Tying in Markets with Network Effects*

Jay Pil Choi[†] Doh-Shin Jeon[‡] Michael D. Whinston[§]

September 9, 2022

Abstract

We develop a leverage theory of tying in markets with network effects. When a monopolist in one market cannot fully extract the whole surplus from consumers, tying can be a mechanism through which the unexploited consumer surpluses are used as a demand-side leverage to create a strategic "quasi installed-base" advantage in another market characterized by network effects. Our mechanism does not require the commitment assumption with technical tying. Tying can lead to the exclusion of more efficient rival firms in the tied market, but expand the tying good market if the latter market is not fully covered with independent pricing. Welfare implications are also discussed.

JEL Codes: D4, L1, L5
Key Words: Tying, Leverage of monopoly power, Network Effects, Imperfect rent extraction

---

*We thank Jacques Crémer, Igal Hendel, Sarit Markovich, and participants in various conferences and seminars for valuable discussions and comments. Choi served as a consultant for the Korea Fair Trade Commission in a case against Google. Whinston has been retained by the Department of Justice in the matter of US v. Google.

[†]Department of Economics, Michigan State University, 220A Marshall-Adams Hall, East Lansing, MI 48824 -1038. E-mail: choijay@msu.edu.

[‡]Toulouse School of Economics, University of Toulouse Capitole. E-mail: dohshin.jeon@tse-fr.eu

[§]Sloan School of Management and Department of Economics, M.I.T., 77 Massachusetts Ave., Cambridge, MA 02139. E-mail: whinston@mit.edu.

1 Introduction

We develop a leverage theory of tying in markets with network effects. More specifically, we consider a situation in which there is a monopolistic firm in one market that is unable to extract fully the whole surplus from consumers. We show that tying allows the monopolist in this market to leverage, in a profitable way, the unextracted "slack" in consumer surplus to monopolize a second market where it faces competition when the second market is characterized by network effects. We also explore welfare implications of this tying mechanism.

The leverage theory of tying typically considers the following scenario: There is a monopolistic firm in one market (say A). This firm, however, faces competition in another market (say B). According to the leverage theory of tying, the monopolistic firm in market A can monopolize market B using the leverage provided by its monopoly power in market A through tying or bundling arrangements. The Chicago School, however, criticized this theory and proposed instead price discrimination as the main motivation for tying. The gist of the Chicago school criticism is based on the so-called "one monopoly theorem," which states that "[a] seller cannot get two monopoly profits from one monopoly." (Blair and Kaserman, 1985).

We demonstrate that in the presence of imperfect rent extraction in a monopolized market and network effects in a market where the monopolist faces competition, tying can be a mechanism through which the unexploited consumer surpluses in the monopolized market are used as a demand-side leverage to create a strategic "quasi installed-base" advantage in the competing market.

In markets with network effects, consumer utility consists of stand-alone benefits and network benefits. Under independent pricing, all firms compete on a level playing field. Even though markets with strong network effects are typically characterized by tipping equilibria in which all consumers choose the same product, yielding maximal network benefits, the network-augmented utility component can be competed away in equilibrium to consumers' benefit. With tying, however, the tying firm can use the unexploited consumer surplus in the tying market in competition against a rival firm in a tied market. We show that this advantage allows the tying firm to lock in consumers who have a high value for the tying product, ensuring that it captures the network effect and enabling it to win in the tied market even against a more efficient rival.

More precisely, consider a situation in which there are two markets, A and B. Firm 1 is a monopolist of product A and sells its product $B1$ in market B against a rival, firm 2, that produces product $B2$. Consumers in the monopolized market A are heterogeneous and some consumers receive surplus in this market under independent pricing. In such a scenario, if firm 1 offers only a bundle, consumers with high valuations for product A may prefer to purchase the bundle even if all other consumers purchase the rival firm's product $B2$. The existence of such consumers ensures a guaranteed market share in market B for firm 1, which is akin to firm 1 having an installed base. This advantage in terms of the quasi-installed base can in turn induce low valuation consumers to purchase the bundle instead of buying $B2$. We show that a process of iterated elimination of dominated strategies can lead to tipping toward the monopolist's bundle.

We first develop our theory in the context of independent products to illustrate how network effects in the tied good market may provide incentives to tie. To illustrate our mechanism, we first consider a situation in which the tying good market is covered (i.e., all consumers purchase the tying product) under independent pricing. In this case, we show that pure bundling is an optimal strategy. In general, when the tying good market is not covered under independent pricing, firm 1 finds it optimal to use a mixed bundling strategy in which consumers can choose between buying the bundle $(A - B1)$ and buying product $B1$ only. This mixed bundling enables firm 1 to screen consumers with respect to their willingness to pay for the monopolized product A while maximizing the network effects for its product $B1$. When the number of consumers buying the bundle is large enough, firm 1 is able to sell even its inferior $B1$ at a profit as a stand-alone product against product $B2$, leading to further extraction of consumer surplus.

We then extend our analysis to the case of complementary products because most tying cases involve products that are complementary. With pure monopoly in the tying product market, we confirm the Chicago School critique that tying cannot be a leverage mechanism even with network effects. However, pure monopoly with absolutely no competitive products is rare. We show that in the presence of an inferior alternative to the tying good market we can restore our mechanism with parallel results to the independent products case; we formally demonstrate the equivalence of the complementary products case to the independent products case, with the inferior alternative in the tying market playing the same role as does the no purchase option in the independent products case.

Our analysis can be used to develop a theory of harm for tying cases when network effects are critical in the determination of the market winner. As we discuss in more detail in Section 7, our model can shed light on the recent EU *Android* case concerning Google's tying practice that requires Android OEM manufacturers to pre-install the Google search app as a condition for licensing Google's app store (the Play Store). This example may be considered a situation with independent products. In contrast, the *Microsoft* case in Europe (IP/04/382) in 2004 can be considered as a situation with complementary products in which Microsoft tied its Windows Media Player (WMP) to its dominant Windows operating system.

The literature on tying as an anticompetitive foreclosure mechanism has focused most on situations in which a monopolist firm commits to use of a tying strategy, as first developed in Whinston (1990).[1] If the market structure in the tied good market is oligopolistic with scale economies, tying can be an effective and profitable strategy to alter market structure by making continued operation unprofitable for tied good rivals. This occurs because a commitment to tying leads the monopolist to price aggressively in order to ensure sales of the valuable product A. However, in Whinston (1990), inducing the exit of the rival firm is essential for the profitability of tying arrangements. Thus, if the competitor has already paid the sunk cost of entry and there is no avoidable fixed cost, tying cannot be a profitable strategy. In contrast, our mechanism requires neither commitment power of the tying firm nor exit of the rival.[2] Most other papers in the tying literature, such as Carlton and

[1] Fumagalli et al. (2018) provide an excellent survey of tying as an exclusionary practice along with discussions on major antitrust cases.

[2] As we can easily verify, if network effects are absent, we replicate his result that bundling is not profitable if firm 2's exit is not induced.

Waldman (2002) and Choi and Stefanadis (2001), have made similar commitment assumptions.[3]

While the commitment assumption makes sense when firms employ technological ties, in many tying cases the tie is a pricing choice that seems to involve little commitment.[4] In those situations, tying must be a best response to the prices of the monopolist's tied good market rivals. The literature on bundling makes clear that with heterogeneous valuations tying can indeed be a best response as a price discrimination mechanism, and when it is it can have effects on the profitability of tied good rivals. However, in these cases tying may be viewed as "innocent" and the effects on rivals inadvertent. What differs in our theory, however, is that tying can be a best response precisely because it *lowers the perceived quality of the monopolist's tied market rivals* by reducing the network benefits they can provide.

The idea of using unexploited consumer surplus as a leverage mechanism appears in some other papers. Burstein (1960) and Greenlee et al. (2008) analyze a setting in which the monopolist in the tying product market sells to consumers with multiunit demands and is unable to fully extract consumer surplus with linear pricing. By tying, even to competitively-supplied tied goods, the monopolist can require buyers to purchase additional products at elevated prices. In essence, tying serves as a substitute for a fixed fee.[5] In contrast, in our model consumers have single-unit demands for the tying good and so tying cannot serve this function.

Calzolari and Denicolo's (2015) theory of exclusive dealing is also based on uncaptured consumer surplus with multiunit buyers. They consider a single-market situation in which there is a dominant firm with a competitive advantage over a fringe of rivals, but buyers are able to obtain information rents due to private information even if they deal exclusively with the dominant firm. Without exclusive dealing, the dominant firm needs to compete for each marginal unit of a buyer's de-

[3]Nalebuff (2004) also shows how bundling can be used as an effective strategy to deter entry with an assumtion that the the incumbent can commit to its prices prior to the challenger's entry decision.

[4]In some cases, repputational concerns might lead to an element of commitment.

[5]However, tying is less efficient than a fixed fee since it causes distortions in the tied good markets.

mand; in contrast with exclusive dealing, the dominant firm competes for the entire volume demanded by a buyer. This change enables the dominant firm to exclude rivals by leveraging on the information rents left on inframarginal units. Thus, the dominant firm is able to exclude rivals with a lower discount with the imposition of exclusive deals. Exclusive dealing serves as a more profitable pricing mechanism despite the fact that it has no effects on the prices or qualities offered by the dominant firm's rivals. In contrast, in our model with heterogeneous consumers with single-unit demands, our mechanism leverages the network effect provided by inframarginal tying good consumers who are "committed" to the bundle to monopolize the tied good market.

Carlton and Waldman (2002) are closely related to our paper in that their theory is also based on network effects in the tied good market. They consider a dynamic two-step entry process for a potential entrant to complementary markets. In their two-period model, only market B is under the threat of entry in the first period. If the entrant successfully enters that market and builds its installed base, it is able to enter the primary market (A) in the second period. A commitment to tying deprives the entrant of the ability to build an installed base and safeguards the tying firm's monopoly position in the primary market from future entry. The purpose of tying is to preserve its market power in the primary market, even though it may entail short-run losses, rather than extend its monopoly power to an adjacent market as in our model. Our leverage mechanism also does not rely on dynamic arguments nor on commitment because there are short-run incentives to foreclose as a best response.

The rest of the paper is organized in the following way. In Section 2, we illustrate the main intuitions behind our tying mechanism through a simple example with discrete consumer types. In Section 3, we describe our baseline model for independent products with a more general demand structure. In Section 4, we analyze the baseline model in which the tying market is fully covered under independent pricing. In Section 5, we extend the analysis to the case in which the tying market is partially covered under independent pricing and uncover the role of mixed bundling as a screening device, which can further increase the tying firm's profits. In Section 6, we consider complementary products, and show that we can derive parallel results to the independent products case if we assume an inferior alternative to the monopolized tying good. We discuss the recent antitrust cases involving Google and Microsoft and offer concluding remarks in Section 7. Detailed proofs are relegated to the Appendix.

2 An Illustrative Example

To explain the main mechanism and intuition behind our model, we provide an illustrative example. There are two markets A and B. Market A is served by a monopolist called firm 1. In market B, firm 1 and firm 2 compete. These two products are independent. Firms' production costs are normalized to zero in all markets. There are two consumers.

2.1 Market A

The two consumers are heterogeneous in terms of their valuations for product A. One is a high (H) type consumer and the other is a low (L) type consumer. Each type consumer's willingness to pay for product A is given by u_k, where $k = H, L$, with $u_H = u_L + s > u_L > 0$ so $s > 0$. We assume that $u_L > s$. This implies that the optimal monopoly price is $p^{A*} = u_L$ and the high type consumers receive a surplus of s. An important feature of market A is that firm 1 is unable to extract the whole surplus in the market despite its monopoly power.

