Tax Competition with Heterogeneous Capital Mobility

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March 16, 2014

*We are grateful to IEB for its financial support. We would like to thank May Elsayyad, Leonzio Rizzo, and Tanguy van Ypersele, as well as participants at the 2011 Journée Louis André Gérard Varet, the IEB IV Workshop on Fiscal Federalism, the GREQAM seminar, the 2013 Southern Economic Association Meetings, and a seminar at Florida International University, for useful comments on an earlier version of this paper. The usual disclaimer applies.
1 Introduction

A controversial issue in the study of tax competition is whether it is desirable for countries or regions to agree not to provide preferential treatment to different forms of capital. The common view is that without such restrictions, countries will aggressively compete for capital that is relatively mobile across different locations, resulting in taxes that are far below their efficient level. By eliminating such preferential treatment, no capital will be taxed at very low rates, because doing so would sacrifice too much tax revenue from the relatively immobile capital. But this solution is not without cost: in an attempt to attract mobile capital, governments can be expected to reduce the common tax rate below the tax at which relatively immobile capital would be taxed in the preferential case. In an important paper, Keen (2001) analyzes this tradeoff using a model in which two identical regions compete over two tax bases that exhibit different degrees of mobility. He finds that governments raise more revenue when the more mobile tax base gets preferential treatment. On the other hand, Haupt and Peters (2005) introduce a preference for investing in the home country, referred to as "home bias," and show that non-preferential regimes lead to higher tax revenue. It is surprising that this seemingly minor assumption changes completely the desirability of one regime versus the other.

Attacking the issue from a different angle, Janeba and Peters (1999) show that the elimination of preferential treatment leads to higher total tax revenues in a context where one of the tax bases is infinitely elastic with respect to cross-country differences in tax rates, in contrast to the finite elasticity assumptions employed by Keen (2001) and Haupt and Peters (2005). The importance of this tax-base elasticity is also apparent in the subsequent papers that have generalized and extended the comparison between preferential and non-preferential regimes, including Wilson (2005), Konrad (2007), and Marceau, Mongrain and Wilson (2010).

Keen (2001) and Haupt and Peters (2005) cannot analyze the case where
one of the tax base elasticities approaches infinity, because pure-strategy equilibria do not exist under their assumption of identical regions. In contrast, Wilson (2005) and Marceau, Mongrain and Wilson (2010) address the existence problem by considering mixed-strategy equilibria, and obtain results supporting Janeba and Peters (1999). In particular, the non-preferential regime raises more revenue than the preferential regime. This result further demonstrates the importance of tax base elasticities, because both of these papers share the assumption that the mobile tax base is infinitely elastic, in contrast to Keen (2001). Janeba and Smart (2003) investigate a more general model than is typically found in the literature on tax-base discrimination, allowing them relate the comparison of the two regimes not only to how the tax bases respond to differences in tax rates across regions, but also how these tax bases respond to a uniform increase in both regions’ tax rates.

In the current paper, we further investigate the conditions under which limiting preferential treatment of particular tax bases is desirable. But we depart from much of the literature in regimes in three important ways, and we obtain new results that differ significantly from those in the literature. First, we replace the assumption that regions seek to maximize tax revenue, which is assumed in all of the papers reviewed above, with the more balanced view that regions also care about the "surplus" obtained in the private sector. Our second departure from the usual framework is shared by the Haupt-Peters paper: firms are distinguished by their region of origin, producing a "home bias effect." But we fill in the micro-foundations for this home bias effect by assuming that firms differ in their cost of relocating from one region to another. In doing so, we are able to demonstrate how the ranking of the two regimes depends critically on the distribution of moving costs. Finally, we not only rank the two regimes in cases where the two competing regions are identical, but we also devote considerable attention to cases where they differ in size.

We consider a two-region world in which each region initially possesses a stock of “domestic firms,” which must incur a cost to relocate to the other
region. The “foreign firms” that the region seeks to attract are the other region’s domestic firms. In the special case of uniform moving costs, we not only find that the non-preferential regime is preferred, but we are also able to quantify how much more tax revenue it raises. If we further specialize the model by assuming revenue-maximizing regions, this difference in revenues becomes very large. However, it declines when private surplus receives significant weight in the regional objective function.

Perhaps our most surprising finding involves the conditions under which the preferential regime is preferred, as in Keen (2001) but in contrast to Haupt and Peters (2005). The main surprise is that these conditions are not that tax bases are sufficiently inelastic with respect to interregional differences in tax rates – recall the message from the previous literature that the desirability of the preferential regime depended strongly on tax bases not being highly elastic – but that the tax bases are sufficiently elastic. This result is proved for the case of two identical regions. In this case, the preferential regime turns out to be preferable when there are a large number of firms with low moving costs, implying that these firms are highly responsive to small differences in tax rates between regions. In the non-preferential case, both regions set the same tax rates in the Nash equilibrium, so no firms move in equilibrium. However, each region has a large incentive to reduce its tax rate by a small amount, since it can then obtain the large number of firms with low moving costs; that is, the tax-base elasticity is high. This undercutting drives down the common equilibrium tax rate. In contrast, a significant number of firms move between regions in the equilibrium for preferential case, because each region has an incentive to set its rate on foreign firms discretely below the tax rate on its domestic firms, in an effort to induce some foreign firms to operate within its borders. Thus, the marginal firm is no longer a firm with small moving costs. Without a relatively large number of firms at the margin, there is less downward pressure on tax rates in the preferential case.

A crucial insight here is that the relevant responsiveness of firms to small changes in tax rates from their equilibrium levels can differ signifi-
cantly between the two tax regimes. If there are relatively many firms with low moving costs, then firm location is very sensitive to small tax changes around the symmetric equilibrium for the non-preferential regime. But this responsiveness is then relatively low at the margin in the preferential case, where each region sets different tax rates on domestic and foreign tax firms, implying that the marginal firm does not have a low moving cost.

From a policy perspective, these results call into question the view that preferential tax treatment of particular types of firms or capital should be limited as a result of the increasing integration of the world economy, given that this integration includes lots of firms with low moving costs.

We also investigate the effects of asymmetries in the sizes of regions, measured by their relative numbers of domestic firms, along with how the results depend on the relative weights given to tax revenue and private surplus in the welfare function. An important insight here is that the difference in equilibrium welfare levels for the two regimes disappears as one region becomes infinitesimally small relative to the other. This result suggests that the problems created by the existence of tiny tax havens cannot be solved by requiring the non-preferential treatment of different tax bases. In addition we uncover a conflict between regional preferences over the regimes, indicating an additional complication in attempts to control wasteful tax competition.

The plan of this paper is as follows. First, we describe the basic features of the model. We then analyze the properties of a non-preferential regime, followed by the properties of a preferential regime. Finally, we compare the two systems. A final section provides concluding remarks. All proof are in the appendix, except for the one which restates already shown results.

