{\{q_2 b_2 < b_1 < q_2^* b_2\}} + b_2 \cdot \mathbb{1}_{\{b_1 < q_2 b_2\}} \right] \\
&= \mathbb{E}\left[b_1 \cdot \mathbb{1}_{\{b_1 > q_2 b_2\}} + b_2 \cdot \mathbb{1}_{\{b_1 < q_2 b_2\}} \right] = \Pi(\mathbf{q}),
\end{aligned}
$$

where the strict inequality holds because $\Pr\{\mathbf{b} : q_2 b_2 < b_1 < q_2^* b_2\} > 0$ and in this event, $b_2 > b_1$ because $q_2^* \leq 1$.

STEP 6: Finally, we address the comparative statics when b_0 increases within the region (b_0^*, b_0^{**}). It is clear that q_2^* strictly decreases in b_0 by its construction in Steps 3 and 4. Given this, the same payoff argument as in the final part of Step 4 shows that the DM's expected payoff strictly decreases in b_0. Plainly, the agent's interim expected payoff is weakly smaller for all $\mathbf{b}$ and strictly so for some $\mathbf{b}$ when b_0 is larger. $\qquad$ Q.E.D.

Proof of Theorem 2. If $\tilde{F}_i$ is a degenerate distribution at zero, the conclusions of Theorem follow from the observations that if $\mathbb{E}[\tilde{b}_j] < b_0$ then $\tilde{\mathbf{q}}^* = \mathbf{0}$, and if $\mathbb{E}[\tilde{b}_j] \geq b_0$ then $\mathbf{q}^* = \mathbf{1}$ because $F_j = \tilde{F}_j$. So assume for the rest of the proof that $\tilde{F}_i$ is not degenerate, i.e. case (a) applies. The Theorem holds trivially if $\tilde{\mathbf{q}}^* = \mathbf{0}$, so also assume $\tilde{\mathbf{q}}^* > \mathbf{0}$, hence $\tilde{\mathbf{q}}^* \gg \mathbf{0}$. Let $\mathbf{b}$ and $\tilde{\mathbf{b}}$

be the random vectors of the project values corresponding to $\mathbf{F}$ and $\tilde{\mathbf{F}}$, respectively. Given the assumptions, we claim that for $m \in \{1, 2\}$

$$\mathbb{E}[b_m | \tilde{q}_m^* b_m = \max_{k \in \{1,2\}} \tilde{q}_k^* b_k] \geq \mathbb{E}[\tilde{b}_m | \tilde{q}_m^* \tilde{b}_m = \max_{k \in \{1,2\}} \tilde{q}_k^* \tilde{b}_k]. \tag{10}$$

For $m = i$, inequality (10) follows from the likelihood-ratio dominance hypothesis. For $m = j$, the argument is as follows. We can write

$$\mathbb{E}[b_j \mid \tilde{q}_j^* b_j = \max_{k \in \{1,2\}} \tilde{q}_k^* b_k] = \frac{\int_0^\infty b_j F_i(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} b_j) f_j(b_j) db_j}{\int_0^\infty F_i(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} b_j) f_j(b_j) db_j} = \int_0^\infty b k_j(b) db,$$

where $k_j(z) := F_i(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} z) f_j(z) / \int_0^\infty F_i(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} \tilde{z}) f_j(\tilde{z}) d\tilde{z}$. Likewise, $\mathbb{E}[\tilde{b}_j \mid \tilde{q}_j^* \tilde{b}_j = \max_{k \in N} \tilde{q}_k^* \tilde{b}_k] = \int_0^\infty b \tilde{k}_j(b) db$, where $\tilde{k}_j(z) := \tilde{F}_i(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} z) f_j(z) / \int_0^\infty \tilde{F}_i(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} \tilde{z}) f_j(\tilde{z}) d\tilde{z}$. To prove inequality (10), it suffices to show that k_j likelihood-ratio dominates $\tilde{k}_j$. Consider any $b' > b$. Algebra shows that

$$\frac{k_j(b')}{k_j(b)} \geq \frac{\tilde{k}_j(b')}{\tilde{k}_j(b)} \Leftrightarrow \frac{F_i\left(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} b'\right)}{F_i\left(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} b\right)} \geq \frac{\tilde{F}_i\left(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} b'\right)}{\tilde{F}_i\left(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} b\right)}.$$

But the right-hand side of the above equivalence is implied by the hypothesis that F_i likelihood-ratio dominates $\tilde{F}_i$.[31]

It now follows that $\mathbf{q}^* \geq \tilde{\mathbf{q}}^*$: strong ordering of $\mathbf{F}$ combined with (10) for each $m \in \{1, 2\}$ implies that there is a weakly larger equilibrium in $\mathbf{F}$ than $\tilde{\mathbf{q}}^*$ (one just raises the second component of the acceptance vector as high as possible so long as it remains optimal for the DM to accept project two when recommended).

For the second part of the Theorem, assume $\mathbf{0} < \tilde{\mathbf{q}}^* < \mathbf{1}$, for if not the conclusion is trivial. So $\tilde{q}_2^* \in (0, 1)$. The argument used above to prove (10) also reveals that since the likelihood-ratio domination of F_i over $\tilde{F}_i$ is strict, k_j strictly likelihood-ratio dominates $\tilde{k}_j$, hence the inequality in (10) must hold strictly for $m \in \{1, 2\}$. But then $\tilde{\mathbf{q}}^*$ cannot be an equilibrium in environment $\mathbf{F}$ because randomization would not be optimal for the DM following a recommendation of project two. It follows from the first part of the Theorem that $\mathbf{q}^* > \tilde{\mathbf{q}}^*$. *Q.E.D.*

Proof of Theorem 4. Assume $\mathbf{q}^*$ is non-zero and pick any non-zero communication equilibrium $\mathbf{q}$. Theorem 1(b) has established that the DM's expected utility from $\mathbf{q}$ is no larger than that from $\mathbf{q}^*$, so it suffices to show that delegation is weakly preferred to $\mathbf{q}^*$, and strictly so if $\mathbf{q}^* < \mathbf{1}$. If

---

[31] To see this, note that it suffices to show that $\frac{d}{dx}[F(x)/\tilde{F}(x)] \geq 0$. This derivative is proportional to $\tilde{F}(x) f(x) - F(x) \tilde{f}(x)$, which we will argue is non-negative. The likelihood-ratio domination hypothesis implies that for any $y < x$, $f(x) \tilde{f}(y) \geq f(y) \tilde{f}(x)$. Integrating this inequality over y from the lower endpoint up to x yields $f(x) \tilde{F}(x) \geq F(x) \tilde{f}(x)$, as desired.

$\mathbf{q}^* = \mathbf{1}$, then the outcome of delegation is identical to that of $\mathbf{q}^*$ and the result is trivially true. So assume $\mathbf{q}^* < \mathbf{1}$, in which case $q_1^* = 1 > q_2^* > 0$ by Theorem 1. Since the DM is indifferent between the outside option and project two when the latter is recommended, the DM's expected payoff is the same as it would be if she always adopted a recommended project, keeping the agent's strategy fixed. Therefore, the DM's expected payoff is $\Pi(\mathbf{q}^*) := \mathbb{E}\left[\sum_{i=1}^{2} b_i \cdot \mathbb{1}_{\{q_i^* b_i > q_j^* b_j, j \neq i\}}\right] < \mathbb{E}\left[\max\{b_1, b_2\}\right]$, where the inequality holds since $q_1^* > q_2^*$. Since this latter payoff is what the DM achieves under delegation, delegation is strictly preferred. *Q.E.D.*

Proof of Theorem 5. The proof consists of three steps.

STEP 1: *If $\mathbf{q}^* < \mathbf{1}$, then $\mathbf{q}^c < \mathbf{1}$.*

Proof: Let $\Pi(\mathbf{q})$ denote the value of committing to $\mathbf{q}$ as given by the maximand in (7). Then,

$$
\begin{aligned}
\left.\frac{\partial \Pi(\mathbf{q})}{\partial q_2}\right|_{\mathbf{q}=\mathbf{1}} &= \mathbb{E}[(b_2 - b_0) \cdot \mathbb{1}_{\{b_2 \geq b_1\}}] + \sum_{i,j \in \{1,2\}} \left.\frac{\partial \mathbb{E}[(b_i - b_0) \cdot \mathbb{1}_{\{q_i b_i \geq q_j b_j, j \neq i\}}]}{\partial q_2}\right|_{\mathbf{q}=\mathbf{1}} \\
&= \mathbb{E}[(b_2 - b_0) \cdot \mathbb{1}_{\{b_2 \geq b_1\}}] + \lim_{\delta \downarrow 0} \frac{\mathbb{E}[(b_2 - b_1) \cdot \mathbb{1}_{\{b_1 > b_2 > (1-\delta)b_1\}}]}{\delta} \\
&= \mathbb{E}[(b_2 - b_0) \cdot \mathbb{1}_{\{b_2 \geq b_1\}}] < 0,
\end{aligned}
$$

where the strict inequality follows from the assumption that $\mathbf{q}^* < \mathbf{1}$, which implies that $\mathbb{E}[b_2 | b_2 \geq b_1] < b_0$. ||

