Trade Protection with Two-Sidedness

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Abstract
The traditional model of trade protection against a foreign monopolist suggests that a home country should often impose a tariff on the foreign firm, even though the this introduces an additional distortion. In this paper, I generalize the traditional model to include a basic network externality, e.g., the foreign firm also generates third party advertisement revenue from its product, and I determine that the network externality will result in a positive optimal tariff under a larger set of convex functions than in the traditional model. In this case, the firm has an additional gain from sales and so it is willing to bare a larger burden of the tax to maintain sales. Extending the model to include two-sided products (e.g., smartphones with consumers and app providers or video game consoles with consumers and game developers) leads to further results. With two-sidedness, the firm is able to alleviate some of the burden of the tariff through rent transfers from one side of the market to the other which makes a positive tariff policy optimal for these industries.

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1 Introduction

I recent years, the increased consumption of new products that have network externalities or some kind of two-sidedness has lead to the development of new market structures. For example, some products are sold to consumers but also contain advertisements which provide the seller with an additional revenue stream; apps, some eReaders and digital devices, internet radio, and other web based content. Alternatively, smartphones with consumers and app providers, video game consoles with consumers and game developers, and credit cards with consumers and merchants are all example of two-sided markets. Many of these products are traded internationally and it is often the case that competition is imperfect within these industries. However, the previous literature has not considered international trade policies for these kinds of products.

This paper determines the optimal trade protection tariff policy when a foreign firm sells a product that has some form of network externality or two-sidedness. First the traditional trade protection model, where a foreign monopoly firm sells in the home country and the home country considers a tariff policy that provides an added distortion but also generates tax revenue, is generalized to include a network externality. I show that with the network externality there is a wider class of demand functions, ones that are more convex, where the optimal tariff policy is a positive tariff. That is, with network externalities it is more likely that a positive tariff is optimal. With the network externality, the foreign firm has an additional incentive to maintain sales to consumers which implies that the foreign firm is willing to bare more of the tariff burden than in the traditional model.

This basic externality model is then generalized to investigate a model of two-sidedness where the foreign firm sells its product, a platform, to consumers and also contracts with content providers. Participation on one side effects the gains on the other side. For consumers, more content makes the platform more desirable. For content providers, more consumers owning the platform implies more potential buyers for their content. This interaction between the two sides generates the network externalities. With two-sidedness, a tariff affects
the platform’s price to consumers and the platform’s fee to content sellers. I show that two-sidedness alleviates the tariff distortion and that the optimal tariff policy will be a positive tariff on sales of the platform to consumers.

These results generalize results found in the previous literature on trade protection from a foreign monopoly firm. Thus, this paper contributes a generalized analysis of trade protection against a foreign monopoly firm to include network externalities and two-sidedness. Katrak (1977), Svedberg (1979), Brander and Spencer (1984a,b) and subsequent literature analyze such a tariff protection policy. Results show that an optimal tariff policy is a positive tariff on the foreign firm so long as demand is not to convex. With network externalities, a positive tariff policy is optimal for demand functions that are more convex. That is, the set of convex functions where a positive tariff is optimal is larger when there are network externalities.

The literature on platforms and two-sided markets originated with Caillaud and Jullien (2003), Ellison and Fudenberg (2003), Rochet and Tirole (2003), Armstrong (2006) and Hagiu (2006); however, consideration of the international trade of these products has yet to be analyzed. With two-sidedness the platform is able to decide which side of the market to extract the most rent from. Thus, two-sidedness allows the foreign platform seller to alleviate much of the burden of the tariff resulting in a positive optimal tariff policy. This result also relates to the growing literature on platform taxation within a single economy without trade. Tremblay and Wilson (2015) show how the social planner will optimally use subsidies in platform sales and in content sales to increase the network externalities and the resulting welfare within the economy. However with imports from a foreign monopoly, the home country does not care about foreign welfare and they know that a platform, with network externalities, has a greater willingness to maintain participation levels than a traditional firm. These differences in incentives, depending on whether the product is imported, lead to the different tax/tariff policy results.

In Section 2 the traditional trade protection model is extended to allow for a simple network externality. I show how this externality leads to a wider range of demand functions
that result in a positive optimal tariff by the home country. The model is then generalized to a two-sided platform marketplace model in Section 3. The foreign firm sells its platform to consumers and also contracts with content providers; this structure is developed in Section 3. The general two-sided model is then solved in Section 4 and it is shown that setting a positive tariff for foreign platform sales leads to greater welfare in the home country. Finally, Section 5 concludes followed by an appendix containing the proofs of the theorems in this paper.