2 The Model

The economy contains two regions, indexed by \( i \in \{1, 2\} \), and a mass of firms of size \( 2N \). In each region, there are \( N_i \) domestic firms. Each firm in region \( i \) generates \( \gamma \geq 1 \) of before-tax profits for its owners. Profits are taxed where they are earned. All firms have the possibility of moving,
but face different moving costs $c$. Moving costs are distributed between zero and one according to a cumulative distribution function, $F(c)$, and the corresponding density distribution function, $f(c)$. Thus, any movement of firms from one region to another is based on tax considerations, and will therefore result in the expenditure of socially wasteful moving costs. One could also interpret those moving costs as location-specific productivity. A firm with a zero moving cost would be equally productive in both regions, while a firm in region $i$ with high moving cost would correspond to a firm with a high suitability toward region $i$.

Regions care about both public funds generated by tax revenue and private surplus. Define $W^i(R^i, \Pi^i)$ as the region $i$ objective function, where $R^i$ is total tax revenue and $\Pi^i$ is total private surplus generated by firms located in region $i$, given the tax policy. We assume that this objective function is the same across regions and displays constant marginal benefits in both arguments. Consequently, we can define $W(R^i, \Pi^i) = \omega R^i + \Pi^i$, where $\omega \geq 1$ is the marginal benefit of tax revenue, and the marginal benefit of the private surplus is normalized to equal one. Under the government’s optimal tax policy, $\omega$ will equal the marginal cost of government revenue, in units of numeraire private surplus. Using common terminology, $\omega$ then equals the marginal cost of public funds, and the excess of $\omega$ over one is the excess burden associated with raising revenue. Tax revenue maximization is a special case where $\omega$ goes to infinity, in which case private sector income receives no weight in the government’s objective function.

We will also consider regional size differences. Region 1 is initially endowed with at least as many firms as region 2, so $N_1 \geq N_2$. We define $n \in [1/2, 1]$ as an index of size heterogeneity between the two regions, where $N_1 = n2N$, and $N_2 = (1 - n)2N$. If both regions have the same size, then $n = 1/2$; heterogeneity grows as $n$ increases.

Some restrictions on the distribution of moving costs are desirable. More specifically, $f'(c)/f(c) \in [-1/\gamma, 1/\gamma]$ guarantees the existence and the unique-
ness of an equilibrium. This condition is sufficient, but not necessary, and simply excludes distribution functions with large peaks and valleys. As a source of examples, we will use the density function, \( f(c) = (1 - \beta) + 2\beta c \), where \( \beta \in [-1, 1] \). Note that \( f'(c) = 2\beta \), so \( \beta > 0 \) represents increasing density functions, while \( \beta < 0 \) represents decreasing density function. The uniform distribution function is represented by \( \beta = 0 \). The cumulative distribution function, \( F(c) = (1 - \beta)c + \beta c^2 \), is quadratic, with \( F(0) = 0 \) and \( F(1) = 1 \). Finally, existence and uniqueness of a pure strategy equilibrium is guaranteed if \( \beta \) rests between plus and minus \( \frac{1}{1+2\gamma} \).

The timing is as follows. First, regions choose their tax rates and all firms draw a moving cost. Then firms chose whether to move or remain in their initial location. Finally, production occurs and taxes are collected.

3 Non- Preferential Regime

Under a non-preferential regime, each region \( i \in \{1, 2\} \) sets a unique tax rate \( t_i \) for all firms, regardless of whether a firm is already in the region (domestic firms) or just moved to the region (foreign firms). For any given \( t_i \geq t_j \), a firm in region \( i \) stays in region \( i \) as long as \([1 - t_i] \gamma \geq [1 - t_j] \gamma - c\). Thus, only firms with \( c \geq (t_i - t_j) \gamma \) stay in region \( i \). If \( t_i < t_j \), firms in region \( j \) move to region \( i \) whenever \( c < (t_i - t_j) \gamma \). Total tax revenue in region \( i \) is denoted by \( R^i(t_i, t_j) \), and is given by:

\[
R^i(t_i, t_j) = \begin{cases} 
\gamma t_i N_i \left[ 1 - F \left((t_i - t_j) \gamma \right) \right] & \text{if } t_i \geq t_j; \\
\gamma t_i N_i + \gamma t_i N_j F \left((t_i - t_j) \gamma \right) & \text{if } t_i < t_j.
\end{cases}
\]

Total surplus \( \Pi^i(t_i, t_j) \) from domestic and foreign firms located in region \( i \) is given by

\[
\Pi^i(t_i, t_j) = \begin{cases} 
\gamma_i (1 - t_i) N_i \left[ 1 - F \left((t_i - t_j) \gamma \right) \right] & \text{if } t_i \geq t_j; \\
\gamma_i (1 - t_i) \left[N_i + N_j F \left((t_i - t_j) \gamma \right) \right] - N_j \int_{0}^{(t_j - t_i) \gamma} c f(c) dc & \text{if } t_i < t_j.
\end{cases}
\]

As stated before, we assume that governments care about private surplus generated by firms (domestic or foreign) who are in or just moved to the

\(^1\)See Lemma 1 and 2 for formal proofs
region for a given set of policies. This implies that the set of firms generating private surplus is taken as given by each governments. This assumption is well motivated in Gordon and Cullen (2012). If the government in region \( i \) maximizes \( W(R^i, \Pi^i) = \omega R^i + \Pi^i \) by choosing \( t_i \) for a given \( t_j \), the best-response function, \( t_i(t_j) \), is given by:

\[
\gamma t_i f((t_j - t_i) \gamma) = \begin{cases} 
\frac{\omega - 1}{\omega} [1 - F((t_i - t_j) \gamma)] & \text{if } t_i \geq t_j; \\
\frac{\omega - 1}{\omega} \left[N_i/N_j + F((t_j - t_i) \gamma)\right] & \text{if } t_i < t_j.
\end{cases}
\] (3)

The left-hand side of each equation represents the net marginal benefit (public minus private) associated with an increase in the tax rate for a given allocation of firms, while the right-hand side represents the marginal cost of losing firms as a result of the same increase in tax rate. If \( \omega = 1 \), then taxing firms is undesirable. It is helpful to interpret a government’s tax-setting rule in terms of the elasticity of its tax base \( B_i \) with respect to tax rate, where \( \epsilon_i = -\frac{t_i}{B_i} \frac{\partial B_i}{\partial t_i} \) is given by:

\[
\epsilon_i = \begin{cases} 
\frac{\gamma t_i f((t_i - t_j) \gamma)}{1 - F((t_i - t_j) \gamma)} & \text{if } t_i \geq t_j; \\
\frac{\gamma t_i f((t_j - t_i) \gamma)}{(N_i/N_j) + F((t_j - t_i) \gamma)} & \text{if } t_i < t_j.
\end{cases}
\] (4)

The equilibrium is characterized by tax base elasticity equal to \( \epsilon_i = \frac{\omega - 1}{\omega} \). Elasticities would equal one if governments where to pursue a revenue-maximizing tax policy. With some weight placed on private surplus, however, a region’s optimal tax rate is kept below this revenue-maximizing rate.

The following lemma provides a condition under which an equilibrium exists.