2.2 Market B

Market B is characterized by network effects. The two products $B1$ and $B2$ are not compatible with each other. In this market, we assume that the two consumers have the same preference. More specifically, firm i's product provides a stand-alone value of v_i to consumers, where $v_2 > v_1 > 0$. If the two consumers purchase the same product i, there are additional network benefits of n with a total value of $v_i + n$. In other words, given that a consumer buys product Bi, her gross surplus is v_i (respectively, $v_i + n$) if she is the only consumer buying the product (respectively, if the other consumer buys also the same product).

2.3 Independent Pricing Equilibrium

We first analyze the market equilibrium when the two products are sold independently by firm 1. In this regime, the two markets can be analyzed independently. As usual in a market with network effects, there can be multiple equilibria in market B due to positive consumption network effects if $\Delta \equiv v_2 - v_1 < n$, which we assume. Note that under this assumption, there is no equilibrium in which the two consumers choose different products given any configurations of prices (p_1^B, p_2^B); in equilibrium, either both consumers purchase $B1$ or $B2$.

2.3.1 Equilibrium with Coordination on B1

Due to consumers' coordination failure, there is an equilibrium in which the inferior product $B1$ wins in the market. The possibility of coordination failure also admits multiple pricing equilibria.[6] More specifically, any price $p_1^B \in [0, n - \Delta]$ can be sustained as an equilibrium price of $B1$ with all consumers purchasing $B1$ because such prices satisfy the following condition which implies that no consumer has an incentive to change his purchasing behavior even if firm 2 offers product $B2$ at a zero price.

$$v_1 + n - p_1^B \geq v_2$$

In an equilibrium in which consumers coordinate on $B1$, firm 1's overall profit is given by

$$\Pi_1^* = \pi_1^{A*} + \pi_1^{B*} \in 2 \cdot [u_L, u_L + (n - \Delta)],$$

with the maximum possible profit being $2 \cdot [u_L + (n - \Delta)]$.

[6] If we make an assumption that consumers can coordinate on the Pareto-optimal outcome, there is no equilibrium in which consumers coordinate on $B1$.

2.3.2 Equilibrium with Coordination on B2

In this equilibrium, consumers coordinate on the superior product $B2$. Once again, there can be multiple pricing equilibria. By the same logic above, any $p_2^B \in [0, n+\Delta]$ can be sustained as an equilibrium price of $B2$ with all consumers purchasing $B2$. In any equilibrium where consumers coordinate on $B2$, firm 1's profit in market B is zero. The overall profit for firm 1 is given by

$$\Pi_1^* = \pi_1^{A*} + \pi_1^{B*} = 2u_L + 0 = 2u_L.$$

2.4 Equilibrium with Tying

Now suppose that firm 1 engages in tying: it bundles its monopolized product A with product $B1$, and sells the bundle at the price of $\widetilde{P}$. Let $\widetilde{p}_2^B$ denote firm 2's price for product B ($B2$) under the tying regime. To show that tying can be profitable for firm 1 and to reduce the number of cases to consider, we make the following assumption:

$$s > 2n \tag{1}$$

We show that under the assumption above, firm 1 is able to use tying as a leverage mechanism to monopolize market B with a divide-and-conquer strategy. The argument shows that there is a unique equilibrium in which consumers' choices are pinned down by iterated dominance. The leverage mechanism with two discrete type consumers operate in two steps. First, tying allows firm 1 to leverage the surplus slack from the monopoly product A that is enjoyed by the high type consumer to gain purchases of $B1$. Once the high type consumer is secured to buy the bundle, the tying firm achieves a strategic advantage for the low type consumer as if the tying firm had the high type consumer as an installed-base of its product. These network benefits allow the tying firm to induce the low type consumer to buy the bundle as well.

To illustrate the existence of a profitable bundling deviation from the independent pricing equilibrium, we first show that firm 1 can strictly win both consumers by switching to a bundle at price $\widetilde{P} = u_L$, giving it the same profit as in the independent pricing equilibrium with consumers coordinating on $B2$. With $\widetilde{P} = u_L$, it is a dominant strategy for the H consumer to purchase the bundle because the bundle is preferred even under the most unfavorable condition that $B2$ is offered free and the other consumer purchases $B2$.

$$H : (u_H + v_1) - \widetilde{P} = s + v_1 > v_2 + n,$$

which is satisfied under our assumption (1). Given that the consumer H purchases the bundle, it is also optimal strategy for the consumer L to purchase the bundle even if $B2$ is offered free.

$$L : (u_L + v_1) + n - \widetilde{P} = v_1 + n > v_2$$

under our assumption $n > \Delta$. If the equilibrium under independent pricing is the one with consumers coordinating on $B2$, the logic above makes it clear that firm 1 can actually deviate and win everyone at a higher profit because both incentive compatibility conditions are satisfied with strict inequality.

We now derive the equilibrium outcome under tying and show that tying is always more profitable than independent pricing unless the equilibrium outcome under independent pricing is coordination on $B1$ with the maximum price of $p_1^B = (n - \Delta)$, in which case tying yields the same profit for firm 1. More precisely, we analyze the best response price $\widetilde{P}$ for firm 1 given $\widetilde{p}_2$. We show below that for any given $\widetilde{p}_2 \geq 0$, it is optimal for firm 1 to sell the bundle to both types of consumers instead of selling it to the high type consumer only. This implies that there is a unique equilibrium which involves tipping toward the bundle.

Define $\widetilde{P}_H$ ($\widetilde{P}_L$) as the bundle price which makes the high type consumer (low type consumer) indifferent between buying the bundle and buying $B2$:

$$\begin{aligned} H &: (u_H + v_1) - \widetilde{P}_H = v_2 + n - \widetilde{p}_2; \\ L &: (u_L + v_1) + n - \widetilde{P}_L = v_2 - \widetilde{p}_2. \end{aligned}$$

Note that in the first equality, we assume that the low type consumer buys $B2$ while in the second equality, we assume that the high type consumer buys the bundle. Under our assumption (1), we can easily verify that $\widetilde{P}_H > \widetilde{P}_L$. This implies that if $\widetilde{P} = \widetilde{P}_L$, it is a strictly dominant strategy for the H-type consumer to buy the bundle, which also induces the L-type consumer to buy the bundle. Then, for any $\widetilde{p}_2 \geq 0$, the profit from choosing $\widetilde{P} = \widetilde{P}_L$ and selling the bundle to both consumers is higher than the profit from choosing $\widetilde{P} = \widetilde{P}_H$ and selling the bundle to the H-type consumer only:

$$2\left[u_L + n - \Delta + \widetilde{p}_2\right] > \left[u_H - n - \Delta + \widetilde{p}_2\right],$$

which is satisfied for any $\widetilde{p}_2 \geq 0$.[7] Hence, for any $\widetilde{p}_2 \geq 0$, firm 1's best response consists in choosing $\widetilde{P} = \widetilde{P}_L$.

In the unique equilibrium with tipping toward the bundle, $\widetilde{p}_2^* = 0$ and $\widetilde{P}^*$ is chosen to make the low type consumer indifferent between buying the bundle and buying $B2$:

$$(u_L + v_1) + n - \widetilde{P}^* = v_2.$$

Hence, the tying firm's profit is

$$\begin{aligned} \widetilde{\Pi}^* &= \widetilde{P}^* = 2[u_L + \underbrace{n}_{\substack{\text{Network Effects Advantage} \\ \text{Due to Demand Leverage via Tying}}} - \Delta] \\ &\geq \Pi_1^* \in 2 \cdot [u_L, u_L + (n - \Delta)] \end{aligned}$$

We can conclude that tying is more profitable than independent pricing except in the most favorable case for firm 1 that arises due to consumers' coordination failure under independent pricing (in which case tying provides the same profit). As market B always tips toward product $B1$ under tying, social welfare decreases by Δ if the equilibrium under independent pricing is coordination on $B2$.[8]

[7] The condition is satisfied for all $\widetilde{p}_2 \geq 0$ if $u_L - s + 3n - \Delta > 0$, which always hold under our assumption that $u_L > s$ and $n > \Delta$.

[8] By the same token, if we consider a case in which $B1$ is superior to $B2$ with $v_1 > v_2$, tying also allows a better product $B1$ to break an equilibrium in which consumers fail to properly coordinate on $B1$ with independent pricing. In this case, tying is socially optimal.

In the next sections, we show that this mechanism applies more broadly with a general demand function. In addition, we show how mixed bundling can be used to screen consumer types to further increase the tying firm's profits.

3 The Baseline Model

We lay out a more general model of tying in markets with network effects. As in the illustrative example in the previous section, we study the case where products A and B are independent and can be used separately. Market A is monopolized by firm 1. In market B, there exist direct network effects, firm 1 and firm 2 compete, and consumers have homogenous valuations: their willingness to pay for each firm's product is given by $v_1+\beta N_1 > 0$ and $v_2+\beta N_2 > 0$, respectively, where $v_1 > 0, v_2 > 0$, $\beta > 0$, and N_i represents the number of consumers using firm i's product B. We normalize the total number of consumers to 1. All marginal costs are zero.

At the heart of our leverage mechanism is "unexploited consumer surplus" in the tying market which can be used in competition with a competitor in another market. If there exist high valuation consumers who receive sufficiently large consumer surpluses, they may be willing to purchase the bundle (rather than product $B2$ only) even if all other consumers purchase $B2$. The existence of such high valuation consumers in market A provides a demand-side leverage for firm 1 akin to having an installed base. If network effects are sufficiently strong, this strategic advantage would more than make up for any quality disadvantage of firm 1 and enables firm 1 to extract surplus from network effects, which makes tying profitable. The mechanism we have is very robust and can be applied to any tying market with unexploited consumer surplus.

More specifically, we consider heterogeneous consumers in market A. We assume that consumers' valuations for product A, denoted u, are distributed on $[\alpha, \alpha + \overline{u}]$, where α represents the lower bound for the consumers' valuations.[9] Let us define a

[9]Consider a product that has a basic functionality plus some additional features. We can imagine a situation in which the basic functionality provides the same utility of α to all consumers, but additional features may generate different levels of extra utility to consumers, which is distributed on $[0, \overline{u}]$.

consumer's type as $x = u - \alpha$, which is assumed to be distributed on $[0, \overline{u}]$ according to a c.d.f. G with a strictly positive density g.[10]

Let p_A be the price of product A. With a change of variables of $p = p_A - \alpha$,we have a demand function $D(p) = 1 - G(p)$ in market A. We assume that $G(.)$ satisfies the monotone hazard rate condition, that is, $\frac{g}{1-G}$ is strictly increasing. In market A, with independent pricing firm 1 chooses p on the consumer side to maximize

$$\max_p (p + \alpha) [1 - G(p)] .$$

Remark 1. *Our model can also be applied to two-sided markets where in market A firm 1 is a two-sided platform that receives advertising revenue whenever it is chosen by a consumer. If we assume that there is an associated advertising revenue of $\alpha > 0$ for each consumer in market A and consumers' valuations for product A is distributed on $[0, \overline{u}]$,our one-sided market model is isomorphic to a two-sided model with additional advertising revenue per consumer.*

In the baseline model, we assume that in market A, α is sufficiently large so that firm 1 serves all consumers with the price of $p^A = \alpha$ (or equivalently, $p = 0$). This condition is given by Assumption 1.