STEP 2: *If $q_i^c < q_j^c$, for $i, j \in \{1,2\}$ with $i \neq j$, then $\mathbb{E}[(b_i - b_0) \cdot \mathbb{1}_{\{q_i^c b_i > q_j^c b_j\}}] < 0$.*

Proof: Suppose to the contrary that $q_i^c < q_j^c$ and that $\mathbb{E}[(b_i - b_0) \cdot \mathbb{1}_{\{q_i^c b_i > q_j^c b_j\}}] \geq 0$. We derive a contradiction by showing that the DM benefits from raising q_i above q_i^c:

$$
\begin{aligned}
\left.\frac{\partial \Pi(\mathbf{q})}{\partial q_i}\right|_{\mathbf{q}=\mathbf{q}^c} &= \mathbb{E}[(b_i - b_0) \cdot \mathbb{1}_{\{b_i \geq b_j\}}] + \sum_{k,l \in \{1,2\}} \left.\frac{\partial \mathbb{E}[(b_k - b_0) \cdot \mathbb{1}_{\{q_k b_k \geq q_l b_l\}}]}{\partial q_i}\right|_{\mathbf{q}=\mathbf{q}^c} \\
&= \mathbb{E}[(b_i - b_0) \cdot \mathbb{1}_{\{q_i^c b_i \geq q_j^c b_j\}}] \\
&\quad + \lim_{\delta \downarrow 0} \left(\frac{\mathbb{E}[q_i^c(b_i - b_0) \cdot \mathbb{1}_{\{(q_i^c + \delta)b_i > q_j^c b_j > q_i^c b_i\}}] - \mathbb{E}[q_j^c(b_j - b_0) \cdot \mathbb{1}_{\{(q_i^c + \delta)b_i > q_j^c b_j > q_i^c b_i\}}]}{\delta}\right) \\
&= \mathbb{E}[(b_i - b_0) \cdot \mathbb{1}_{\{q_i^c b_i \geq q_j^c b_j\}}] + (q_j^c - q_i^c)b_0 \left(\lim_{\delta \downarrow 0} \frac{\Pr\{(q_i^c + \delta)b_i > q_j^c b_j > q_i^c b_i\}}{\delta}\right) > 0,
\end{aligned}
$$

where the inequality holds because the first term of the last line is nonnegative by the hypothesis, the second term is strictly positive since $q_i^c < q_j^c$, and the limit point is well defined and is strictly positive given the assumption of there being positive density on $\{(b_i, b_j) | q_i^c b_i = q_j^c b_j\}$. ||

STEP 3: *If $\mathbf{q}^* > 0$, then $\mathbf{q}^c > \mathbf{q}^*$.*

Proof: Since $\mathbf{q}^* > 0$, we can assume that $\mathbf{q}^c > 0$, as $\mathbf{q}^*$ does at least as well as $\mathbf{0}$. Step 2 implies that $q_1^c \geq q_2^c$, since otherwise $\mathbf{q}^* > \mathbf{0}$ implies $\mathbb{E}[(b_1 - b_0) \cdot \mathbb{1}_{\{q_1^c b_1 > q_2^c b_2\}}] \geq 0$, a contradiction to Step 2. Given $q_1^c \geq q_2^c$, we can now assume that $q_1^c = 1$, since otherwise by proportionally raising both q_1^c and q_2^c until q_1^c reaches 1, the DM cannot be worse off.

Next, we know that $q_2^c < 1$ by Step 1. We cannot have $q_2^c \leq q_2^*$, because if this were the case then the equilibrium condition for $\mathbf{q}^*$ implies $\mathbb{E}[(b_2 - b_0) \cdot \mathbb{1}_{\{q_2^c b_2 > q_1^c b_1\}}] \geq 0$, which contradicts Step 2 since $q_2^c < 1 = q_1^c$. $\|$ *Q.E.D.*

Proof of Theorem 6. By Theorem 4, whenever $\mathbf{q}^* > \mathbf{0}$, the DM is weakly better off from full delegation than from communication. Since under full delegation the outside option is never chosen, it follows that choosing any b_0 that gives rise to $\mathbf{q}^* > 0$ is strictly dominated for the DM by choosing optimally between b_0 and playing $\mathbf{q}^* = \mathbf{1}$, given that $c(\cdot)$ is strictly increasing. Among the set of b_0's such that $q^* = 0$, the DM is just maximizing $b_0 - c(b_0)$. The statement of the theorem now follows immediately. *Q.E.D.*

Proof of Theorem 7. Assume the signal is value-neutral. Suppose first the DM has learned some signal $s \in S$, and a communication equilibrium $\mathbf{q}(s) = (q_A(s), q_B(s))$ ensues, where $q_i(s)$ denotes the probability of project $i = A, B$ being accepted by the DM when the agent recommends i given signal s. There are two possibilities. First, if $\mathbf{q}(s) = \mathbf{0}$, then the DM's payoff will be b_0. Suppose next $\mathbf{q}(s) > \mathbf{0}$. Then following the argument of Theorem 4, the DM's payoff is no higher than it is under delegation. The latter payoff is $\mathbb{E}[\max\{b_A, b_B\}|s]$, which by the value-neutrality assumption is independent of the signal realization and hence is equal to $\mathbb{E}[\max\{b_A, b_B\}]$. Thus, regardless of $\mathbf{q}(s)$, the DM's expected payoff from having learned s is no greater than $\max\{b_0, \mathbb{E}[\max\{b_A, b_B\}]\}$. But the latter is exactly the DM's expected payoff under "no information." More precisely, since b_A and b_B are identically distributed, $\mathbb{E}[b_A|b_A \geq b_B] = \mathbb{E}[b_B|b_A \leq b_B] = \mathbb{E}[\max\{b_A, b_B\}]$. Hence, if $\mathbb{E}[\max\{b_A, b_B\}] \geq b_0$, then a truthful equilibrium arises under no information, and if $\mathbb{E}[\max\{b_A, b_B\}] < b_0$, only the zero equilibrium arises under no information. Since the preceding argument applies to any signal realization, the first statement of the Theorem follows.

Suppose next that the signal is also non-trivial. This implies that

$$\hat{b}_0 := \min_{s \in S} \left(\min\{\mathbb{E}[b_A|b_A \geq b_B, s], \mathbb{E}[b_B|b_A < b_B, s]\} \right) < \mathbb{E}[\max\{b_A, b_B\}] =: \bar{b}_0. \qquad (11)$$

Consider any $b_0 \in (\hat{b}_0, \bar{b}_0)$. (11) implies that there is a truthful equilibrium under "no information," which gives the DM a payoff of $\mathbb{E}[\max\{b_A, b_B\}]$. (11) also implies that there is no truthful equilibrium following any observed signal; hence, following any observed signal, the DM's payoff is strictly less than $\mathbb{E}[\max\{b_A, b_B\}]$. Integrating over all possible signals, the second statement of the Theorem follows. *Q.E.D.*

The remaining appendices are supplementary and not intended for publication.

B   Omitted Proofs

This Appendix provides proofs for Lemmas 1 and 2.

Proof of Lemma 1. Consider any equilibrium (α, μ) with more than n on-path messages. Without loss, we can assume that all of them have positive ex-ante probability, since there is clearly an outcome-equivalent equilibrium where this is the case. Let M^* be the set of on-path messages. First suppose that $\alpha(m) = 0$ for all $m \in M^*$. Then for any $m \in M^*$ and any $i \in N$, we must have $\mathbb{E}[b_i|m] \leq b_0$, so that $\mathbb{E}[b_i] \leq b_0$, and it follows that there is an outcome-equivalent "babbling" or pooling equilibrium with only one on-path message.

Next, consider the case where some $m^* \in M^*$ leads to an alternative project with positive probability, i.e. $Support[\alpha(m^*)] \cap N \neq \emptyset$. This requires that $Support[\alpha(m)] \cap N \neq \emptyset$ for all $m \in M^*$, since for almost all types, the agent would never use a message that fails this property given the availability of m^*. We will argue that there is an outcome-equivalent equilibrium with one fewer message used. This is sufficient to prove the Lemma because by repeating the argument as many times as needed, there will be an outcome-equivalent equilibrium with exactly n messages.

