Information Aggregation through Informal Elections on Slippery Slopes

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Abstract

A policymaker may have concerns about “slippery slope” when she evaluates a reform: for some unknown distribution of agents’ preferences, the reform paves the way to future undesirable outcomes. We propose a simple model to investigate whether informal elections, including protests, polls, and non-binding voting, can aggregate dispersed information about the desirability of reform when the policymaker has such concerns. We find that the policymaker uses a non-monotonic rule in response to an informal election: she reforms if the turnout in the informal election is neither too low nor too high. This non-monotonicity may be obstructive: we identify conditions under which effective information aggregation is impossible when the population approaches infinity.

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1 Introduction

Motivation. It is widely believed that, compared with democratic regimes, authoritarian regimes govern in a persistent style. A prominent example is the sequence of five-year plans of the Soviet Union, which was first introduced by Joseph Stalin in the late 1920s and continued till the dissolution of the Union. One potential reason for such persistence is that the ruling parties (or persons) is not bound by any formal rules so that their policies need not be subject to changes of people’s wills. This is a key feature of the so-called informal institutions and has been intensively discussed by North (1990) and many others.

However, such persistent governance is treading on thin ice. The policymakers of authoritarian regimes still have incentive to find and implement policies that increase economic surplus. Nevertheless, such policies inevitably change the distribution of political power. For instance, reforms that lean towards a market economy may not only create more economic surplus but also redistribute political power to entrepreneurs or middle-class citizens. The latter may result in future undesirable outcomes for the rulers themselves, such as further loss of political power or even regime changes.

Some scholars use the phrase “slippery slope” to capture this dilemma faced by policymakers; see for example Bai and Lagunoff (2011) on fiscal conservatism or Acemoglu et al. (2012) on constitutions. On the one hand, the policymakers enjoy the power to pursue their private interests and extract rents from the economy. On the other hand, policies that yield more economic surplus may twist the distribution of political power and create risks to the regime.

Since policymakers are not omniscient, they need information about two aspects of a reform: first, whether the reform is economically beneficial; and (perhaps more importantly) whether the accompanied redistribution of political power is safe. The modern Condorcet Jury Theorem(s), which state that some formal elections (e.g. a majority voting system) can effectively aggregate dispersed information, may not be applicable to authoritarian regimes.

As we have emphasized, formal channels are invalidated by the nature of authoritarian
regimes. The lack of formal institutions in authoritarian regimes not only frees the hands of their rulers but also shuts down the bottom-up channel of information aggregation and transmission. For instance, if congress is only a rubber stamp, then policymakers may find it hard to get any information from the results of the voting. Thus, for information aggregation, policymakers may resort to informal elections, including protests, polls, and non-binding voting. To understand authoritarian regimes, it is crucial to understand whether dispersed information can be aggregated through informal elections when there is slippery-slope concern.

Model and results. In this paper, we propose a simple model to investigate when a policymaker has concerns about the slippery slope, whether informal elections can achieve information aggregation. Since our model builds upon Battaglini (2017), it is framed using protest, an interaction between citizens and government, as the theme:

Each citizen has a private signal about the economic impact of the reform. The citizens simultaneously decide whether to participate in a protest for reform. To capture the slippery-slope concern, we assume that there are two ideological types of citizens: conservatives and radicals. The reform, besides its economic impact, redistributes political power from the government to all citizens, and the radicals would like to use their newly acquired power against the government. The government dislikes the reform if the proportion of radicals (in the population) is too large. Thus, we frame the question “whether a redistribution of political power is safe” into “whether the distribution of citizens’ ideological preferences is favorable”.

The government is not omniscient: he knows neither the economic impact of the reform nor the proportion of radicals in the population. He observes the number of protesters,

\footnote{Following Battaglini (2017), we assume that the number of citizens follows a Poisson distribution. This assumption allows us to use the powerful techniques developed by Myerson (1998, 2000).}

\footnote{Acemoglu et al. (2012) discuss de-secularization in Turkey. In their example, whether empowering religious groups would ultimately reduce the rights of secular groups depends on whether a significant proportion of the religious groups is “fundamentalist”. Similar statements can be made about “perestroika and glasnost” in USSR or Yushin Constitution in South Korea, if a people-enemy dichotomy is more than pure rhetoric.}
makes inferences accordingly, and decides whether to implement the reform or maintain the status quo. This political process is “informal” in the sense that the government does not abide by any pre-committed rule.

More precisely, there are three fundamental states: In the first state, the reform is not economically beneficial, although the proportion of radicals is small; in the second state, the reform is economically beneficial, and the proportion of radicals is medium; in the third (and slippery-slope) state, the reform is economically beneficial, but the proportion of radicals is large. The conservatives prefer the reform at the second and the third states, whereas the radicals prefer the reform regardless of the fundamental states. The government prefers the reform only at the second state.\(^3\)

Conditional on a fundamental state, each citizen receives an i.i.d private signal which is “economic”: the signal is uninformative about whether the fundamental state is the second or the third. This assumption allows us to see the pure effect of the slippery slope; on the other hand, such political “blindness” may result from government censorship and citizen self-censorship. We also assume that there is a positive probability that the signal almost fully reveals whether the fundamental state is the first or not. Under this assumption, we show that the protest can achieve information aggregation when there is no slippery-slope concern, i.e., when the ex-ante probability of the third and slippery-slope state is zero. Thus, if we find a failure of information aggregation when there is the slippery-slope concern, i.e., when the ex-ante probability of the third and slippery-slope state is positive, then the failure cannot be attributed to a lack of informative signals as in Battaglini (2017).

With the slippery-slope concern, our results on players’ equilibrium strategies are threefold. First, the government’s decision rule is non-monotonic and features double pivots: he implements the reform only when the number of protesters is neither too small nor too large; in particular, he infers from a very large number of protesters that the reform is politically dangerous. Second, anticipating the double pivots, conservatives participate

\(^3\)When the population is large, our results extend to the case where the conservatives are aligned with the government: they prefer the reform only at the second state.
in the protest if they believe that the reform is economically beneficial with sufficiently high probability, whereas radicals participate in the protest if they believe that the reform is economically beneficial with sufficiently low probability. In short, each citizen plays a cutoff protest strategy, but the strategies of two types have reverse monotonicity. Finally, we formulate a pivot information correspondence which maps citizens’ protest strategies to plausible likelihood ratios about the fundamental states conditional on a citizen’s being pivotal and taking in account of the government’s best response. This tool is indeed simple yet powerful for asymptotic analysis as the population grows to infinity.

Turning to information aggregation, we formulate a notion of limit equilibrium (as the population grows to infinity) based on the pivot information correspondence. We focus on a “stable” subclass of such limit equilibria, with “stability” understood as the negation of the following notion of instability: an equilibrium is unstable if the best response to a perturbation of this equilibrium is typically far from this equilibrium. Each stable limit equilibrium has the property of full information equivalence: the ex ante probabilities that a citizen participates in the protest are different across three fundamental states.

We show that each stable limit equilibrium is the limit of a sequence of responsive equilibria as the population grows to infinity. Thus, if a stable limit equilibrium exists, then the protest can aggregate information. However, we identify conditions under which there is no stable limit equilibrium under certain conditions: for example, the conservatives’ value of a “wise” reform is too high or the government’s value from a “wise” reform is too low. Moreover, these conditions are independent of the ex ante probability of the slippery-slope state. Slippery-slope concern obstructs “stable” information aggregation through the citizens’ equilibrium assessment of the relative likelihood of the double pivots.

Back to our research question, there are non-trivial conditions under which protest cannot aggregate information effectively (if “effectiveness” requires “stability” ). On the other hand, we may observe a government refusing to reform in response to a very large-scale protest. It

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4The same idea of “stability” appears in Ekmekci and Lauermann (2022) and Fey (1997).
is not because the protest is uninformative, but rather the opposite is true: the government
learns (almost) perfectly that the reform is politically dangerous.

Related literature. Our modeling of the slippery-slope state is inspired by previous works
on dynamic institutional changes, especially Acemoglu et al. (2012). Most of the studies
in this stream of literature focus on the dynamic evolution of formal institutions, such as
constitutions or laws that are determined by legislative bargaining (e.g. Acemoglu et al.,
2010; Bai and Lagunoff, 2011), without dispersed information. Departing from their focus
on democracies, we study authoritarian regimes that are not subject to votes but need to
aggregate dispersed information. The slippery-slope concern prevents the government from
implementing the reform when the number of protesters is too large.

This paper also belongs to the literature on information aggregation in collective decision-
making. With binding voting, the famous Condorcet jury theorem(s) say that information
aggregation is always achievable when the number of participants is sufficiently large (e.g.
Ladha, 1992; Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1997, 1999). With
non-binding voting, some recent studies from the literature on cheap talk with multiple
senders demonstrate the possibility of complete unraveling (e.g. Wolinsky, 2002; Morgan
and Stocken, 2008; Levit and Malenko, 2011; Chen, 2022). As far as we know, Chen (2022)
is the only one among them who assumes that the principal and the agents have ex post
misaligned interest at some fundamental state; however, the government-principal in his
model always uses a monotonic decision rule.

The Poisson protest formulated by Battaglini (2017) is the skeleton of our model. His
model may be regarded as cheap talk with a stochastic number of senders, and his result on
the failure of information aggregation through protest can be attributed to a lack of very
informative signals. Our model, however, allows each citizen to receive an informative signal
with a small but positive probability, Moreover, we show that information can be aggregated

\footnote{For policy dynamics without explicit and direct redistribution of political power, see for example Duggan
and Kalandrakis (2012) and Dziuda and Loeper (2016).}
when there is no slippery-slope concern. Therefore, the failure of information aggregation in our model is entirely driven by the presence of the slippery-slope state.

**Structure of the paper.** The paper is organized as follows. Section 2 introduces the model settings. Section 3 analyzes a finite-population benchmark. Section 4 presents a general Poisson-game framework with a large population. Section 5 discusses several extensions and model variations.

# 2 Model

In this section, we set up a game of Poisson protest on slippery slope and formulate a non-trivial subclass of sequential equilibria.

A population of citizens simultaneously decide whether to participate in a protest. Participation is costless. After observing the number of protesters, the government chooses between a reform $A$ and a status quo $B$.

**Ideological types and fundamental states.** Each citizen has two ideological types, either conservative or radical. Let $\Omega \equiv \{b, a_1, a_0\}$ be the space of fundamental states. The fundamental states are understood in relation to both the players’ *ex-post* policy preferences and the proportion of conservatives (in the population):

<table>
<thead>
<tr>
<th>state</th>
<th>supporter of reform</th>
<th>proportion of conservatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>radicals</td>
<td>high</td>
</tr>
<tr>
<td>$a_1$</td>
<td>all citizens, government</td>
<td>medium</td>
</tr>
<tr>
<td>$a_0$</td>
<td>all citizens</td>
<td>low</td>
</tr>
</tbody>
</table>

The conservatives call for reform when the fundamental state is $a_1$ or $a_0$. The radicals crave the reform regardless of the fundamental state. The government regard $a_0$ as a *slippery slope*: when the proportion of conservatives is low, the reform is “unwise” from the government’s perspective.
Cardinal policy preference. For each $\omega \in \Omega$, let all players’ payoffs from $B$ be normalized to 0. Each player’s payoff from $A$ is as follows:

<table>
<thead>
<tr>
<th></th>
<th>$b$</th>
<th>$a_1$</th>
<th>$a_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>conservative</td>
<td>$-1$</td>
<td>$\beta$</td>
<td>$\beta$</td>
</tr>
<tr>
<td>radical</td>
<td>$1$</td>
<td>$1$</td>
<td>$1$</td>
</tr>
<tr>
<td>government</td>
<td>$-1$</td>
<td>$\gamma$</td>
<td>$-1$</td>
</tr>
</tbody>
</table>

Here, $\beta, \gamma > 0$. $\beta$ (or $\gamma$) is the value of a wise reform for the conservatives (or the government).

Poisson population. The number of (active) citizens follows Pois($n$), the Poisson distribution with mean $n > 0$. Given Pois($n$), let $\pi(q; n)$ be the probability that there are $q$ citizens. We refer to $n$ as the mean population. Conditional on the fundamental state being $\omega \in \Omega$, the citizens’ ideological types are i.i.d: each citizen is conservative with probability $z_\omega$ and radical with probability $1 - z_\omega$. $(z_\omega)_{\omega \in \Omega}$ represents the proportion of conservatives: conditional on $\omega$, the number of conservative citizens follows Pois($nz_\omega$). We assume

A1. $z_b \geq z_1 > z_0$,

in which we write $z_0, z_1$ instead of $z_{a_0}, z_{a_1}$ for notational simplicity. Under (A1), the proportion of conservatives is the lowest at $a_0$ and the highest at $b$. Note that the ideological type is informative about the fundamental state.