Assumption 1 (Full Market Coverage)**.**

$$\alpha \geq \frac{1 - G(0)}{g(0)} = \frac{1}{g(0)}.$$

Section 5 analyzes the case in which Assumption 1 is not satisfied. In market B, we make the following assumption:

Assumption 2.

$$\Delta \equiv v_2 - v_1 > 0, \Delta < \beta < \frac{1}{2g(x)} \textit{ for all } x \in [0, \overline{u}].$$

[10] We admit the possibility that $\overline{u} = \infty$.

$\Delta > 0$ means that firm 2's product ($B2$) has higher quality than firm 1's ($B1$). $\beta > \Delta$ means that network effects are sufficiently important relative to the quality differential Δ: if all consumers buy product B from firm 1, then its (network-augmented) quality $v_1 + \beta$ becomes higher than that of the rival v_2. $\beta < 1/\left[2g(x)\right]$ is a stability condition in the tying regime; otherwise, the network effects may make the demand tip to one product and an interior equilibrium, if it exists, can be unstable.[11] Our analysis is robust to alternative assumptions. After analyzing our model under Assumption 2, we observe that our results do not change when β is large enough that Assumption 2 is violated.

We consider two simultaneous pricing games and compare them. In the absence of tying, firm 1 chooses p_1^A for product A and p_1^B for product $B1$ and firm 2 chooses p_2^B for product $B2$. With tying, firm 1 chooses $\widetilde{P}$ for the bundle of product A and $B1$ and firm 2 chooses $\widetilde{p}_2^B$ for product B.

4 Analysis of the Baseline Model

We here analyze the baseline model. In the rest of the paper, we restrict attention to coalition-proof Nash equilibrium (CPNE) of the consumer response, a stronger notion of self-enforceability that accounts for coalitional deviations [Bernheim, Peleg, and Whinston (1987)]. We use this refinement of Nash equilibrium to simplify exposition and avoid the issue of multiplicity of equilibrium. One implication of the "coalition-proofness" in consumer response is that when players have identical preferences they coordinate on the Pareto-optimal outcome.

4.1 Independent Pricing Equilibrium

In the absence of tying, the two markets can be analyzed independently as we assume independent products.

[11] If G is uniform, $\beta < \frac{1}{2g(x)} = \frac{\overline{u}}{2}$ is the necessary and sufficient condition for an interior equilibrium to be stable. For general distributions, it is a sufficient, but not necessary, condition for the stability of an interior equilibrium because the violation of the condition implies only local instability.

In market B, all consumers have the same preference. Since we restrict our attention to CPNE of the consumer response, there is a unique equilibrium which consists in tipping toward firm 2's product.[12] In the equilibrium, firm 1 charges zero price ($p_1^B = 0$) and firm 2 charges $p_2^B = \Delta$. With independent pricing, B1 and B2 compete on a level playing field with no firm having advantage in network effects. As a result, the network-augmented utility component is competed away; the equilibrium prices do not contain parameter β.

Let π_1^{A*} (π_2^{B*}) represent firm 1's (firm 2's) profit from market A (B). Without tying, each firm's profit is given by

$$\begin{aligned} \Pi_1^* &= \pi_1^{A*} = \alpha; \\ \pi_2^{B*} &= \Delta. \end{aligned} \tag{2}$$

Summarizing, we have:

Proposition 1. *Suppose Assumptions 1 and 2 hold. Consider the case without tying.*
(i) In market A, firm 1 charges $p_1^{A*} = \alpha$ and receives a profit of $\pi_1^{A*} = \alpha$.
(ii) In market B, firm 1 charges zero price ($p_1^{B*} = 0$) and firm 2 charges $p_2^{B*} = \Delta$; firm 2's profit is $\pi_2^{B*} = \Delta$.

4.2 Tying

Market A is covered under independent pricing in the baseline model analyzed in this section. We thus consider only tying with pure bundling. As will be clear from the analysis of Section 5, limiting our attention to pure bundling is without any loss of generality.[13] In the next section, we consider a case where market A is not

[12] As is standard in the literature, we make the tie-breaking assumption in favor of the firm that can offer the highest consumer surplus to avoid the open set problem.

[13] Section 5 shows that when the monopolized market is not fully covered under independent pricing, firm 1's use of mixed bundling expands the number of consumers buying the monopolized product (by purchasing the bundle). Here, in Section 4, we consider the case in which the monopolized market is fully covered under independent pricing and pure bundling leads to tipping toward the bundle as the unique outcome. Since there is no market expansion, even if firm 1 uses mixed bundling, it will obtain the same tipping outcome.

fully covered under independent pricing and derive conditions under which mixed bundling can be more profitable.

In the presence of tying, let $\widetilde{P}$ be the price of the bundle of firm 1 and let $\widetilde{p}_2^B$ be the price of firm 2's product B. Given a pair of prices $\left(\widetilde{P}, \widetilde{p}_2^B\right)$, any equilibrium in consumers' choices has the cut-off property because if a consumer prefers the bundle to B2, any consumer whose valuation for A is higher would also prefer the bundle: there will be a critical type with the property that all higher types will purchase the bundle. If we have an interior equilibrium in which both firms have positive market shares, the critical consumer type (represented by $\widetilde{x}$) is indifferent between the bundle $(A - B1)$ and 2's product $(B2)$ with $G(\widetilde{x})$ representing the market share of firm 2 (i.e., $1 - G(\widetilde{x})$ is the market share of the bundle). The next lemma characterizes the equilibrium outcome in consumers' choices with using the change of variables $P = \widetilde{P} - \alpha$. In particular, we show that an interior equilibrium exists only when the price pair $\left(P, \widetilde{p}_2^B\right)$ satisfies the following condition.[14]

$$\beta - \Delta < \left(P - \widetilde{p}_2^B\right) < \overline{u} - \beta - \Delta \tag{3}$$

Lemma 1. *Given $\left(P, \widetilde{p}_2^B\right)$, the unique outcome in consumers' choices that survives iterated deletion of dominated strategies is as follows:*
(i) If (3) holds, consumers whose valuation for A is higher than $\widetilde{x}^* \in (0, \overline{u})$ purchase the bundle while consumers whose valuation is lower than $\widetilde{x}^*$ purchase B2, where $\widetilde{x}^*$ satisfies

$$\widetilde{x}^* + v_1 + \beta(1 - G(\widetilde{x}^*)) - P = v_2 + \beta G(\widetilde{x}^*) - \widetilde{p}_2^B. \tag{4}$$

(ii) If $\left(P - \widetilde{p}_2^B\right) \leq \beta - \Delta$, all consumers purchase the bundle (i.e., $\widetilde{x}^* = 0$).
(iii) If $\left(P - \widetilde{p}_2^B\right) \geq \overline{u} - \beta - \Delta$, all consumers purchase B2 only (i.e., $\widetilde{x}^* = \overline{u}$) .

[14] Note that the existence of such price pair is guaranteed because $\overline{u} - \beta - \Delta > \beta - \Delta > 0$. To see this, note that $\overline{u} \geq \frac{1}{\overline{g}}$, where $\overline{g} = \max_x g(x)$ for $x \in [0, \overline{u}]$ because $\int_0^{\overline{u}} g(x)dx = 1 \leq \overline{g}\overline{u}$. This implies that $\overline{u} > 2\beta$ by Assumption 2.

Proof. Let $\psi(t, x)$ be the payoff gain from purchasing the bundle over purchasing $B2$ for a type t consumer (i.e., whose willingness to pay for A is $\alpha + t$) if all other players whose types are higher than x choose the bundle.

$$\psi(t, x) = t + \beta(1 - 2G(x)) - \Delta - \left(P - \widetilde{p}_2^B\right).$$

Notice that $\psi(t, x)$ is continuous in t and x, increasing in t, and decreasing in x. As in the analysis of global games, we can use an induction argument to set in motion the process of iterated deletion of dominated strategies.[15]

(i) If $\left(P - \widetilde{p}_2^B\right) < \overline{u} - \beta - \Delta$, we can easily check that even when all other consumers are expected to choose $B2$ (i.e., $\overline{x}^0 = \overline{u}$) , it is optimal to choose the bundle for any consumers whose type is higher than $\overline{x}^1 = \beta + \Delta + \left(P - \widetilde{p}_2^B\right) < \overline{u} = \overline{x}^0$. Given that at least a measure of $1 - G(\overline{x}^1)$ consumers choose the bundle, we can derive another cut-off value $\overline{x}^2 < \overline{x}^1$. Note that $\overline{x}^n$ is a decreasing sequence. Similarly, if $\beta - \Delta < \left(P - \widetilde{p}_2^B\right)$, then even when all other consumers are expected to choose the bundle (i.e., $\underline{x}^0 = 0$), it is optimal to choose $B2$ for any consumers whose type is lower than $\underline{x}^1 = -\beta + \Delta + \left(P - \widetilde{p}_2^B\right) > 0 = \underline{x}^0$. Given that at least a measure of $G(\underline{x}^1)$ consumers choose $B2$, we can derive another cut-off value $\underline{x}^2 > \underline{x}^1$. Note that $\underline{x}^n$ is an increasing sequence. Thus, when $\beta - \Delta < \left(P - \widetilde{p}_2^B\right) < \overline{u} - \beta - \Delta$, the continuity of $\psi(t, x)$ and the way the two sequences $\overline{x}^n$ and $\underline{x}^n$ are constructed imply that $\psi(\underline{x}, \underline{x}) = \psi(\overline{x}, \overline{x}) = 0$, where $\overline{x} = \lim\limits_{n\to\infty} \overline{x}^n$ and $\underline{x} = \lim\limits_{n\to\infty} \underline{x}^n$. Define $\Psi(x)$ as follows:

$$\Psi(x) \equiv \psi(x, x) = x + \beta(1 - 2G(x)) - \Delta - \left(P - \widetilde{p}_2^B\right).$$

Note that under Assumption 2, $\Psi(x)$ is increasing in x because

$$\Psi'(x) = 1 - 2\beta g(x) > 0.$$

When $\beta - \Delta < \left(P - \widetilde{p}_2^B\right) < \overline{u} - \beta - \Delta$, we have $\Psi(0) < 0 < \Psi(\overline{u})$. Therefore, $\widetilde{x}^*$ is the unique solution to $\Psi(x) = 0$, which is an equivalent condition to (4).

(ii) If $\left(P - \widetilde{p}_2^B\right) \leq \beta - \Delta$, the process of iterated deletion of dominated strategies

[15] For an excellent survey of global games, see Morris and Shin (2010).

leads to $\widetilde{x}^* = 0$ because $\Psi(0) > 0$.

(iii) Similarly, if $\left(P - \widetilde{p}_2^B\right) \geq \overline{u} - \beta - \Delta$, the process of iterated deletion of dominated strategies leads to $\widetilde{x}^* = \overline{u}$ because $\Psi(\overline{u}) < 0$. ■

Lemma 1 immediately indicates that there is a profitable deviation for firm 1 from the independent pricing equilibrium in which all consumers buy $B2$. Consider a strategy for firm 1 to tie the two products and sell the bundle at a price of $\widetilde{P} = \alpha + (\beta - \Delta)$ (i.e., $P = \beta - \Delta$). Then, for any price $\widetilde{p}_2^B \geq 0$, all consumers buy the bundle and firm 1 receives a profit of $\alpha + (\beta - \Delta) > \alpha = \pi_1^{A*}$.