Since $|M^*| > n$, there is some project $i^* \in N$ and at least two distinct messages $m_1 \in M^*$ and $m_2 \in M^*$ such that $Support[\alpha(m_1)] \subseteq \{0, i^*\}$ and $Support[\alpha(m_2)] \subseteq \{0, i^*\}$. Letting $\alpha(i|m)$ be the probability that $\alpha(\cdot)$ puts on any project i following message m, we must have $\alpha(i^*|m_1) = \alpha(i^*|m_2) > 0$, since otherwise one of the two messages would never be used. Let $B_1 := \{\mathbf{b} : \mu(\mathbf{b}) = m_1\}$ and $B_2 := \{\mathbf{b} : \mu(\mathbf{b}) = m_2\}$. Optimality of α implies

$$\text{for } k \in \{1, 2\}, \quad \mathbb{E}[b_{i^*}|\mathbf{b} \in B_k] \geq \max\{b_0, \max_{j \in N} \mathbb{E}[b_j|\mathbf{b} \in B_k]\}, \tag{12}$$

$$\text{for } k \in \{1, 2\}, \quad \mathbb{E}[b_{i^*}|\mathbf{b} \in B_k] = b_0 \text{ if } \alpha(i^*|m_k) < 1. \tag{13}$$

Now consider a strategy $\tilde{\mu}$ defined as follows: for any $\mathbf{b} \notin B_1 \cup B_2$, $\tilde{\mu}(\mathbf{b}) = \mu(\mathbf{b})$; for any $\mathbf{b} \in B_1 \cup B_2$, $\tilde{\mu}(\mathbf{b}) = m_1$. So $\tilde{\mu}$ is identical to μ except that all types using m_2 switch to m_1, so that m_2 is not used in $\tilde{\mu}$. It is immediate that

$$\text{for } j \in N \text{ and } m \in M^* \setminus \{m_1, m_2\}, \mathbb{E}[b_j|m; \tilde{\mu}] = \mathbb{E}[b_j|m; \mu]. \tag{14}$$

Moreover, by iterated expectations,

$$\text{for } j \in N, \mathbb{E}[b_j|m_1; \tilde{\mu}] = \Pr(B_1)\mathbb{E}[b_j|\mathbf{b} \in B_1] + \Pr(B_2)\mathbb{E}[b_j|\mathbf{b} \in B_2]. \tag{15}$$

(12), (13), (14), and (15) imply the desired conclusion that $(\alpha, \tilde{\mu})$ is an equilibrium, and by construction, it is outcome-equivalent to (α, μ) but uses one fewer message. $\hspace{2cm}$ Q.E.D.

Proof of Lemma 2. The first statement is immediate. For sufficiency, fix any **q** satisfying (2) and (3) for all i with $\Pr\{\mathbf{b} : q_i b_i = \max_{j \in N} q_j b_j\} > 0$. We consider two cases:

(i) Suppose first there is some i with $q_i > 0$. Then the agent has a best response, μ, that satisfies (1) and also has the property that any project that is recommended on path has positive ex-ante probability of being recommended. Such a μ and **q** are mutual best responses and Bayes Rule is satisfied. The only issue is assigning an appropriate out-of-equilibrium belief when any off-path project j is recommended; one can specify the off-path belief that $b_k = b_0$ for all k, which clearly rationalizes q_j.

(ii) Now suppose $q_i = 0$ for all i. Then for all i, $\Pr\{\mathbf{b} : q_i b_i = \max_{j \in N} q_j b_j\} > 0$ and $\mathbb{E}[b_i \mid q_i b_i = \max_{j \in N} q_j b_j] = \mathbb{E}[b_i]$. It follows from (3) that for all i, $\mathbb{E}[b_i] \leq b_0$, and hence there is an equilibrium where the DM always chooses the outside option with "passive beliefs" of maintaining the prior no matter the recommendation, and the agent always recommends project one. $\hspace{2cm}$ Q.E.D.

C   More than Two Projects

This Appendix shows how our main results extend to multiple projects, $n > 2$, under the strong ordering condition of Definition 3. We first generalize Theorem 1:

Theorem 8. *Assume strong ordering, as stated in Definition 3.*

1. *For any equilibrium* **q**, *for any* $i < j$, *if* $q_i > 0$, *then* $q_i \geq q_j$, *and if* $q_i > 0$ *and* $q_j < 1$, *then* $q_i > q_j$.

2. *There is a largest equilibrium,* **q*** *such that:*

   (a) *If* $b_0 \leq b_0^* := \mathbb{E}[b_n | b_n = \max_{j \in N} b_j]$, *then there exists a truthful equilibrium* **q**$^* = \mathbf{1}$.

   (b) *If* $b_0 \in (b_0^*, b_0^{**})$ *for some* $b_0^{**} \geq b_0^*$, *then* **q**$^* \gg \mathbf{0}$ *and* $q_1^* = 1$; *that is, the largest equilibrium is a pandering equilibrium. Moreover, for any* $\tilde{b}_0 > b_0$ *in this interval,* **q**$^* > \tilde{\mathbf{q}}^*$, *where these are the largest equilibria respectively for* b_0 *and* $\tilde{b}_0$.

   (c) *If* $b_0 > b_0^{**}$, *then only the zero equilibrium exists,* **q**$^* = \mathbf{0}$.

Proof. The proof is in several steps.

STEP 1: *Fix any equilibrium* **q** *and any* $i < j$. *If* $q_i > 0$, *then* $q_i \geq q_j$, *and if in addition* $q_j < 1$, *then* $q_i > q_j$.

Proof: Fix any equilibrium $\mathbf{q}$ and any projects $i < j$. Suppose to the contrary that $q_j > q_i > 0$. Then

$$
\begin{aligned}
\mathbb{E}\left[b_i \,\middle|\, q_i b_i \geq \max\left\{q_j b_j, \max_{k \neq i,j} q_k b_k\right\}\right]
&= \mathbb{E}\left[b_i \,\middle|\, b_i \geq \max\left\{\left(\frac{q_j}{q_i}\right) b_j, \max_{k \neq i,j}\left(\frac{q_k}{q_i}\right) b_k\right\}\right] \\
&> \mathbb{E}\left[b_j \,\middle|\, \left(\frac{q_j}{q_i}\right) b_j \geq \max\left\{b_i, \max_{k \neq i,j}\left(\frac{q_k}{q_i}\right) b_k\right\}\right] \\
&= \mathbb{E}\left[b_j \,\middle|\, q_j b_j \geq q_k b_k, \forall k \neq j\right] \geq b_0,
\end{aligned}
$$

where the strict inequality is because of strong ordering and the weak inequality is because $q_j > 0$. But this implies that $\mathbb{E}[b_i \,|\, q_i b_i = \max_{k \in N} q_k b_k] > b_0$, which is contradiction with $q_i \in (0,1)$. This proves that $q_i \geq q_j$. For the second statement, notice that $0 < q_i = q_j < 1$ implies that the final inequality above must hold with equality. Since the strict inequality above still applies, the DM's optimality requires $q_i = 1$, a contradiction. $\|$

For the remaining results, we consider a mapping $\psi : [0,1]^n \to [0,1]^n$ such that for each $\mathbf{q} = (q_1, ..., q_n) \in [0,1]^n$,

$$
\psi_i(q_1, ..., q_n) := \max\left\{q_i' \in [0,1] \,\middle|\, \mathbb{E}[b_i \,|\, q_i' b_i \geq q_j b_j, \forall j \neq i] \geq b_0\right\} \tag{16}
$$

with the convention that $\max \emptyset := 0$. The mapping ψ_i calculates the highest probability with which the DM is willing to accept project i when it is recommended according to (1) subject to the constraint that the posterior belief does not fall below b_0.

STEP 2: *The mapping ψ has a largest fixed point $\mathbf{q}^*$.*

Proof: It suffices to prove that the mapping is monotonic, since Tarski's fixed point theorem then implies that the set of fixed points is nonempty and contains a largest element. Fix any $\mathbf{q}' \geq \mathbf{q}$. We will prove that $\psi(\mathbf{q}') \geq \psi(\mathbf{q})$. If $\psi_i(\mathbf{q}) = 0$ for some i then clearly $\psi_i(\mathbf{q}') \geq \psi_i(\mathbf{q})$. So suppose $\psi_i(\mathbf{q}) > 0$ for some i. Then $\mathbb{E}[b_i \,|\, \psi_i(\mathbf{q}) b_i \geq q_j b_j, \forall j \neq i] \geq b_0$. Since $\mathbf{q}' \geq \mathbf{q}$, for any such i, (R2′) implies that $\mathbb{E}[b_i \,|\, \psi_i(\mathbf{q}) b_i \geq q_j' b_j, \forall j \neq i] \geq \mathbb{E}[b_i \,|\, \psi_i(\mathbf{q}) b_i \geq q_j b_j, \forall j \neq i]$. Putting the two facts together, we have $\mathbb{E}[b_i \,|\, \psi_i(\mathbf{q}) b_i \geq q_j' b_j, \forall j \neq i] \geq b_0$, from which it follows that $\psi_i(\mathbf{q}') \geq \psi_i(\mathbf{q})$. $\|$

STEP 3: *The largest fixed point $\mathbf{q}^*$ of ψ is an equilibrium.*

Proof: By Lemma 2, it suffices to prove that $\mathbf{q}^*$ satisfies (2) and (3). To begin, suppose $q_i^* > 0$, then, since $q_i^* = \psi_i(\mathbf{q}^*) > 0$, we have

$$
\mathbb{E}[b_i \,|\, q_i^* b_i = \max_{k \in N} q_k^* b_k] \geq b_0. \tag{17}
$$

Now consider any project $j \neq i$ with $q_j^* > 0$. If $q_j^* = 1$, then $q_j^* \geq q_i^*$, so $q_i^* b_i \geq q_j^* b_j$ implies

$b_i \geq b_i$. It thus follows that

$$\mathbb{E}[b_j \,|\, q_i^* b_i = \max_{k \in N} q_k^* b_k] \leq \mathbb{E}[b_i \,|\, q_i^* b_i = \max_{k \in N} q_k^* b_k]. \tag{18}$$