Information structure. Let $\mu \in \text{int } \Delta(\Omega)$ be a common prior about the fundamental states. We assume

A2. $\frac{\mu_0}{\mu_1} > \gamma$.

Under (A2), the government does not reform at $\mu$ even if the concern about slippery slope is negligible in the sense that $\mu_0 \approx 0$.

Conditional on $\omega$, each citizen receives an i.i.d signal $s \in S \equiv [s, \bar{s}]$; the distribution of her signal is independent of her ideological type and given by a density function $f(s|\omega)$. We make the following assumptions on the state-dependent densities:
A3. For each $s \in S$, $f(s|a_0) = f(s|a_1)$, and we write $f(s|a) \equiv f(s|a_1)$ henceforth.

A4. $f(s|a)$ and $f(s|b)$ are positive on $(s, \bar{s})$; $0 = f(s|a) < f(\bar{s}|b)$ and $0 = f(\bar{s}|b) < f(s|a)$.

A5. $f(s|a)$ and $f(s|b)$ has the monotone likelihood ratio property (MLRP); i.e., $\varphi(s) \equiv \frac{f(s|b)}{f(s|a)}$ is weakly decreasing.

Under (A3), a citizen cannot infer from her signal whether the fundamental state is $a_1$ or $a_0$. (A3) is more appealing in authoritarian regimes where (self-)censorship and punishment make it difficult to learn about the distribution of ideology of the population. Under (A4), $s$ fully reveals that the fundamental state is $b$, whereas $\bar{s}$ fully reveals that the fundamental state is not $b$. (A4) requires that very informative signals be possible, although they may be very rare in a probabilistic sense.

For each citizen, her ideological type and signal are private information. All other features of the protest game is common knowledge.

**Solution concept.** Let $p_c : S \to [0, 1]$ represent a (signal-symmetric) protest strategy of the conservatives: each conservative citizen participates in the protest with probability $p_c(s)$ when her signal is $s \in S$. We use “$s$” and its variants to denote a conservative’s signals. Similarly, let $p_r : S \to [0, 1]$ represent a protest strategy of the radicals. We use “$t$” and its variants to denote a radical’s signals.

Given a protest strategy profile $(p_c, p_r)$ and conditional on $\omega$, the number of protesters follows $\text{Pois}(nm_\omega(p_c, p_r))$, in which

$$m_\omega(p_c, p_r) \equiv z_\omega \int_S p_c(s)f(s|\omega)ds + (1-z_\omega)\int_S p_r(t)f(t|\omega)dt.$$  

We refer to $m_\omega$ as the mean scale (of the protest) at $\omega$.

Let $\tau_g : \mathbb{Z}_+ \to [0, 1]$ represent a reform strategy of the government: the government reforms with probability $\tau_g(q)$ when $q \in \mathbb{Z}_+$ protesters are observed.
There is always a trivial sequential equilibrium: all citizens abstain from the protest, regardless of their ideological types and signals; the protest is thus uninformative, and the government maintains the status quo. For non-triviality, we are interested in sequential equilibria in which the protest is informative and responsive.

**Definition 1.** A protest strategy profile \((p_c, p_r)\) is informative if \(\omega \neq \omega'\) implies \(m_\omega(p_c, p_r) \neq m_{\omega'}(p_c, p_r)\). A sequential equilibrium of the protest game is informative if the equilibrium protest strategy profile \((p_c, p_r)\) is informative.

**Definition 2.** A sequential equilibrium of the protest game is responsive if the equilibrium reform strategy \(\tau_g\) is non-zero: there is \(q \in \mathbb{Z}_+\) such that \(\tau_g(q) > 0\).

Note that an informative equilibrium may not be responsive. Our notion of informativeness might also seem stronger than necessary: an equilibrium might be “semi-informative” in the sense that the mean scales at two states are identical and different from the third state. However, we shall show in the next section that a responsive equilibrium must be informative in the sense of Definition 1.

### 3 Finite mean population

In this section, we characterize responsive equilibria of the protest game for a fixed mean population \(n > 0\).

A responsive equilibrium is informative in the sense of Definition 1. In a responsive equilibrium, the government reforms with positive probability only when a medium number of protesters are observed. Thus, each citizen faces double pivotal events: her participation in the protest increases/decreases the chance of reform when there are a small/large number of other protesters.

A responsive equilibrium is “reformative” if a large number of protesters indicate that the reform is optimal from the conservatives’ perspective.\(^6\) In a reformative equilibrium,\(^6\)

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\(^6\)Our approach is also applicable to “anti-reformative” equilibria in which a large number of protesters
the protest strategy profile can be represented by a pair of cutoff strategies with “reverse monotonicity”: a conservative participates in the protest only if her signal is sufficiently high, whereas a radical participates in the protest only if her signal is sufficiently low.

Based on this cutoff structure and the correspondence from mean scale to a citizen’s information conditional on being pivotal, we provide a fixed-point representation of reformative equilibria.

3.1 Government decision and double pivots

In this subsection, we characterize the government’s reform strategy in a responsive equilibrium. Except for Lemma 3, we use a weaker version of (A2): \( \frac{\mu_b + \mu_0}{\mu_1} > \gamma \).

Suppose the protest in a sequential equilibrium has a (vector) mean scale \( m \in [0,1]^\Omega \). Let

\[
    h_n(q, m) \equiv \frac{\mu_b}{\mu_1} e^{-n(m_b-m_1)} \left( \frac{m_b}{m_1} \right)^q + \frac{\mu_0}{\mu_1} e^{-n(m_0-m_1)} \left( \frac{m_0}{m_1} \right)^q.
\]

Observing \( q \in \mathbb{Z}_+ \) protesters, the government’s posterior belief about state \( a_1 \) is simply \( \Pr(a_1|q, m) = \frac{1}{1+h_n(q,m)} \). The government is willing to reform if and only if \( \Pr(a_1|q, m) \geq \frac{1}{1+\gamma} \), or equivalently, \( h_n(q, m) \leq \gamma \). Since \( h_n(q, m) \) is convex in \( q \), the following lemma is immediate.

**Lemma 1.** Let \( Q(m) \equiv \{ q \in \mathbb{Z}_+ : \Pr(a_1|q, m) \geq \frac{1}{1+\gamma} \} \). Then \( Q(m) \) is an interval.

Note that an equilibrium is responsive only if \( Q(m) \) is non-empty.

By Lemma 1, the government may play a monotone cutoff reform strategy. \( \tau_g \) is an increasing cutoff reform strategy if there is \( \hat{q} \geq 0 \) such that \( q > \hat{q} \) implies \( \tau_g(q) = 1 \) and \( q < \hat{q} \) implies \( \tau_g(q) = 0 \). \( \tau_g \) is a decreasing cutoff reform strategy if there is \( \tilde{q} \geq 0 \) such that \( q < \tilde{q} \) implies \( \tau_g(q) = 1 \) and \( q > \tilde{q} \) implies \( \tau_g(q) = 0 \).

\[\text{indicate that the status quo is optimal.} \]
Since the radicals crave reform regardless of the fundamental state, they response uninformatively to monotone cutoff reform strategies. Jointly with the assumption \( z_1 > z_0 \) (A1), we can rank \( m_1 \) and \( m_0 \).

**Lemma 2.** Suppose \( p_r \) and \( \tau_g \) constitute a responsive equilibrium. If \( \tau_g \) is an increasing cutoff reform strategy, \( p_r(t) \equiv 1 \) and thus \( m_1 \leq m_0 \). If \( \tau_g \) is a decreasing cutoff reform strategy, \( p_r(t) \equiv 0 \) and thus \( m_1 \geq m_0 \).

Next we show that a responsive equilibrium is informative in the sense of Definition 1. The main idea is that if an equilibrium is not informative, then it is either not responsive or the government plays a monotone cutoff reform strategy. In the latter case, we apply Lemma 2 to uncover some contradiction.

**Proposition 1.** In a responsive equilibrium, \( \omega \neq \omega' \) implies \( m_\omega \neq m_{\omega'} \).

**Proof.** Consider different ways of pooling multiple fundamental states.

1. **Uninformative protest.** When \( m_b = m_1 = m_0 \) and \( \frac{\mu_b + \mu_0}{\mu_1} > \gamma \), \( \Pr(a_1 | q, m) \equiv \mu_1 < \frac{1}{1+\gamma} \). Thus, \( Q(m) = \emptyset \) and the equilibrium is not responsive.

2. **Pooling \{b, a_0\}.** When \( m_b = m_0 < m_1 \), \( h_n(q, m) \) is strictly decreasing in \( q \), and \( h_n(0, m) > \gamma \). If \( Q(m) \) is non-empty, then the government plays an increasing cutoff reform strategy. By Lemma 2, \( m_1 \leq m_0 \), which is a contradiction. When \( m_b = m_0 > m_1 \), \( h_n(q, m) \) is strictly increasing in \( q \). If \( Q(m) \) is non-empty, then the government plays a decreasing cutoff reform strategy. By Lemma 2, \( m_1 \geq m_0 \), which is a contradiction.

3. **Pooling \{b, a_1\}.** When \( m_b = m_1 > m_0 \), \( h_n(q, m) \) is strictly decreasing in \( q \), and \( h_n(0, m) > \gamma \). If \( Q(m) \) is non-empty, then the government plays an increasing cutoff reform strategy. By Lemma 2, \( m_1 \leq m_0 \), which is a contradiction. When \( m_b = m_1 < m_0 \), \( h_n(q, m) \) is strictly increasing in \( q \). If \( Q(m) \) is non-empty, then the government plays a decreasing cutoff reform strategy. By Lemma 2, \( m_1 \geq m_0 \), which is a contradiction.

4. **Pooling \{a_1, a_0\}.** When \( m_b < m_1 = m_0 \), \( h_n(q, m) \) is strictly decreasing in \( q \), and \( h_n(0, m) > \gamma \). If \( Q(m) \) is non-empty, then the government plays an increasing cutoff reform
strategy. By Lemma 2 and \( m_1 = m_0, p_c(s) \equiv 1 \). Thus, \( m_b = m_1 = m_0 \), which is a contradiction. When \( m_b > m_1 = m_0 \), \( h_n(q, m) \) is strictly increasing in \( q \). If \( Q(m) \) is non-empty, then the government plays a decreasing cutoff reform strategy. By Lemma 2 and \( m_1 = m_0, p_c(s) \equiv 0 \). Thus, \( m_b = m_1 = m_0 \), which is a contradiction.

By Proposition 1, the mean scales at different fundamental states are different. The next proposition fully characterizes their ranking.

**Proposition 2.** In a responsive equilibrium, \( m_1 \) is the median of \( m \).

**Proof.** The argument is almost the same as “Pooling \( \{b, a_0\} \)” in the proof of Proposition 1, and we don’t repeat the details. If \( m_1 = \max m \), then we deduce that \( m_1 < m_0 \). If \( m_1 = \min m \), then we deduce that \( m_1 > m_0 \). In either case, we find a contradiction.

By Proposition 2, a responsive equilibrium has either \( m_b < m_1 < m_0 \) or \( m_0 < m_1 < m_b \). When \( m_b < m_1 < m_0 \), a large number of protesters indicate that the reform is optimal from the conservatives’ perspective. From now on, we focus on such “reformative” equilibria.

**Definition 3.** A responsive equilibrium is reformative if \( m_b < m_1 < m_0 \).

It is easy to see that the government cannot play a monotone cutoff cutoff strategy in a reformative equilibrium: \( m_b < m_1 < m_0 \) implies \( \gamma < \lim_{q \to \infty} h_n(q, m) = \infty \), and thus \( \tau_g \) cannot be an increasing cutoff strategy; by Lemma 2 and \( m_1 < m_0 \), \( \tau_g \) cannot be a decreasing cutoff strategy either.

**Proposition 3.** In a reformative equilibrium, there are \( \hat{q}, \check{q} \in \mathbb{Z}_+ \) such that \( q \in (\hat{q}, \check{q}) \) implies \( \tau_g(q) = 1 \) and \( q \notin [\hat{q}, \check{q}] \) implies \( \tau_g(q) = 0 \). Moreover, \( \tau_g(0) < 1 \) and \( \frac{\partial h}{\partial q}(0, m) < 0 \).