In what follows, we show that there is a unique equilibrium, which involves tipping toward the bundle.

Lemma 2. *Under Assumptions 1 and 2, there is no interior equilibrium in which both firms have positive market shares.*

Proof. Suppose that there is an interior equilibrium with $\widetilde{x} \in (0, \overline{u})$. In such an equilibrium, we have

$$\widetilde{x} + v_1 + \beta(1 - G(\widetilde{x})) - P = v_2 + \beta G(\widetilde{x}) - \widetilde{p}_2^B. \tag{5}$$

Then, (5) can be written as follows:

$$P = \phi(\widetilde{x}) + \widetilde{p}_2^B, \tag{6}$$

where $\phi(x) = x + \beta(1 - 2G(x)) - \Delta$. Given $\widetilde{p}_2^B$, there is one-to-one relationship between P and $\widetilde{x}$ because $\phi(x)$ is increasing in x with $\phi'(x) = 1 - 2\beta g(x) > 0$. Thus, we can write firm 1's maximization problem as

$$\underset{\widetilde{x}}{Max} \widetilde{\Pi}_1(\widetilde{x}; \widetilde{p}_2^B) = \left(\phi(\widetilde{x}) + \widetilde{p}_2^B + \alpha\right) \cdot (1 - G(\widetilde{x}))$$

Similarly for firm 2, given P, there is one-to-one relationship between $\widetilde{p}_2^B$ and $\widetilde{x}$. Firm 2 solves

$$\underset{\widetilde{x}}{Max} \widetilde{\pi}_2(\widetilde{x}; P) = [P - \phi(\widetilde{x})] \cdot G(\widetilde{x})$$

The best responses for each firm are characterized by the following first order conditions.

$$\frac{\partial \widetilde{\Pi}_1}{\partial \widetilde{x}} = \phi'(\widetilde{x})(1 - G(\widetilde{x})) - \left(\phi(\widetilde{x}) + \widetilde{p}_2^B + \alpha\right) g(\widetilde{x}) = 0; \tag{7}$$

$$\frac{\partial \widetilde{\pi}_2}{\partial \widetilde{x}} = -\phi'(\widetilde{x})G(\widetilde{x}) + [P - \phi(\widetilde{x})] \cdot g(\widetilde{x}) = 0. \tag{8}$$

By adding (7) and (8) along with (6), we can derive the following condition:

$$P - \widetilde{p}_2^B = \frac{(1-2G)\phi'}{g} - \alpha = \frac{(1-2G)(1-2\beta g)}{g} - \alpha. \tag{9}$$

For the existence of an interior equilibrium, we need to have $\widetilde{x} \in (0, \overline{u})$. Note that $\phi(0) = \beta - \Delta > 0$, which implies that $\phi(\widetilde{x}) > 0$ for any $\widetilde{x} \in (0, \overline{u})$ because $\phi(x)$ is increasing in x. This, in turn, implies that we need to have

$$P - \widetilde{p}_2^B (= \phi(\widetilde{x})) > 0.$$

However, condition (9) implies that $P - \widetilde{p}_2^B < 0$. This is trivially so if $1 \leq 2G(\widetilde{x})$. If $1 - 2G(\widetilde{x}) > 0$, then we have

$$P - \widetilde{p}_2^B = \frac{(1-2G)(1-2\beta g)}{g} - \alpha < \frac{(1-G)}{g} - \alpha < \frac{1}{g(0)} - \alpha < 0$$

by Assumption 1. We thus have a contradiction; there is no interior equilibrium. ■

Lemma 2 implies that the market equilibrium cannot be an interior equilibrium with both firms having positive market shares. The following lemma shows that there is no tipping equilibrium towards firm 2's product, either.

Lemma 3. *There is no equilibrium with tipping toward product B of firm 2.*

Proof. We find that under Assumption 2, the surplus slack from A is such that the consumer with the highest willingness to pay strictly prefers the bundle to product B of firm 2 at price $P = 0$ (i.e., $\widetilde{P} = \alpha$) even if the rival chooses $\widetilde{p}_2^B = 0$:

$$\overline{u} > \Delta + \beta.$$

The condition above holds by Assumption 2 because $\overline{u} \geq \frac{1}{\overline{g}} > 2\beta > \Delta + \beta$, where $\overline{g} = \max_x g(x)$ for $x \in [0, \overline{u}]$ (see footnote 9). Hence, the market share of the bundle is strictly positive at $\widetilde{P} = \alpha$ and $\widetilde{p}_2^B = 0$; therefore, there is no equilibrium with tipping toward product B of firm 2. ■

We now look for a tipping equilibrium toward the bundle. In such an equilibrium, we have $\widetilde{p}_2^B = 0$. When $\widetilde{p}_2^B = 0$, the first-order derivative of firm 1's profit is

$$\left.\frac{\partial \widetilde{\Pi}_1}{\partial P}\right|_{\widetilde{p}_2^B=0} = \left[1 - G\left(\phi^{-1}\left(\Delta - \beta + P\right)\right)\right] - \left(P + \alpha\right) g\left(\phi^{-1}\left(\Delta - \beta + P\right)\right) \phi'^{-1},$$

where $\phi(x) = x + \beta(1 - 2G(x)) - \Delta$.

From Lemma 1, $P \equiv \beta - \Delta (> 0)$ generates tipping when $\widetilde{p}_2^B = 0$. We can verify that

$$\left.\frac{\partial \widetilde{\Pi}_1}{\partial P}\right|_{P=\beta-\Delta, \widetilde{p}_2^B=0} < 0.$$

In other words, increasing P above $\beta - \Delta$ reduces firm 1's profit, which establishes it as a local maximizer. In the Appendix, we prove that $P = \beta - \Delta$ is also the price that achieves the global maximum. We thus conclude that firm 1's best response consists in $P = \beta - \Delta$; at $P = \beta - \Delta$, the bundle's market share is one and therefore lowering further the price does not increase the demand for the bundle.

Proposition 2. *Suppose that Assumptions 1 and 2 hold. Under tying, we have a unique equilibrium which involves tipping toward the bundle.*
(i) The equilibrium prices are given by

$$\widetilde{P}^* = \alpha + P^* = \alpha + (\beta - \Delta),\ \widetilde{p}_2^{B*} = 0.$$

(ii) Tying is profitable:

$$\widetilde{\Pi}_1^* = \alpha + (\beta - \Delta) > \alpha = \pi_1^{A*}$$

(iii) Both consumer surplus and social welfare decrease:

$$\begin{aligned}\widetilde{CS}^* &= CS^* - (\beta - \Delta) < CS^* \\ \widetilde{SW}^* &= SW^* - \Delta < SW^*\end{aligned}$$

where CS^* (SW^*) is consumer surplus (welfare) without tying and $\widetilde{CS}^*$ ($\widetilde{SW}^*$) is consumer surplus (welfare) under tying.

Firm 1 faces a trade-off. On the one hand, tying enables it to leverage the surplus slack from the tying product to the tied product such that consumers coalesce around its bundle. Hence it can expropriate the surplus from the network effect, which is equal to β. On the other hand, tying induces an aggressive response of firm 2, which lowers the price from Δ to zero. Then, as firm 1's product B is inferior by Δ, it should lower its price by Δ. To understand the negative price effects of tying (in the absence of network effects), notice that consumers are indifferent between $B1$ and $B2$ at independent pricing equilibrium. Holding p_2^B *fixed*, a change to bundling with a bundle price equal to the independently-priced price of A yields the same utilities for all consumers when they all buy the bundle and the same profit for firm 1. But once firm 2 lowers its price, firm 1 is worse off.

In the presence of network effects, tying is profitable as we assume $\beta > \Delta$. Notice the role network effects play in our model. Without network effects (i.e., $\beta = 0$), tying is not profitable unless it leads to the exclusion of firm 2. If firm 2 has already paid its sunk cost of entry or it has no avoidable fixed cost, tying will reduce the tying firm's profits by Δ, replicating Whinston's (1990) result.

Carbajo *et al.* (1990) develop a model with heterogeneous consumers in the tying market and focus on the case where the rivals' entry or exit decisions are not affected by tying as in our paper. In their model, tying can be profitable because tying serves as a mechanism to differentiate the monopolist's product in the tied good market from that of its rival. Tying thus softens price competition and increases both the tying firm's and rival firm's profits. However, in our model tying intensifies price competition and the effects on the profit from the altered behavior of the tied market rival is negative. Nonetheless, tying can be profitable through the changes in the behavior of inframarginal consumers. In the presence of network effects, the existence of inframarginal consumers operates as a quasi-installed base that bestows a strategic advantage in the positive feedback process, which eventually leads to profitable foreclosure of the rival firm.

Tying reduces welfare by Δ as consumers adopt the inferior product B. Finally, regarding consumer surplus, as tying reduces welfare by Δ and changes the total industry profit from $\alpha + \Delta$ to $\alpha + (\beta - \Delta)$, it reduces consumer surplus by $\beta - \Delta$:

$$\begin{aligned}\widetilde{CS}^* - CS^* &= \underbrace{\left(\widetilde{SW}^* - SW^*\right)}_{=-\Delta} - \underbrace{\left(\widetilde{\Pi}_1^* - \pi_1^{A*} - \pi_2^{B*}\right)}_{=(\beta-2\Delta)} \\ &= -(\beta - \Delta) < 0\end{aligned}$$

Remark 2. *We have analyzed the effects of tying under Assumption 2 that ensures the stability if interior equilibrium. Suppose that network effects are strong enough that the demand system under tying is not stable and only the tipping equilibrium exists under tying. We can derive the same result as long as we assume that the consumer with the highest willingness to pay for product A strictly prefers the bundle to only product $B2$ when prices are $P = \widetilde{p}_2^B = 0$:*

$$\overline{u} > \Delta + \beta.$$

Since this inequality implies that there is no tipping equilibrium toward product $B2$, the only equilibrium is tipping toward the bundle at the prices $\widetilde{P}^* = \alpha + \beta - \Delta$ and $\widetilde{p}_2^{B*} = 0$, replicating the result in Proposition 2.

5 Partial Coverage and Mixed Bundling

In the previous section, we considered the case where the tying good market (A) is covered under independent pricing. In that case, we showed that the market equilibrium under pure bundling entails all consumers purchasing the bundle, leading to foreclosure of firm 2 in the tied market (B). In this section, we consider an alternative scenario in which Assumption 1 is not satisfied and thus market A is not fully covered under independent pricing. In this alternative scenario, we cannot rule out the possibility that the equilibrium under pure bundling may lead to an interior equilibrium in market B with high type consumers purchasing the bundle while low type consumers purchasing $B2$ only. In that case, we show that mixed bundling can dominate pure bundling. The analysis in this section also indicates that our focus on pure bundling in the previous section is without any loss of generality.

Note that our model does not require any commitment assumption about bundling, in contrast to Whinston (1990) and most models of strategic leverage theory. This means mixed bundling is a priori (weakly) the best strategy because it has more pricing instruments; independent pricing and pure bundling pricing strategies can be replicated with mixed bundling with a suitable choice of prices for the bundle and separate products. We delineate conditions under which mixed bundling is strictly better than pure bundling and independent pricing. In particular, we analyze the following mixed bundling strategy: in addition to selling the bundle at $\widetilde{P}$, it keeps selling product $B1$ at $\widetilde{p}_1^B$, which could be sold to those whose willingness to pay for product A is not high.[16] This mixed bundling strategy enables firm 1 to screen consumers with respect to their willingness to pay for product A while maximizing the leveraged network effects for its product $B1$.