If $q_j^* \in [0, 1)$, then we have

$$\mathbb{E}[b_j \,|\, q_i^* b_i = \max_{k \in N} q_k^* b_k] \leq \mathbb{E}[b_j \,|\, q_j^* b_j = \max_{k \in N} q_k^* b_k] \leq b_0, \tag{19}$$

where the second inequality follows from $q_j^* = \psi_j(\mathbf{q}^*)$ and from the construction of ψ for the case $q_j^* < 1$, and the first inequality is explained as follows: Define $x := \max_{k \neq i, j} q_k^* b_k$, and let G be its cumulative distribution function. Then, the middle term of (19) can be written as

$$\mathbb{E}[b_j \,|\, q_j^* b_j = \max_{k \in N} q_k^* b_k] = \frac{\int_0^\infty b_j G(q_j^* b_j) F_i(\frac{q_j^*}{q_i^*} b_j) f_j(b_j) db_j}{\int_0^\infty G(q_j^* b_j) F_i(\frac{q_j^*}{q_i^*} b_j) f_j(b_j) db_j} = \int_0^\infty b \hat{f}_j(b) db,$$

where

$$\hat{f}_j(z) := \frac{G(q_j^* z) F_i(\frac{q_j^*}{q_i^*} z) f_j(z)}{\int_0^\infty G(q_j^* \tilde{z}) F_i(\frac{q_j^*}{q_i^*} \tilde{z}) f_j(\tilde{z}) d\tilde{z}}.$$

Likewise, the left-most term of (19) can be written as

$$\mathbb{E}[b_j \,|\, q_i^* b_i = \max_{k \in N} q_k^* b_k] = \frac{\int_0^\infty b_j \left(\int_{\frac{q_j^*}{q_i^*} b_j}^\infty G(q_i^* b_i) f_i(b_i) db_i \right) f_j(b_j) db_j}{\int_0^\infty \left(\int_{\frac{q_j^*}{q_i^*} b_j}^\infty G(q_i^* b_i) f_i(b_i) db_i \right) f_j(b_j) db_j} = \int_0^\infty b \tilde{f}_j(b) db,$$

where

$$\tilde{f}_j(z) := \frac{\left(\int_{\frac{q_j^*}{q_i^*} z}^\infty G(q_i^* b_i) f_i(b_i) db_i \right) f_j(z)}{\int_0^\infty \left(\int_{\frac{q_j^*}{q_i^*} z}^\infty G(q_i^* b_i) f_i(b_i) db_i \right) f_j(\tilde{z}) d\tilde{z}}.$$

Note that $\tilde{\eta}(z) := \int_{\frac{q_j^*}{q_i^*} z}^\infty G(q_i^* b_i) f_i(b_i) db_i$ is non-increasing in z, while $\hat{\eta}(z) := G(q_j^* z) F_i(\frac{q_j^*}{q_i^*} z)$ is nondecreasing in z. Hence, for any $z' > z$,

$$\frac{\tilde{f}_j(z')}{\tilde{f}_j(z)} = \frac{\tilde{\eta}(z') f_j(z')}{\tilde{\eta}(z) f_j(z)} \leq \frac{f_j(z')}{f_j(z)} \leq \frac{\hat{\eta}(z') f_j(z')}{\hat{\eta}(z) f_j(z)} = \frac{\hat{f}_j(z')}{\hat{f}_j(z)}, \tag{20}$$

whenever the left-most and right-most terms are well defined.

The inequality (20) means that $\hat{f}$ likelihood-ratio dominates $\tilde{f}$, which proves the first inequality of (19). When combined, (17), (18), and (19) imply that $\mathbf{q}^*$ satisfies (2). The construction of ψ implies that $\mathbf{q}^*$ satisfies (3). $\|$

STEP 4: *The largest fixed point $\mathbf{q}^*$ of ψ is the largest equilibrium.*

Proof: Suppose to the contrary that there is an equilibrium $\hat{\mathbf{q}} \not\leq \mathbf{q}^*$. Define a mapping $\hat{\psi} : \prod_{i \in N}[\hat{q}_i, 1] \to \prod_{i \in N}[\hat{q}_i, 1]$ such that for each $\mathbf{q} = (q_1, ..., q_n) \in \prod_{i \in N}[\hat{q}_i, 1]$,

$$\hat{\psi}_i(q_1, ..., q_n) := \max \left\{ q_i' \in [\hat{q}_i, 1] \,\middle|\, \mathbb{E}[b_i \mid q_i' b_i \geq q_j b_j, \forall j \neq i] \geq b_0 \right\},$$

again with the convention that $\max \emptyset := 0$. Since $\hat{\mathbf{q}}$ is an equilibrium, it must satisfy (2), so $\hat{\psi}_i(\hat{\mathbf{q}}) \geq \hat{q}_i$. Hence the mapping is well defined on the restricted domain. Further, since $\psi_i(\hat{\mathbf{q}}) \geq \hat{q}_i$ for each i, it must be that $\hat{\psi}(\mathbf{q}) = \psi(\mathbf{q})$ for any $\mathbf{q} \in \prod_{i \in N}[\hat{q}_i, 1]$. Hence $\hat{\psi}$ is monotonic, and Tarski's fixed point theorem implies existence of a fixed point, say $\hat{\mathbf{q}}^+$. By construction, $\hat{\mathbf{q}}^+ \geq \hat{\mathbf{q}}$. Evidently, $\hat{\mathbf{q}}^+$ is a fixed point of ψ as well (in the unrestricted domain). Since $\mathbf{q}^*$ is the largest fixed point, we must have $\mathbf{q}^* \geq \hat{\mathbf{q}}^+ \geq \hat{\mathbf{q}}$, a contradiction. The result follows since $\mathbf{q}^*$ is an equilibrium by Step 3. $\|$

STEP 5: *If $\mathbf{q}^* \neq \mathbf{0}$, then $\mathbf{q}^* \gg \mathbf{0}$ and $q_1^* = 1$.*

Proof: Suppose $\mathbf{q}^* \neq \mathbf{0}$. Then, there must exist $k \in N$ such that $q_k^* > 0$. Fix any $i \neq k$. By (A3), there exists $\alpha > 0$ such that

$$b_0 \leq \mathbb{E}\left[b_i \,\middle|\, b_i > \alpha b_k\right] = \mathbb{E}\left[b_i \,\middle|\, \left(\frac{q_k^*}{\alpha}\right) b_i > q_k^* b_k\right] \leq \mathbb{E}\left[b_i \,\middle|\, \left(\frac{q_k^*}{\alpha}\right) b_i > q_j^* b_j, \forall j\right],$$

which implies that, for $q_i = \frac{\overline{q}^*}{\alpha} > 0$, $\mathbb{E}\left[b_i \,\middle|\, q_i b_i > q_j^* b_j, \forall b_j\right] \geq b_0$. It follows that $q_i^* = \psi_i(\mathbf{q}^*) \geq q_i > 0$. We have thus proven $\mathbf{q}^* \gg \mathbf{0}$. Step 1 then implies that $q_i^* \geq q_j^*$ for any $i < j$. Suppose $q_1^* < 1$. Then, it must be that $\mathbb{E}[b_i \mid q_i^* b_i = \max_{k \in N} q_k^* b_k] = b_0$ for all $i \in N$. Now consider $\overline{\mathbf{q}}^* = \left(\frac{1}{q_i^*}\right) \mathbf{q}^*$. Clearly, $\overline{\mathbf{q}}^*$ is also an equilibrium and $\overline{\mathbf{q}}^* \gg \mathbf{q}^*$, which contradicts Step 4. $\|$

STEP 6: *Let $\mathbf{q}^*(b_0)$ denote the largest equilibrium under outside option b_0. Then, $\mathbf{q}^*(b_0) \geq \mathbf{q}^*(b_0')$ for $b_0 < b_0'$. If $b_0 < b_0'$ and $\mathbf{q}^*(b_0') \in (\mathbf{0}, \mathbf{1})$, then $\mathbf{q}^*(b_0) > \mathbf{q}^*(b_0')$.*

Proof: Write $\psi(\mathbf{q}; b_0)$ in (16) to explicitly recognize its dependence on b_0. It is easy to see that $\psi_i(\mathbf{q}; b_0)$ is nonincreasing in b_0. It follows that the largest fixed point $\mathbf{q}^*(\mathbf{b}_0)$ is nonincreasing in b_0, proving the first statement. To prove the second, let $b_0 < b_0'$ and $\mathbf{q}^*(b_0') \in (\mathbf{0}, \mathbf{1})$. The statement holds trivially if $\mathbf{q}^*(b_0) = \mathbf{1}$. Hence, assume $\mathbf{q}^*(b_0) < \mathbf{1}$. By Step 1 and Step 5, we must have $q_n^*(b_0) \in (0, 1)$, and this implies that $\mathbb{E}[b_n | q_n^*(b_0) b_n = \max_{k \in N} q_k^*(b_0) b_k] = b_0 < b_0'$. Clearly, $\mathbf{q}^*(b_0) \neq \mathbf{q}^*(b_0')$. By the first statement, it follows that $\mathbf{q}^*(b_0) > \mathbf{q}^*(b_0')$. $\|$