**Lemma 1:** Whenever \( f'(c)/f(c) \in [-1/\gamma, 1/\gamma] \) for both \( i \), the best-response functions are monotonically upward slopping, with a slope less than one. Then an equilibrium exists and is unique.

Note that the conditions in Lemma 1 are sufficient, but not necessary. The conditions ensure the existence and uniqueness for all possible tax rates up to 100\%, and for all positive values of the exogenous welfare weight, \( \omega \). We next examine the equilibrium tax rates when regions have the same size.
**Proposition 1:** Under a non-preferential regime, if regions are identical, there exists a unique Nash equilibrium where \( t_1 = t_2 = t^{np} = \frac{\omega - 1}{\omega} \frac{1}{f(0)} \).

**Corollary to Proposition 1:** Under a non-preferential regime, if regions are identical and if \( f(c) = (1 - \beta) + 2\beta c \), there exists a unique symmetric Nash equilibrium, where \( t^{np} = \frac{\omega - 1}{\omega} \frac{1}{\gamma(1-\beta)} \).

An increase in either \( \gamma \) or \( f(0) \) generates more elastic tax bases, and so leads to a reduction in tax rates. As \( \gamma \) increases, moving costs become smaller relative to the fiscal benefit of moving. Similarly, if \( f(0) \) is large, many firms are able to move at no cost. Note also that tax-base elasticities are the same, regardless of whether the tax base changes because new firms are coming in or existing firms are retained. This is an important departure from the rest of the literature which assumes different tax bases with different elasticities. As anticipated, taxes are high when the net marginal benefit of public spending (\( \omega \)) is high. Large marginal benefits for public spending means tax-base elasticity closer to one in equilibrium, as \( \epsilon_i = \frac{\omega - 1}{\omega} \). Figure 1 illustrates the case where \( F(c) \) is a uniform distribution (\( \beta = 0 \)), and the government gives positive weight only to tax revenue (\( \omega \rightarrow \infty \)).

![Figure 1](image_url)

Given the equilibrium tax rates defined above, total tax revenue in each region is given by \( R^i = \frac{\omega - 1}{\omega} \frac{N}{f(0)} \), and the corresponding private surpluses are given by \( \Pi^i = \left[ 1 - \frac{\omega - 1}{\omega} \frac{1}{\gamma f(0)} \right] \gamma N \). Note that with the specific distribution function, \( f(0) \) is simply equal to \( \frac{1}{\gamma(1-\beta)} \). Consequently, total welfare is given by:

\[
W^i(R^i, \Pi^i) = \gamma N + \frac{(\omega - 1)^2}{\omega} \frac{N}{f(0)}
\]

As we can see, total welfare is compose of two terms. The first term represents the maximal private surplus, achievable if both regions were to set taxes to zero. The second term represents the gain from public revenues. More specifically, we can re-write the second term as \( (\omega - 1)\gamma N t_i \epsilon_i = (\omega - 1)R^i \).
3.1 Heterogenous Regions

We close this section by discussing how size differences between regions affect the equilibrium. Whenever \( n > \frac{1}{2} \), the smallest region is more aggressive with its tax rate. This observation confirms similar results in the literature on tax competition, like in Bucovetsky (1991). In our context, however, small is defined as the region with the least number of domestic firms. The next proposition relates the equilibrium tax rates to differences in regional size.

**Proposition 2:** Under a non-preferential regime, if region 1 is larger than region 2, there exist a unique Nash equilibrium with \( t_{np}^1(n) > t_{np}^2(n) \), where

\[
\frac{t_{np}^1(n)}{t_{np}^2(n)} = \frac{1 - F(γ[t_1(n) - t_2(n)])}{\frac{1-n}{n} + F(γ[t_1(n) - t_2(n)])} > 1. \tag{6}
\]

This difference in tax rates can be explained by looking at the tax base elasticities for both regions. In equilibrium, both elasticities are equalized, so \( \epsilon_1 = \epsilon_2 \). Having fewer domestic firms gives the small region a strategic advantage, attracting new firms has a proportionally bigger impact on the small region’s tax revenue. Consequently, the two elasticities are equalized when the small region sets the lower tax rate. For the case of a uniform cost distribution, the best-response functions given by equation (3) can be used to state the equilibrium tax rates as follows:

**Corollary to Proposition 2:** Under a non-preferential regime, if \( n > 1/2 \) and moving costs are uniformly distributed, there exists a unique Nash equilibrium where:

\[
t_1(n) = \left[ \frac{ω - 1}{3ω - 2} \right] \left[ \frac{nω + (ω - 1)}{ωγn} \right]
\]

and

\[
t_2(n) = \left[ \frac{ω - 1}{3ω - 2} \right] \left[ \frac{(1-n)ω + (ω - 1)}{ωγn} \right].
\]

When governments simply maximizes tax revenue (\( ω = \infty \)), tax rates becomes \( t_1(n) = \frac{1+n}{3γn} \) and \( t_2(n) = \frac{2-n}{3γn} \). Figure 2 represents the asymmetric
Nash equilibrium when moving costs are uniformly distributed. The best-response functions are discontinuous at $t_1 = t_2$, and can only cross where $t_1 > t_2$.

Figure 2

Corollary to Proposition 2 helps us identify some important patterns. First, if governments place additional weight on tax revenue relative to private surplus (higher $\omega$), tax rates in both regions increase. We can also see that both regions set lower tax rates as heterogeneity increases. This negative relation between heterogeneity and tax rates can be generalized. More heterogeneity makes the smaller region more aggressive, and since tax rates are strategic complements, both regions set lower tax rates. Moreover, as heterogeneity increases, the difference in tax rates, as defined as $\Delta t_{12} = t_1 - t_2$, grows. This implies that more heterogeneity also leads to more movement of firms.

**Proposition 3:** More heterogeneity (higher $n$) leads to lower tax rates $t_1$ and $t_2$, and greater difference in tax rates, $\Delta t_{12}$.

We will now look at tax revenue and welfare with asymmetric regions. For any difference in tax rates, total tax revenues for both regions are given by:

$$R^1 = 2nN \frac{\omega - 1}{\omega} \left[ 1 - F \left( \frac{\omega \Delta t_{12}}{\omega} \right) \right]^2,$$

$$R^2 = \frac{2N}{n} \frac{\omega - 1}{\omega} \left[ (1 - n) + nF \left( \frac{\omega \Delta t_{12}}{\omega} \right) \right]^2.$$

With uniform moving cost, the difference in tax rates becomes $\Delta t_{12} = \frac{\omega^p - \omega^q}{\omega^p - \omega^q} \frac{2n-1}{\gamma n}$, and so the two regions’ tax revenues are given by:

$$R^1 = \frac{2N}{n} \left[ \frac{\omega - 1}{\omega} \right] \left[ n - (2n - 1) \frac{\omega - 1}{\omega - 2} \right]^2;$$

$$R^2 = \frac{2N}{n} \left[ \frac{\omega - 1}{\omega} \right] \left[ (1 - n) + (2n - 1) \frac{\omega - 1}{\omega - 2} \right]^2. \quad (7)$$

When governments maximizes tax revenue ($\omega = \infty$), the expressions above can further simplified to $R^1 = 2N \frac{(1+n)^2}{9n}$ and $R^2 = 2N \frac{(2-n)^2}{9n}$. Under this special case, tax revenues diminish in both regions with an increase in size.
heterogeneity. This is not necessarily true in general. For example, under
the uniform distribution, when $\omega$ is sufficiently close to one, the large re-
gion’s tax revenue increases when regions become more heterogenous. More
heterogeneity leads to lower tax rates for both region, but the large region
increases its tax base by construction.