2 Trade with a Network Externality

In this section, the basic model of trade protection against a foreign monopoly firm is extended to include a simple network externality. In the traditional model developed by Branden and Spencer (1984a), the foreign firm sells a standard good to home consumers. Now suppose there exists a network benefit from the sale of the firm’s product. For example, for each sale of a digital device the firm is also able to generate some advertising revenues from each consumer that purchases the device; alternatively, the product could include a same side network effect that the firm is able to charge its customers for, Xbox Live for video games consoles. Another example is that this revenue from the network benefit is generated from an additional side of the market, the content side.

To formalize this, consider the following profit function for the foreign firm’s sales in the home country:

$$\Pi(N_1) = N_1 \cdot p(N_1) - C_1 \cdot N_1 - T \cdot N_1 + A(N_1),$$  \hspace{1cm} (1)

where $N_1$ is the quantity sold, $p(N_1)$ is the inverse demand function, $C_1$ is the marginal cost, $T$ is the tariff, and $A(N_1)$ is the network benefit with $A'(N_1) > 0$. For example, $A(N_1)$ is the third party advertising revenue generated from product sales to consumers. The foreign

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1Two-sided markets will be focused on specifically in the next section. However, this base model provides some general insights across many possible types of network benefit structures, including two-sidedness.

2This notation will be more clear in the next section, $N_1$ is the quantity sold which is equivalent to the number of consumers that purchase the firm’s product.
firm maximizes profits with respect to $N_1$ which gives the following first and second order conditions:

$$\Pi'(N_1) = p(N_1) + N_1 \cdot p'(N_1) - C_1 - T + A'(N_1) = 0, \quad (2)$$

$$\Pi''(N_1) = 2p'(N_1) + N_1 \cdot p''(N_1) + A''(N_1) < 0. \quad (3)$$

By totally differentiating the first order condition, $\Pi'(N_1) = 0$, with respect to $N_1$ and $T$ we have the following comparative static:

$$\frac{\partial N_1}{\partial T} = \frac{1}{\Pi''(N_1) + A''(N_1)}. \quad (4)$$

The change in quantity sold from a change in the tariff, $\frac{\partial N_1}{\partial T}$, depends on the relative convexity of demand. This relative convexity measure is given by $R$ where

$$R = \frac{N_1 \cdot p''(N_1)}{p'(N_1)}. \quad (5)$$

From Equations (3) and (5) it follows that $\Pi''(N_1) = p'(N_1)(R + 2) + A''(N_1)$. Furthermore, since $\frac{\partial p}{\partial T} = p'(N_1) \cdot \frac{\partial N_1}{\partial T}$, Equation (4) implies that

$$\frac{\partial p}{\partial T} = \frac{p'(N_1)}{p'(N_1)(R + 2) + 2A''(N_1)}. \quad (6)$$

In solving the home country’s welfare maximization problem with respect to the tariff, $T$, it follows that the home’s optimal tariff policy is

$$T^* = N_1 \cdot \left( \frac{\partial p}{\partial T} - 1 \right) \cdot \left( \frac{\partial N_1}{\partial T} \right)^{-1}. \quad (7)$$

The traditional result without network externalities is that the tariff is positive when the increase in price from an increase in the tariff is less than one, $T > 0$ when $\frac{\partial p}{\partial T} < 1$. That is, the optimal tariff is positive when consumers do not bare all or more of the tariff through the increase in price. This occurs when demand is not to convex, (i.e., when $R > -1$).
In this model with network externalities, the result differs. Equations (4), (5), (6), and (7) imply that

\[ T^* = -N_1 \cdot \left( \frac{p'(N_1) \cdot (R + 1) + 2A''(N_1)}{[p'(N_1)(R + 2) + 2A''(N_1)]^2} \right). \]  

Thus, \( T^* > 0 \) if and only if \( R > \frac{-2A''(N_1)}{p'(N_1)} - 1 \). This implies that the change in the marginal network benefit, \( A''(N_1) \), and the convexity of demand, \( R \), determine the optimal tariff policy. This leads to the following result:

**Theorem 1** (The Optimal Tariff with a Network Externality). *When consumers provide a decreasing marginal benefit to the foreign firm, \( A''(N_1) < 0 \), then the convexity requirement on demand is relaxed; it now must be that \( R > R^* \) where \( R^* < -1 \).*