STEP 7: *The truthful equilibrium exists if and only if $b_0 \leq b_0^* := \mathbb{E}[b_n | b_n = \max_{j \in N} b_j]$.*

Proof: If $b_0 \leq b_0^*$, then $(R1')$ implies that $b_0 \leq \mathbb{E}[b_i | b_i = \max_{j \in N} b_j]$ for all $i \in N$, so there is a truthful equilibrium. If $b_0 > b_0^*$, $\mathbf{q} = \mathbf{1}$ clearly violates (2), so there cannot be a truthful equilibrium. ‖

STEP 8: *There exists $b_0^{**} \geq b_0^*$ such that the largest equilibrium is $\mathbf{q}^*(b_0) \in (\mathbf{0}, \mathbf{1})$—it is a pandering equilibrium—if $b_0 \in (b_0^*, b_0^{**})$ and it is zero equilibrium if $b_0 > b_0^{**}$. For any $b_0, b_0' \in (b_0^*, b_0^{**})$ such that $b_0 < b_0'$, $\mathbf{q}^*(b_0) > \mathbf{q}^*(b_0')$.*

Proof: The first statement follows directly from Steps 1, 5, 6, and 7. The second statement follows directly from Step 7 by noting that $\mathbf{q}^*(b_0') \in (\mathbf{0}, \mathbf{1})$. ‖ 　　　　　　　　*Q.E.D.*

Remark 1. Unlike with Theorem 1, the largest equilibrium may not be the best equilibrium when there are many projects.[32] Yet, it is compelling to focus on. First, it clearly maximizes the agent's (interim) payoff. Second, there is a sense in which any non-zero equilibrium $\mathbf{q}$ where $q_i = 0$ for some i must be supported with "unreasonable" off-path beliefs. Informally, a forward-induction logic goes as follows: by proposing a project i when $q_i = 0$ (which is off the equilibrium path in a non-zero equilibrium), the agent must be signaling that i is sufficiently better than all the projects that he could get implemented with positive probability. So the DM should focus her beliefs on those types that would have the most to gain from such a deviation. Naturally, the agent has more to gain the higher is b_i. But then, with enough weight of beliefs on high b_i's, the DM should accept i with probability one, contradicting $q_i = 0$. This intuition is formalized in Appendix G, where we show that the D1 criterion of Cho and Kreps (1987) rules out any equilibrium $\mathbf{q} \neq \mathbf{0}$ with $q_i = 0$ for some i.[33] Given that when $\mathbf{q}^* \neq \mathbf{0}$ it will generically be the only equilibrium where all projects are implemented with positive probability on the equilibrium path,[34] and $\mathbf{q}^*$ is obviously better for both players than the zero equilibrium, we find it reasonable to focus on $\mathbf{q}^*$.

Focusing on the largest equilibrium, the comparison pitching result of Theorem 2 can also be generalized to the multi-project environment:

Theorem 9. *Fix b_0 and an environment $\mathbf{F} = (F_1, \ldots, F_i, \ldots, F_n)$ that satisfies strong ordering (Definition 3). Let $\tilde{\mathbf{F}} = (F_1, \ldots, \tilde{F}_i, \ldots, F_n)$ be a new environment such that either*

　　(a) $\tilde{\mathbf{F}}$ satisfies strong ordering and F_i likelihood-ratio dominates $\tilde{F}_i$; or

---

[32] To see why, suppose $n = 4$ and the largest equilibrium is $\mathbf{q}^* = (1, q_2^*, q_3^*, q_4^*) \gg 0$ while another equilibrium is $\mathbf{q} = (1, q_2, q_3, 0)$ with $q_2 > 0$ and $q_3 > 0$. Even if $q_2^* > q_2$ and $q_3^* > q_3$, so that $\mathbf{q}^*$ has less pandering than $\mathbf{q}$ toward project one, it could be that $\mathbf{q}^*$ has more pandering toward project two over three than $\mathbf{q}$, i.e. $1 > q_3/q_2 > q_3^*/q_2^*$. If projects two and three are ex-ante significantly more likely to be better than projects one and four, it is possible that the DM could prefer $\mathbf{q}$ over $\mathbf{q}^*$.

[33] To circumvent the usual problems of refinement in cheap-talk games, we apply the refinement in a restricted "veto game" where the DM can only choose between the proposed project and the outside option. This is no longer a cheap-talk game, but as mentioned previously in fn. 9, is almost equivalent for our purposes. Specifically, any equilibrium we consider in the cheap-talk game has a counterpart in the veto game.

[34] A proof of this statement is available on request.

(b) $\tilde{F}_i$ is a degenerate distribution at zero.

In either case, let $\mathbf{q}^*$ and $\tilde{\mathbf{q}}^*$ denote the largest equilibria respectively under $\mathbf{F}$ and $\tilde{\mathbf{F}}$. Then $\mathbf{q}^* \geq \tilde{\mathbf{q}}^*$; moreover, $\mathbf{q}^* > \tilde{\mathbf{q}}^*$ if $\tilde{\mathbf{q}}^* \neq \mathbf{1}$ and $\mathbf{q}^* > \mathbf{0}$ and either (b) holds or the likelihood-ratio dominance in (a) is strict.

Proof. The proof is very similar to that of Theorem 2, so we do not reproduce the entire argument. The key difference is that instead of inequality (10), we must now show that for any $j \in N$,

$$\mathbb{E}[b_j | \tilde{q}_j^* b_j = \max_{k \in N} \tilde{q}_k^* b_k] \geq \mathbb{E}[\tilde{b}_j | \tilde{q}_j^* \tilde{b}_j = \max_{k \in N} \tilde{q}_k^* \tilde{b}_k]. \tag{21}$$

(As before, case (b) is straightforward, so we focus on case (a) of the Theorem so that $\tilde{F}_i$ is not degenerate at zero, and moreover, we can assume $\tilde{q}^* \gg \mathbf{0}$.) For $j = i$, (21) follows from likelihood-ratio dominance of F_i over $\tilde{F}_i$. For $j \neq i$, (21) is proven as follows. Define $x := \max_{k \neq i,j} \tilde{q}_k^* b_k$, and let G be its cumulative distribution function. We can write

$$\mathbb{E}[b_j \mid \tilde{q}_j^* b_j = \max_{k \in N} \tilde{q}_k^* b_k] = \frac{\int_0^\infty b_j G(\tilde{q}_j^* b_j) F_i(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} b_j) f_j(b_j) db_j}{\int_0^\infty G(\tilde{q}_j^* b_j) F_i(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} b_j) f_j(b_j) db_j} = \int_0^\infty b k_j(b) db,$$

where $k_j(z) := \dfrac{G(\tilde{q}_j^* z) F_i(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} z) f_j(z)}{\int_0^\infty G(\tilde{q}_j^* \tilde{z}) F_i(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} \tilde{z}) f_j(\tilde{z}) d\tilde{z}}$. Likewise,

$$\mathbb{E}[\tilde{b}_j \mid \tilde{q}_j^* \tilde{b}_j = \max_{k \in N} \tilde{q}_k^* \tilde{b}_k] = \int_0^\infty b \tilde{k}_j(b) db,$$

where $\tilde{k}_j(z) := \dfrac{G(\tilde{q}_j^* z) \tilde{F}_i(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} z) f_j(z)}{\int_0^\infty G(\tilde{q}_j^* \tilde{z}) \tilde{F}_i(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} \tilde{z}) f_j(\tilde{z}) d\tilde{z}}$. To prove inequality (21), it suffices to show that k_j likelihood-ratio dominates $\tilde{k}_j$. Consider any $b' > b$. Algebra shows that

$$\frac{k_j(b')}{k_j(b)} \geq \frac{\tilde{k}_j(b')}{\tilde{k}_j(b)} \iff \frac{F_i\left(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} b'\right)}{F_i\left(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} b\right)} \geq \frac{\tilde{F}_i\left(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} b'\right)}{\tilde{F}_i\left(\frac{\tilde{q}_j^*}{\tilde{q}_i^*} b\right)},$$

which is the same inequality as we had in the proof of Theorem 2, so again the right-hand side of the equivalence is implied by the hypothesis that F_i likelihood-ratio dominates $\tilde{F}_i$ (see the earlier proof for additional details).

To finish the proof of the first part of the Theorem, let ψ and $\tilde{\psi}$ denote the mappings (16) for environments $\mathbf{F}$ and $\tilde{\mathbf{F}}$, respectively. Then, (21) means that $\psi(\tilde{\mathbf{q}}^*) \geq \tilde{\psi}(\tilde{\mathbf{q}}^*)$. This implies that there exists a fixed point of ψ weakly greater than $\tilde{\mathbf{q}}^*$. It follows that $\mathbf{q}^* \geq \tilde{\mathbf{q}}^*$.

Just as in the proof of Theorem 2, the second part of the current Theorem follows from the fact that inequality (21) has to hold strictly when the likelihood-ratio domination of F_i over $\tilde{F}_i$ is strict; hence $\psi(\tilde{\mathbf{q}}^*) > \tilde{\psi}(\tilde{\mathbf{q}}^*)$, whereby $\mathbf{q}^* > \tilde{\mathbf{q}}^*$.   *Q.E.D.*

D   Leading Examples

This Appendix provides detailed computations for the leading examples with $n = 2$. We prove that they satisfy strong ordering and verify the expressions provided in Examples 1 and 2.