More specifically, consider an alternative case where Assumption 1 is violated and thus market A is not fully covered under independent pricing: $\alpha < \frac{1-G(0)}{g(0)} = \frac{1}{g(0)}$. In this case, firm 1 sets a price of $p_1^{A*} = \alpha + p^*$, where p^* satisfies the following condition.

$$p^* = \frac{1 - G(p^*)}{g(p^*)} - \alpha \ (> 0). \tag{10}$$

The mass of consumers buying product A without tying is given by $1 - G(p^*)$. We maintain Assumption 2.

[16] In other words, $B1$ can be purchased independently. However, the purchase of $B1$ is required to purchase A.

Then, without tying, firm 1 receives a profit of

$$\Pi_1^* = \pi_1^{A*} = (\alpha + p^*)(1 - G(p^*)) = \frac{[1 - G(p^*)]^2}{g(p^*)}.$$

Firm 2's profit is the same as in the previous section:

$$\pi_2^{B*} = \Delta.$$

We replace Assumption 1 with Assumption 3.

Assumption 3 (Partial Market Coverage)**.**

$$\frac{1}{g(0)} - G^{-1}\left(\frac{\beta - \Delta}{2\beta}\right) < \alpha < \frac{1}{g(0)}.$$

The second inequality in Assumption 3 is simply the maintained assumption in this section that market A is not fully covered. The first inequality is a sufficient condition for mixed bundling to dominate independent pricing as shown in Lemma 4 below. It guarantees that if all consumers of A under independent pricing purchase the bundle, $B1$ offers a higher utility than $B2$ even if all remaining consumers purchase $B2$. That is, it is a dominant strategy to choose $B1$ over $B2$ if they are offered at the same price; $\beta \cdot (1 - G(p^*)) > \Delta + \beta \cdot G(p^*)$, which can be rewritten as $\beta \cdot (1 - 2G(p^*)) > \Delta$. This condition which ensures that enough consumers buy A under independent pricing is more likely to be satisfied if α is large and $\frac{\Delta}{\beta}$ is small.[17]

We first show that mixed bundling leads to the foreclosure equilibrium in which firm 2 attracts no consumer even if it charges $\widetilde{p}_2^B = 0$. Due to network effects in market B, we can have multiple equilibria with mixed bundling depending on the coordination assumption adopted by consumers. Nonetheless, we demonstrate that under Assumption 3 mixed bundling with foreclosure is profitable even under the most pessimistic coordination assumption (i.e., stack the deck) against the tying

[17] See the proof of Lemma 4 in the Appendix for the derivation of the first inequality. If x is uniformly distributed with $g = 1/\overline{u}$ over $[0, \overline{u}]$, the condition in Assumption 3 can be written as $\frac{(\beta+\Delta)}{2\beta}\overline{u} < \alpha < \overline{u}$.

firm. The pricing strategy we consider thus is robust in the sense that it guarantees profitable market foreclosure regardless of coordination assumptions in consumers' purchase decisions; our foreclosure equilibrium is derived only with iterated dominance.[18]

Given $\widetilde{p}_2^B = 0$, consider a mixed bundling strategy $(\widetilde{P}, \widetilde{p}_1^B)$ that induces foreclosure of $B2$, with consumers whose types are higher than $\widetilde{x}$ purchasing the bundle and the remainder purchasing $B1$ only. First, with a change of variables $\widetilde{P} = \alpha + P$, we find P that ensures that all consumers whose types are higher than $\widetilde{x}$ purchase the bundle with the process of iterated dominance as in the pure bundling case.

$$\widetilde{x} + \beta\left[1 - G(\widetilde{x})\right] + v_1 - P = \beta G(\widetilde{x}) + v_2 \tag{11}$$

Note that condition (11) is derived under the most pessimistic assumption for the tying firm that all consumers who do not purchase the bundle buy the rival firm's product $B2$.[19] This means that given the bundle price of $\widetilde{P}(= \alpha + P) = \alpha + \widetilde{x} + \beta(1 - 2G(\widetilde{x})) - \Delta$, there is at least a network size of $[1 - G(\widetilde{x})]$ for $B1$.

Now let us analyze the decision of consumers whose types are lower than $\widetilde{x}$. If the following condition holds, it is a dominant strategy to purchase $B1$ instead of $B2$; it is better to purchase $B1$ even if all other consumers below $\widetilde{x}$ purchase $B2$:

$$v_1 + \beta(1 - G(\widetilde{x})) - \widetilde{p}_1^B \geq v_2 + \beta G(\widetilde{x}) \tag{12}$$

or $\widetilde{p}_1^B \leq \beta(1 - 2G(\widetilde{x})) - \Delta$. With the price of $\widetilde{p}_1^B = \beta(1 - 2G(\widetilde{x})) - \Delta$,we can also verify that the type $\widetilde{x}$ is also indifferent between the bundle and $B1$ only, as we assumed.

$$P - \widetilde{p}_1^B = \widetilde{x}$$

[18] If we adopt a different coordination assumption that is more favorable towards the tying firm (such as CPNE or coordination on the Pareto-superior outcome), the profit from tying is higher and the condition in Assumption 3 can be further relaxed.

[19] We can easily verify that at the bundle price derived under condition (11) the highest type has a weakly dominant strategy to buy the bundle which start the process of iterated dominance.

To summarize, the following prices guarantee market foreclosure of $B2$ with market segmentation in which consumers whose types are higher than $\widetilde{x}$ purchase the bundle while those whose types are lower than $\widetilde{x}$ purchase $B1$ only.

$$P(\widetilde{x}) = \widetilde{x} + \beta(1 - 2G(\widetilde{x})) - \Delta, \qquad \widetilde{p}_1^B(\widetilde{x}) = \beta(1 - 2G(\widetilde{x})) - \Delta.$$

Hence,

$$\widetilde{\Pi}_1(\widetilde{x}) = \underbrace{(1 - G(\widetilde{x}))\left[P(\widetilde{x}) + \alpha\right]}_{\text{Profit from Bundle Sale}} + \underbrace{G(\widetilde{x})\widetilde{p}_1^B(\widetilde{x})}_{\text{Profit from Sale of B1}}$$

Given a mixed bundling strategy of $(P(\widetilde{x}), \widetilde{p}_1^B(\widetilde{x}))$, it is useful to think of a *fictitious* price of A because the consumers who purchase the bundle are effectively paying the following price for A.

$$\underbrace{\widetilde{p}_1^A(\widetilde{x})}_{\text{fictitious price of } A} = \widetilde{P}(\widetilde{x}) - \widetilde{p}_1^B(\widetilde{x}) = \alpha + \widetilde{x}$$

This exercise facilitates the comparison of profits under mixed bundling and independent pricing. With the apparatus of a fictitious price for A, we can rewrite the tying firm's profit (when the marginal type for the bundle purchase is $\widetilde{x}$) as follows. Note that product B can be sold as part of the bundle as well as a stand-alone product with all consumers purchasing it.

$$\widetilde{\Pi}_1(\widetilde{x}) = \underbrace{(1 - G(\widetilde{x}))\widetilde{p}_1^A(\widetilde{x})}_{\text{Profit from Sale of A}} + \underbrace{\widetilde{p}_1^B(\widetilde{x})}_{\text{Profit from Sale of B}} \tag{13}$$

$$= [(1 - G(\widetilde{x}))(\alpha + \widetilde{x})] + [\beta(1 - 2G(\widetilde{x})) - \Delta] \tag{14}$$

For instance, if firm 1 chooses $\widetilde{P}$ and $\widetilde{p}_1^B$ to implement $\widetilde{x} = x^*(= p^*)$, where x^* is the marginal type in market A under independent pricing, firm 1's profit with mixed bundling is

$$\widetilde{\Pi}_1(x^*) = \pi_1^{A*} + \widetilde{p}_1^B(x^*)$$

Therefore, tying that implements $\widetilde{x} = x^*$ is profitable if $\widetilde{p}_1^B(x^*) = [\beta(1 - 2G(x^*) - \Delta] >$

0. To gain insight on the sign of $\widetilde{p}_1^B(x^*)$, let us decompose $\widetilde{p}_1^B(x^*)$ as

$$\widetilde{p}_1^B(x^*) = \underbrace{\beta(1 - G(\widetilde{p}_1^B(x^*)))}_{\substack{\text{Leverage of Network Effects} \\ \text{from Consumers Purchasing the Bundle}}} - \underbrace{\left(\Delta + \beta G(\widetilde{p}_1^B(x^*))\right)}_{\substack{\text{Price Concession Needed} \\ \text{to Win in Market B}}} \qquad (15)$$

In competition against $B2$, the tying firm receives a strategic advantage in terms of the quasi-installed base effect from the consumers who purchase the bundle, which is represented by the first term in (15). However, to win against a superior product $B2$, the price needs to be reduced by Δ. To make the purchase of B1 to be a dominant strategy and guarantee market foreclosure of B2 under any coordination assumptions, a further concession of $\beta G(\widetilde{p}_1^B(x^*)$ is needed.[20] These two negative effects are captured by the second term in (15). The next lemma shows that the first term dominates the second one under Assumption 3, which makes mixed bundling profitable.

Lemma 4. *Under Assumptions 2 and 3,*

(i) Without tying, firm 1 chooses $x^* = p^* > 0$ and hence some consumers are excluded from the consumption of the monopoly product.

(ii) Firm 1 can strictly increase its profit with mixed bundling at the prices $\widetilde{P} = (\alpha + p^*) + \beta(1 - 2G(p^*)) - \Delta$ and $\widetilde{p}_1^B = \beta(1 - G(p^*)) - \Delta$ even if firm 2 chooses $\widetilde{p}_2^B = 0$.

Proof. See the Appendix. ■

More generally, the optimal choice of $\widetilde{x}^*$ is characterized by the following first order condition, which is obtained by maximizing $\widetilde{\Pi}_1(\widetilde{x})$:

$$\widetilde{x}^* = \left[\frac{(1 - G(\widetilde{x}^*))}{g(\widetilde{x}^*)} - \alpha\right] - 2\beta, \qquad (16)$$

[20] If we adopt an alternative coordination assumption such CPNE, market foreclosure can be achieved with a higher price of $\widetilde{p}_1^B(x^*)$ and mixed bundling can be more profitable.

where the terms in the square bracket appear under independent pricing. The term -2β is new in comparison to independent pricing and its presence induces firm 1 to expand the market coverage of A relative to independent pricing: $\widetilde{x}^* < x^*$. This is because a larger market coverage in market A through bundling can be leveraged to market B with network effects. The larger market share enhances the size of the quasi-installed base for $B1$ at the expense of potential network size for $B2$.

From (16) we can conclude that the equilibrium is interior in the sense that not all consumers buy the bundle (i.e. $\widetilde{x}^* > 0$) if and only if $\alpha + 2\beta < \frac{1}{g(0)}$. Otherwise, we have a tipping equilibrium ($\widetilde{x}^* = 0$) toward the bundle with $\widetilde{P}^* = \alpha + P^* = \alpha + (\beta - \Delta)$. In both cases, tying expands market A because $\widetilde{x}^* < x^*$ under our assumptions.