4 Preferential Regime

Under a preferential tax regime, each region $i$ taxes its existing domestic
firms and newly-arrived foreign firms at different rates, $t_i$ for domestic firms
and $\tau_i$ for the foreign firms. When $t_i \geq \tau_j$, a firm in region $i$ will stay in
region $i$ if $[1 - t_i] \gamma \geq [1 - \tau_j] \gamma - c$, or $c > (t_i - \tau_j) \gamma$. On the other hand, if
$t_i \leq \tau_j$, all domestic firms stay. In addition, firms in region $j$ move to region
$i$ whenever $c < (t_j - \tau_i) \gamma$. Total tax revenue in region $i$, $R^i(t_i, \tau_i, t_j, \tau_j)$, is
given by:

$$
R^i(\cdot) = \begin{cases}
N_i[1 - F((t_i - \tau_j)\gamma)] \gamma t_i & \text{if } t_i > \tau_j \& \tau_i \geq t_j; \\
N_i[1 - F((t_i - \tau_j)\gamma)] \gamma t_i + N_j F((t_j - \tau_i)\gamma) \gamma \tau_i & \text{if } t_i > \tau_j \& \tau_i < t_j; \\
N_i \gamma t_i & \text{if } t_i \leq \tau_j \& \tau_i \geq t_j; \\
N_i \gamma t_i + N_j F((t_j - \tau_i)\gamma) \gamma \tau_i & \text{if } t_i \leq \tau_j \& \tau_i < t_j.
\end{cases} (8)
$$

Total surplus from domestic and foreign firms $\Pi^i(t_i, \tau_i, t_j, \tau_j)$ located in
region $i$ is given by

$$
\Pi^i(\cdot) = \begin{cases}
N_i[1 - F((t_i - \tau_j)\gamma)] \gamma (1 - t_i) & \text{if } t_i > \tau_j \& \tau_i \geq t_j; \\
N_i[1 - F((t_i - \tau_j)\gamma)] \gamma (1 - t_i) + N_j \int_{0}^{(t_j - \tau_i)\gamma} cf(c)dc & \text{if } t_i > \tau_j \& \tau_i < t_j; \\
N_i \gamma (1 - t_i) & \text{if } t_i \leq \tau_j \& \tau_i \geq t_j; \\
N_i \gamma t_i + N_j F((t_j - \tau_i)\gamma) \gamma (1 - t_i) - N_j \int_{0}^{(t_j - \tau_i)\gamma} cf(c)dc & \text{if } t_i \leq \tau_j \& \tau_i < t_j.
\end{cases} (9)
$$

The government in region $i$ maximizes $W(R^i, \Pi^i) = \omega R^i + \Pi^i$ by choosing
t$_i$ and $\tau_i$ for given values of $t_j$ and $\tau_j$. Note that the overall Nash equilib-
rium can be characterized as two independent Nash equilibria. Governments
compete for firms initially located in region $i$ through the choices of $t_i$ and
\( \tau_j \), and compete for firms initially located in region \( j \) through the choice of \( t_j \) and \( \tau_i \). For region \( i \), the best-response functions, \( t_i(\tau_j) \) and \( \tau_i(t_j) \), are given by:

\[
t_i(\tau_j) = \begin{cases} 
\frac{\omega-1}{\gamma} \frac{1-F((t_i(\tau_j)-\tau_j)\gamma)}{\gamma} \frac{f((t_i(\tau_j)-\tau_j)\gamma)}{\tau_j} & \text{if } t_i > \tau_j; \\
0 & \text{if } t_i \leq \tau_j \leq 1.
\end{cases}
\]

(10)

\[
\tau_i(t_j) = \begin{cases} 
\frac{\omega-1}{\gamma} \frac{F((t_j-\tau_i(\tau_j))\gamma)}{f((t_j-\tau_i(\tau_j))\gamma)} & \text{if } \tau_i < t_j; \\
0 & \text{if } \tau_i \geq t_j.
\end{cases}
\]

(11)

First-order conditions for the foreign tax rates are similar. To interpret these conditions, recall that each region sets its tax base elasticity equal to \( \frac{\omega-1}{\omega} \). Denote by \( \epsilon_i^d(t_i) = -(t_i/B_i^d)(\partial B_i^d/\partial t_i) \) the elasticity of the domestic tax base \( B_i^d \) with respect to tax \( t_i \), and by \( \epsilon_i^f(\tau_i) = -(\tau_i/B_i^f)(\partial B_i^f/\partial \tau_i) \) the elasticity of the foreign tax base \( B_i^f \) with respect to tax \( \tau_i \). These tax base elasticities are given by:

\[
\epsilon_i^d(t_i) = \begin{cases} 
\gamma t_i \frac{f((t_i-\tau_i)\gamma)}{1-F((t_i-\tau_i)\gamma)} & \text{if } t_i > \tau_j; \\
0 & \text{if } t_i \leq \tau_j.
\end{cases}
\]

(12)

\[
\epsilon_i^f(\tau_i) = \begin{cases} 
\gamma \tau_i \frac{f((t_j-\tau_i)\gamma)}{F((t_j-\tau_i)\gamma)} & \text{if } \tau_i < t_j; \\
0 & \text{if } \tau_i \geq t_j.
\end{cases}
\]

(13)

When \( t_i \leq \tau_j \), region \( i \)'s tax base becomes perfectly inelastic, as region \( i \) retains all its firms. Region \( i \) therefore raise its tax rate \( t_i \) until it equals \( \tau_j \). Whether it raises it further will depend on the value of \( \tau_j \). See Figure 3, for the case where \( t_i(\tau_j) > \tau_j \), except for high values of \( \tau_j \). In this latter case, the high values of \( t_i \) needed to exceed \( \tau_j \) imply a domestic tax base elasticity above \( \frac{\omega-1}{\omega} \), which implying that \( t_i \) must be lowered to satisfy the first-order condition for \( t_i \).

Figure 3

To guarantee the existence and the uniqueness of an equilibrium tax rates, we must look at the slopes of the best-response functions. Lemma 2 states those conditions.
Lemma 2: Whenever \( f'(c)/f(c) \in [-1/\gamma, 1/\gamma] \), the best-response functions are monotonically upward sloping, with a slope less than one. Then, an equilibrium exists and is unique.

Unlike the non-preferential regime, the best-response functions do not depend on regional sizes in this case. Thus, heterogeneity in size has no influence on the equilibrium tax rates, it obviously influence tax revenues directly however. The unique equilibrium involves identical tax policies between regions. An important feature of the equilibrium is that a region’s foreign firms will enjoy lower tax rates than its domestic firms, even though there is no heterogeneity between domestic and foreign firms. The next proposition identifies the equilibrium tax rates.