D.1   Scale-invariant Uniform Distributions

Assume that b_2 is uniformly distributed on $[0, 1]$, while b_1 is uniformly distributed on $[v, 1 + v]$ with $v > 0$. Assumption (A1) requires $b_0 < 1$; this also guarantees (A3).

Strong ordering: We compute

$$\mathbb{E}[b_2 | b_2 > \alpha b_1] = \frac{\int_v^{1+v} \int_{\alpha b_1}^1 b_2 db_2 db_1}{\int_v^{1+v} \int_{\alpha b_1}^1 db_2 db_1} = \frac{3v^2\alpha^2 + 3v\alpha^2 + \alpha^2 - 3}{3\alpha + 6v\alpha - 6} \text{ for } \alpha \in \left[0, \frac{1}{1+v}\right],$$

and

$$\mathbb{E}[b_2 | b_2 > \alpha b_1] = \frac{\int_v^{1/\alpha} \int_{\alpha b_1}^1 b_2 db_2 db_1}{\int_v^{1/\alpha} \int_{\alpha b_1}^1 1 db_2 db_1} = \frac{2}{3} + \frac{\alpha}{3}v \text{ for } \alpha \in \left(\frac{1}{1+v}, \frac{1}{v}\right]. \tag{22}$$

Both expressions are increasing in α in the relevant range.   Note that $\mathbb{E}[b_2 | b_2 > \alpha b_1]$ is not defined for $\alpha > 1/v$.

Similarly, it can be computed that

$$\mathbb{E}[b_1 | b_1 > \alpha b_2] = \begin{cases} v + \frac{1}{2} & \text{if } \alpha \leq v \\ \frac{-2v^3 + 3v^2\alpha + 6v\alpha - \alpha^3 + 3\alpha}{-3v^2 + 6v\alpha - 3\alpha^2 + 6\alpha} & \text{if } \alpha \in (v, 1+v) \\ \frac{6v^2 + 6v + 2}{6v + 3} & \text{if } \alpha \geq 1 + v \end{cases} \tag{23}$$

and hence is non-decreasing in α.

The Largest Equilibrium: From Theorem 1 and formula (22),

$$b_0^* = \mathbb{E}[b_2 | b_2 > b_1] = \frac{2}{3} + \frac{v}{3}.$$

For $b_0 > b_0^*$, a pandering equilibrium $\mathbf{q}^* = (1, q_2^*)$ with $q_2^* \in (0, 1)$ requires $\mathbb{E}[b_2 | b_2 > b_1/q_2^*] = b_0$.

Substituting from (22) yields the solution

$$q_2^*(b_0) = \frac{v}{3b_0 - 2} \tag{24}$$

so long as the right hand side above is larger than v, which is guaranteed since $b_0 < 1$. That $q_1^* = 1$ implies $\mathbb{E}[b_1|b_1 > q_2^*(b_0)b_2] \geq b_0$, into which we substitute (24) to obtain

$$\mathbb{E}\left[b_1 \Big| b_1 > \frac{v}{3b_0 - 2}b_2\right] \geq b_0.$$

By substituting from (23), it can be verified that the left-hand side of the above expression is continuous and weakly decreasing in b_0, while the right-hand side is, obviously, strictly increasing. Moreover, by the definition of b_0^*, $\mathbb{E}\left[b_1 \Big| b_1 > \frac{v}{3b_0^* - 2}b_2\right] = \mathbb{E}[b_1|b_1 > b_2] > b_0^*$. Therefore, there is a unique b_0^{**} such that

$$\mathbb{E}\left[b_1 \Big| b_1 > \frac{v}{3b_0^{**} - 2}b_2\right] = b_0^{**},$$

and $b_0^{**} > b_0^*$. It can be verified that $b_0^{**} < 1$ if and only if $v < \frac{1}{2}$. It follows that a pandering equilibrium $\mathbf{q}^* = (1, q_2^*)$ with $q_2^* \in (0, 1)$ exists if and only if $b_0 \in (b_0^*, \min\{1, b_0^{**}\})$. If $b_0^{**} < 1$ (i.e., $v < 1/2$), then for $b_0 \in (b^{**}, 1)$ the only equilibrium is $\mathbf{q} = (0, 0)$.

Remark 2. Finally, what happens if $b_0 > 1$, so that Assumptions (A1) and (A3) fail? If $v < 1/2$, then $\mathbb{E}[b_1] = v + \frac{1}{2} < b_0$, hence $\mathbf{q} = (0, 0)$ is the only equilibrium. If $v > \frac{1}{2}$, then for $b_0 \in \left(1, \frac{1}{2} + v\right)$, $\mathbf{q} = (1, 0)$ is the only equilibrium, whereas for $b_0 > \frac{1}{2} + v$, $\mathbf{q} = (0, 0)$ is the only equilibrium. Thus, a violation of (A1) and (A3) allow for the non-influential equilibrium $(1, 0)$ to be the largest equilibrium for certain values of b_0.

D.2 Exponential Distributions

Assume that b_1 and b_2 are exponentially distributed with respective means v_1 and v_2, where $v_1 > v_2 > 0$. Assumption (A1) is obviously satisfied for any $b_0 \in \mathbb{R}_{++}$; we will show below that (A3) requires $b_0 < 2v_2$.

Strong Ordering: Denoting the project densities respectively by $f_1(\cdot)$ and $f_2(\cdot)$, we have that

$$\begin{aligned}
\mathbb{E}[b_1|b_1 > b_2] &= \frac{\int_0^\infty (\mathbb{E}[b_1 > b_2|b_2]\Pr(b_1 > b_2|b_2)f_2(b_2)db_2}{\int_0^\infty \Pr(b_1 > b_2|b_2)f_2(b_2)db_2} \\
&= \frac{v_1 + v_2}{v_1}\int_0^\infty (v_1 + b_2)e^{-\frac{1}{v_1}b_2}\left(\frac{1}{v_2}e^{-\frac{1}{v_2}b_2}\right)db_2 \\
&= \int_0^\infty (v_1 + b_2)\left(\frac{v_1 + v_2}{v_1 v_2}\right)e^{-\left(\frac{v_1 + v_2}{v_1 v_2}\right)b_2}db_2 \\
&= v_1 + \frac{v_1 v_2}{v_1 + v_2}.
\end{aligned}$$

Similarly,
$$\mathbb{E}[b_2|b_2 > b_1] = v_2 + \frac{v_1 v_2}{v_1 + v_2}.$$

Moreover, since αb_i is exponentially distributed with mean αv_i, the above calculations imply

$$\mathbb{E}[b_i|b_i > \alpha b_j] = v_i + \frac{\alpha v_i v_j}{v_i + \alpha v_j}. \tag{25}$$

Since the right-hand side above is strictly increasing in α for any $\alpha \in \mathbb{R}_+$, strong ordering is satisfied. Note that since $\lim_{\alpha \to \infty} \mathbb{E}[b_i|b_i > \alpha b_j] = 2v_i$, Assumption (A3) requires $b_0 < 2v_2$.

The Largest Equilibrium: From Theorem 1 and equation (25),

$$b_0^* = \mathbb{E}[b_2|b_2 > b_1] = v_2 + \frac{v_1 v_2}{v_1 + v_2}.$$

For $b_0 > b_0^*$, a pandering equilibrium $\mathbf{q} = (1, q_2^*)$ with $q_2^* \in (0,1)$ requires $\mathbb{E}[b_2|b_2 > b_1/q_2^*] = b_0$. Substituting from (25) yields the solution

$$q_2^*(b_0) = \frac{v_1}{v_2}\left(\frac{2v_2 - b_0}{b_0 - v_2}\right). \tag{26}$$

That $q_1^* = 1$ implies $\mathbb{E}[b_1|b_1 > q_2^*(b_0)b_2] \geq b_0$, into which we substitute (25) to obtain

$$3v_1 - b_0\frac{v_1}{v_2} \geq b_0.$$

Since the left-hand side of the this inequality is decreasing in b_0 and the right-hand side is increasing in b_0, the inequality is satisfied if and only if

$$b_0 \leq b_0^{**} = \frac{3v_1 v_2}{v_2 + v_1}$$

Note that $b_0^{**} < 2v_2$ if and only if $v_1 < 2v_2$. It follows that a pandering equilibrium $\mathbf{q}^* = (1, q_2^*)$ with $q_2^* \in (0,1)$ exists if and only if $b_0 \in (b_0^*, \min\{b_0^{**}, 2v_2\})$. If $b_0^{**} < 2v_2$ (i.e., if $v_1 < 2v_2$), then for $b_0 \in (b_0^{**}, 2v_2)$, the only equilibrium is $\mathbf{q} = (0,0)$.