**Proof.** \( m_b < m_1 < m_0 \) implies that \( Q(m) \) is bounded above. \( \check{q} \equiv \min Q(m) \) and \( \hat{q} \equiv \max Q(m) \) are the integers that we seek.

If \( \tau_g(0) = 1 \) or \( \frac{\partial h}{\partial q}(0, m) \geq 0 \), the convexity of \( h_n \) in \( q \) implies that \( \tau_g \) is a decreasing cutoff strategy, which contradicts \( m_1 < m_0 \).
We refer to \( \hat{q} \) as the *lower pivot* and \( \check{q} \) as the *upper pivot*. Given the double pivots, whether a citizen’s participation in the protest increases the chance of reform is ambiguous. In the next subsection, we characterize the citizens’ best response to the double pivots using the following lemma.

**Lemma 3.** Suppose \( \frac{m_1}{\mu_1} > \gamma \) (A2). In a reformative equilibrium, \( \hat{q} > nm_b \). Thus, \( \hat{q} - 1 \leq q < q' \implies \pi(q; nm_b) > \pi(q'; nm_b) \).

**Proof.** Suppose \( \frac{\hat{q}}{n} \leq m_b \). Note that \( m_b < m_1 \) implies \( m_b < \frac{m_b - m_1}{\ln m_b - \ln m_1} \). Given \( \frac{\hat{q}}{n} < \frac{m_b - m_1}{\ln m_b - \ln m_1} \), we have \( h_n(\hat{q}, \textbf{m}) > \frac{\mu_b}{\mu_1} > \gamma \), which is a contradiction.

Since \( \hat{q} > nm_b \), we have \( \frac{\pi(q; nm_b)}{\pi(q'; nm_b)} = \frac{q+1}{nm_b} \times \ldots \times \frac{q'}{nm_b} > 1 \).

### 3.2 Citizen decision and reverse monotonicity

In this subsection, we characterize the citizens’ protest strategies in a reformative equilibrium.

A conservative with signal \( s \in S \) assigns probability
\[
\frac{\mu_\omega z_\omega f(s|\omega)}{\sum_{\Omega} \mu_{\omega'} z_{\omega'} f(s|\omega')} \pi(q; nm_\omega)
\]
to the event “the state being \( \omega \) and there being \( q \) other protesters”, whereas a radical with signal \( t \in S \) assigns probability
\[
\frac{\mu_\omega (1 - z_\omega) f(t|\omega)}{\sum_{\Omega} \mu_{\omega'} (1 - z_{\omega'}) f(t|\omega')} \pi(q; nm_\omega)
\]
to the event “the state being \( \omega \) and there being \( q \) other protesters”.

Suppose \( \hat{q} \leq \check{q} - 1 \) are the double pivots in a reformative equilibrium. Let \( \hat{\tau} \equiv \tau_{g}(\hat{q}) \) and \( \check{\tau} \equiv \tau_{g}(\check{q}) \) be the probabilities of reform at the double pivots. Let
\[
\hat{\pi}_{n;\omega} \equiv \hat{\tau} \pi(\hat{q} - 1; nm_\omega) + (1 - \hat{\tau}) \pi(\hat{q}; nm_\omega)
\]
\[
\check{\pi}_{n;\omega} \equiv (1 - \check{\tau}) \pi(\check{q} - 1; nm_\omega) + \check{\tau} \pi(\check{q}; nm_\omega).
\]
$\hat{\pi}_{n,\omega} - \bar{\pi}_{n,\omega}$ is the net pivot probability at $\omega$. A conservative with signal $s \in S$ participates only if

$$\mu_b z_b (\hat{\pi}_{n,b} - \bar{\pi}_{n,b}) \varphi(s) \leq \sum_{\omega \neq b} \mu_\omega z_\omega (\hat{\pi}_{n,\omega} - \bar{\pi}_{n,\omega}) \beta; \quad \text{(BR}_c)$$

a radical citizen with signal $t \in S$ participates only if

$$\mu_b (1 - z_b) (\hat{\pi}_{n,b} - \bar{\pi}_{n,b}) \varphi(t) \geq \sum_{\omega \neq b} \mu_\omega (1 - z_\omega) (\hat{\pi}_{n,\omega} - \bar{\pi}_{n,\omega}). \quad \text{(BR}_r)$$

From (BR$_c$) and (BR$_r$), it follows almost immediately that the citizens’ protest strategy profile consists of a pair of cutoff strategies with reverse monotonicity.

**Proposition 4.** Suppose $p_c$ and $p_r$ constitute a reformative equilibrium. There is $(s^*, t^*) \in S \times S$ such that $p_c(s) = 1_{s \geq s^*}$ and $p_r(t) = 1_{t \leq t^*}$; i.e., the conservatives play an increasing cutoff protest strategy, whereas the radicals play a decreasing cutoff protest strategy.

**Proof.** When $\hat{\tau} \leq \tilde{\tau} - 1$, it suffices to show $\hat{\pi}_{n,b} > 1$. When $\hat{\tau} = \tilde{\tau}$, we give a separate treatment.

1. Suppose $\hat{\tau} < \tilde{\tau} - 1$. By Lemma 3, $\hat{\pi}_{n,b} \geq \pi(\hat{\tau}; nm) / \pi(\tilde{\tau}; nm) > 1$.

2. Suppose $\hat{\tau} = \tilde{\tau} - 1$. Since the equilibrium is responsive, at least one of $\hat{\tau}$ and $\bar{\tau}$ is positive. Thus, $\hat{\pi}_{n,b} / \bar{\pi}_{n,b} > \pi(\hat{\tau}; nm) / \pi(\tilde{\tau}; nm) = 1$.

3. Suppose $\hat{\tau} = \tilde{\tau}$. Since the equilibrium is responsive, $\hat{\tau} = \bar{\tau} > 0$. A conservative with signal $s \in S$ participates only if

$$\hat{\tau} \mu_b z_b (\pi(\hat{\tau} - 1; nm) - \pi(\hat{\tau}; nm)) \varphi(s) \leq \hat{\tau} \left[ \sum_{\omega \neq b} \mu_\omega z_\omega (\pi(\hat{\tau} - 1; n\omega) - \pi(\hat{\tau}; n\omega)) \right] \beta;$$

When $\hat{\tau} = \tilde{\tau} - 1$, the net pivot probability at $\omega$ is

$$\hat{\tau} \pi(\hat{\tau} - 1; n\omega) + (\hat{\tau} - \tilde{\tau}) \pi(\hat{\tau}; n\omega) - \bar{\tau} \pi(\hat{\tau}; n\omega) = \hat{\pi}_{n,\omega} = \tilde{\pi}_{n,\omega}$$
a radical with signal $s \in S$ participates only if

$$
\hat{\tau}_b (1 - z_b) \left( \pi(\hat{q} - 1; nm_b) - \pi(\hat{q}; nm_b) \right) \varphi(s)
\geq \hat{\tau} \left[ \sum_{\omega \neq b} \mu_\omega (1 - z_\omega) \left( \pi(\hat{q}; nm_\omega) - \pi(\hat{q} - 1; nm_\omega) \right) \right].
$$

By Lemma 3, $\frac{\pi(\hat{q} - 1; nm_b)}{\pi(\hat{q}; nm_b)} > 1$. \qed

The intuition behind Proposition 4 is simple. Suppose the fundamental state is $b$. The lower pivot is closer to $nm_b$, the mean of $\text{Pois}(nm_b)$, and more likely than the upper pivot. Thus a citizen’s participation increases the chance of reform. Increasing the chance of reform (at $b$) incurs a loss for a conservative but a gain for a radical. Given MLRP, a conservative with a higher signal believes that her participation is less likely to incur the loss, whereas a radical with a lower signal believes that her participation is more likely to incur the gain.

### 3.3 Reformative domain and pivot information correspondence

In this subsection, we briefly study the domain of protest strategy profiles for reformative equilibria. Our main task is to define and investigate the pivot information correspondence: for each mean scale $m \in [0, 1]^{\Omega}$, the government’s best response to $m$ induces a set of posterior likelihood ratios of fundamental states conditional on each of the double pivots. A reformative equilibrium is then represented as a fixed point based on a pivot information correspondence.

By Proposition 4, the protest strategy profile of a reformative equilibrium can be represented by a pair of cutoff signals $(s, t) \in S \times S$. For each $\omega \in \Omega$, let

$$
m_\omega(s, t) \equiv z_\omega (1 - F(s|\omega)) + (1 - z_\omega) F(t|\omega)
$$

be the mean scale at $\omega$ when the protest strategy profile consists of $p_c(s') \equiv 1_{s' \geq s}$ and $p_r(t') \equiv 1_{t' \leq t}$. 16
**Definition 4.** The asymptotic reformative domain of the protest game is

\[ R \equiv \{(s, t) \in S \times S : m_b(s, t) < m_1(s, t) < m_0(s, t)\} \]

For a fixed mean population \( n > 0 \), the \( n \)-reformative domain is

\[ R_n \equiv R \cap \left\{(s, t) \in S \times S : \min_{q \in \mathbb{Z}_+} h_n(q, m(s, t)) \leq 0\right\} \]

\((s, t) \in R\) does not imply \( Q(m(s, t)) \neq \emptyset\), and \( R_n \) is the adequate domain for a fixed \( n \). However, for each \((s, t) \in R\), there is \( n^\dagger \in \mathbb{R}_{++} \) such that \( n > n^\dagger \) implies \( (s, t) \in R_n \), and thus \( R \) is appropriate in an “asymptotic” sense.\(^8\)

Let

\[ C_{b,1} \equiv \{(s, t) \times S \times S : m_b(s, t) = m_1(s, t)\} \]

\[ C_{1,0} \equiv \{(s, t) \times S \times S : m_1(s, t) = m_0(s, t)\} \]

The geometry of \( R \) can be studied through analyzing the “curves” \( C_{b,1} \) and \( C_{1,0} \). A simple but useful property of \( R \) is as follows.

**Lemma 4.** There is \( t^\dagger > s \) such that \((s, t) \in R\) implies \( t > t^\dagger \).

**Proof.** Note that \((\bar{s}, \bar{s}) \in C_{b,1} \cap C_{1,0}\). Taking total derivatives for \( C_{b,1} \) yields

\[ (z_b f(s|b) - z_1 f(s|a))ds = [(1 - z_0) f(t|b) - (1 - z_1) f(t|a)]dt; \]

by (A4), we have \( \frac{dt}{ds}|_{(\bar{s}, \bar{s})} = -\frac{z_1 f(\bar{s}|a)}{(1 - z_0) f(\bar{s}|b)} < 0 \). Taking total derivatives for \( C_{1,0} \) yields

\[ f(s|a)ds = -f(t|a)dt. \]

\(^8\)We don’t know whether \( R_n \) is increasing in \( n \).
Thus, $C_{1,0}$ is strictly decreasing; by (A4), we have $\frac{d}{ds}|_{(s, \bar{s})} = -\infty$. It follows that $(\bar{s}, \bar{s})$ is not in the closure of $R$. Therefore,

$$t^\dagger \equiv \inf \{ t \in S : \exists s \in S, (s, t) \in R \} > \bar{s}.$$ 

In fact, $t^\dagger$ can be chosen such that $m_b = m_1 = m_0$ at $(s, t^\dagger)$ for some $s \in S$.

Now we turn to the pivot information correspondence.