Proposition 3. *Suppose Assumptions 2 and 3 hold. Then there is a unique equilibrium which involves mixed bundling and a zero market share for firm 2.*[21]

(i) If $\alpha + 2\beta \geq \frac{1}{g(0)}$, the equilibrium involves tipping toward the bundle at the equilibrium prices given by

$$\widetilde{P}^* = \alpha + (\beta - \Delta),\ \widetilde{p}_2^{B*} = 0.$$

(ii) Otherwise, only a measure of $1 - G(\widetilde{x}^*)$ consumers buy the bundle whereas the rest buy product B of firm 1 at the prices given by

$$\begin{aligned} \widetilde{P}^* &= \alpha + \widetilde{x}^* + \beta(1 - 2G(\widetilde{x}^*)) - \Delta = \frac{(1 - G(\widetilde{x}^*))}{g(\widetilde{x}^*)} - \beta(1 + 2G(\widetilde{x}^*)) - \Delta \\ \widetilde{p}_1^{B*} &= \beta(1 - 2G(\widetilde{x}^*)) - \Delta,\ \widetilde{p}_2^{B*} = 0. \end{aligned}$$

$\widetilde{x}^*$ satisfies (16) and the bundling expands the monopolized market as $\widetilde{x}^* < x^*$.

(iii) Mixed bundling is always profitable.

It is worth discussing the mechanism and intuition behind our results. In our model, mixed bundling enables firm 1 to screen consumers with more price instru-

[21] Althoguh here we explicitly analyze only pricing options for firm 1 that involve sale of the bundle and product $B1$, it can be shown that firm 1 has no profitable deviation including to strategies that also offer separate sales of product A.

ments while still maintaining the ability to leverage surplus of inframarginal consumers for its monopoly product to the competing product: as in the case of pure bundling, it is as if firm 1 already had an installed base advantage in competition for product B, which ensures firm 1's market dominance and enables it to expropriate the resulting network benefits for consumers in product B. By contrast, under independent pricing with tipping, the equilibrium market prices are independent of network effects and all benefits from network effects are competed away. We can contrast this result with the results in most models of strategic leverage theory including Whinston (1990). In these models, mixed bundling replicates the outcome under independent pricing by undoing the strategic effects of pure bundling. This is because pure bundling is *ex post* suboptimal due to its price-intensifying effects; it is optimal only ex ante when the tying firm is able to commit to pure bundling. Without such commitment, mixed bundling has no bite as a strategic instrument. In the presence of network effects, in contrast, mixed bundling retains the strategic value as a leverage mechanism even without any commitment assumption as it is ex post optimal.

We now investigate welfare implications of mixed bundling in our model. In the case where market A is covered under independent pricing (i.e., $x^* = 0$), bundling is profitable, but always welfare-reducing. When market A is not covered (i.e., $x^* > 0$), however, there is an opposing welfare effect of bundling: it expands the coverage of market A ($\widetilde{x}^* < x^*$). Welfare impacts thus can be ambiguous. More precisely, the loss in welfare in market B, which is equal to Δ, should be compared to the increase in welfare in market A:

$$\widetilde{SW}^* - SW^* = \underbrace{\int_{\widetilde{x}^*}^{x^*} (x+\alpha) g(x) dx}_{\text{Market Expansion Effect in A}} - \underbrace{\Delta}_{\text{Efficiency Loss in B}}$$

Proposition 4. *The welfare effect of the mixed bundling is ambiguous as the negative effect of efficiency loss by Δ in market B needs to be compared with the positive effect of expansion in market A.*

To explore further how the market expansion effect depends on key parameters of the model, consider first the case in which market A is not fully covered even with mixed bundling (i.e., $\widetilde{x}^* > 0$ with $\alpha + 2\beta < \frac{1}{g(0)}$). In this case, note that $\widetilde{x}^*$ is decreasing in β, whereas x^* is independent of β. This implies that the market expansion effect is positively related to β. In contrast, if market A is fully covered with mixed bundling (i.e., $\widetilde{x}^* = 0$), $\widetilde{x}^*$ is invariant in both α and β as long as $\alpha + 2\beta \geq \frac{1}{g(0)}$, whereas x^* is decreasing in α. As a result, the extent of market expansion, $x^* - \widetilde{x}^*$, decreases in α when market A is fully covered with mixed bundling. We illustrate these effects with a uniform distribution of x.

Example 1. *For the uniform distribution case with $\overline{u} = 1$, we can derive closed form expressions for welfare analysis. More specifically, Assumptions 2 and 3 can be written as $\Delta < \beta < \frac{1}{2}$ and $\frac{(\beta+\Delta)}{2\beta} < \alpha < 1$, respectively. Under independent pricing we have $x^* = (1-\alpha)/2$ and*

$$SW = \underbrace{\frac{3}{8}(\alpha+1)^2}_{SW_A} + \underbrace{v_2 + \beta}_{SW_B}$$

With bundling, we consider two cases.

(i) If $\alpha + 2\beta < 1$, only some consumers buy the bundle and market A is not fully covered with $\widetilde{x}^* = \frac{1-\alpha-2\beta}{2} (< x^* = \frac{1-\alpha}{2})$.

$$\widetilde{SW} = \underbrace{\frac{(3+3\alpha-2\beta)(1+\alpha+2\beta)}{8}}_{\widetilde{SW}_A} + \underbrace{v_1 + \beta}_{\widetilde{SW}_B}$$

(ii) If $\alpha + 2\beta \geq 1$, all consumers buy the bundle and market A is fully covered (i.e., $\widetilde{x}^* = 0$).

$$\widetilde{SW} = \underbrace{\alpha + \frac{1}{2}}_{\widetilde{SW}_A} + \underbrace{v_1 + \beta}_{\widetilde{SW}_B}$$

Taken together, we have

$$\widetilde{SW} - SW = \underbrace{\begin{cases} \frac{\beta(1+\alpha-\beta)}{2} & \text{if } \alpha + 2\beta < 1 \\ \frac{(1+3\alpha)(1-\alpha)}{8} & \text{if } \alpha + 2\beta \geq 1 \end{cases}}_{\textit{Market Expansion Effect in A}} - \underbrace{\Delta}_{\textit{Efficiency Loss in B}}$$

We can easily confirm that the positive market expansion effect is increasing in β when market A is not covered with mixed bundling and decreases in α when market A is covered with mixed bundling under Assumptions 2 and 3.

We now investigate the effects of mixed bundling on consumer welfare. Consumer welfare under independent pricing can be written as the sum of consumer surplus in market A and market B.

$$CS = \underbrace{\int_{x^*}^{\overline{u}} [1 - G(x)]dx}_{CS_A} + \underbrace{v_1 + \beta}_{CS_B = (v_2+\beta)-\Delta}$$

(i) If $\alpha + 2\beta \geq \frac{1}{g(0)}$, market A is covered and all consumers purchase the bundle at the price of $\widetilde{P}^* = \alpha + \beta - \Delta$. In this case, it is useful to think that consumers pay a (fictitious) prices of $\widetilde{p}_1^{A*} = \alpha$ for product A and $\widetilde{p}_1^{B*} = \beta - \Delta$ for product $B1$, with $\widetilde{P}^* = \widetilde{p}_1^{A*} + \widetilde{p}_1^{B*}$. Then,

$$\widetilde{CS}^* = \underbrace{\int_0^{\overline{u}} [1 - G(x)]dx}_{\widetilde{CS}_A} + \underbrace{v_1 + \Delta}_{\widetilde{CS}_B = (v_1+\beta)-(\beta-\Delta)}$$

We have

$$\widetilde{CS}^* - CS = \underbrace{\int_0^{x^*} [1 - G(x)]dx}_{\widetilde{CS}_A - CS_A > 0} + \underbrace{(\Delta - \beta)}_{\widetilde{CS}_B - CS_B < 0}$$

Tying increases consumer surplus in market A by expanding the market size with a lower (fictitious) price, but decreases consumer surplus in market B. The overall effect depends on the relative magnitude of these two opposing effects. Consumers are more likely to suffer from tying if α is higher because it will reduce the positive market expansion effect in market A. A higher β also makes tying less favorable for consumers. With independent pricing, the network-augmented utility term (represented by β) is competed away and passed onto consumers, but is expropriated by the tying firm with mixed bundling. In contrast, an increase in Δ directly increases consumer surplus under mixed bundling. The reason is that Δ (quality advantage of $B2$ over $B1$) is captured as a profit by firm 2 under independent pricing. However, under tying, as firm 2 charges zero price, Δ is fully compensated by the tying firm to induce consumers to purchase the inferior product $B1$ as part of the bundle. Thus, the effects of Δ on social welfare and consumer surplus are opposite.

(ii) If $\alpha + 2\beta < \frac{1}{g(0)}$, only a measure $1 - G(\widetilde{x}^*)$ of consumers buy the bundle (at the price of $P^* = \widetilde{x}^* + \beta(1 - 2G(\widetilde{x}^*)) - \Delta$), whereas the rest buying product B of firm 1 (at the price of $\widetilde{p}_1^{B*} = \beta(1 - 2G(\widetilde{x}^*)) - \Delta$). In this case, we can define a fictitious price of A by firm 1 as $\widetilde{p}_1^{A*} = \widetilde{P}^* - \widetilde{p}_1^{B*}$. In other words, we treat consumers who purchase the bundle at the price of $\widetilde{P}$ as if they pay an effective price of $\widetilde{p}_1^A$ and $\widetilde{p}_1^B$, respectively, for products A and $B1$. With a change of variables, $\widetilde{p}^* = \widetilde{p}_1^{A*} - \alpha$, we have $\widetilde{p}^* = P^* - \widetilde{p}_1^{B*} = \widetilde{x}^*$. Then, we can decompose the total consumer surplus into (fictitious) consumer surplus in market A and consumer surplus in market B.

$$\widetilde{CS}^* = \underbrace{\left[\int_{\widetilde{x}^*}^{\overline{u}} [1 - G(x)]dx\right]}_{\widetilde{CS}_A} + \underbrace{\left[v_1 + \beta - \widetilde{p}_1^{B*}\right]}_{\widetilde{CS}_B}$$

Thus, we have

$$\widetilde{CS}^* - CS = \underbrace{\int_{\widetilde{x}^*}^{x^*} [1 - G(x)]dx}_{\widetilde{CS}_A - CS_A > 0} + \underbrace{[\Delta - \beta(1 - 2G(\widetilde{x}^*)]}_{\widetilde{CS}_B - CS_B < 0}$$

Once again, the effects of tying on total consumer surplus depend on the relative magnitudes of two opposite effects in markets A and B.

Example 2. *For the uniform distribution case with $\overline{u} = 1$,*

$$CS = \underbrace{\frac{(\alpha+1)^2}{8}}_{CS_A} + \underbrace{v_1 + \beta}_{CS_B}$$

(i) If $\alpha + 2\beta \geq 1$,

$$\widetilde{CS}^* = \frac{1}{2} + v_1 + \Delta$$

In this case, we have

$$\widetilde{CS} - CS = \underbrace{\left[\frac{1}{2} - \frac{(\alpha+1)^2}{8}\right]}_{\widetilde{CS}_A - CS_A > 0} + \underbrace{[\Delta - \beta]}_{\widetilde{CS}_B - CS_B < 0}$$

(ii) If $\alpha + 2\beta < 1$,

$$\widetilde{CS}^* = \underbrace{\frac{(1+\alpha+2\beta)^2}{8}}_{\widetilde{CS}_A} + \underbrace{[v_1 + \beta(1-\alpha-2\beta) + \Delta]}_{\widetilde{CS}_B}$$

We thus have

$$\begin{aligned} \widetilde{CS}^* - CS &= \underbrace{\frac{\beta(1+\alpha+\beta)}{2}}_{\widetilde{CS}_A - CS_A > 0} + \underbrace{[\Delta - \beta(\alpha+2\beta)]}_{\widetilde{CS}_B - CS_B < 0} \\ &= \frac{\beta(1-\alpha-3\beta)}{2} + \Delta, \end{aligned}$$

In both cases, $\left(\widetilde{CS}^* - CS\right)$ *is decreasing in* α *and increasing in* Δ*. However, the impact of* β *on the changes in consumer surplus can be ambiguous because an increase in* β *can expand the coverage of market A more when it is not fully covered even under mixed bundling.*

In summary, an increase in α induces firm 1 to serve more consumers in market A and leaves more consumer surplus to inframarginal consumers under independent pricing. This creates more incentives for firm 1 to engage in tying to extend its monopoly power to adjacent markets with network effects. However, social welfare and consumer surplus move in the opposite direction because the room for positive market expansion gets exhausted with an increase in α.