When there exist decreasing marginal network externality profits, which is the case for most platform industries where the additional consumers are the least likely to use the product (they view fewer ads, buy fewer apps, subscribe to less additional content, etc.),

then the network externality makes it more likely that the optimal tariff policy is a positive tariff. In other words, the optimal tariff is positive for more demand functions, demand functions that are more convex, when there are network externalities than in the traditional model without externalities. When there exist increasing marginal benefits, \( A''(N_1) > 0 \), a subsidy is more likely. In this case, a subsidy will decrease the price considerably and increase sales. However, it cannot be the case that \( A''(N_1) \) is too large because then the second order condition for profit maximization will fail; a common problem with network models.

The reason why a positive tariff is more likely with network externalities is since the foreign firm has a greater incentive to make more sales than when there do not exist network externalities. Thus, an increase in the tariff by the home country will result in an increase in price that is lower than in the traditional model because sales now generate an additional

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3For more on decreasing marginal returns from consumers see Lee (2013), Bresnahan et al. (2015), Jeitschko and Tremblay (2015) and Tremblay (2015).

4We focus on the case when marginal benefits are decreasing, \( A''(N_1) < 0 \), as this ensures profit maximization and is seen in existing network industries.
revenue stream to the firm. To this end, the firm responds to the tariff with a lower price than it would without network externalities.

3 Two-Sided Network Externalities

Consider a foreign firm that sells a platform, a smartphone, video game console, or other digital devices, to consumers but also contracts with content providers, app developers or game developers. The platform is sold in the home country by the foreign firm. The platform charges consumers a fixed fee to purchase or become a member of the platform. Sellers generate profits from selling their product or service to the platform’s consumers and the platform charges sellers a percentage of their profits to join the platform and have access to the platform’s consumers. This is the fee structure used by many platform marketplaces; app stores take a percent of app sales, as do video game platforms and online marketplaces.

Consumers care about the amount of content available to them on the platform, while sellers care about the total number of consumers that join the platform as more consumers implies greater demand for their content. This generates the network externalities that exist on the platform. Trade is free, so the platform cannot set different prices to its home consumers and foreign consumers. Similarly, a seller faces the same platform fee regardless of their location.

3.1 The Platform

Consumers pay a fixed membership fee, $P$, to join the foreign platform. For example, this is the retail price that consumers pay to purchase a smartphone or video game console. Similarly, the platform charges an ad valorem fee, $f$, to sellers; that is, the platform takes a percent of each sale. The platform’s fees may be negative in order to subsidize membership.

5Depending on the type of platform market consumers are either members of the platform (e.g., consumers are members of Pandora) or consumers own the platform (e.g., consumers own a smartphone or video game console).
Platform profits are given by:

$$\Pi = N_1 \cdot (P - C) + f \cdot (p \cdot q) \cdot N_2 - T \cdot H_1 \cdot N_1,$$

(9)

where \( N_1 \) \((N_2)\) is the number of consumers (sellers) that join the platform and \( H_1 \) is the proportion of consumers from the home country where the platform must pay a tariff \( T \) for each unit sold in the home country. The constant marginal cost to the platform for an additional consumer is given by \( C \), and the constant marginal cost to the platform for an additional seller is assumed to be zero.\(^6\) For simplicity assume fixed costs are zero. The expected total number of transactions for each content seller is given by \( q \cdot N_2 \) where \( q \) is the expected number of sales made by each of the \( N_2 \) sellers that join the platform. Lastly, the expected price of a product is given by \( p \). Thus, the profits that the platform receives from the consumer side is the number of consumers times the profit per consumer and the profits the platform receives from the seller side is the ad valorem fee times the expected total revenue generated by all of the transactions between consumers and sellers.

The timing of play is as follows. First, given the home country tariff \( T \), the platform sets prices \( P \) and \( f \), either of which can be less than zero. Then agents make their participation decisions.\(^7\)

### 3.2 Consumers and Sellers

Consumers are on Side 1 and sellers on Side 2. There exists a mass of consumers on Side 1, normalized to 1, with individual consumer types indexed by \( \tau \in [0, 1] \). The number of consumers that decide to join the platform is denoted by \( N_1 \in [0, 1] \). The distribution each consumer type \( \tau \) is divide between home and foreign consumers with \( H_1 \) being the total mass of home consumers and \( F_1 \) being the mass of foreign consumers with \( H_1 + F_1 = 1 \).