DM's Expected Payoff: If $b_0 < b_0^*$, the DM's ex-ante expected payoff is

$$
\begin{aligned}
\pi^t &:= \mathbb{E}\left[\max\{b_1, b_2\}\right]\\
&= \left(\frac{v_1}{v_1 + v_2}\right)\left(v_1 + \frac{v_1 v_2}{v_1 + v_2}\right) + \left(\frac{v_2}{v_1 + v_2}\right)\left(v_2 + \frac{v_1 v_2}{v_1 + v_2}\right)\\
&= v_1 + v_2 - \frac{v_1 v_2}{v_1 + v_2}.
\end{aligned}
$$

For $b_0 \in (b_0^*, \min\{b_0^{**}, 2v_2\})$, the DM's expected payoff is

$$
\begin{aligned}
\pi^p \;:=\; & \Pr(b_1 > q_2^*(b_0)b_2)\mathbb{E}[b_1|b_1 > q_2^*(b_0)b_2] + \Pr(q_2^*(b_0)b_2 > b_1)b_0 \\
=\; & \left(\frac{v_1}{v_1 + q_2 v_2}\right)\left(v_1 + \frac{v_1 q_2 v_2}{v_1 + q_2 v_2}\right) + \left(\frac{q_2 v_2}{v_1 + q_2 v_2}\right)b_0 \\
=\; & \left(\frac{v_1}{v_1 + q_2 v_2}\right)\left(3v_1 - b_0 \frac{v_1}{v_2}\right) + \left(\frac{q_2 v_2}{v_1 + q_2 v_2}\right)b_0 \\
=\; & \frac{1}{(v_2)^2}\left(2b_0 (v_2)^2 - (v_2 + v_1)(b_0)^2 + 4b_0 v_1 v_2 - 3v_1 (v_2)^2\right) \\
=\; & \pi^t - \frac{(v_1 + v_2)}{v_2^2}(b_0 - b_0^*)^2 .
\end{aligned}
$$

Finally, if $b_0 > (b_0^{**}, 2v_2)$, the DM's expected payoff is just b_0.

Remark 3. What happens if $2v_2 < b_0$, so that Assumption (A3) fails? Then $q_2 = 0$ in any equilibrium. If $v_1 < 2v_2$, then $E[b_1] = v_1 < b_0$, hence $\mathbf{q} = (0,0)$ is the only equilibrium. If $v_1 > 2v_2$, then for $b_0 \in (2v_2, v_1)$, $\mathbf{q} = (1,0)$ is the unique equilibrium whereas for $b_0 > v_1$, $q = (0,0)$ is the unique equilibrium. Thus, a violation of (A3) allows for the non-influential equilibrium $(1,0)$ to be the largest equilibrium, for certain values of b_0.

E   Simple Randomization by the DM

This Appendix shows that for two projects there is no perfect Bayesian equilibrium where the DM puts positive probability on both projects for a positive measure of agent types, except for knife-edged prior distributions.

To this end, consider any equilibrium (μ, α), where $\mu : \mathcal{B} \to M$ is the agent's pure strategy and $\alpha : M \to \Delta(\{0, 1, 2\})$ is the DM's possibly-mixed strategy. For any $m \in M$, $\alpha(i|m)$ is the probability that the DM chooses project i following message m.

Suppose there is an on-path message m^* such that $\min\{\alpha(1|m^*), \alpha(2|m^*)\} > 0$. (If m^* does not exist, we are done.) We can assume that there is some other on-path message that induces a different action distribution from the DM, because otherwise $\mathbb{E}[b_1] = \mathbb{E}[b_2]$, which is knife-edged. Moreover, no on-path message can lead to the status quo with probability 1, since the agent will never use such a message given the availability of m^*.

STEP 1: There exist constants $q_1 > \alpha(1|m^*)$ and $q_2 > \alpha(2|m^*)$ such that for any on-path message m, either(i) $\alpha(m) = \alpha(m^*)$, or (ii) $\alpha(1|m) = 0$ and $\alpha(2|m) = q_2$, or (iii) $\alpha(1|m) = q_1$ and $\alpha(2|m) = 0$.

To prove this, suppose m is on path and $\alpha(m) \neq \alpha(m^*)$. We cannot have the agent strictly prefer m^* to m or vice-versa independent of his type, so suppose $\alpha(1|m) > \alpha(1|m^*)$ and $\alpha(2|m^*) > \alpha(2|m)$, with the opposite case treated symmetrically below. Then m^* will be used

by the agent only if

$$b_1 \alpha\left(1|m^*\right) + b_2 \alpha\left(2|m^*\right) \geq b_1 \alpha\left(1|m\right) + b_2 \alpha\left(2|m\right),$$

or $b_2 \geq b_1 k$, where $k := \frac{\alpha(1|m)-\alpha(1|m^*)}{\alpha(2|m^*)-\alpha(2|m)}$. If $k \geq 1$, $\mathbb{E}\left[b_2|m^*\right] > \mathbb{E}\left[b_1|m^*\right]$, which cannot be, hence $k < 1$. Analogously, message m will be used by the agent only if $b_1 \geq \frac{b_2}{k}$. Since $k < 1$, $\mathbb{E}\left[b_2|m\right] < \mathbb{E}\left[b_1|m\right]$, which implies that $\alpha\left(2|m\right) = 0 < \alpha\left(1|m\right)$.

A symmetric argument applies to the case of $\alpha\left(1|m\right) < \alpha\left(1|m^*\right)$ and $\alpha\left(2|m^*\right) < \alpha\left(2|m\right)$, establishing that in this case $\alpha\left(2|m\right) > 0 = \alpha\left(1|m\right)$.

Finally, note that all on-path messages that lead to (possibly degenerate) randomization between project 1 and the outside option must put the same probability on project 1, call it q_1, and this must be strictly larger than $\alpha\left(1|m^*\right)$ — otherwise they would not be used. Analogously for project 2 and the outside option.

STEP 2: Suppose there is an on-path message m_1 such that $\alpha\left(1|m_1\right) = q_1$ and an on-path message m_2 such that $\alpha\left(2|m_2\right) = q_2$. We cannot have $q_1 = q_2 = 1$, for then only at most a zero measure of types will induce randomization from the DM. So suppose $q_1 = 1 > q_2$. Then m^* will only be used by types such that $b_2 > b_1$, contradicting $\mathbb{E}\left[b_2|m^*\right] = \mathbb{E}\left[b_1|m^*\right]$. Similarly for $q_1 < 1 = q_2$. Therefore, $\max\{q_1, q_2\} < 1$, which implies

$$\mathbb{E}\left[b_1|m_1\right] = \mathbb{E}\left[b_2|m_2\right] = b_0. \tag{27}$$

Since m_1 is used by the agent when

$$\frac{q_1 - \alpha\left(1|m^*\right)}{\alpha\left(2|m^*\right)} b_1 \geq b_2,$$

and m_2 is used by the agent when

$$\frac{\alpha\left(1|m^*\right)}{q_2 - \alpha\left(2|m^*\right)} b_1 \leq b_2,$$

we can visualize the $b_1 - b_2$ rectangle as being partitioned into three regions by the two line segments $b_2 = xb_1$ and $b_2 = yb_1$ where $x := \frac{q_1-\alpha(1|m^*)}{\alpha(2|m^*)}$ and $y := \frac{\alpha(1|m^*)}{q_2-\alpha(2|m^*)}$. Message m_1 is used in the bottom region, m^* in the middle region, and m_2 in the top region. Hence, any prior distribution that solves (27) and also satisfies $\mathbb{E}\left[b_1|m^*\right] = \mathbb{E}\left[b_2|m^*\right]$ is knife-edged.

STEP 3: Suppose that any on-path message m with $\alpha\left(m\right) \neq \alpha\left(m^*\right)$ has $\alpha\left(2|m\right) = 0$. (A symmetric argument applies to the other case where $\alpha\left(1|m\right) = 0$.) Then there is some m_1 with $\alpha\left(1|m_1\right) = q_1$. We must have $q_1 < 1$ because otherwise the agent will use m_1 whenever $b_1 \geq b_2$, contradicting $\mathbb{E}\left[b_1|m^*\right] = \mathbb{E}\left[b_2|m^*\right]$. Thus $\mathbb{E}\left[b_1|m_1\right] = b_0$. But now analogously to step 2, we can

view the type space as partitioned into two regions by a line segment

$$b_2 = \frac{q_1 - \alpha\left(1|m^*\right)}{\alpha\left(2|m^*\right)} b_1,$$

with message m_1 used in the lower cone and m^* in the upper cone. For non-knife-edge priors, the requirement that $\mathbb{E}\left[b_1|m_1\right] = b_0$ will imply a finite number of candidates for $\frac{q_1 - \alpha(1|m^*)}{\alpha(2|m^*)}$, none of which will also yield $\mathbb{E}\left[b_1|m^*\right] = \mathbb{E}\left[b_2|m^*\right]$.

F   Revelation of Verifiable Information

In this Appendix, we show how revelation of hard information by the agent can lead to asymmetries in soft information about projects. This formalizes the assertion in Section 2.2 that asymmetric distributions for the project values can be viewed as resulting from either asymmetries that are directly observable to the DM or private but verifiable information of the agent that is fully revealed.