Proposition 4: Under a preferential regime, there exists a unique Nash equilibrium where domestic tax rates, \( t_i = t_j = t^p \), are greater than the foreign tax rates, \( \tau_i = \tau_j = \tau^p \), where:

\[
t^p = \left( \frac{\omega - 1}{\omega \gamma} \right) \left( \frac{1 - F(\gamma [t^p - \tau^p])}{f(\gamma [t^p - \tau^p])} \right) \quad (14)
\]

\[
\tau^p = \left( \frac{\omega - 1}{\omega \gamma} \right) \left( \frac{F(\gamma [t^p - \tau^p])}{f(\gamma [t^p - \tau^p])} \right) \quad (15)
\]

\[
\frac{\tau^p}{t^p} = \frac{1 - F(\gamma [t^p - \tau^p])}{F(\gamma [t^p - \tau^p])} < 1. \quad (16)
\]

Corollary to Proposition 4: Under a preferential regime with moving costs uniformly distributed, there exists a unique Nash equilibrium where:

\[
t^p = \left[ \frac{\omega - 1}{\omega \gamma} \right] \left[ \frac{2\omega - 1}{3\omega - 2} \right], \quad \text{and} \quad \tau^p = \left[ \frac{\omega - 1}{\omega \gamma} \right] \left[ \frac{\omega - 1}{3\omega - 2} \right]
\]

As we can see in Figure 3, a region always sets a lower tax rate on foreign firms compared to domestic firms, independently of \( N_1 \) and \( N_2 \). Note that if regions care only about tax revenue, then \( t^p = \frac{2}{3\gamma} \) and \( \tau^p = \frac{1}{3\gamma} \).
Consequently, preferential tax treatment is always used to attract foreign firms. To better understand this result, we should examine the tax-base elasticities. In equilibrium, the domestic and foreign tax base elasticities are positive, and equalized. Imagine that $\tau_i$ was to be smaller than $t_j$, but only by a very small amount. Region $i$ would then attract almost no firms from region $j$. Reducing $\tau_i$ further would then change its foreign tax base by a large proportion. This implies a large foreign tax base elasticity $e^f(\tau_i)$. On the other hand, region $j$ would lose few domestic firms. Increasing its tax rate on domestic firms would only reduce its domestic tax base by a small proportion. This implies a small domestic tax base elasticity $e^d(t_j)$. As the gap in tax rate increases, both elasticities converges to the point where there are equal, and $\tau_i < t_i$.

There is another important difference with the preferential tax treatment. Capital flows in both direction. Firms in region 1 with low moving cost seek low foreign tax rate in region 2, and at the same time, firms in region 2 with low moving costs seek the low foreign tax rate in region 1. For the uniform and homogenous regions case, where only tax revenue receives weight in the objective function, a total of $N/3$ firms move from each regions, creating a sum of moving costs equal to $2N \int_0^{1/3} cdc = \frac{N}{3}$. In the general case, where both tax revenue and private surplus enter the objective function, the difference in tax rates, $t^p - \tau^p = \frac{\omega - 1}{3\omega - 2}$, is smaller, but still positive, producing a sum of moving costs given by:

$$\text{MovingCosts} = N \left( \frac{\omega - 1}{3\omega - 2} \right)^2 \quad (17)$$

Note too that this cost figure does not depend on the size difference between regions; size determines only the proportion of total movers coming from each jurisdiction. We later compare these moving costs with those for the non-preferential regime, where firms move from the large region to the small region, since the latter has the lower tax rate.

With tax rates independent of regional size, size difference influences tax
revenue only directly. For the uniform case, tax revenues are

\[
R^1 = \frac{2N}{\omega^r - \omega^p} \frac{n(2\omega^r - \omega^p)^2 + (1-n)(\omega^r - \omega^p)^2}{(3\omega^r - 2\omega^p)^2};
\]

\[
R^2 = \frac{2N}{\omega^r - \omega^p} \frac{(1-n)(2\omega^r - \omega^p)^2 + n(\omega^r - \omega^p)^2}{(3\omega^r - 2\omega^p)^2}.
\]

(18)

Thus, we find that \( R^1 \) rises with \( n \), whereas \( R^2 \) falls with \( n \). In this sense, an increase in size heterogeneity favors the large region in terms of generating more tax revenues, simply because it is bigger. Both regions are losing the same percentage of firms to the other region, since the loss is based on the common difference in tax rates. The large region ends up with a higher fraction of domestic firms and a lower fraction of foreign firms. Fewer firms are moving from the small region to the large region than the number of firms who are moving in the reverse direction. Letting \( \lambda \) denote the fraction of firms that move from each region, the equilibrium numbers of domestic and foreign firms in each region is:

Number of Domestic Firms = \( \begin{cases} (1 - \lambda)nN & \text{for region 1;} \\ (1 - \lambda)(1 - n)N & \text{for region 2.} \end{cases} \)

Number of Foreign Firms = \( \begin{cases} \lambda(1 - n)N & \text{for region 1;} \\ \lambda n N & \text{for region 2.} \end{cases} \)

With uniform moving costs, the fraction of movers \( \lambda \) is less than one-half. We can then make three observations: i) despite potentially losing many domestic firms and gaining few foreign firms, the large region remains larger, ii) the number of domestic firms facing the higher domestic tax rate generate more tax revenue in the large region, iii) only a small number of foreign firms incur wasteful moving costs in the large region. Putting these three observations together, we can conclude that the weighted sum of tax revenue and private surplus is greater in the large region than in the small one. In general, the first observation may not be true, but the two other ones will always be. Finally, if we look at the value of this weighted sum per firm, it is also higher in the large region, since a higher fraction of that region’s firms are the domestic type, who pay higher tax rates and incur no moving costs.
The aggregate welfare loss arising from tax competition, both in terms of loss revenue and moving costs, does not depend on the relative region sizes. However, our analysis suggests that the large region bears the greater loss, in either absolute or per capita terms.

5 Comparing the two systems

We now compare the preferential and non-preferential regimes. We start with the case of a uniform distribution of moving costs and first show that welfare is always higher under the non-preferential regime when the regions are equal in size. We then investigate size differences, and find that the two regions differ in their preferences over regimes when the size difference is sufficiently large.\(^2\) Next, we consider non-uniform differences in moving costs and explain how they can cause the preferential regime to produce higher welfare than the preferential regime.

