Merger Policy with Merger Choice*

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Abstract

We analyze the optimal policy of an antitrust authority towards horizontal mergers when merger proposals are endogenous and firms choose which of several mutually exclusive mergers to propose.

1 Introduction

The evaluation of proposed horizontal mergers involves a basic trade-off: mergers may increase market power, but may also create efficiencies. Whether a given merger should be approved depends, as first emphasized by Williamson (1968), on a balancing of these two effects.

In most of the literature discussing horizontal merger evaluation, the assumption is that a merger should be approved if and only if it improves welfare, whether that be aggregate surplus or just consumer surplus, as is in practice the standard adopted by most antitrust authorities [see, e.g., Farrell and Shapiro (1993), McAfee and Williams (1992)]. This paper contributes to a small literature that formally derives optimal merger approval rules. This literature started with Besanko and Spulber (1993), who discussed the optimal rule for an antitrust authority who cannot directly observe efficiencies but who recognizes that firms know this information and decide whether to propose a merger based on this knowledge. Other recent papers in this literature include Nocke and Whinston (2008), Ottaviani and Wickelgren (2009), and Armstrong and Vickers (2010).

In this paper, we focus on a setting in which one "pivotal" firm may merge with one of a number of other firms. These mergers are mutually exclusive, and each may result in a different post-merger cost level. The merger that is proposed is the result of a bargaining

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process among the firms. The antitrust authority observes the characteristics of the merger that is proposed, but neither the feasibility nor the characteristics of any mergers that are not proposed. We focus in the main part on an antitrust authority who wishes to maximize expected consumer surplus. Our main result characterizes the form of the antitrust authority's optimal policy, which we show should impose a tougher standard on mergers involving larger acquirers (in terms of their pre-merger share). Specifically, the minimal acceptable level of increase in consumer surplus is strictly positive for all but the smallest acquirer, and is larger the greater is the acquirer's premerger share.

The closest papers to our are Lyons (2003) and Armstrong and Vickers (2010). Lyons first identifies the issue that arises when firms may choose which merger to propose. Armstrong and Vickers (2010) provide an elegant characterization of the optimal policy when mergers (or, more generally, projects that may be proposed by an agent) are ex ante identical in terms of their distributions of possible outcomes. Our paper differs from Armstrong and Vickers (2010) primarily in its focus on the optimal treatment of mergers that differ in this ex ante sense.

The paper is also related to Nocke and Whinston (2008). That paper established conditions under which the optimal dynamic policy for an antitrust authority who wants to maximize discounted expected consumer surplus is a completely myopic policy, in which a merger is approved if and only if it does not lower consumer surplus at the time it is proposed. One of the important assumptions for that result was that potential mergers were "disjoint," in the sense that the set of firms involved in different possible mergers do not overlap. The present paper explores, in a static setting, the implications of relaxing that disjointness assumption, focusing on the polar opposite case in which all potential mergers are mutually exclusive.

The paper proceeds as follows: We describe the model in Section 1. Section 2 derives our main result, which characterizes the optimal policy in the case in which the bargaining between firms proceeds as in the Segal (1999) offer game. In Section 4, we show that our main characterization result extends to some other bargaining models, including the case in which the bargaining is efficient. Section 5 discusses some other extensions of our results, and Section 6 concludes.

2 The Model

We consider a homogeneous goods industry in which firms compete in quantities (Cournot competition). Let $\mathcal{N} = \{0, 1, 2, ..., N\}$ denote the (initial) set of firms. All firms have constant returns to scale; firm i's marginal cost is denoted c_i. Inverse demand is given by $P(Q)$. We impose standard assumptions on demand:

Assumption 1. *For all Q such that $P(Q) > 0$, we have:*

(i) $P'(Q) < 0$;

(ii) $P'(Q) + QP''(Q) < 0$;

(iii) $\lim_{Q\to\infty} P(Q) = 0$.