Fix $(\hat{\tau}, \bar{\tau}, m) \in [0, 1]^2 \times m(R_n)$. We rewrite (BR$_c$) and (BR$_r$) into

$$\varphi(s) \leq \left( 1 - \frac{\hat{\pi}_{n,1}}{\bar{\pi}_{n,1}} \right) + \frac{z_0}{z_1} \left( \frac{\mu_b \hat{\pi}_{n,0} \mu_0 \bar{\pi}_{n,0}}{\mu_1 \bar{\pi}_{n,1}} - \frac{\hat{\pi}_{n,1} \mu_0 \bar{\pi}_{n,0}}{\bar{\pi}_{n,1}} \right) \beta \quad \text{(BR'}_c)$$

$$\varphi(t) \geq -\left( 1 - \frac{\hat{\pi}_{n,1}}{\bar{\pi}_{n,1}} \right) + \frac{1-\gamma}{\bar{\pi}_{n,1}} \left( \frac{\mu_b \hat{\pi}_{n,0} \mu_0 \bar{\pi}_{n,0}}{\mu_1 \bar{\pi}_{n,1}} - \frac{\hat{\pi}_{n,1} \mu_0 \bar{\pi}_{n,0}}{\bar{\pi}_{n,1}} \right). \quad \text{(BR'}_r)$$

In (BR’$_c$) and (BR’$_r$), $\frac{\mu_b \hat{\pi}_{n,0}}{\mu_1 \bar{\pi}_{n,1}}$ is a likelihood ratio conditional on the lower pivot; $\frac{\mu_0 \bar{\pi}_{n,0}}{\bar{\pi}_{n,1}}$ is a likelihood ratio conditional on the upper pivot; $\frac{\hat{\pi}_{n,1}}{\bar{\pi}_{n,1}}$ is the relative magnitude of the double pivots. Denote the right-hand side of (BR’$_c$) by $\beta l_{n,c}(\hat{\tau}, \bar{\tau}; m)$ and the right-hand side of (BR’$_r$) by $l_{n,r}(\hat{\tau}, \bar{\tau}; m)$. $l_{n,c}$ can be interpreted as the conservatives’ pivot likelihood ratio, and $l_{n,r}$ can be interpreted as the radicals’ pivot likelihood ratio.

Fix $(s, t) \in R_n$. If both of the double pivots satisfy $\hat{h}_n(q, m(s, t)) < \gamma$, then $\hat{\tau} = \bar{\tau} = 1$, and $(s, t)$ determines the value of $l_{n,c}$ and $l_{n,r}$. Otherwise, the government has the freedom to choose $\hat{\tau} \in [0, 1]$ or $\bar{\tau} \in [0, 1]$; since $l_{n,c}$ and $l_{n,r}$ are continuous in $(\hat{\tau}, \bar{\tau})$, the government’s mixed reform strategy yields a (non-trivial) compact interval of values for $l_{n,c}$ or $l_{n,r}$. In summary, we can associate $(s, t)$ with a pair of compact intervals in the space of likelihood

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9When $\hat{q} = \bar{q}$, (BR$_c$) and (BR$_r$) fail to represent the citizens’ best response: $\bar{\pi}_{n,\omega} - \hat{\pi}_{n,\omega} = (2\bar{\tau} - 1)\pi_\omega - \pi(\hat{q} - 1; nm_\omega) - \pi(\hat{q}; nm_\omega)$, whereas the net pivot probability is $\pi_\omega - \pi(\hat{q} - 1; nm_\omega) - \pi(\hat{q}; nm_\omega)$. In contrast, (BR’$_c$) and (BR’$_r$), in ratio form, always give a valid representation as $\frac{2\bar{\tau} - 1}{2\bar{\tau} - 1} = \bar{\tau} = 1$. 

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ratios. Formally, let $m_{s,t} \equiv m(s,t)$ and

$$
\bar{L}_{n,c}(s,t) \equiv \begin{cases}
    l_{n,c}(1,1;m_{s,t}), & \text{if } h_n(\bar{q}, m_{s,t}) < \gamma \text{ and } h_n(\hat{q}, m_{s,t}) < \gamma; \\
    l_{n,c}([0,1],1;m_{s,t}), & \text{if } h_n(\bar{q}, m_{s,t}) = \gamma \text{ and } h_n(\hat{q}, m_{s,t}) < \gamma; \\
    l_{n,c}(1,[0,1];m_{s,t}), & \text{if } h_n(\bar{q}, m_{s,t}) < \gamma \text{ and } h_n(\hat{q}, m_{s,t}) = \gamma; \\
    l_{n,c}([0,1],[0,1];m_{s,t}), & \text{if } h_n(\bar{q}, m_{s,t}) = \gamma \text{ and } h_n(\hat{q}, m_{s,t}) = \gamma.
\end{cases}
$$

To incorporate the possibility that a conservative’s best response is uninformative, let

$$
L_{n,c}(s,t) \equiv \begin{cases}
    \bar{L}_{n,c}(s,t), & \text{if } \max \bar{L}_{n,c}(s,t) \geq 0; \\
    [\min \bar{L}_{n,c}(s,t),0], & \text{if } \max \bar{L}_{n,c}(s,t) < 0.
\end{cases}
$$

Similarly, we can define $L_{n,r}(s,t)$.

By the continuity of $h_n$, $l_{n,c}$ and $l_{n,r}$, we have:

**Lemma 5.** The pivot information correspondence $(L_{n,c}, L_{n,r}) : R_n \Rightarrow \mathbb{R} \times \mathbb{R}$ is upper hemi-continuous and compact-and-convex-valued.

The following characterization of reformative equilibria is immediate:

**Proposition 5.** $(s^*, t^*) \in R_n$ constitutes a reformative equilibrium if and only if $(\varphi(s^*), \varphi(t^*)) \in (\beta L_{n,c}, L_{n,r})(s^*, t^*)$.

Proposition 5 is silent about existence of reformative equilibria. Fixed point theorems are not applicable: in Section 4.2, we shall see that the correspondence $(\varphi^{-1} \circ (\beta L_{n,c}), \varphi^{-1} \circ L_{n,r})$ may map some $(s,t) \in R_n$ to the complement of the closure of $R_n$.

**4 Large mean population**

In this section, we are primarily interested in a sequence of reformative equilibria whose mean scales $(m_n)$ converge to $m$ with $m_b < m_1 < m_0$. We say that strong information
aggregation\textsuperscript{10} is achievable if there is such a sequence of reformative equilibria. Using limit pivot information correspondences, we identify conditions under which strong information aggregation is not achievable in a class of stable “limit equilibrium” when the government has slippery-slope concern ($\mu_0 > 0$), whereas information aggregation is always possible when the government has no slippery-slope concern ($\mu_0 = 0$).

4.1 Double pivots, likelihood ratios and relative magnitude

Throughout this subsection, we fix a sequence of reformative equilibria whose mean scales $\left( m_n \right)$ converge to $m$ with $m_b < m_1 < m_0$.

4.1.1 Limit double pivots

Let

$$\hat{x}(m) = \frac{m_b - m_1}{\ln m_b - \ln m_1}$$

$$\bar{x}(m) = \frac{m_0 - m_1}{\ln m_0 - \ln m_1}.$$  

Note that $m_b < \hat{x} < m_1 < \bar{x} < m_0$.

Recall that $h_n(q, m') = \frac{\mu_b}{\mu_1} e^{-n(m_b - m_1)} \left( \frac{m_1}{m_b} \right)^q + \frac{\mu_0}{\mu_1} e^{-n(m_0 - m_1)} \left( \frac{m_1}{m_0} \right)^q$. Let $\hat{q}_n$ and $\bar{q}_n$ be the equilibrium double pivots when the mean population is $n$. The following lemma states that $\hat{x}$ and $\bar{x}$ represent the limit double pivots.

**Lemma 6.** Let $(q_n)$ be a sequence in $\mathbb{Z}_+$ such that $\left( \frac{q_n}{n} \right)$ converges.

1. If $\lim_{n \to \infty} \frac{q_n}{n} \in (\hat{x}, \bar{x})$, then $\lim_{n \to \infty} h_n(q_n, m_n) = 0$.

2. If $\lim_{n \to \infty} \frac{q_n}{n} < \hat{x}$ or $\lim_{n \to \infty} \frac{q_n}{n} > \bar{x}$, then $\lim_{n \to \infty} h_n(q_n, m_n) = \infty$;

3. $\lim_{n \to \infty} \frac{q_n}{n} = \hat{x}$ and $\lim_{n \to \infty} \frac{q_n}{n} = \bar{x}$.

\textsuperscript{10}Our approach relies on distinguishable limit double pivots and is not directly applicable to the case $m_n \to m$ with $m_b = m_1 = m_0$. In the current draft, we argue that $m_b = m_1 = m_0$ is unstable as a “limit equilibrium” but don’t have a proof of whether $m_b = m_1 = m_0$ implies failure of information aggregation.
Proof. The argument for (1) and (2) are similar. (3) follows immediately from (1) and (2).

Here we present the details only for (1).

Suppose \( \lim_{n \to \infty} \frac{q_n}{m_n} \in (\hat{x}, \check{x}) \). Note that

\[
\ln \left( e^{-n(m_{n:b} - m_{n:1})} \left( \frac{m_{n:b}}{m_{n:1}} \right)^{q_n} \right) = -n \left[ (m_{n:b} - m_{n:1}) - \frac{q_n}{n} (\ln m_{n:b} - \ln m_{n:1}) \right].
\]

Since \( \lim_{n \to \infty} \frac{q_n}{m_n} > \hat{x} \), we have

\[
\lim_{n \to \infty} \left[ (m_{n:b} - m_{n:1}) - \frac{q_n}{n} (\ln m_{n:b} - \ln m_{n:1}) \right] > 0.
\]

Thus, \( e^{-n(m_{n:b} - m_{n:1})} \left( \frac{m_{n:b}}{m_{n:1}} \right)^{q_n} \to 0 \). Similarly, since \( \lim_{n \to \infty} \frac{q_n}{m_n} < \check{x} \), we have

\[
\lim_{n \to \infty} \left[ (m_{n:0} - m_{n:1}) - \frac{q_n}{n} (\ln m_{n:0} - \ln m_{n:1}) \right] > 0
\]

and \( e^{-n(m_{n:0} - m_{n:1})} \left( \frac{m_{n:0}}{m_{n:1}} \right)^{q_n} \to 0 \). In summary, \( \lim_{n \to \infty} h_n(q_n, m_n) = 0 \). \( \square \)

### 4.1.2 Limit likelihood ratios

Let \( \hat{x}_n \) be the lower solution to \( h_n(x, m_n) = \gamma \). Recall that

\[
\hat{\pi}_{n:o} \equiv \hat{\pi}_n(\hat{q}_n - 1; nm_\omega) + (1 - \hat{\pi}_n)\pi(\hat{q}_n; nm_\omega).
\]

When \( \hat{x}_n \) is an integer, \( \hat{x}_n = \hat{q}_n \); for each \( \hat{\pi}_n \in [0, 1] \), there is \( \hat{\pi}_n' \in [0, 1] \) such that

\[
\frac{\hat{\pi}_{n:b}}{\hat{\pi}_{n:1}} = \frac{\hat{\pi}_n'(\hat{x}_n - 1, nm_{n:b})}{\pi(\hat{x}_n - 1, nm_{n:1})} + (1 - \hat{\pi}_n') \frac{\pi(\hat{x}_n, nm_{n:b})}{\pi(\hat{x}_n, nm_{n:1})}
\]

\[
= \frac{\pi(\hat{x}_n, nm_{n:b})}{\pi(\hat{x}_n, nm_{n:1})} \left[ \hat{\pi}_n(m_{n:1}; \frac{m_{n:b}}{m_{n:1}}) + (1 - \hat{\pi}_n') \right].
\]
When \( \hat{x}_n \) is not an integer, \( \hat{x}_n - 1 < \hat{q}_n - 1 < \hat{x}_n \), and there is \( \hat{\tau}'_n \in [0, 1] \) such that

\[
\hat{\pi}_{n:b} = \hat{\tau}'_n \frac{\pi(\hat{x}_n - 1, nm_{n;b})}{\pi(\hat{x}_n - 1, nm_{n;b})} + (1 - \hat{\tau}'_n) \frac{\pi(\hat{x}_n, nm_{n;b})}{\pi(\hat{x}_n, nm_{n;b})} = \frac{\pi(\hat{x}_n, nm_{n;b})}{\pi(\hat{x}_n, nm_{n:1})} \left[ \hat{\tau}'_{n:1} + (1 - \hat{\tau}'_n) \right].
\]

Since \( \frac{\hat{x}_n}{n} \to \hat{x} < \hat{x} \), we know from the proof of Lemma 6 that \( \frac{\mu_0}{\mu_1} \frac{\pi(\hat{x}_n, nm_{n:0})}{\pi(\hat{x}_n, nm_{n:1})} \to 0 \). Thus, \( \frac{\mu_0}{\mu_1} \frac{\pi(\hat{x}_n, nm_{n,b})}{\pi(\hat{x}_n, nm_{n:1})} \to \gamma \). In summary, a limit likelihood ratio conditional on the limit lower pivot falls in a compact interval determined by \( \gamma \) and \( m \); a similar claim holds for the limit upper pivot.