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\(^6\)That is, the marginal cost to the platform for providing an additional app or game is zero.

\(^7\)Coordination failure and no-trade equilibrium are not the focus of this paper. In equilibrium, agents must have consistent beliefs that support the equilibrium prices and allocations of agents.
Consumers have unit demands for each product, an app or game, that is available. However, consumers are heterogeneous in their value for a particular product. For a given product, consumers indexed with a lower $\tau$ are more likely to value the product. Thus, some consumers are more likely to value a product than other consumers. For example, teens may have many apps on their smartphone relative to their parents; whereas, their parents may have a few apps that they value. The expected utility for consumer $\tau$ from a product is given by $CS(\tau)$ and $CS(\tau)$ is decreasing in $\tau$ which captures this consumer heterogeneity.\footnote{Consumer heterogeneity of this form is common in these marketplaces as shown by Bresnahan et al. (2015) for smartphone applications and Lee (2013) for video games.}

Consumers also gain utility from the platform that is not generated by the seller side of the market. This utility is denoted by $V$ and can be large, as in the case of smartphones where there are many uses for a smartphone outside of using apps, or close to zero, as in the case with Pandora where there is little gain from internet radio outside of interaction with playing music. Thus, a consumer of type $\tau$ has utility

$$u_1(\tau) = V + CS(\tau) \cdot N_2 - P,$$

where $N_2$ is the number of products that are made available on the platform and $P$ is the price a consumer pays to join the platform. Every consumer has a reservation utility that is normalized to zero. Thus, a consumer $\tau$ joins the platform when $u_1(\tau) \geq 0$.

Individual sellers are indexed by $\theta \in [0, \infty)$ and each seller has one product (e.g., for smartphones this may be a map app, a weather app, a car racing app, etc.). Thus, the number of products available on the platform is the same as the number of sellers that join the platform; denoted by $N_2$. The distribution for each seller type $\theta$ is divide between home and foreign sellers with $H_2$ being the total mass of home sellers and $F_2$ being the mass of foreign sellers with $H_1 + F_1 = 1$.

For simplicity, consumer demand is the same across all products. This implies that each seller will set the same price for their product and make the same number of sales in expec-
tation. The expected profit from product sales is given by \( \pi \) and sellers have heterogeneous sunk costs of product development. Let \( C \cdot \theta \) be the sunk cost to develop product \( \theta \). That is, low \( \theta \)-type sellers have lower sunk costs than high \( \theta \)-sellers. This leads to endogenous entry of sellers on the platform. For simplicity, the marginal cost for every seller is zero\(^9\). Thus, a seller of product \( \theta \) when joining the platform has utility

\[
 u_2(\theta) = (1 - f)\pi - C \cdot \theta, \tag{11}
\]

where \( f \) is the ad valorem fee the platform charges a seller that joins the platform. Every seller has a reservation utility that is normalized to zero. Thus, a seller of type \( \theta \) joins the platform when \( u_2(\theta) \geq 0 \).

4 Two-Sided Equilibrium

To determine the Nash Equilibrium of this game, a further development of the interaction between consumers and sellers is developed. In two-sided markets where consumers purchase products or content from sellers, the consumer demand for the platform and the seller demand for the platform depend on the consumer demand for the products available on the platform. Thus, the equilibrium prices and sales that occur within the platform marketplace are critical in understanding the entire platform game.

To set up the interaction more formally, let \( \upsilon \) be the reservation value that a consumer has for a given product. Suppose that a consumer of type \( \tau \) has a positive reservation for a given product with probability \( (1 - \tau) \); that is, \( \upsilon > 0 \) with probability \( (1 - \tau) \). Thus, \( \tau \) captures the probability that a consumer has any interest in a given product where consumers of a low \( \tau \)-type are more likely to value a given product. This implies that consumers with lower \( \tau \) have a positive value for more products (teens with apps) than consumers with higher \( \tau \) (parents with fewer apps), in expectation.