6 Complementary Products

In the baseline model, we have analyzed independent products. In this section, we consider complementary products. In line with the Chicago school logic, we first show that tying is not a profitable strategy as a leverage mechanism to suppress competition in complementary product markets. However, if we consider inferior and competitively supplied alternatives to the tying product as in Whinston (1990), we can reestablish the possibility of anticompetitive, but profitable, tying for complementary products. One major difference from Whinston (1990) is, once again, we do not rely on the commitment assumption and subsequent exit of the rival firm in the tied product market. The robustness and applicability of our model to the complementary products case is important because most antitrust cases in tying for markets with network effects entail complementary products.

6.1 The Basic Model

We consider a setting that parallels the baseline model except that products are now complementary. For the purpose of exposition, consider product A as the primary product whereas B is an add-on product, that is, for the use of product B, product A is necessary; without A, product B is of no use. For instance, product A can be considered as an operating system whereas B is application software. App stores (A) that are required to download apps (B) can be another example.

When products are sold independently, consumers can use one of the two system products, $(A, B1)$ and $(A, B2)$, depending on which firm's product B is used. To simplify the analysis, let us assume that consumers' valuations for the combined products $A{-}B1$ and $A{-}B2$ are respectively given by $u+(v_1+\beta N_1)$ and $u+(v_2+\beta N_2)$, where $u = (\alpha+x) \in [\alpha, \alpha+\overline{u}]$ with x distributed on $[0, \overline{u}]$ according to a c.d.f. G and a strictly positive density g, and $\Delta \equiv v_2 - v_1 > 0$ as in the independent products case. We first show that for the complementary products case, firm 1 has no incentive to tie for the purpose of eliminating competition in product market B. We maintain the same parametric assumptions (i.e., Assumptions 1 and 2) made in the independent products case.

In the absence of the competitor $B2$, firm 1's optimal price would be any combinations of p_1^A and p_1^B such that $p_1^A + p_1^B = \alpha + v_1 + \beta$ and $p_1^A \geq \alpha$ under Assumption 1.[22] Firm 1's profit in the absence of any competitors is $\alpha + v_1 + \beta$. With complementary products, tying automatically forecloses firm 2 because $B2$ has no value as a stand-alone product. Therefore, firm 1 can behave as if it is a monopolist in both markets under tying. The tying profit is the same as the one without any competitors, i.e., $\widetilde{\Pi}_1 = \alpha + v_1 + \beta$.

Now consider the effects of firm $B2$'s presence on firm 1's profit under independent pricing. In the presence of a more efficient rival firm in market B, firm 1 can actually charge a price below its cost for product $B1$ with a corresponding price increase for the complementary product A, which leaves the total price for $A - B1$ combination constant at $\alpha + v_1 + \beta$. Given this pricing strategy, firm 2's optimal response is to reduce its own price below Δ to be able to sell to consumers. This leads to an outcome in which firm 1 can extract the more efficient rival firm's efficiency benefits through a higher price of the monopolized complementary product.

In fact, there is a continuum of Nash equilibria due to firm 1's ability to "price squeeze" and extract a portion of the surplus Δ (Choi and Stefanadis, 2001). More precisely, there is a continuum of equilibria parameterized by $\lambda \in [0,1]$, which represents the degree of price squeeze exercised by firm 1:

$$\widehat{p}_1^A = \alpha + v_1 + \beta + \lambda\Delta,\ \widehat{p}_1^B = -\lambda\Delta,\ \widehat{p}_2^B = (1-\lambda)\Delta$$

Without tying, firm 1's profit is given by $\Pi_1 = \alpha + v_1 + \beta + \lambda\Delta \geq \widetilde{\Pi}_1$; all equilibria under independent pricing yields a higher profit than that under tying unless $\lambda = 0$ (in which case the profits are the same), establishing the Chicago school argument.

[22] For this result, we actually need a less stringent assumption, which is $\alpha + v_1 + \beta > 1/g(0)$.

Proposition 5. *Consider the case of perfect complements where firm 1 is a monopolist in market A. There is no incentives for tying as a mechanism to monopolize market B.*

6.2 An Inferior Alternative Product in the Tying Market

Pure monopoly with absolutely no competitive products is rare. We thus consider a more realistic case where there is an inferior alternative product that is competitively supplied at the marginal cost of zero.[23] We now call firm 1's product in the tying market $A1$ while the alternative product is called $A2$. To maintain mathematical isomorphism between the complementary and independent product cases, we normalize consumers' valuations for the combined products that include this alternative $A2-B1$ and $A2-B2$ to $(v_1+\beta N_1)$ and $(v_2+\beta N_2)$, respectively.[24]

6.2.1 No tying

Consider the following price configurations in which firm 1 charges $p_1^A = \alpha$ and $p_1^B = 0$ for its two component products whereas firm B2 charges $p_2^B = \Delta$. Given these prices, all consumers purchase $A1$ and $B2$ and firm 1's profit is given by α. We now demonstrate that in the presence of an alternative product in market A, the strategy of "price squeeze" is not profitable for firm 1. To see this, consider the best scenario of perfect price squeeze for firm 1 with $p_1^B = -\Delta$ and $p_2^B = 0$ and firm 1 charges p_1^A. Consumers of type x chooses $A1$ over $A2$ if

$$\alpha + x - p_1^A \geq 0$$

With a change of variables $p = p_1^A - \alpha$, the market share for firm 1 in market A

[23] The assumption of competitvely supplied alternative is for simplicity. It can be supplied by a firm with market power.

[24] We can allow a more general utility specificaton by assuming that consumers' valuations for the combined products $A2-Bj$ is given by $u'+(v_j+\beta N_j)$ for $j=1,2$ with $u'<u$. For instance, we can assume that $u' = \alpha' + (1-\theta)x$ with $\alpha' < \alpha$ and $1 > \theta \geq 0$ without qualitatively changing any results, where $(\alpha - \alpha')$ and θ represent the degree of quality inferiority for the alternative product.

is given by

$$s_1^A = \begin{cases} 0 \text{ if } p > \overline{u} \\ 1 - G(p) \text{ if } 0 \leq p \leq \overline{u} \\ 1 \text{ if } p \leq 0 \end{cases}$$

It can be easily verified that the optimal price under Assumption 1 is that $p_1^A = \alpha$ with a profit of α. Even if firm 1 succeeds in price squeeze in market B, it is unable to raise the price of its complementary product in the presence of an inferior alternative; only consumers reap the benefit of the price squeeze. In fact, the strategy of price squeeze is a weakly dominated strategy for firm 1 because in the (off-the-equilibrium) event of selling its own component $B1$, its overall profit is reduced; it is selling $B1$ at a loss without any offsetting benefits from market A. The idea of profitable "price squeeze" for complementary products case relies on the monopoly producer's ability to increase the price of its monopolized product when the complementary product price is squeezed. However, this ability is constrained in the presence of an alternative product A.

Proposition 6. *Suppose Assumption 1 holds. In the presence of an inferior alternative, the market equilibrium for the complementary products case is identical to the one for the independent products case.*

(i) In market A, firm 1 charges $p_1^{A*} = \alpha$ and receives a profit of $\pi_1^{A*} = \alpha$.

(ii) In market B, firm 1 charges $p_1^{B*} = 0$ and firm 2 charges $p_2^{B*} = \Delta$; firm 2's profit is $\pi_2^{B*} = \Delta$.

Our analysis indicates that the analysis of complementary products case closely parallels with that of independent products, with the inferior alternative in the tying good market playing the role of no purchase option in the independent products case. We next show that the same logic applies in the tying case.

6.3 Tying

In the presence of tying, let $\widetilde{P}$ be the price of the bundle of firm 1 and let $\widetilde{p}_2^B$ be the price of firm 2's product $B2$. Note that $A2$ is provided competitively at the price of zero. In the complementary products case, we can interpret tying as firm 1's decision to make its product $A1$ incompatible with product $B2$. With a change of variables, define $P = \widetilde{P} - \alpha$. Given a pair of prices $\left(P, \widetilde{p}_2^B\right)$, any equilibrium in consumers' choices has the cut-off property as the independent products case. With this formulation, we can immediately see that in the presence of an alternative inferior product in the tying market, the case of complementary products is isomorphic to the case of independent products. There is a unique equilibrium, which involves tipping toward the bundle. We thus can derive the following proposition that parallels an earlier result for the independent products case.[25]

Proposition 7. *Suppose assumptions 1 - 2 hold. Under tying with complementary products, we have a unique equilibrium which involves tipping toward the bundle.*

(i) The equilibrium prices are given by

$$\widetilde{P}^* = \alpha + P^* = \alpha + (\beta - \Delta),\ \widetilde{p}_2^{B*} = 0.$$

(ii) Tying is profitable:

$$\widetilde{\Pi}_1^* = \alpha + (\beta - \Delta) > \alpha = \pi_1^{A*}$$

(iii)Both consumer surplus and social welfare decrease:

$$\begin{aligned} \widetilde{CS}^* &= CS^* - (\beta - \Delta) < CS^* \\ \widetilde{SW}^* &= SW^* - \Delta < SW^* \end{aligned}$$

where CS^* (SW^*) is consumer surplus (welfare) without tying and $\widetilde{CS}^*$ ($\widetilde{SW}^*$) is consumer surplus (welfare) under tying.

[25] All proofs follow verbatim from those in the independent products case. Even if we use a more general utility specification for bundles that include an inferior alternative, we require only minor modifications in the proofs.

7 Applications and Conclusion

In this paper, we have developed a leverage theory of tying in markets with network effects. We first analyze incentives to tie for independent products. When a monopolist in one market cannot fully extract the whole surplus from consumers, tying can be a mechanism through which unexploited consumer surpluses in one market are used as a demand-side leverage to create a strategic "quasi installed-base" advantage in another market characterized by network effects. Our mechanism does not require the commitment assumption with technological tying. Tying can lead to the exclusion of more efficient rival firms in the tied market, but can also in some cases expand the tying good market if the latter market is not fully covered with independent pricing. We also extend our analysis to the complementary products case. By allowing the existence of inferior alternatives as in Whinston (1990), we show that the setup of complementary products is mathematically identical to that of independent products. We also discuss welfare implications of tying.