Formally, suppose that all projects are ex-ante identical. Each project i independently draws from a distribution $G(\cdot)$ a verifiable component, $v_i \in V$, where V is a compact subset of $\mathbb{R}$. Thereafter, each project draws it's value b_i independently from a family of distributions $F(b_i|v_i)$ with density $f(b_i|v_i)$. The agent privately observes the vector $(\mathbf{v}, \mathbf{b})$ and then communicates with the DM in two stages. First, he sends a vector of messages $\mathbf{r} := (r_1, \ldots, r_n)$ about $\mathbf{v}$ subject to the constraint that for each i,

$$r_i \in \{X : X \subseteq V, X \text{ is closed}, v_i \in X\}.$$

This formulation captures that each v_i is hard information: the agent can claim that v_i lies in any subset of V so long as the claim is true. Thereafter, the agent sends a cheap-talk message just as in our baseline model. Finally, the DM implements a project or the outside option.

The key assumption we make is that the distributions $F(b|v)$ satisfy the monotone likelihood-ratio property (MLRP): if $v > v'$, then for all $b > b'$, $\frac{f(b|v)}{f(b'|v)} > \frac{f(b|v')}{f(b'|v')}$. Moreover, assume that for any vector of hard information, $\mathbf{v}$, the project distributions $(F(\cdot|v_1), \ldots, F(\cdot|v_n))$ satisfy strong ordering.

Proposition 1. *In this extended model with privately observed hard information, there is an equilibrium where the agent fully reveals his hard information by sending $r_i = v_i$, and the subsequent cheap-talk subgame outcome is identical to the largest equilibrium, q^*, of our baseline model where each $F_i = F(\cdot|v_i)$.*

Proof. Consider a skeptical posture by the DM, where for any hard information report $r_i \subseteq V$, the DM believes that $v_i = \min r_i$. Then for any profile $\mathbf{r}$, the DM plays the q^* associated with

our baseline model where each $F_i = F(\cdot | \min r_i)$. Since $F(b|v)$ has the MLRP, Theorem 9 implies that if the agent deviates from $\mathbf{r} = \mathbf{v}$ to any other hard information report, he only induces a weakly smaller acceptance profile from the DM in the ensuing cheap-talk game. Thus the agent can do no better than playing $r_i = v_i$ and then playing according to q^* of the game where $F_i = F(\cdot | v_i)$. Plainly, the DM is playing optimally as well. $\qquad\qquad$ *Q.E.D.*

G   Equilibrium Refinement

This Appendix shows how a formal refinement can be used to justify focussing on the largest equilibrium, $\mathbf{q}^*$, even when $n > 2$. Recall from Remark 1 that although q^* is the best equilibrium when $n = 2$, this need not be the case when $n > 2$. We show here that if an equilibrium $\mathbf{q}$ survive an appropriate belief-based refinement, then either $\mathbf{q} = \mathbf{0}$ or $\mathbf{q} \gg \mathbf{0}$. Since $\mathbf{q}^*$ is generically the only equilibrium where all projects are accepted with positive probability when recommended (assuming a non-zero equilibrium exists), and it is better than the zero equilibrium, this provides a rationale to focus on the largest equilibrium.

G.1   Preliminaries

The refinement we use is the **D1** criterion of Cho and Kreps (1987); some other criteria based on strategic stability such as *universal divinity* (Banks and Sobel, 1987) would yield the same conclusion. Since standard belief-based refinements are powerless in cheap-talk games,[35] we apply the D1 refinement to a slightly restricted "veto game" where the DM must choose between the project proposed by the agent and the outside option. To be clear: the only difference between the veto game and the equilibrium class we study in the cheap-talk game is that in the latter, the DM is allowed to choose one of the alternative projects that are *not* proposed by the agent. Any of the equilibria we study in the cheap-talk game is an equilibrium of the veto game,[36] and accordingly we believe that focussing on the veto game to apply belief-based refinements is reasonable.

One final note before introducing the refinement: we will assume here that $\underline{b}_i > 0$ for all i. This is to ensure that no matter the realization of $\mathbf{b}$, the agent strictly prefers getting any project implemented over the outside option. The case of $\underline{b}_i = 0$ for some i can be accommodated along similar lines, but requires some additional care.[37]

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[35]Cheap-talk refinements such as Farrell's (1993) *neologism proofness* or Chen, Kartik and Sobel's (2008) *no incentive to separate* are not sufficient here either.

[36]The converse is not always true, because one can support "perverse" equilibria in the veto game that would not be equilibria in the cheap-talk game. But since our goal is to refine the set of equilibria of the cheap-talk game, this is immaterial.

[37]To elaborate: we would then have to assume that the agent's utility from project i is $\bar{u} + b_i$ for some $\bar{u} > 0$. While this would alter the exact equilibrium vector $\mathbf{q}^*$ compared to the baseline model where $\bar{u} = 0$, it does not

G.2   The D1 refinement

Given any equilibrium $\mathbf{q}$, define the sets

$$
\begin{aligned}
D_i\left(\mathbf{b}\right) &:= \left\{x \in [0,1] : x b_i \geq \max_k q_k b_k\right\}, \\
D_i^+\left(\mathbf{b}\right) &:= \left\{x \in [0,1] : x b_i > \max_k q_k b_k\right\}.
\end{aligned}
$$

Either of these sets may be empty. In words, relative to a given equilibrium, $D_i\left(\mathbf{b}\right)$ is the set of acceptance probabilities of project i that would make the agent of type $\mathbf{b}$ weakly prefer proposing project i compared to following the equilibrium strategy, and similarly for $D_i^+\left(\mathbf{b}\right)$ with strict preference.

We can now state Cho and Kreps's (1987) D1 criterion adapted to our framework. Recall that in any equilibrium, if $q_i > 0$, then i is proposed on the equilibrium path; on the other hand, if $\mathbf{q} > 0$, $q_i = 0$ implies that project i is never proposed and hence is an out-of-equilibrium project.

Definition 4. An equilibrium with $\mathbf{q} > 0$ satisfies the D1 criterion if for any i such that $q_i = 0$,

$$
\mu\left(\mathbf{b}|i\right) = 0 \text{ if there exists } \mathbf{b}' \text{ s.t. } D_i\left(\mathbf{b}\right) \subseteq D_i^+\left(\mathbf{b}'\right),
$$

where $\mu\left(\cdot|i\right)$ is the DM's equilibrium belief about project values when project i is proposed.

To interpret, note that an equilibrium with $\mathbf{q} > 0$ and $q_i = 0$ must have $\mathbb{E}_{\mu(\cdot|i)}\left[b_i\right] \leq b_0$; otherwise, sequential rationality requires $q_i = 1$. D1 requires that we support this behavior of the DM with beliefs that put zero probability on any type $\mathbf{b}$ such that there is some other type, $\mathbf{b}'$ that would strictly prefer to propose the project for any acceptance probability for which $\mathbf{b}$ weakly prefers proposing it to equilibrium. Intuitively, in such case, $\mathbf{b}'$ is "more likely" to deviate to project i than $\mathbf{b}$, and the D1 criterion requires that $\mathbf{b}$ be eliminated from the support of the DM's beliefs when i is proposed.

G.3   Selection

The following result shows that any non-zero equilibrium that satisfies the D1 criterion must have all projects proposed in equilibrium.

Proposition 2. *Any equilibrium* $\mathbf{q} > 0$ *satisfies the D1 criterion only if* $\mathbf{q} \gg 0$.

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qualitatively change any of the results. The refinement analysis below would go through with this specification even if $\underline{b}_i = 0$ for any i.

Proof. Suppose not, toward a contradiction. Then there is D1 equilibrium where $\mathbf{q} > 0$ but $q_i = 0$ for some $j \neq i$. We will argue that for any $\mathbf{b}$ with $b_i \leq b_0$, D1 requires that $\mu(\mathbf{b}|i) = 0$; this proves the proposition because it implies that $\mathbb{E}_{\mu(\cdot|i)}[b_i] > b_0$, contradicting $q_i = 0$. Note that since $\underline{b}_i > 0$ for all i, $\max_k q_k b_k > 0$.

First consider any $\mathbf{b}$ such that $b_i \leq b_0$ and $b_i \leq \max_k q_k b_k$. Then either $D_i(\mathbf{b}) = \{1\}$ or $D_i(\mathbf{b}) = \emptyset$. On the other hand, for any $\mathbf{b}'$ such that $b_i' > \max_{k \neq i} b_k'$, $D_i^+(\mathbf{b}') \supseteq \{1\}$. Hence D1 requires that $\mu(\mathbf{b}|i) = 0$.

Now consider any $\mathbf{b}$ such that $b_i \leq b_0$ and $b_i > \max_k q_k b_k$. Then $D_i(\mathbf{b}) = \left[\max_k \frac{q_k b_k}{b_i}, 1\right] \subsetneq [0,1]$. Consider $\mathbf{b}'$ such that $b_k' = b_k$ for all $k \neq i$, and $b_i' > b_0$. For any $x \in D_i(\mathbf{b})$, $x b_i' > \max_k q_k b_k'$, and hence $D_i(\mathbf{b}) \subseteq D_i^+(\mathbf{b}')$. So again, D1 requires that $\mu(\mathbf{b}|i) = 0$. $\qquad$ *Q.E.D.*

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