\(^9\)In the case for app or video game sellers, marginal cost is nearly zero.
Given a consumer values a product, their reservation value is distributed uniformly between zero and $\sigma$; hence, $v \sim U[0, \sigma]$. This implies that conditional on consumers having a positive reservation value for a product, that value is drawn from the same uniform distribution. Sellers take the number of consumers that join the platform, $N_1$, as given. Thus, demand for a product is given by:

$$q \equiv \int_0^{N_1} Pr(\tau \text{ buys}|p)d\tau = \int_0^{N_1} \left(1 - \frac{p}{\sigma}\right)(1 - \tau)d\tau = \left(1 - \frac{p}{\sigma}\right) \left(1 - \frac{N_1}{2}\right) \cdot N_1. \quad (12)$$

where $p$ is the price of product. Thus, assuming consumer’s valuations follow the uniform distribution results in consumers having linear demand for a given product. The corresponding inverse demand is:

$$p = \sigma \left(1 - \frac{1}{\left(1 - \frac{N_1}{2}\right) \cdot N_1}\right) \cdot q. \quad (13)$$

Seller profits from product sales are given by:

$$\pi = p \cdot q = p \cdot \left(1 - \frac{p}{\sigma}\right) \left(1 - \frac{N_1}{2}\right) \cdot N_1. \quad (14)$$

Maximizing seller profits gives the following solution for seller $\theta^{10}$:

$$p^* = \frac{\sigma}{2}, \quad (15)$$

$$q^* = \frac{1}{4} (2 - N_1) \cdot N_1, \quad (16)$$

$$\pi^* = \frac{\sigma}{8} \cdot (2 - N_1) \cdot N_1. \quad (17)$$

Given product prices from Equation $^{(15)}$, the consumer surplus from a product for a consumer of type $\tau$ is given by:

$$CS(\tau) \equiv E[v - p^*|v \geq p^*] \cdot Pr(\tau \text{ buys}|p^*) = \frac{\sigma}{16} (1 - \tau). \quad (18)$$

$^{10}$The second-order conditions hold for profit maximization.
This implies we have agent utilities given by:

\[ u_1(\tau) = V + \frac{\sigma}{16}(1 - \tau) \cdot N_2 - P, \quad (19) \]

\[ u_2(\theta) = (1 - f)\frac{\sigma}{8} \cdot (2 - N_1) \cdot N_1 - F \cdot \theta, \quad (20) \]

Notice that \( u_1(N_1) = 0 \) and \( u_2(N_2) = 0 \) identify the marginal agents for the home and foreign countries that join the platform, \( \tau = N_1 \) and \( \theta = N_2 \). Thus, Equations (19) and (20) imply that the platform can solve for \( P \) and \( N_2 \) as functions of \( N_1 \) and \( f \):

\[ P = V + \frac{\sigma}{16}(1 - N_1) \cdot N_2 = V + \frac{\sigma^2}{128F}(1 - N_1)(1 - f)(2 - N_1) \cdot N_1, \]

\[ N_2 = (1 - f)\frac{\sigma}{8} \cdot (2 - N_1) \cdot N_1. \]

Hence, the platform’s profit is a function of \( N_1 \) and \( f \). To simplify calculations, let \( V = C \).\(^{11}\)

In solving the platform’s problem, we have the following theorem.

**Theorem 2** (Platform Sales with a Tariff). An increase in the home tariff implies fewer consumers, home and foreign, join the platform in equilibrium, \( \frac{\partial N_1}{\partial T} < 0 \). On the seller side, when the tariff exists, \( T > 0 \), an increase in the tariff also results in fewer sellers joining the platform, \( \frac{\partial N_2}{\partial T} < 0 \) for \( T > 0 \). Furthermore, the home tariff policy has a stronger impact on the platform when there are more home consumers.

This is a standard result in trade: A tariff reduces consumption of the good being taxed, in this case the platform. However, with platforms there are additional effects from tariff policies. Through the network benefits we have a feedback effect resulting in fewer sellers joining the platform. In order to determine the optimal tariff policy, the effect of tariffs on prices to each side of the market must be considered.

In the traditional one-sided model of trade protection, the home country should implement a tariff policy when \( \frac{\partial P}{\partial T} < 1 \). That is, so long as the tax is not transferred to the consumers entirely. With two-sided markets, the tariff effects the platform’s prices to each side of the market. Furthermore, the network benefits imply that sales of the platform to

\(^{11}\)This assumption is not critical and it makes computations straightforward: In the market for smartphones and video game consoles it would be the case that the marginal cost to produce the platform and the membership gains consumers receive are positive and approximately equal.
consumers have an additional benefit to the platform on the seller side of the market. Thus, with network benefits the platform has a greater demand for supplying consumers, selling its platform to consumers, than with the traditional good. This implies that the increase in the consumer price from a tariff will be less with network benefits than in the tradition model. Furthermore, a tariff reduces the platform fee to sellers which further suggests that the use of tariffs is beneficial for the home country.