**Lemma 7.** (1) \( \lim_{n \to \infty} \frac{\mu_0 \pi(\hat{x}_n, nm_{n,b})}{\mu_1 \pi(\hat{x}_n, nm_{n:1})} \in \gamma \left[ 1, \frac{m_1}{m_0} \right] \) and \( \lim_{n \to \infty} \frac{\mu_0 \pi(\hat{x}_n, nm_{n:0})}{\mu_1 \pi(\hat{x}_n, nm_{n:1})} = 0 \).

(2) \( \lim_{n \to \infty} \frac{\mu_0 \pi_{n,b}}{\mu_1 \pi_{n,1}} = 0 \) and \( \lim_{n \to \infty} \frac{\mu_0 \pi_{n,0}}{\mu_1 \pi_{n,1}} \in \gamma \left[ \frac{m_1}{m_0}, 1 \right] \).

By Lemma 7, a citizen knows for sure that the fundamental state is not \( a_0 \) conditional on the limit lower pivot, and she knows for sure that the fundamental state is not \( b \) conditional on the limit upper pivot.

### 4.1.3 Limit relative magnitude

The technique here is developed in Myerson (2000). By the Stirling’s formula, we have

\[
q! \approx \sqrt{2\pi q} \left( \frac{q}{e} \right)^q
\]

when \( q \) is sufficiently large.\(^{11}\) Applying this approximation to \( \pi(q; nm_{n:1}) \) yields

\[
\frac{1}{n} \ln \pi(q; nm_{n:1}) \approx -\ln(2\pi q) - \frac{q}{2n} - nm_{n:1} \left[ \frac{q}{nm_{n:1}} \ln \frac{q}{nm_{n:1}} - \frac{q}{nm_{n:1}} + 1 \right].
\]

\(^{11}\) \( \ln(q!) - \ln \left( \sqrt{2\pi q} \left( \frac{q}{e} \right)^q \right) \in \left( \frac{1}{12q+1}, \frac{1}{12q} \right) \).
Let

\[ \hat{\psi}(m) \equiv \int_{1}^{\frac{\hat{x}(m)}{m_1}} \ln x \, dx \]

\[ \tilde{\psi}(m) \equiv \int_{1}^{\frac{\tilde{x}(m)}{m_1}} \ln x \, dx. \]

We have

\[ \frac{1}{n} \ln \frac{\hat{\pi}_{n,1}}{\tilde{\pi}_{n,1}} \rightarrow \hat{\psi}(m) - \tilde{\psi}(m). \]

When \( \hat{\psi}(m) \neq \tilde{\psi}(m) \), the limit relative magnitude is well-specified.

**Lemma 8.** (1) If \( \hat{\psi}(m) < \tilde{\psi}(m) \), then \( \lim_{n \to \infty} \frac{\hat{\pi}_{n,1}}{\tilde{\pi}_{n,1}} = 0 \). (2) \( \hat{\psi}(m) \leq \tilde{\psi}(m) \).

**Proof.** (1) follows from our argument prior to this lemma.

To show (2), suppose \( \hat{\psi}(m) > \tilde{\psi}(m) \). Then \( \lim_{n \to \infty} \frac{\hat{\pi}_{n,1}}{\tilde{\pi}_{n,1}} = 0 \). Dividing both sides of (BR_r) by \( \mu_1(1 - z_1)\hat{\pi}_{n,1} \) yields

\[ \frac{\mu_b(1 - z_b)}{\mu_1(1 - z_1)} \left( \frac{\hat{\pi}_{n,b}}{\hat{\pi}_{n,1}} - \frac{\tilde{\pi}_{n,b}}{\tilde{\pi}_{n,1}} \right) \varphi(t) \geq \left( 1 - \frac{\hat{\pi}_{n,1}}{\tilde{\pi}_{n,1}} \right) + \frac{\mu_0(1 - z_0)}{\mu_1(1 - z_1)} \left( \frac{\tilde{\pi}_{n,0}}{\tilde{\pi}_{n,1}} - \frac{\hat{\pi}_{n,0}}{\hat{\pi}_{n,1}} \right) \]

By Lemma 7, this inequality converges to \( 0 \cdot \varphi(t) \geq 1 \). Let \( t_n \) be the radicals’ equilibrium cutoff signal at \( n \). Then \( t_n \to \tilde{g} \), which contradicts Lemma 4.

Next, we divide our analysis into two cases: (1) \( \hat{\psi} < \tilde{\psi} \), the case with *dominated (upper) pivot*; (2) \( \hat{\psi} = \tilde{\psi} \), the case with *undominated pivots*. Our focus is on the first case, as the second case is “unstable”.

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4.2 Dominated pivots

Suppose $\hat{\psi}(m) < \tilde{\psi}(m)$ and consider $n \to \infty$. By Lemma 8, the lower pivot becomes infinitely more likely than the upper pivot. Let $\hat{l} \equiv \lim_{n \to \infty} \frac{\mu_{b} \hat{\pi}_{n,b}}{\mu_{1} \hat{\pi}_{n,1}} \in \left[1, \frac{m_{1}}{m_{b}}\right]$. $(BR'_c)$ converges to

$$\varphi(s) \leq \frac{\beta z_{1}}{\gamma z_{b} \hat{l}}.$$

Thus, the conservatives play a non-trivial increasing cutoff protest strategy. Turning to the radicals, $(BR'_r)$ converges to

$$\varphi(t) \geq -\frac{1}{1 - \frac{z_{1}}{z_{b}}} \gamma \hat{l}.$$

Thus, the radicals participate in the protest regardless of their signals when $n$ is sufficiently large.

Using the conservatives’ cutoff signals as domain, we formulate our first notion of limit reformative equilibrium.

**Definition 5.** $s^* \in S$ is a limit reformative equilibrium with dominated pivot if: (1) $\varphi(s^*) \in \frac{\beta z_{1}}{\gamma z_{b}} \left[\frac{m_{b}(s^*, \overline{s})}{m_{1}(s^*, \overline{s})}, 1\right]$; (2) $\hat{\psi}(m(s^*, \overline{s})) < \tilde{\psi}(m(s^*, \overline{s}))$.

When there is a limit reformative equilibrium with dominated pivot, we can say that information aggregation with dominated pivot is achievable.

The following proposition is immediate.

**Proposition 6.** Let $(s_{n}, t_{n})$ be the citizens’ protest strategy of a reformative equilibrium when the mean population is $n$. Suppose $(s_{n}, t_{n}) \to (s^*, t^*)$ such that $m^* \equiv m(s^*, t^*)$ satisfies both $m_{b}^* < m_{1}^* < m_{0}^*$ and $\hat{\psi}(m^*) < \tilde{\psi}(m^*)$. Then $t^* = \overline{s}$ and $s^*$ is a limit reformative equilibrium with dominated pivot.

A converse of Proposition 6 is also true.
Proposition 7. Suppose $s^* \in S$ is a limit reformative equilibrium with dominated pivot. Then there is $s_n \to s^*$ and $n^1 > 0$ such that for each $n > n^1$, $(s_n, \bar{s})$ constitutes a reformative equilibrium when the mean population is $n$.

By Proposition 7, if there is a limit equilibrium with dominated pivot, then strong information aggregation is achievable. We postpone the proof of Proposition 7 until we show some key properties of limit reformative equilibria with dominated pivot.

By definition, the set of limit reformative equilibria with dominated pivot is the intersection of 
\[
S^* \equiv \{ s \in S : \varphi(s) \in [0, \infty) \}
\]
and
\[
S^< \equiv \{ s \in S : \hat{\psi}(m(s, \bar{s})) < \hat{\psi}(m(s, \bar{s})) \}.
\]

Properties of $S^*$. Note that $S^*$ is non-empty: by (A4), $\varphi(S) = [0, \infty]$; $\frac{m_b(s, \bar{s})}{m_1(s, \bar{s})} \in [1 - z_b, 1]$. Let
\[
H_b(s) \equiv \frac{z_b m_1(s, \bar{s})}{m_b(s, \bar{s})} \varphi(s) = \frac{z_b f(s|b)}{1 - z_b F(s|b)} \cdot \left( \frac{z_1 f(s|a)}{1 - z_1 F(s|a)} \right)^{-1}.
\]

Note that $H_b$ can be regarded as a hazard ratio. It is straightforward to identify the minimum and the maximum of $S^*$.

Lemma 9. $\min S^* = \varphi^{-1} \left( \frac{z_1}{z_b} \right)$, $\max S^* = \max H_b^{-1} \left( \frac{\beta}{\gamma} \right) < \bar{s}$. Both $\min S^*$ and $\max S^*$ are strictly decreasing in $\frac{\beta}{\gamma}$.

Proof. Note that $S^* = \{ s \in S : \varphi(s) \leq \frac{\beta}{\gamma} \} \cap \{ s \in S : H_b(s) \geq \frac{\beta}{\gamma} \}$. Since $\varphi(s) \geq \frac{\beta}{\gamma}$ implies $H_b(s) > \frac{\beta}{\gamma}$, $\min S^* = \varphi^{-1} \left( \frac{z_1}{z_b} \right)$, $\{ s \in S : \varphi(s) \leq \frac{\beta}{\gamma} \} = [\varphi^{-1} \left( \frac{z_1}{z_b} \right), \bar{s}]$ implies $\max S^* = \max H_b^{-1} \left( \frac{\beta}{\gamma} \right)$.
max \{ s \in S : H_b(s) \geq \frac{\beta}{\gamma} \}. Since \( H_b(\bar{x}) = 0 \), max\( S^* = \max H_b^{-1} \left( \frac{\beta}{\gamma} \right) < \bar{x} \) and is strictly decreasing in \( \frac{\beta}{\gamma} \).

By Lemma 9, the conservatives’ equilibrium cutoff signal cannot exceed \( \max S^* \) given a dominated (upper) pivot.

**Properties of \( S^< \).** To characterize \( S^< \) per se is complicated. We first identify a necessary condition for \( s \in S^< \).

**Lemma 10.** (1) If \( s \in S^< \), then \( \frac{F(s|b)}{F(s|a)} < \frac{2z_1-z_0}{z_b} \).

(2) When \( \frac{2z_1-z_0}{z_b} \leq 1 \), \( S^< = \emptyset \).

(3) When \( \frac{2z_1-z_0}{z_b} > 1 \), let \( s^\dagger \) be the unique solution to \( \frac{F(s|b)}{F(s|a)} = \frac{2z_1-z_0}{z_b} \), then \( s \in S^< \) implies \( s > s^\dagger \).

**Proof.** We start with decomposing \( \hat{\psi}(m) \) and \( \tilde{\psi}(m) \). Let \( \eta_1(x) \equiv \int_1^x \ln \tilde{\alpha} dx \) and \( \eta_2(x) \equiv \frac{x-1}{\ln x} \). Then \( \hat{\psi}(m) = \eta_1 \left( \eta_2 \left( \frac{m_b}{m_1} \right) \right) \) and \( \tilde{\psi}(m) = \eta_1 \left( \eta_2 \left( \frac{m_0}{m_1} \right) \right) \). Since \( x-1 > \ln x = \frac{dn}{dx} \), we have

\[
\frac{1}{2} \left( 1 - \eta_2 \left( \frac{m_b}{m_1} \right) \right)^2 < \hat{\psi}(m)
\]

\[
\tilde{\psi}(m) < \frac{1}{2} \left( \eta_2 \left( \frac{m_0}{m_1} \right) - 1 \right)^2.
\]

Thus, \( \hat{\psi}(m) < \tilde{\psi}(m) \) implies \( \eta_2 \left( \frac{m_b}{m_1} \right) + \eta_2 \left( \frac{m_0}{m_1} \right) > 2 \). Since \( \eta_2(x) < \frac{1}{2} (x-1) + 1 \), \( \eta_2 \left( \frac{m_b}{m_1} \right) + \eta_2 \left( \frac{m_0}{m_1} \right) > 2 \) implies

\[
\frac{m_b}{m_1} + \frac{m_0}{m_1} > 2 \quad (\star)
\]

Evaluating \( (\star) \) at \( (s, \bar{x}) \) completes the proof of (1) and (2). By Lemma 0 in Duggan and Martinelli (2001), \( \frac{F(s|b)}{F(s|a)} \) is strictly decreasing in \( s \). When \( \frac{2z_1-z_0}{z_b} > 1 \), \( s^\dagger \) is well-defined, and (3) follows from (1).