5.1 Uniform Moving Costs

Adding up the revenue for the preferential case, given by (18), we obtain:

\[
R^1 + R^2 = 2N \left( \frac{\omega - 1}{\omega} \right) \left[ \frac{(2\omega - 1)^2 + (\omega - 1)^2}{(3\omega - 2)^2} \right] < 2N \left( \frac{\omega - 1}{\omega} \right) \tag{19}
\]

From (7), total revenue under the non-preferential regime is

\[
R^1 + R^2 = \frac{2N}{n} \left[ \frac{\omega - 1}{\omega} \right] \left\{ \left[ n - (2n - 1) \frac{\omega - 1}{3\omega - 2} \right]^2 + \left[ (1 - n) + (2n - 1) \frac{\omega - 1}{3\omega - 2} \right]^2 \right\}, \tag{20}
\]

where the terms involving \((2n - 1)\) for \(n > 1/2\) reflect the shift in tax base from region 1 to region 2 as a result of region 1’s higher tax rate. If the two regions have the same size, then this expression reduces to

\[
R^1 + R^2 = 2N \left( \frac{\omega - 1}{\omega} \right). \tag{21}
\]

\(^2\)Note too that governments in our model do not take into account this wasteful mobility, because they care only about the surplus of their existing firms in equilibrium. Thus, the surplus of new firms that might be induced to enter the region in not counted, and the exit of firms from the region in response to a marginal policy change does not impact surplus because these marginal firms are indifferent about where to locate.
Comparing (21) with (19), we see that tax revenues are higher under the non-preferential case, at least with identical regions. Since revenue receives more weight than private surplus, and moving costs are positive only in the preferential case, we have–

**Proposition 5:** If regions are identical and moving costs are uniformly distributed, total welfare is higher under the non-preferential regime than under the preferential regime.

In fact, the difference becomes substantial when governments care only about tax revenue, which is the case on which the literature has focused. Setting $\omega = \infty$ in (19) then gives the following revenue for the preferential case:

$$R_1 + R_2 = 2\frac{N}{9}$$

(22)

In other words, moving from the non-preferential regime to the preferential regime reduces revenue by four-ninths, while also introducing wasteful commuting costs. This is a substantial loss. Recall also that there is only wasteful mobility of firms in the preferential case when regions are identical. On the other hand, the term in the square brackets in eq. (19) goes to zero as $\omega$ goes to one. Thus, the relative revenue loss from moving to a preferential regime goes to zero as the weight given to tax revenue goes to the weight on private surplus. Of course, all tax rates are converging to zero in this case. This means that wasteful difference in tax rates under the preferential regime is also becoming small, thereby making the two tax regime similar.

Let us now vary regional sizes. An interesting question is, what happens in the limit as $n$ goes to one, implying that the number of domestic firms in region 2 is going to zero? If we compare (19) with (20) in this limiting case, we see that they are identical. This result is easy to explain. When region 2 has almost no domestic firms, then its optimal strategy under the non-preferential regime is to set almost the same tax it would choose under the preferential regime, where it can distinguish between domestic
and foreign firms for tax purposes. Meanwhile, region 1 attracts almost no foreign firms from region 2, because there are almost none, so its non-preferential tax is basically a tax on only domestic firms. Thus, region 1 sets this tax at almost the same level as its tax on domestic firms in the preferential regime. Gathering our results about the non-preferential tax rates, we have—

**Proposition 6:** With uniform moving costs, and for symmetric regions, tax rates are such that \( t_1(.5) = t_2(.5) > t^p > \tau^p \). When \( n > .5 \), we have that \( t_1(n) \) and \( t_2(n) \) are both declining with \( n \). Moreover, \( t_1(n) - t_2(n) \) is increasing with \( n \), and

\[
\lim_{n \to 1} t_1(n) = t^p > \lim_{n \to 1} t_2(n) = \tau^p.
\]

With the tax difference between region 1 and region 2 nearly the same as the difference between domestic and foreign tax rates under the preferential regime, the total number of firms that are switching regions is almost the same as under the preferential regime. Thus, we may conclude that tax revenue and wasteful moving costs are almost the same, implying almost identical private surpluses.

To conclude, while we saw that the welfare gain from switching to a non-preferential regime can be be substantial in the case of identical regions, this gain completely disappears in the limit as one of the regions becomes infinitesimally small.\(^3\) Since this latter case corresponds to small tax havens, the suggestion here is that limiting preferential treatment of tax bases may not be adequate to significantly control tax evasion in practice.

But is there always at least some positive welfare gain from the switch to a non-preferential regime, at least for the case of uniform moving costs? We have not ruled out the possibility that the welfare gain not only goes to zero as \( n \) goes to one, but that the welfare gain may actually become negative on the way to \( n = 1 \). Our calculations show that it is possible for

\[^3\]This result does not not require uniform moving costs.
the switch to a non-preferential regime to lower tax revenue, but only for values of the welfare weight $\omega$ close to one. For there to be a drop in tax revenue at $n = .8$, for example, $\omega$ must be below 1.14.

Let us now compare the welfare levels for each region under the two regimes. We first show that any welfare difference is not the result of a difference in the number of firms occupying the region.

**Proposition 7:** Assume that moving costs are uniformly distributed. For any given $n \geq .5$, each region’s equilibrium sum of domestic and foreign firms does not depend on whether the preferential or non-preferential regime is in place.

Thus, the choice of regimes does not affect either region’s ability to induce firms to locate their operations there. Rather, this choice affects regional welfare levels through differences in the equilibrium tax rates. For any $n$ where $.5 < n < 1$, region 1 has higher taxes in the non-preferential case, because its equilibrium tax rate, $t_1$, exceeds the tax rates on domestic and foreign firms in the preferential case. With revenue valued more highly than private surplus, it follows that region 1 always prefers the non-preferential regime. Region 2 will also prefer the non-preferential regime if it is not much smaller than region 1, but an interesting conflict emerges when region 2 becomes sufficiently small. In this case, we next prove that region 2 will now prefer the preferential regime, in contrast to 1’s preference for the non-preferential regime. We summarize these results for regional welfare as follows:

**Proposition 8:** Assume that moving costs are uniformly distributed. Then for all $n$ where $.5 \leq n < 1$, region 1 prefers the non-preferential regime, whereas region 2 prefers the non-preferential regime only for $n$ sufficiently close to $.5$; that is, there exists an $\alpha < 1$ such that if $\alpha \leq n < 1$, then region 2 prefers the preferential regime.

Finally, let us return to the case of revenue-maximizing regions and examine how differences in regional size affect regional welfare. Setting $n = 1$ and
\( \omega = \infty \), we know that total revenue under either regime is given by (22), whereas region 2’s revenue under either regime is obtained by evaluating (35) found in the appendix. In this way, we obtain the limiting values of each region’s revenue for both regimes as \( n \) goes to one:

\[ R^1 = \frac{8}{9} N, \text{ and } R^2 = \frac{2}{9} N. \]

Thus, total revenue under the non-preferential regime drops from \( 2N \) to \( (10/9)N \) as \( n \) rises from .5 to 1. The drop in both \( t_1 \) and \( t_2 \) as \( n \) rises from 1/2 to 1 is responsible for this fall. Most of this revenue loss is occurring in region 2, which is losing its domestic tax base. But it is still impressive that region 2 is managing to end up with 20 percent of the economy’s total tax base, although its share of domestic firms is going to zero. Evidently, tiny countries can inflict sizable harm on large countries, in their ability to siphon off the latter’s tax base.