**Theorem 3** (Effect of the Tariff on Platform Prices). *An increase in the home tariff implies a weak increase or a decrease in the platform price to consumers so that \( \frac{\partial P}{\partial T} < 0.2 \), and a decrease in the platform’s fee to sellers, \( \frac{\partial f}{\partial T} < 0 \).*

With two-sidedness we see that the network externalities increase the platforms need to maintain participation on each side of the market. As a result, a tariff on platform sales to consumers does not increase the consumer price by much, \( \frac{\partial P}{\partial T} < 0.2 \). This reduces consumer participation but the effect of higher prices to consumers is dampened by the increased supply of sellers on the platform, as noted by Theorem 2. This increase in sellers due to the tariff occurs because the platform lowers its fee to seller. Thus, the platform uses its two-sidedness to reduce the total burden of the tax. First, it places some of the burden on consumers through a higher price. However, the platform also lowers its fee to sellers which dampens this burden to consumers and maintains the platform’s overall participation. This implies that the tariff to consumer sales effectively pushes much of the platform profits from the consumer side of the market to the seller side of the market.

To complete the analysis on the tariff policy for the home country, the optimal home tariff that maximizes the home countries welfare is determined. In this case, the home country does not take into account the welfare for foreign agents, (foreign consumers, foreign sellers, or foreign platform profits) in determining the home country welfare and the optimal tariff. In solving for the optimal tariff we have the following:

**Theorem 4** (Effect of the Tariff on Platform Prices). *In equilibrium, the home country implements a positive tariff policy so that \( T > 0 \).*
Given our previous findings on trade protection with network externalities, a positive tariff policy against a monopoly platform may not be surprising. However, Tremblay and Wilson (2015) show that the optimal tax policy within an economy is a subsidy policy to increase participation and the number of interactions on a platform. This generates large network externality gains that the platform would otherwise not capture. In the case with trade, the home country does not need to account for the effect on platform profits and the home country knows that the platform has a greater need to support each of its two sides to maximize profits. Thus, a tariff results in minor losses of home consumer and seller surplus while generating significant tariff revenues. Hence, a positive tariff is optimal.

5 Conclusion

In recent years the emergence of new products with network externalities has lead to the development of research on industries where network effects and two-sidedness are present. To this point, the literature has not considered issues regarding the international trade of these products. In many of these industries one firm has the majority of the market share and so I consider the optimal trade protection policy for a home country that imports a product with network externalities from a foreign monopoly firm. In generalizing the traditional trade protection model to include a network externality, I find that the optimal tariff policy will be a positive tariff for a larger class of demand functions than for the traditional model without the network externality. This is since the firm now has an additional incentive to bare more of the tariff burden as maintaining a high level of sales generates additional revenues from the network benefits.

In expanding the model to consider two-sidedness, the product being sold by the firm is a platform, a smartphone or video game console, that is sold to consumers. For this product, the firm also makes contracts with content providers. Consumers want more content and providers want the platform to be sold to as many consumers as possible to increase demand
for their content. This interaction develops the network externalities that exist in this two-sided market. With two-sidedness, the firm is able to shift revenue generation from one side of the market to the other. Thus, with the implementation of the tariff, the distortion is partially neutralized by the firm. Thus, consumers and sellers bare very little burden from the tariff and the optimal tariff policy is a positive tariff on the two-sided product.

Appendix

Proof of Theorem 1. Equation (8) implies that \( T^* \) is positive when \( p'(N_1) \cdot (R + 1) + 2A''(N_1) < 0 \). This occurs when \( R > \frac{-2A'(N_1)}{p'(N_1)} - 1 := R^* \). Since \( \frac{-2A''(N_1)}{p'(N_1)} < 0 \), it follows that \( R^* < -1 \).