By Lemma 10, the conservatives’ cutoff signal must exceed \( s^\dagger \) to support a dominated
(upper) pivot. On the other hand, when $n$ is sufficiently large and $s < s < s^\dagger$, we have $(s, \bar{s}) \in R_n$ but $\varphi^{-1} \circ L_{n,r}(s, \bar{s}) \approx s$ by Lemma 8: the correspondence $(\varphi^{-1} \circ (\beta L_{n,c}), \varphi^{-1} \circ L_{n,r})$ maps $(s, \bar{s})$ to the complement of the closure of $R_n$.

4.2.1 Conditions for existence

From Lemmas 9 and 10, we see a tension between the conservatives’ equilibrium response and a dominated (upper) pivot. Now we are ready to identify non-trivial necessary conditions for the existence of limit reformative equilibrium with dominated pivot.

**Proposition 8.** If there is a limit reformative equilibrium with dominated pivot, then (1) $\frac{2z_1 - z_0}{z_b} > 1$ and (2) $\frac{\beta}{\gamma} < l^\dagger$ for some $l^\dagger > 0$.

Let $\sigma^\dagger : [1, \infty] \to S$ be the inverse of $F(\cdot \mid b) / F(\cdot \mid a)$. When the hazard ratio $H_b$ is strictly decreasing in $s$, Proposition 8 can be written more compactly.

**Corollary 1.** Suppose $H_b$ is strictly decreasing in $s$. If there is a limit reformative equilibrium with dominated pivot, then (1) $\frac{2z_1 - z_0}{z_b} > 1$ and (2) $\frac{\beta}{\gamma} < H_b \left( \sigma^\dagger \left( \frac{2z_1 - z_0}{z_b} \right) \right)$.

The necessary condition identified in Proposition 8 or Corollary 1 are independent of $\mu \in \text{int } \Delta(\Omega)$, the common prior over fundamental states. As long as there is a slippery-slope concern, information aggregation with dominated pivot is impossible given certain primitives, for example, when $\frac{\beta}{\gamma}$ is too large.

For a sufficient condition, recall the decomposition of $\hat{\psi}$ and $\tilde{\psi}$ in the proof of Lemma 10: $\eta_1(x)$ is $\cup$-shaped with minimizer at $x = 1$, $\eta_2(x)$ is strictly increasing in $x$. It can be shown that $(\hat{\psi} - \tilde{\psi})(m(s, \bar{s})$ is decreasing in $s$ around $\bar{s}$. When $\hat{\psi}(m(\bar{s}, \bar{s})) < \tilde{\psi}(m(\bar{s}, \bar{s}))$, or equivalently, $\eta_1(\eta_2(\frac{1-z_b}{1-z_1})) < \eta_1(\eta_2(\frac{1-z_b}{1-z_1}))$, there is $s^\dagger \in \text{int } S$ such that $(s^\dagger, \bar{s}) \subset S^<$. The following sufficient condition for the existence of limit reformative equilibrium with dominated pivot is immediate.

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Proposition 9. Suppose \( \eta_1 \left( \eta_2 \left( \frac{1-z_2}{1-z_1} \right) \right) < \eta_1 \left( \eta_2 \left( \frac{1-z_0}{1-z_1} \right) \right) \). Then there is \( l^4 > 0 \) such that \( \frac{\beta}{\gamma} < l^4 \) implies \( S^* \cap S^c \neq \emptyset \); i.e., there is a limit reformative equilibrium with dominated pivot.

Our conditions about \( \frac{\beta}{\gamma} \) is similar to those in Battaglini (2017). However, information aggregation in Battaglini (2017) is obstructed by the lack of signals with high precision. In our model, there is no lack of signals with high precision by (A4), and information aggregation is obstructed by “rational expectation” about relative magnitude of the double pivots. This point will become clearer in Section 4.2.3.

Before we proceed to the case with \( \hat{\psi}(m) = \tilde{\psi}(m) \). We present a proof of Proposition 7 using pivot information correspondence. By an almost identical argument, it is shown that information aggregation is always achievable when there is no slippery-slope concern.

4.2.2 Proof of Proposition 7

Let \( L_c(s) \equiv \frac{\beta z_1}{\gamma z_0} \left[ \frac{m_b(s^*, \bar{s})}{m_1(s^*, \bar{s})}, 1 \right] \). Consider \( s^* \in S^* \cap S^c \) such that \( \varphi(s^*) \in \text{int} \ L_c(s^*) \). When \( \varphi(s^*) \) coincides with the boundaries of \( L_c(s^*) \), it suffices to use a sequence of vanishing perturbations toward the interior.

Choose \( \delta > 0 \) such that \( K \equiv [s^*, s^* + \delta] \) has the following property: \( K \subset S^c \); with the domain restricted to \( K \), the graph of \( \varphi \) is in the interior of the graph of \( L_c \). For each \( s \in K \), let \( \hat{x}_n(s) \) (or \( \check{x}_n(s) \)) be the lower (or upper) solution to \( h_n(x, m(s, \bar{s})) = \gamma \). By Lemmas 6, 7 and 8, when \( \hat{x}_n(s) \) is an integer and regardless of whether \( \check{x}_n(s) \) is an integer, (BR\(_c^r\)) and (BR\(_r^l\)), evaluated at \( (s, \bar{s}) \), can be written as

\[
\varphi(s) \leq \beta \left( \frac{1-O(e^n)}{2 \frac{z_1}{z_0} \gamma - O(e^n)} \right) \left[ \frac{m_b}{m_1}, 1 \right] \\
\varphi(\bar{s}) \geq -\left( \frac{1-O(e^n)}{1-\frac{z_2}{z_1} \gamma - O(e^n)} \right) \left[ \frac{m_b}{m_1}, 1 \right].
\]

Since \( \left( \frac{1-O(e^n)}{2 \frac{z_1}{z_0} \gamma - O(e^n)}, \frac{1-O(e^n)}{1-\frac{z_2}{z_1} \gamma - O(e^n)} \right) \) converges uniformly to \( \left( \frac{1}{2 \frac{z_1}{z_0} \gamma}, \frac{1}{1-\frac{z_2}{z_1} \gamma} \right) \) on \( K \), there is \( n_K > 0 \)
such that: for each \( n > n K \), if \( \hat{x}_n(s) \) is an integer, then \((s, \bar{s})\) constitutes a reformative equilibrium when the mean population is \( n \).

It remains for us to find \( s_n \in K \) such that \( \hat{x}_n(s_n) \) is an integer and \( s_n \to s^* \). This task is accomplished through a simple lemma: when \( n \) is sufficiently large, a small increase in \( s \) leads to a large decrease in \( \hat{x}_n \), and thus there is \( s \) close to \( s^* \) and “integerizing” \( \hat{x}_n \).

Lemma 11. \( \lim_{n \to \infty} \left( \frac{1}{n} \frac{d \hat{x}_n}{ds} \right) = \left( \frac{\frac{d}{m_b} - 1}{\ln \left( \frac{m}{m_1} \right)} \right) z_b f(s|b) + \left( 1 - \frac{1}{m_1} \right) z f(s|a) > 0 \).

Proof. We start with studying \( \hat{x}_n \) as the lower solution to \( h_n(x, m) = \gamma \). Consider the first order partial derivatives of \( h_n \):

\[
\frac{\partial h_n}{\partial x}|_{y=\hat{x}_n} = \sum_{\omega \neq a_1} \frac{\mu_\omega}{\mu_1} e^{-n(m_\omega-m_1)} \left( \frac{m_\omega}{m_1} \right)^x \ln \left( \frac{m_\omega}{m_1} \right); \\
\frac{\partial h_n}{\partial m_\omega}|_{y=\hat{x}_n} = \frac{\mu_\omega}{\mu_1} e^{-n(m_\omega-m_1)} \left( \frac{m_\omega}{m_1} \right)^x \left( -n + \frac{x}{m_\omega} \right); \\
\frac{\partial h_n}{\partial m_1}|_{y=\hat{x}_n} = \sum_{\omega' \neq a_1} \frac{\mu_{\omega'}}{\mu_1} e^{-n(m_{\omega'}-m_1)} \left( \frac{m_{\omega'}}{m_1} \right)^x \left( n - \frac{x}{m_1} \right).
\]

By definition of \( \hat{x}_n \) and Lemma 6, we have

\[
\frac{\partial h_n}{\partial x}|_{y=\hat{x}_n} = (\gamma - O(e^n)) \ln \left( \frac{m_b}{m_1} \right) + O(e^n) \ln \left( \frac{m_0}{m_1} \right) \\
\frac{\partial h_n}{\partial m_b}|_{y=\hat{x}_n} = n (\gamma - O(e^n)) \left( \frac{\hat{x}}{m_b} - 1 + o(n) \right) \\
\frac{\partial h_n}{\partial m_1}|_{y=\hat{x}_n} = n \gamma \left( 1 - \frac{\hat{x}}{m_1} + o(n) \right) \\
\frac{\partial h_n}{\partial m_0}|_{y=\hat{x}_n} = nO(e^n) \left( \frac{\hat{x}}{m_b} - 1 + o(n) \right).
\]

Since \( \frac{\partial m_\omega}{\partial s}|_{(s, \bar{s})} = -z_\omega f(s|\omega) \), the lemma follows from the chain rule. \( \square \)
By Lemma 11, there is $\delta_n \to 0$ such that: when $n$ is sufficiently large, $s^* + \delta_n < s^* + \delta$, and $\hat{x}_n(s^* + \delta_n)$ is an integer. The proof of Proposition 7 is complete.

4.2.3 No slippery-slope concern

Suppose $\mu_0 = 0$. Our protest game becomes Battaglini’s model with radical-partisan. Focusing on “reformative” equilibrium in which $m_b < m_1$, we have:

- the government plays an increasing cutoff reform strategy;
- the conservatives play an increasing cutoff protest strategy;
- the radicals participate in the protest regardless of their signals.

Consider the fixed-point formulation

$$\varphi(s) \in \frac{\beta_{z_1}}{\gamma_{z_b}} \left[ \frac{m_b(s, \bar{s})}{m_1(s, \bar{s})}, 1 \right].$$

Regarding $(FP_{\mu_0=0})$ in relation to reformative equilibrium, we have:

1. since $\varphi(S) = [0, \infty]$ by (A4), there is $s^* \in \text{int } S$ satisfying $(FP_{\mu_0=0})$;

2. fix a mean population $n > 0$; if we formulate pivot information correspondence as in Section 3.3, then a counterpart of Lemma 5 implies the existence of reformative equilibrium;

3. if $s_n \in S$ satisfies $(FP_{\mu_0=0})$ and the unique solution to $h_n(x, m(s_n, \bar{s}))$ is an integer, then $(s_n, \bar{s})$ constitutes a reformative equilibrium.

Since Lemma 11 also holds for $\mu_0 = 0$, there is a sequence of reformative equilibria whose mean scales $m_n \to m$ with $m_b < m_1$;\footnote{$s^* \in \text{int } S$ implies $m_b(s^*, \bar{s}) < m_1(s^*, \bar{s})$.} i.e., information aggregates. To summarize:
Proposition 10. Suppose $\mu_0 = 0$. There is $s^* \in \text{int} S$ such that $\varphi(s^*) \in \frac{\beta z_1}{\gamma z_b} \left[ \frac{m_0(s^*, \bar{\pi})}{m_1(s^*, \bar{\pi})}, 1 \right]$. For each such $s^*$, there is a sequence $s_n \to s^*$ such that for each $n$, $(s_n, \bar{\pi})$ constitutes a reformative equilibrium when the mean population is $n$.

Comparing Proposition 10 to Definition 5 and Section 4.2.1, we see how slippery slope forces the citizens to assess relative magnitude of the double pivots and may obstruct information aggregation.

### 4.3 Undominated pivots

Suppose $\hat{\psi}(m) = \check{\psi}(m)$.