### 5.2 Non-uniform Moving Costs

We have seen that a region may prefer the preferential regime over the non-preferential regime if it is sufficiently small. In this section, we depart from the assumption of a uniform distribution of moving costs, and identify distributions under which both regions prefer the preferential regime, although they are identical in size. Moreover, we provide a straightforward economic interpretation of these distributions.

In the symmetric case, each region gets half the firms under both regimes. Thus, it will prefer the regime that generates the most tax revenue, since a dollar of revenue is more valuable than a dollar of private surplus. Our main result is as follows:

**Proposition 9:** With identical regions, the preferential tax regime generates more tax revenues if the distribution of moving costs features a sufficiently decreasing density distribution function; more precisely, if and only if:

\[
\frac{f(\gamma[t^P - \tau^P])}{f(0)} < [1 - F(\gamma[t^p - \tau^P])]^2 + F(\gamma[t^p - \tau^P])^2. \tag{23}
\]
A sufficient condition for the preferential tax regime to generate more tax revenues is that \( f'(c) < 0 \) for all \( c \), and

\[
\frac{f(0)}{f(1)} \geq 2.
\]  

(24)

Note that the uniform distribution of moving costs definitively does not satisfy this condition. With the distribution function, \( f(c) = (1 - \beta) + 2\beta c \), which we specified earlier, it would be sufficient that \( \beta < -1/3 \) for the preferential regime to generate more tax revenue. At the same time, it is sufficient that \( \beta > -1/(1 + 2\gamma) \) to ensure the existence and the uniqueness of all equilibria. These conditions are very restrictive sufficient conditions. Thus, it is clear that the necessary conditions can be satisfied in cases where \( \gamma > 1 \).

When the distribution of moving costs features a decreasing density, many firms are easily attracted, even for small differences in tax rates between two regions. Home bias behavior in investment decisions, as described in Haupt and Peters (2005), would correspond to a distribution function which does not satisfy this condition, because few firms would be willing to move under this assumption. Many other reasons can account for distribution functions that would either satisfy or not satisfy the condition stated in Proposition 9. Consequently, this model can nest both Keen (2001) and Haupt and Peters (2005) models.

6 Concluding Remarks

In this paper, we have investigated the relative merits of preferential vs. non-preferential tax regimes in a model of tax competition. The literature on this topic contains two views of the meaning of preferential tax treatment. The more common view is that governments distinguish between different types of capital, or firms, according to their mobility characteristics. But the literature on optimal taxation in an open economy emphasizes the difficulties involved in making such distinctions. Preferential treatment must
be based on observable characteristics of firms that may be only loosely associated with mobility differences.\textsuperscript{4} Thus, preferential tax regimes often consist of the foreign-owned portion of a tax base being taxed at a lower rate than the domestic-owned portion, a behavior that is also labeled “discrimination.” Some countries – e.g. Canada and the US – have signed mutually advantageous tax treaties, which would be jeopardized if one or the other actor were to start discriminating. In addition, the prohibition of the asymmetric treatment of foreign and domestic firms has been included in treaties in the EU and the OECD. Both the EU and the OECD are active in trying to reduce the extent of discrimination among their members.\textsuperscript{5}

We have adopted this second view in a 2-region model with domestic firms and foreign firms, distinguished by their region of origin. Using this model, we have found that the non-preferential regime can yield substantially more tax revenue than the preferential regime. But we have also seen that more revenue will be raised in the preferential case when the number of firms with low moving costs is relatively high. Since this is thought to be increasingly the case in the modern world economy, our results call into question the benefits of the nondiscrimination principal in OECD guidelines for international taxation, at least as a method for controlling wasteful tax competition. In any case, we also find that any benefits of nondiscrimination disappear when the size difference between the competing regions becomes large, as in the case tiny tax havens. Finally, our results point to a conflict between large and small regions in their preferences between the regimes: the large region may have difficulty implementing a non-discrimination principal because the small region prefers to discriminate.

\begin{footnotesize}
\textsuperscript{4}Hong and Smart (2010) assume that all firms must face the same statutory tax rates, and they analyze the use of tax havens to achieve desirable differences in effective marginal tax rates. Hagen, Osmundsen, and Schjelderup (1998) work with a model where a firm’s mobility is related to the size of its investment, in which case it is optimal to impose a nonlinear tax on investment.

\textsuperscript{5}On this, see OECD (1998).
\end{footnotesize}
7 References


8 Appendix

Proof of Lemma 1: By differentiating the first-order condition, we obtain the slope of the best-response function:

\[
\frac{\partial t_i(t_j)}{\partial t_j} = \begin{cases} 
\frac{(\omega-1)f((t_i-t_j)\gamma)+\omega\gamma t_i f'(t_i-t_j)\gamma}{(2\omega-1)f((t_i-t_j)\gamma)+\omega\gamma t_i f'(t_i-t_j)\gamma} & \text{if } t_i \geq t_j; \\
\frac{(\omega-1)f((t_j-t_i)\gamma)-\omega\gamma t_i f'(t_j-t_i)\gamma}{(2\omega-1)f((t_j-t_i)\gamma)-\omega\gamma t_i f'(t_j-t_i)\gamma} & \text{if } t_i < t_j.
\end{cases}
\]

The second-order conditions are both satisfied if and only if the denominators are positive, or equivalently, if:

\[
t_i \frac{f'(c)}{f(c)} \in \left[-\frac{\omega}{2\omega-1}, \frac{\omega}{2\omega-1}\right].
\]

If this condition is satisfied at \( t_i = 1 \) and \( \omega = \infty \), then it is always satisfied. We can then conclude that if the sufficient condition, \( f'(c)/f(c) \in [-1/2\gamma, 1/2\gamma] \), is satisfied, the second-order conditions are also satisfied. Existence of an equilibrium is guaranteed when the second-order conditions are satisfied. For any value of \( t_i \geq t_j \), region i’s best-response function is then positively sloped, only if the numerator is also positive. This conditions holds if \( f'(c)/f(c) > -1/\gamma \), and the reverse condition applies when \( t_i < t_j \). Consequently, a sufficient condition for the best-response functions to be positively sloped is, \( f'(c)/f(c) \in [-1/\gamma, 1/\gamma] \). Moreover, it is easy to see that if the best-response functions are upward sloping, then their slopes are less than one. This guarantees a unique solution. QED

Proof of Proposition 1: Given Lemma, 1 both reaction functions cross only once. Solving equation (3) reveals that \( t_1 = t_2 = \frac{\omega-1}{\omega} \frac{1}{\gamma f(0)}. \) QED

Proof of Proposition 2: First, we prove by contradiction that in any equilibrium, whenever \( n > 1/2 \), it must be the case that \( t_1 > t_2 \). Imagine that a combination of tax rates, \( t_2 > t_1 \), solves both best response functions. From (3), we could then show that

\[
\frac{1-F(\gamma |t_2-t_1|)}{t_2} = n + (1-n)F(\gamma |t_2-t_1|).
\]

\[
(1-n)t_1
\]
Re-writing the equation above, we obtain:

\[
\frac{t_1}{t_2} = \frac{n}{1-n} + \frac{F(\gamma[t_2 - t_1])}{1 - F(\gamma[t_2 - t_1])} > 1.
\] (28)