Proof of Theorem 2. In solving the platform’s problem we have the following first order conditions for \( f \) and \( N_1 \):

\[
f = \frac{(3 - N_1)}{4(2 - N_1)},
\]

\[
T \cdot F_1 \cdot \frac{512C}{\sigma^2} = N_1 \cdot (5 - 3N_1) \cdot (5 - 6N_1).
\]

Equation (22) implicitly provides a function for the number of consumers for a given tariff rate, \( N^*_1(T) \). This can easily be seen in Figure 2. The x-axis is the left hand side of Equation (22). Thus, profit maximization fails for all unless the tariff is not too severe, \( T \in \left[ -\frac{\sigma^2}{H_2256C}, -\frac{\sigma^2}{H_1128C} \right] \) is required.\(^1\) Notice, that when there is no tariff, \( T = 0 \), the equilibrium number of consumers is \( N^*_1 = 5/6 \). Total differentiating Equation (22) implies that

\[
\frac{dN_1}{dT} = \frac{512C}{\sigma^2} H_1 (25 - 90N_1 + 54N_1^2)^{-1}.
\]

This is less than zero for all \( N_1 \) such that \( T \in \left[ -\frac{\sigma^2}{H_2256C}, -\frac{\sigma^2}{H_1128C} \right] \). This confirms the clear downward slope in the graph so that \( \frac{\partial N_1}{\partial T} < 0 \).

By solving for \( N_2 \) from Equation (20) with \( u_2(N_2) = 0 \) and Equation (21), we have that

\(^{12}\)Such a restriction is typically required with two-sided markets.
Figure 1: Equilibrium Consumers.

\[ N_2 = \frac{\sigma}{32C}(5 - 3N_1)N_1 \]. This implies that

\[
\frac{\partial N_2}{\partial T} = \frac{\partial N_2}{\partial N_1} \cdot \frac{\partial N_1}{\partial T} = \frac{\sigma}{32C}(5 - 6N_1) \cdot \frac{\partial N_1}{\partial T}.
\]

Thus, we have \( \frac{\partial N_2}{\partial T} < 0 \) when \( N_1 < \frac{5}{6} \) which occurs when \( T > 0 \).

Lastly, from Equation 22 it is clear that an increase in \( H_1 \) magnifies the effect of \( T \) on \( N_1 \), and so the home tariff policy has a stronger impact on the platform when there are more home consumers.

**Proof of Theorem 3:** For the seller fee, Equation (21) implies that

\[
\frac{\partial f^*}{\partial T} = \frac{\partial f^*}{\partial N_1} \cdot \frac{\partial N_1}{\partial T} = \frac{1}{2(2 - N_1)} \cdot \frac{\partial N_1}{\partial T} < 0.
\]

Thus, \( \frac{\partial f^*}{\partial T} < 0 \).

By solving for \( P \) from Equation (19) with \( u_1(N_1) = 0 \) and since \( N_2 = \frac{\sigma}{32C}(5 - 3N_1)N_1 \) from the proof of Theorem 2 we have that:

\[
P = V + \frac{\sigma^2}{512C}(5 - 3N_1)(1 - N_1)N_1.
\]
This implies that
\[
\frac{\partial P}{\partial T} = \frac{\partial P}{\partial N_1} \cdot \frac{\partial N_1}{\partial T} = H_1 \cdot \frac{5 - 16N_1 + 9N_1^2}{25 - 90N_1 + 54N_1^2}
\]
Showing the result is best seen graphically. For the tariff levels such that the equilibrium still exists, Figure 2 displays how the change in consumer price from a change in the tariff, \( \frac{\partial P}{\partial T} \), varies across all possible levels of the number of consumers \( N_1 \) when \( H_1 = 1 \). Thus, this is the highest level of \( \frac{\partial P}{\partial T} \) across \( H_1 \). Notice that it is always the case that \( \frac{\partial P}{\partial T} < .2 \) and the change in price is usually positive unless the tariff rate is very high, i.e. low levels of \( N_1 \). □

**Proof of Theorem 4:** Home country welfare is given by
\[
W = H_1 \cdot \int_0^{N_1} u_1(\tau)d\tau + H_2 \cdot \int_0^{N_2} u_2(\theta)d\theta + T \cdot H_1 \cdot N_1.
\]
Substituting, using Equations (19) and (19), and maximizing welfare with respect to the tariff implies that for the equilibrium when the platform does not face a tariff, \( N_1 = N_1^*(T = 0) \), the first order condition for welfare maximization implies that
\[
\frac{\partial W}{\partial N_1} \bigg|_{N_1^*} = H_1 \frac{120.2\sigma^2}{36,864C} + T \cdot H_1 = 0.
\]
This only occurs when \( T > 0 \) for all \( H_1 \in [0, 1] \). Thus, it is optimal for the home country to impose a tariff on the foreign monopoly platform. □

References


18