Our analysis can be used to develop a theory of harm for tying cases when network effects are critical in the determination of the market winner. For instance, our model can shed light on the recent antitrust investigation concerning Google's practices in its MADA (Mobile Application Distribution Agreement) contracts. In particular, the EC decision has concluded that Google has engaged in illegal tying by requiring Android OEM "manufacturers to pre-install the Google search app ..., as a condition for licensing Google's app store (the Play Store)."[26] Google's Play Store can be considered the tying product as a "must-have" app, with other third party app stores being inferior alternatives. In the search market, Google faces competition from other search engines and there are network effects, in particular, stemming from the fact that search results quality increases with the scale of queries received by a search engine.[27] One might wonder why Google's tie would be any different

[26] See the European Commission Press Release "Antitrust: Commission fines Google €4.34 billion for illegal practices regarding Android mobile devices to strengthen dominance of Google's search engine," released on July 18, 2018. Available at http://europa.eu/rapid/press-release_IP-18-4581_en.htm.

[27] See Schäfer and Sapi (2020) for empirical evidence of network effects in Internet search.

from Google simply paying for preinstallation of Google's search app. Our model suggests that bundling may be an optimal way for Google to lock in part of the search market, reducing the quality of rivals.[28]

The model may also be applicable to the Microsoft case in Europe (IP/04/382) in 2004. The European Commission held Microsoft guilty of an abuse of dominant position by "tying its Windows Media Player (WMP), a product where it faced competition, with its ubiquitous Windows operating system."[29] Microsoft had a near monopoly position in the PC operating system market with over 90 percent of market share. We can consider Linux as an inferior alternative to Microsoft's Windows OS in the tying market. With respect to the WMP case, the media player market can be considered as the tied market in which Microsoft faced competition (from firms such as RealPlayer) and network effects are critical. More precisely, the media player market can be considered a two-sided market with indirect network effects. If more content is provided in the format of a particular company, then more consumers will use the company's Media Player to access such content. Moreover, if more consumers select a particular company's Media Player, then content providers will obviously have an incentive to make their content available in the format of the company.[30] Our model assumes direct network effects in the tied market, but can be considered capturing such feedback effects of two-sided markets in a reduced form.[31]

We developed our model in the context of one-sided markets. However, as shown in the Appendix, our model is mathematically equivalent to the one with two-sided

[28] One limitation in application of our model to this Google example, hopwever, is that the "buyers" are distributors (phone OEMs/carriers) not consumers. Thus, while the search "purchase" can be interpreted as determining which search engine gets to be the preinstalled default on the distributor's device, distributors are not single-unit consumers and more complicated pricing than linear pricing may be possible.

[29] The case also involved Microsoft's conduct of "deliberately restricing interoperability between Windows PCs and non-Microsoft work group servers." https://ec.europa.eu/commission/presscorner/detail/en/IP_04_382

[30] The Korean Fair Trade Commission also fined Microsoft 33 billion won (US$32 million) for abusing its market dominant position by bundling Windows OS with its instant messaging (IM) program as well as WMP. For the messenger case, the tied market market is characterized by direct network effects as in our model.

[31] See Choi and Jeon (2021) for an analysis of tying that explicitly accounts for indirect network effects in two-sided markets.

tying market with advertising revenues (with a reinterpretation of α as per-consumer advertising revenue). This may have important implications for recent antitrust debates on two-sided digital platforms. We showed that welfare impacts of tying depend on the relative magnitudes of positive market expansion effects and negative market foreclosure effects of more efficient firms. When advertising revenue is important (i.e., α is high) and services are already provided for free, as is common for many digital platforms, our model indicates that there are more incentives to engage in tying to leverage unexploited consumer surplus. In that case, however, there are no more socially beneficial market expansion effects. Therefore, the effects of tying are more likely to be anticompetitive in such a case. In addition, the negative effects on consumer surplus will be more pronounced as network effects in the tied market become more important. This implies that more scrutiny may be warranted when ad-financed digital platforms engage in tying with other products or services characterized by network effects.

References

[1] Adams, Walter, J. and Janet L. Yellen, "Commodity Bundling and the Burden of Monopoly," *Quarterly Journal of Economics*, 1976, 90 (Aug.), pp. 475–498.

[2] Bernheim, B. Douglas, Bezalel Peleg, and Michael Whinston, "Coalition-Proof Nash Equilibria I. Concepts," *Journal of Economic Theory*, 1987, pp. 1-12.

[3] Bork, Robert, "Vertical Integration and the Sherman Act: The Legal History of an Economic Misconception," *University of Chicago Law Review*, 1954, 22 (1), pp. 157-201

[4] Bowman, Ward S. Jr., "Tying Arrangements and the Leverage Problem," *Yale Law Journal*, 1957, 67, pp. 19-36.

[5] Burstein, Meyer L., "The Economics of Tie-in Sales," *Review of Economics and Statistics*, 1960, 42, pp. 68-73.

[6] Carbajo, Jose, David De Meza, and Daniel J. Seidman, "A Strategic Motivation for Commodity Bundling," *Journal of Industrial Economics*, March 1990, *38*, pp. 283-298.

[7] Carlton, Dennis, W. and Michael Waldman, "The Strategic Use of Tying to Preserve and Create Market Power in Evolving Industries," *Rand Journal of Economics*, Summer 2002, pp. 194-220.

[8] Carlton, Dennis, W. and Michael Waldman, "Upgrades, Switching Costs and the Leverage Theory of Tying," *Economic Journal*, June 2012, pp. 675-706.

[9] Calzolari, Giacomo and Vincenzo Denicolò, "Exclusive Contracts and Market Dominance," *American Economic Review*, 2015, 105 (11), pp. 3321-3351.

[10] Choi, Jay Pil and Doh-Shin Jeon, ""A Leverage Theory of Tying in Two-Sided Markets with Non-Negative Price Constraints," *American Economic Journal: Microeconomics*, 2021, pp. 283-337.

[11] Choi, Jay Pil and Christodoulos Stefanadis, "Tying, Investment and the Dynamic Leverage Theory," *Rand Journal of Economics*, Spring 2001, pp. 52-71.

[12] Director, Aaron and Edward H. Levi, "Law and the Future: Trade Regulation," Northwestern University Law Review, 1956, 51, pp. 281-296

[13] Fumagalli, Chiara, Massimo Motta, and Claudio Calcagno, *Exclusionary Practices: The Economics of Monopolisation and Abuse of Dominance*, Cambridge University Press, 2018.

[14] Greenlee, Patrick, David Reitman, and David S. Sibley, "An Antitrust Analysis of Bundled Loyalty Discounts," *International Journal of Industrial Organization*, 2008, pp. 1132-1152.

[15] Katz, Michael and Carl Shapiro, "Technology Adoption in the Presence of Network Externalities," *Journal of Political Economy*, 1986, *94*, pp.822-841.

[16] Morris, Stephen and Hyun Song Shin, "Global Games: Theory and Applications," in *Advances in Economics and Econometrics: Theory and Applications, Eighth World Congress*, edited by Mathias Dewatripont, Lars Peter Hansen and Stephen J. Turnovsky, Cambridge University Press, 2010, pp. 56 - 114.

[17] Nalebuff, Barry, "Bundling as an Entry Barrier," *Quarterly Journal of Economics*, February., 2004, pp. 159-187.

[18] Posner, Richard A., *Antitrust Law: An Economic Perspective*. Chicago: University of Chicago Press, 1976.

[19] Schaefer, Maximilian and Geza Sapi, "Learning from Data and Network Effects: The Example of Internet Search," DIW Berlin Discussion Paper No. 1894, September 2020.

[20] Stigler, George, J., "United States v. Loew's Inc.: A Note on Block-Booking," *The Supreme Court Review*, 1963, pp. 152-157.

[21] Whinston, Michael D., 'Tying, Foreclosure and Exclusion,' *American Economic Review*, 1990, *80*, pp. 837–859.

Appendix

An Equivalent Model with Two-Sided Tying Market

We show that our model is isomorphic to the one with two-sided tying market with advertising revenues. More precisely, consider market A where consumers' valuations (u) are distributed on $[\alpha, \alpha+1]$ according to G. In the absence of tying, the two markets can be analyzed independently as we assume independent products. Let us use a change of variable such that $\widehat{u} = u - \alpha$, and $\widehat{p}_1^A = p_1^A - \alpha$ with $\widehat{G}(u-\alpha) = G(u)$. In market A, firm 1 solves the following problem:

$$\max_{\widehat{p}_1^A} \left[\widehat{p}_1^A + \alpha\right] (1 - \widehat{G}(\widehat{p}_1^A))$$

With this formulation, we have mathematically the same problem as the advertising model we analyzed. For instance, the condition that firm 1 serves all consumers with the price of $\widehat{p}_1^A = 0$ (i.e., $p_1^A = \alpha$) is given by

$$\alpha \geq \frac{1 - \widehat{G}(0)}{\widehat{g}(0)} = \frac{1}{\widehat{g}(0)}.$$

The rest of the analysis is the same with a change of variables.

Proof of Lemma 4

We need to show that $\beta(1 - 2G(p^*) > \Delta$, where p^* satisfies condition (10). The required condition is equivalent to

$$G(p^*) < \frac{1}{2}\left(1 - \frac{\Delta}{\beta}\right) = \frac{\beta - \Delta}{2\beta}.$$

As G is an increasing function, this condition is equivalent to

$$p^* = \frac{1 - G(p^*)}{g(p^*)} - \alpha < G^{-1}\left(\frac{\beta - \Delta}{2\beta}\right)$$

This inequality is always satisfied under Assumption 3 because $\frac{1-G(p^*)}{g(p^*)} < \frac{1-G(0)}{g(0)} = \frac{1}{g(0)}$ due to the monotone hazard rate assumption.

Proof of $P = \beta - \Delta$ as the Global Maximizer

If $\widetilde{p}_2^B = 0$, firm 1's FOC is

$$\begin{aligned}
\left.\frac{\partial \widetilde{\Pi}_1}{\partial \widetilde{x}}\right|_{\widetilde{p}_2^B=0} &= \phi'(\widetilde{x})(1-G(\widetilde{x})) - (\phi(\widetilde{x})+\alpha)\, g(\widetilde{x}) \\
&= (1-2\beta g(\widetilde{x}))(1-G(\widetilde{x})) - [(\widetilde{x}+\beta(1-2G(\widetilde{x}))-\Delta+\alpha]g(\widetilde{x}) \\
&= [\underbrace{(1-G(\widetilde{x}))-(\widetilde{x}+\alpha)g(\widetilde{x})}_{(-)\text{ at all }\widetilde{x}>0}] - 2\beta g(\widetilde{x})(1-G(\widetilde{x})) - [\beta(1-2G(\widetilde{x}))-\Delta]g(\widetilde{x}) \qquad \text{(A1)}
\end{aligned}$$

Suppose that there is a profitable deviation against $\widetilde{p}_2^B = 0$ to some $\widetilde{x} > 0$. By (A1) the best such deviation must have

$$\beta(1-2G(\widetilde{x})) - \Delta < 0. \qquad \text{(A2)}$$

Note that it requires the following condition to implement $\widetilde{x}$ when $\widetilde{p}_2^B = 0$:

$$P = \phi(\widetilde{x}) = \widetilde{x} + \underbrace{\beta(1-2G(\widetilde{x}))-\Delta}_{(-)}$$

So by (A2), we have

$$P < \widetilde{x}$$

This implies that firm 1 could make more profit by deviating to independent pricing at price $p^A = \widetilde{x} + \alpha$ since it would sell the same number of units at a higher price. But the best independent pricing yields profits of α, which is a lower profit than $\widetilde{x}$ with tying. We thus have a contradiction.