**Limit reformative equilibrium.** Given $\omega \in \Omega$, $(s, t) \in R$ and $(\hat{\tau}, \check{\tau}) \in [0, 1]^2$, let

\[ \hat{T}_\omega = (1 - \hat{\tau}) + \hat{\tau} \frac{\check{x}(m(s, t))}{m_\omega(s, t)} \]
\[ \check{T}_\omega = (1 - \check{\tau}) \frac{\check{x}(m(s, t))}{m_\omega(s, t)} + \check{\tau}. \]

$(\hat{\tau}, \check{\tau})$ is the limit of $(\hat{\tau}_n, \check{\tau}_n)$, the government’s “mixed” reform strategy when the mean population is $n$.\(^{14}\) We introduce an additional variable $\xi \in [0, \infty)$\(^{15}\) to capture the limit relative magnitude of the double pivots. Let

\[ l_c(s, t; \hat{\tau}, \check{\tau}; \xi) \equiv \frac{\beta z_1}{\gamma z_b} \left[ \hat{T}_1 - \xi \left( \hat{T}_1 + \frac{z_0}{z_1} \gamma \check{T}_0 \right) \right] \]
\[ l_r(s, t; \hat{\tau}, \check{\tau}; \xi) \equiv \frac{1 - z_1}{(1 - z_b) \gamma} \left[ \frac{\check{T}_1 + \frac{1 - z_0}{z_1} \gamma \hat{T}_0}{\hat{T}_1} \right]. \]

$(l_c, l_r)$ represents the citizens’ pivot information in the limit. Here is a subtlety about $\xi$: it is not the limit of $\frac{\check{x}_n}{x_n}$ but the limit of $\frac{\check{x}(x_n, nm_n)}{\hat{x}(x_n, nm_n)}$, in which $\hat{x}_n$ (or $\check{x}_n$) is the lower (or upper)

\(^{14}\)See also Section 4.1.2.

\(^{15}\)By Lemma 4, we can choose a compact domain for $\xi$. 

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solution to \( h_n(x, \mathbf{m}_n) \).\footnote{The Poisson probabilities are replaced by their smooth approximation based on the Stirling’s formula.} The relation between \( \hat{\pi}_{n,1} \) and \( \hat{\pi}_{n,1}^{(x;m_n,n)} \) is represented by

\[
\lim_{n \to \infty} \frac{\hat{\pi}_{n,1}}{\hat{\pi}_{n,1}^{(x;m_n,n)}} = \lim_{n \to \infty} \left[ \frac{\pi(x_n;n,m_{n,1}) (1 - \hat{\tau}_n) \frac{q_0}{m_{n,1}} + \hat{\tau}_n}{\pi(x_n;n,m_{n,1}) (1 - \hat{\tau}_n) + \hat{\tau}_n \frac{q_0}{m_{n,1}}} \right] = \frac{\tilde{T}_1}{\tilde{T}_1}.
\]

Thus, the government’s “mixed” reform strategy is isolated from the limit relative magnitude. By examining \( \lim_{n \to \infty} \frac{\pi(x_n;n,m_{n,1})}{\pi(x_n;n,m_{n,1})} \), we see that \( \xi \) cannot be identified solely by the the limit mean scale \( \mathbf{m} \).

Using the citizens’ cutoff signals as domain, we formulate a notion of limit reformative equilibrium more general than that in Section 4.2.

**Definition 6.** \((s^*, t^*) \in R\) is a limit reformative equilibrium if there is \((\hat{\tau}, \tilde{\tau}; \xi) \in [0, 1]^2 \times [0, \infty)\) such that: (1) \( \varphi(s^*) = \max \{ l_c(s^*, t^*; \hat{\tau}, \tilde{\tau}; \xi), 0 \} \) and \( \varphi(t^*) = \max \{ l_r(s^*, t^*; \hat{\tau}, \tilde{\tau}; \xi), 0 \} \); (2) \( \hat{\psi}(m(s^*, t^*)) \leq \tilde{\psi}(m(s^*, t^*)) \) and \( \xi \left( \tilde{\psi}(m(s^*, t^*)) - \hat{\psi}(m(s^*, t^*)) \right) = 0 \).

By Definition 6, \( \xi = 0 \) when \( \hat{\psi}(m(s^*, t^*)) < \tilde{\psi}(m(s^*, t^*)) \), and we obtain Definition 5, the notion of limit reformative equilibrium with dominated pivot, as a special case. When \( \hat{\psi}(m(s^*, t^*)) = \tilde{\psi}(m(s^*, t^*)) \), we call \((s^*, t^*)\) a limit reformative equilibrium with undominated pivots. A counterpart of Proposition 6 is immediate:

**Proposition 11.** Let \((s_n, t_n)\) be the citizens’ protest strategy of a reformative equilibrium when the mean population is \( n \). Suppose \((s_n, t_n) \to (s^*, t^*)\) such that \( m^* = m(s^*, t^*) \) satisfies both \( m_0^* < m_1^* < m_0^* \). Then \((s^*, t^*)\) is a limit reformative equilibrium.

We abstain from stating and proving a counterpart of Proposition 7 in this draft.

**Instability of limit equilibrium with undominated pivots** As equality is, at least heuristically, non-generic, limit equilibria with undominated pivots are *unstable* in the following sense.

Suppose \( \hat{\psi}(m(s,t)) = \tilde{\psi}(m(s,t)) \) defines a (one-dimensional) curve in \( R \). \((s^*, t^*)\), being a limit reformative equilibrium with undominated pivots, is on this (\( \hat{\psi} = \tilde{\psi} \))-curve. For an
arbitrarily small \( \varepsilon > 0 \), \( \hat{\psi} \neq \check{\psi} \) on the \( \varepsilon \)-ball centered at \((s^*, t^*)\), except for the intersection of this ball with the \((\hat{\psi} = \check{\psi})\)-curve. When \( \hat{\psi} \neq \check{\psi} \), we have argued in Lemma 8 and Section 4.2 that the radicals plays uninformatively as (limit) best response: \( \hat{\psi} < \check{\psi} \) implies \( t = \bar{s} \), whereas \( \hat{\psi} > \check{\psi} \) implies \( t = s \). Therefore, for almost all perturbations of \((s^*, t^*)\), the best response is far from \((s^*, t^*)\).

For a more formal argument, see Appendix A.1.

**Conditions for existence.** Despite the instability, limit reformative equilibria with undominated pivots worth our further investigation, as they may aggregate information when there is no limit reformative equilibrium with dominated pivot. For sufficiently large or small \( \xi \), we can identify conditions under which there is no corresponding limit reformative equilibrium with undominated pivots. For medium \( \xi \), we have a full characterization in the form of a system of inequalities in model primitives (independent of \((\hat{\tau}, \check{\tau}; \xi)\)). But limited by our knowledge about the geometry of the \((\hat{\psi} = \check{\psi})\)-curve, we don’t have a clear-cut answer about (non-)existence. See Appendix A.2.

**Semi-pooling limit.** Proposition 1 states that there is no “semi-pooling” responsive equilibrium for each finite mean population \( n \). By Lemma 8, there is no sequence of reformative equilibria whose mean scales converge to \( m \) with \( m_b = m_1 < m_0 \) or \( m_b < m_1 = m_0 \): in the first case, we have \( \hat{\psi} < \check{\psi} \); in the second case, we have \( \hat{\psi} > \check{\psi} \); thus either \( t = \bar{s} \) or \( t = s \).

5 Discussion

5.1 Anti-reformative equilibrium

In contrast to the notion of reformative equilibrium, responsive equilibrium with \( m_0 < m_1 < m_b \) may be called *anti-reformative* equilibrium. Consider a sequence of anti-reformative equilibria whose mean scales converge to \( m \) with \( m_0 < m_1 < m_b \). For this sequence, here are the anti-reformative counterparts of some previous results:
• For $n$ sufficiently large, the conservatives play a decreasing cutoff strategy, whereas the radicals play an increasing cutoff strategy.

• The asymptotic anti-reformative domain coincides with the asymptotic reformative domain $R$.

• There is $l^t > 0$ such that a limit anti-reformative equilibrium with dominated (lower) pivot exists if and only if $\eta_1 \left( \eta_2 \left( \frac{z_0}{z_1} \right) \right) > \eta_1 \left( \eta_2 \left( \frac{z_0}{z_1} \right) \right)$ and $\frac{\eta}{\eta} < l^t$: for each such equilibrium, the radicals abstain regardless of their signals, and $\frac{m_0(s, s)}{m_1(s, s)} = \frac{z_0}{z_1}$ simplifies the analysis.

5.2 Minor variations

In a limit reformative equilibrium with dominated (upper) pivot, the citizens essentially ignore the upper pivot. Thus, if we consider those variants of our model concerning only the upper pivot in the limit, then our analysis in Section 4.2 is immediately applicable. Here are some examples.

**Reactionary policy.** Suppose the government will choose a reactionary policy when she believes that the fundamental state is very likely to be $a_0$. The reactionary policy is worse than the status quo for both types of citizens regardless of the fundamental states. When $m_b < m_1 < m_0$ in the limit, the reactionary policy concerns only the upper pivot, and our analysis in Section 4.2 applies. If strong information aggregation is achievable with dominated pivot (see also Propositions 7 and 9), then the citizens protest as if they are unaware of the possibility of reaction, and the reactionary policy is chosen whenever the fundamental state is $a_0$.

**Aligned conservatives.** Suppose the conservatives also receive $-1$ from the reform if the fundamental state is $a_0$. When $m_b < m_1 < m_0$ in the limit, the conservatives’ caution about
unwise reform concerns only the upper pivot. Thus, such aligned conservatives behave like the previous “misaligned” conservatives in the limit.
Appendix A  More on undominated pivots

A.1 Instability

Here we formalize the idea that a perturbation of \((s, t)\) on \(\hat{\psi}(m(s, t)) = \tilde{\psi}(m(s,t))\) typically leads to \((s', t')\) with \(\hat{\psi}(m(s', t')) \neq \tilde{\psi}(m(s', t'))\).

Recall that \(\hat{\psi}(m) = \eta_1 \left( \eta_2 \left( \frac{m_b}{m_1} \right) \right)\) and \(\tilde{\psi}(m) = \eta_1 \left( \eta_2 \left( \frac{m_a}{m_1} \right) \right)\). Let \(\hat{m} = \frac{m_b}{m_1}\) and \(\tilde{m} = \frac{m_a}{m_1}\).

The mapping from \((\hat{m}, \tilde{m})\) to \((\hat{\psi}, \tilde{\psi})\) has Jacobian

\[
\begin{pmatrix}
\ln(\eta_2(\hat{m}))\eta_2'(\hat{m}) & 0 \\
0 & \ln(\eta_2(\tilde{m}))\eta_2'(\tilde{m})
\end{pmatrix}
\]

which has full rank when \(\hat{m}, \tilde{m} \neq 1\). Let’s write \(f_{s,\omega}\) in place of \(f(s|\omega)\) and \(F_{s,\omega}\) in place of \(F(s|\omega)\). The mapping from \((s, t)\) to \((\hat{m}, \tilde{m})\) has Jacobian rank-equivalent to

\[
J_1 \equiv \begin{pmatrix}
-z_b m_1 f_{s,b} + z_1 m_b f_{s,a} & (1 - z_b) m_1 f_{t,b} - (1 - z_1) m_b f_{t,a} \\
-f_{s,a} F_{t,a} & (1 - F_{s,a}) f_{t,a}
\end{pmatrix}
\]

If \(J_1\) has full rank, then for each \((s, t) \in R\), the image of the mapping from a small neighborhood \((s, t)\) to the space of \((\hat{\psi}, \tilde{\psi})\) is an open set; in particular, for “almost all” \((s', t')\) in this neighborhood, \(\hat{\psi} \neq \tilde{\psi}\). Contrapositively, the stability of \((s, t)\) on \(\hat{\psi} = \tilde{\psi}\) requires that \(J_1\) be not full-ranked.

Let \(M^\dagger\) be the set of \((3 \times 2)\)-matrices of which the first two rows are collinear, corresponding to \(J_1\), whereas the third row has identical entries, corresponding to \(\hat{\psi} = \tilde{\psi}\). \(M^\dagger\) has co-dimension 2.