Since \(n > 1/2\), it must be the case that the left-hand side of the equation above is greater than 1. Consequently, we must have that \(t_1 > t_2\), which is a contradiction. The same contradiction does not apply for values of \(t_1 > t_2\). Moreover, when \(t_2 = 0\), then \(t_1(0) > 0\). We also know that \(\frac{\partial t_1(t_2)}{\partial t_2} < 1 < \frac{1}{\frac{n}{2}}\), so consequently there exist a unique Nash equilibrium where \(t_1(n) > t_2(n)\). The ratio \(\frac{t_1(n)}{t_2(n)}\) can be derived as above. QED

**Proof of Proposition 3:** For any value of \(t_1 > t_2\), the first-order condition for region 1 is independent of \(n\). Using equation (6), we can show that

\[
\frac{\partial t_2(t_1)}{\partial n} = \frac{-(\omega - 1)/w'(1-n)^2}{(2\omega - 1)f(\gamma[t_1 - t_2]) - \omega\gamma t_2 f'(\gamma[t_1 - t_2])} < 0.
\] (29)

Since \(t_1(t_2)\) is increasing in \(t_2\), this result implies that \(t_1\) and \(t_2\) are both decreasing with \(n\). We can define \(\Delta t_{12} = t_1 - t_2\) as

\[
\Delta t_{12} = \frac{1 - F(\gamma\Delta t_{12})}{\gamma f(\gamma\Delta t_{12})} - \frac{1 - n + F(\gamma\Delta t_{12})}{\gamma f(\gamma\Delta t_{12})} = \frac{2n-1 - 2F(\gamma\Delta t_{12})}{\gamma f(\gamma\Delta t_{12})}.
\] (30)

Comparative static reveals that:

\[
\frac{\partial \Delta t_{12}}{\partial n} = \frac{1/\gamma n^2}{3f(\gamma\Delta t_{12}) + \gamma\Delta t_{12} f'(\gamma\Delta t_{12})}.
\] (31)

The expression above is always positive when \(f'(c)/f(c) \in [-2/\gamma, 2/\gamma]\). QED

**Proof of Lemma 2:** The slopes of the best-response functions are given by:

\[
\frac{\partial t_i(\tau_j)}{\partial \tau_j} = \begin{cases} 
\frac{f((t_i-\tau_j)\gamma) + \gamma t_i f'((t_i-\tau_j)\gamma)}{2f((t_i-\tau_j)\gamma) + \gamma t_i f'((t_i-\tau_j)\gamma)} & \text{if } t_i > \tau_j; \\
0 & \text{if } t_i \leq \tau_j.
\end{cases}
\] (32)

\[
\frac{\partial \tau_i(t_j)}{\partial t_j} = \begin{cases} 
\frac{f((t_j-\tau_i)\gamma) - \gamma \tau_j f'((t_j-\tau_i)\gamma)}{2f((t_j-\tau_i)\gamma) - \gamma \tau_j f'((t_j-\tau_i)\gamma)} & \text{if } \tau_i < t_j; \\
0 & \text{if } \tau_i \geq t_j.
\end{cases}
\] (33)
See Lemma 1 for the rest of the proof, as both proofs are almost identical. QED

**Proof of Proposition 4:** Given the first-order conditions, no solution can be found for value of \( t_i < \tau_j \) or \( t_j < \tau_i \). Consequently, the domestic tax rate for region \( i \), and the foreign tax rate for region \( j \) are such that \( t_i > \tau_j \). Equations (14) and (15) are derived directly from the first-order conditions, and (16) follows. QED

**Proof of Proposition 7:** Consider region 1. Under the non-preferential regime, it obtains no foreign firms, but its loss in foreign firms is proportional to \( n(2 - (1/n)) = 2n - 1 \), where \( n \) measures the relative size of region 1’s initial domestic tax base, and \( 2 - (1/n) \) shows how the difference in regional tax rates depends in \( n \). For the same factor of proportionality, under the preferential regime, region 1’s loss of domestic firms is proportional to \( n \), whereas its gain of foreign firms is proportional to \( 1 - n \), resulting in a net loss equal also equal to \( 2n - 1 \). Thus, the number of firms that occupy region 1 in equilibrium does not depend on the tax regime. A symmetrical argument applies to region 2. QED

**Proof of Proposition 8:** The remaining result that we need to prove is that region 2 is better off under the preferential regime if \( n \) is close to one. Region 2 prefers the preferential regime if tax revenue is higher. Given Proposition 6, any difference in tax revenue between the two regimes disappears as \( n \) goes to 1. Thus, we can compare revenue for high \( N \) by differentiating the revenue function with respect to \( n \) for each regime, and evaluating the derivatives at \( n = 1 \). For the preferential regime, (18) shows that a marginal reduction in \( n \) from 1 raises revenue by

\[
- \frac{dR^2}{dn} = 2N \frac{\omega - 1}{\omega} \left[ \frac{(2\omega - 1)^2 - (\omega - 1)^2}{(3\omega - 2)^2} \right].
\]

For the non-preferential case, note that (7) may be used to write \( R^2 \) as follows:

\[
R^2 = \frac{2N}{n} \frac{\omega - 1}{\omega} \left[ \frac{(2 - n)\omega - 1)^2}{(3\omega - 2)^2} \right].
\]
Differentiating this expression with respect to $n$ and evaluating the derivative at $n = 1$ gives

$$\frac{dR^2}{dn} = 2N \frac{\omega - 1}{\omega} \left[ \frac{(\omega - 1)^2 - 2(\omega - 1)}{(3\omega - 2)^2} \right].$$  \hspace{1cm} (36)

Comparing the numerators in the square brackets in (34) and (36), we see that (34) exceeds (36) for $\omega = 1$, and the gap in the numerators widens as $\omega$ increases. \textbf{QED}

**Proof of Proposition 9:** With homogenous regions, tax revenue for region $i$ under a preferential tax regime is given by

$$R^i = \frac{\omega - 1}{\omega} \left[ 1 - F(\gamma[t_p - \tau_p]) \right]^2 + F(\gamma[t_p - \tau_p])^2. \hspace{1cm} (37)$$

Tax revenue $R^i$ is larger than tax revenue under a non-preferential tax regime, $R^i = (\omega - 1)/(\omega f(0))$, only if condition (23) is satisfied. Since the right-hand side of the condition is less one, it must be the case that $f(\gamma[t_p - \tau_p]) < f(0)$, and so the density function must be sufficiently decreasing. Since, $[1 - F(\gamma[t_p - \tau_p])]^2 + F(\gamma[t_p - \tau_p])^2 \geq 1/2$, a sufficient sufficient condition is that $\frac{f(0)}{f(1)} \geq 2$. \textbf{QED}
Figure 1: Symmetric equilibrium with uniform distribution, and homogenous regions
Figure 2: Asymmetric equilibrium with uniform distribution when $n < 1$
Figure 3: Preferential taxes $t_1$ and $\tau_2$ for uniform moving costs.