Let \(D\) represent the domain consisting of \((f_{\theta,\omega}, F_{\theta,\omega})_{\theta \in \{s,t\}, \omega \in \{a,b\}}\) such that \(F_{t,a}, F_{t,b} > 0\) (by Lemma 4) and \(F_{s,a}, F_{s,b} < 1\).\(^{17}\) To save space, let \(z'_\omega = 1 - z_\omega\). The mapping from \(x \in D\) to \(J_1 \times (\hat{\psi}, \tilde{\psi})\) has Jacobian rank-equivalent to

\(^{17}\)See “Large relative magnitude” in Appendix A.2 for an analysis of equilibrium with \(s = \pi\).
<table>
<thead>
<tr>
<th></th>
<th>$F_{s,a}$</th>
<th>$f_{s,a}$</th>
<th>$F_{s,b}$</th>
<th>$f_{s,b}$</th>
<th>$F_{t,a}$</th>
<th>$f_{t,a}$</th>
<th>$F_{t,b}$</th>
<th>$f_{t,b}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$z_b z_1 f_{s,b}$</td>
<td>$z_1 m_b$</td>
<td>$-z_1 z_b f_{s,a}$</td>
<td>$-z_b m_1$</td>
<td>$-z_b z'<em>1 f</em>{s,b}$</td>
<td>0</td>
<td>$z_1 z'<em>b F</em>{t,b}$</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>$-z'<em>b z_1 f</em>{t,b}$</td>
<td>0</td>
<td>$z_b z'<em>1 f</em>{t,a}$</td>
<td>0</td>
<td>$z'_b z'<em>1 f</em>{t,b}$</td>
<td>$-z'_1 m_b$</td>
<td>$z'_1 z'<em>b f</em>{t,a}$</td>
<td>$z'_b m_1$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>$F_{t,a}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>$-f_{t,a}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$1 - F_{s,a}$</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>$z_1 m_b$</td>
<td>0</td>
<td>$-z_b m_1$</td>
<td>0</td>
<td>$-z'_1 m_b$</td>
<td>0</td>
<td>$z'_b m_1$</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>$F_{t,a}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1 - $F_{s,a}$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Row 0 in gray is the coordinates of $x \in D$, rows 1 to 4 are gradients of $J_1$, and rows 5 and 6 are gradients of $(\hat{\psi}, \check{\psi})$. This $(6 \times 8)$-Jacobian has full rank on $D$. Thus, for a "generic" subset of the space of $(F_a, F_b)$ which is $C^1$ and has strict MLRP, the subset of $(s, t) \in R \cap (\text{int} S \times \text{int} S)$, at which $J_1$ has no full rank and $\hat{\psi} = \check{\psi}$, has co-dimension 2. In other words, the set of such $(s, t)$ has zero dimension and is discrete.

### A.2 Conditions for existence

Suppose $(s^*, t^*) \in R$ is a limit reformative equilibrium with undominated pivots. Given $(s^*, t^*)$ and the corresponding mean scale, the range of $\hat{T}_\omega$ and $\check{T}_\omega$ are well-specified for each $\omega \in \Omega$. Our analysis is organized along different values of $\xi$.

Recall that in the limit, the citizens’ best response can be represented by

\[
\varphi(s) \leq \frac{\beta z_1}{\gamma z_b} \frac{\hat{T}_1 - \xi \left( \hat{T}_1 + \frac{\omega}{z_1} \gamma \hat{T}_0 \right)}{\hat{T}_b} \tag{BR_c^\infty}
\]

\[
\varphi(t) \geq \frac{1 - z_1}{(1 - z_b) \gamma} \frac{\xi \left( \hat{T}_1 + \frac{1 - \omega}{1 - z_1} \gamma \hat{T}_0 \right) - \hat{T}_1}{\hat{T}_b} \tag{BR_r^\infty}
\]

When $\xi$ is sufficiently small, the radicals participate regardless of their signals; when $\xi$ is large, the conservatives abstain regardless of their signals; when $\xi$ is medium, the citizens’ protest strategy profile exhibits proper reverse monotonicity.
Small relative magnitude. Suppose \( \xi \left( \hat{T}_1 + \frac{z_0}{z_1} \gamma \hat{T}_0 \right) < \hat{T}_1 \). Then \( t^* = \bar{s} \). Since \( \frac{1 - z_0}{1 - z_1} > \frac{z_0}{z_1} \), we have \( \xi \left( \hat{T}_1 + \frac{z_0}{z_1} \gamma \hat{T}_0 \right) < \hat{T}_1 \), and thus

\[
\varphi(s^*) = \frac{\beta z_1 \hat{T}_1}{\gamma z_b \hat{T}_b} \left( 1 - \frac{\hat{T}_1 + \frac{z_0}{z_1} \gamma \hat{T}_0}{\hat{T}_1 + \frac{z_0}{z_1} \gamma \hat{T}_0} \right) \\
= \beta \frac{z_1 - z_0}{z_b(1 - z_1)} \frac{\hat{T}_1}{\hat{T}_b} + \frac{1 - z_0 \gamma}{1 - z_1} \\
\geq \frac{\beta}{1 + \gamma z_b(1 - z_0)} m_b(s^*, \bar{s}).
\]

The first inequality uses the hypothesis \( \xi \left( \hat{T}_1 + \frac{z_0}{z_1} \gamma \hat{T}_0 \right) < \hat{T}_1 \), and the second inequality relies on \( \frac{\hat{T}_1}{\hat{T}_b} \in \left[ \frac{m_b}{m_1}, 1 \right] \) and \( \frac{\hat{T}_1}{\hat{T}_b} < \frac{1 - z_0}{1 - z_1} \). Using the hazard ratio \( H_b \) defined in Section 4.2, the previous inequality can be written as

\[
H_b(s^*) > \frac{\beta}{1 + \gamma z_1(1 - z_0)}.
\]

By Lemma 10, we have \( s^* > z \). Therefore, \( (\text{Nec}_s) \) fails when \( \beta \) is sufficiently large, and it becomes more stringent when \( \gamma \) is smaller.

Large relative magnitude. Suppose \( \xi \left( \hat{T}_1 + \frac{z_0}{z_1} \gamma \hat{T}_0 \right) > \hat{T}_1 \). Then \( s^* = \bar{s} \). Since \( \frac{1 - z_0}{1 - z_1} > \frac{z_0}{z_1} \), we have \( \xi \left( \hat{T}_1 + \frac{1 - z_0}{1 - z_1} \gamma \hat{T}_0 \right) > \hat{T}_1 \). With an argument similar to the case with small \( \xi \), we obtain

\[
\varphi(t^*) > \frac{F(t^*|b)}{F(t^*|a)} \frac{z_1 - z_0}{z_1(1 - z_0)} \frac{1}{1 + \frac{(1 - z_1)z_0}{(1 - z_0)z_1}}.
\]

Thus, \( (\text{Nec}_{t;1}) \) is not affected by \( \beta \) as only the radicals participate, and it is more stringent when \( \gamma \) is smaller. Since \( \varphi(\bar{s}) = 0 \) and \( \frac{F(\bar{s}|b)}{F(\bar{s}|a)} = 1 \), \( (\text{Nec}_{t;1}) \) also requires \( t^* \) be sufficiently small.

To characterize \( t^* \), note that it is the solution to \( \hat{\psi}(m(\bar{s}, t)) = \hat{\psi}(m(s, t)) \). Using \( \eta_1 \) and \( \eta_2 \) defined in the proof of Lemma 10, the equality can be written as \( \eta_1 \left( \eta_2 \left( \frac{z_0}{z_1} \frac{F(t^*|b)}{F(t^*|a)} \right) \right) = \eta_1 \left( \eta_2 \left( \frac{1 - z_0}{1 - z_1} \right) \right) \). We see that \( \hat{\psi} \) is strictly increasing in \( t \) and \( \hat{\psi} \) is constant in \( t \). Thus, there is
at most one solution to the equality. Let $R_{s=s} = \{ t \in S : (s, t) \in R \}$. Since $0 = \hat{\psi}(m(s, t)) < \tilde{\psi}(m(s, t))$ when $t = \inf R_{s=s}$, the existence of $t^*$ is equivalent to

$$\eta_1 \left( \eta_2 \left( \frac{1 - z_b}{1 - z_1} \right) \right) \geq \eta_1 \left( \eta_2 \left( \frac{1 - z_0}{1 - z_1} \right) \right).$$

(Nec$_{l;2}$) is violated, for example, when $z_b$ and $z_1$ are close.

Note that (Nec$_{l;2}$) is solely about the proportion of conservatives $(z_\omega)_{\omega \in \Omega}$, whereas (Nec$_{l;1}$) also depends on the signal structure and $\gamma$.

**Medium relative magnitude.** Suppose

$$\xi \left( \frac{\tilde{T}_1 + z_0 \gamma \tilde{T}_0}{z_1} \right) \leq \tilde{T}_1 \leq \xi \left( \tilde{T}_1 + \frac{1 - z_0}{1 - z_1} \gamma \tilde{T}_0 \right).$$

Then (BR$^\infty_c$) and (BR$^\infty_r$) hold with equality at $(s^*, t^*)$. Consider the system

$$\xi \left( \tilde{T}_1 + \frac{z_0 \gamma \tilde{T}_0}{z_1} \right) \leq \tilde{T}_1 \leq \xi \left( \tilde{T}_1 + \frac{1 - z_0}{1 - z_1} \gamma \tilde{T}_0 \right)$$

$$\varphi(s) = \frac{\beta z_1}{\gamma z_b} \frac{\tilde{T}_1 - \xi \left( \tilde{T}_1 + \frac{z_0 \gamma \tilde{T}_0}{z_1} \right)}{\tilde{T}_b}$$

$$\varphi(t) = \frac{1 - z_1}{(1 - z_b) \gamma} \frac{\xi \left( \tilde{T}_1 + \frac{1 - z_0}{1 - z_1} \gamma \tilde{T}_0 \right) - \tilde{T}_1}{\tilde{T}_b}.$$

The existence of $(\hat{\tau}, \tilde{\tau}; \xi)$ solving this system at $(s^*, t^*)$ is equivalent to the existence of $(\hat{\tau}, \tilde{\tau})$ solving the following equality (obtained by eliminating $\xi$ from the two equalities in the system) at $(s^*, t^*)$:

$$\frac{\tilde{T}_1 - z_0}{\tilde{T}_b z_1 (1 - z_1)} = \frac{1}{\beta} \left( \frac{\tilde{T}_1 + 1 - z_0}{\tilde{T}_0 + 1 - z_1} \right) \frac{z_b}{z_1} \varphi(s) + \left( \frac{\tilde{T}_1 + z_0}{\tilde{T}_0 + z_1} \right) \frac{1 - z_b}{1 - z_1} \varphi(t).$$

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Since \( \frac{\tau_1}{\tau_b} \in \left[ \frac{m_b}{m_1}, 1 \right] \) and \( \frac{\tau_0}{\tau_b} \in \left[ 1, \frac{m_0}{m_1} \right] \), the existence of \((\hat{\tau}, \hat{\tau})\) solving this equality at \((s^*, t^*)\) is equivalent to whether the following system

\[
\frac{m_b}{m_1} \frac{z_1 - z_0}{z_1(1 - z_1)} \leq \frac{1}{\beta} \left( \frac{m_0}{m_1} + \frac{1 - z_0}{1 - z_1} \right) \frac{z_b}{z_1} \varphi(s) + \left( \frac{m_0}{m_1} + \frac{z_0}{z_1} \gamma \right) \frac{1 - z_b}{1 - z_1} \varphi(t) \\
\frac{z_1 - z_0}{z_1(1 - z_1)} \geq \frac{1}{\beta} \left( 1 + \frac{1 - z_0}{1 - z_1} \gamma \right) \frac{z_b}{z_1} \varphi(s) + \left( 1 + \frac{z_0}{z_1} \gamma \right) \frac{1 - z_b}{1 - z_1} \varphi(t)
\]

holds at \((s^*, t^*)\). As \( \beta \to \infty \) and \( \gamma \to 0 \), this system converges to

\[
\varphi(t) \leq \frac{z_1 - z_0}{z_1(1 - z_b)} \leq \frac{m_0(s, t)}{m_b(s, t)} \varphi(t). \tag{**}
\]

By Lemma 4, let \( m_b = m_1 = m_0 \) at \((s_1, t^\dagger)\). Suppose a \((\hat{\psi} = \hat{\psi})\)-curve starts at \((s_0, \bar{s})\) and ends at \((s_1, t^\dagger)\). At \((s_0, \bar{s})\), \((**)\) is violated. If \( \frac{z_1 - z_0}{z_1(1 - z_b)} \leq \frac{m_0(s_1, t^\dagger)}{m_b(s_1, t^\dagger)} \varphi(t^\dagger) \), then \((**)\) is satisfied by a section of the \((\hat{\psi} = \hat{\psi})\)-curve; if \( \frac{z_1 - z_0}{z_1(1 - z_b)} > \frac{m_0(s_1, t^\dagger)}{m_b(s_1, t^\dagger)} \varphi(t^\dagger) \), \((**)\) may be violated on the whole curve but we don’t have a clear-cut answer given our very limited knowledge about the geometry.
References