It is well known that under these conditions there exists a unique Nash equilibrium in quantities. Moreover, this equilibrium is "stable" (each firm i's best-response function $b_i(Q_{-i}) \equiv$

$\arg\max_{q_i}[P(Q_{-i}+q_i)-c_i]q_i$ satisfies $b_i'(Q_{-i}) \in (-1,0)$, where $Q_{-i} \equiv \sum_{j\neq i} q_j$) so that comparative statics are "well behaved" (if a subset of firms jointly produce less [more] because of a change in their incentives to produce output, then equilibrium industry output will fall [rise]). The vector of output levels in the pre-merger equilibrium is given by $q^0 \equiv (q_0^0, q_1^0, ..., q_n^0)$, where q_i^0 is firm i's quantity. For simplicity, we assume that pre-merger marginal costs are such that all firms in $\mathcal{N}$ are "active" in the pre-merger equilibrium, i.e., $q_i^0 > 0$ for all i. Aggregate output, price, consumer surplus, firm i's profit and aggregate profit in the pre-merger equilibrium are denoted $Q^0 \equiv \sum_i q_i^0$, $P^0 \equiv P(Q^0)$, CS^0, $\pi_i^0 \equiv [P(Q^0)-c_i]q_i^0$, and $\sum_{i\in\mathcal{N}} \pi_i^0$, respectively.

Suppose that there is a set of K potential mergers, each between firm 0 (the"target") and a single merger partner (an "acquirer") $k \in \mathcal{K} \subseteq \mathcal{N}$. There is a random variable $\phi_k \in \{0,1\}$ that determines the feasibility of the merger between firm 0 and firm k. If $\phi_k = 1$, the merger is feasible. A feasible merger is described by $M_k = (k, \bar{c}_k)$, where k is the identity of the acquirer and $\bar{c}_k$ the (realized) post-merger marginal cost, which is drawn from distribution function G_k with support $[l, h_k]$ and no mass points. The random draws of ϕ_k and $\bar{c}_k$ are independent across mergers. If merger M_k, $k \geq 1$, is implemented, the vector of outputs in the resulting post-merger equilibrium is denoted $q(M_k) \equiv (q_1(M_k), ..., q_N(M_k))$, where $q_k(M_k)$ is the output of the merged firm, aggregate output is $Q(M_k) \equiv \sum_i q_i(M_k)$, and firm i's market share is $s_i(M_k) \equiv q_i(M_k)/Q(M_k)$. The post-merger profit of non-merging firm i is given by $\pi_i(M_k) \equiv [P(Q(M_k)) - c_i]\, q_i(M_k)$, the merged firm's profit by $\pi_k(M_k) \equiv [P(Q(M_k)) - \bar{c}_k]\, q_k(M_k)$, and aggregate profit by $\sum_{i\in\mathcal{N}\setminus\{0\}} \pi_i(M_k)$. The induced change in consumer surplus is

$$\Delta CS(M_k) \equiv \left\{ \int_0^{Q(M_k)} P(s)ds - P(Q(M_k))Q(M_k) \right\} - CS^0.$$

If no merger is implemented, the status quo (or "null merger") M_0 obtains, resulting in outcome $q(M_0) \equiv q^0$, $s_i(M_0) \equiv q_i^0/Q^0$, and $\Delta CS(M_0) = \Delta\Pi(M_0) = 0$. The realized set of feasible mergers is denoted $\mathfrak{F} \equiv \{M_k : \phi_k = 1\} \cup M_0$.

As these mergers are mutually exclusive, at most one merger can be proposed to the antitrust authority. If merger M_k, $k \in \mathfrak{F}$, is proposed, the antitrust authority can observe all aspects of that merger. We assume that the antitrust authority can commit ex ante to a merger-specific approval policy by specifying an approval set $\mathcal{A} \equiv \{M_k : \bar{c}_k \in \mathcal{A}_k\} \cup M_0$, where $\mathcal{A}_k \subseteq [l, h_k]$ for $k \in \mathcal{K}$ are the post-merger marginal cost levels that would lead to approval of merger k. Because of our assumption of full support and no mass points, we can without loss of generality restrict attention to the case where each $\mathcal{A}_k$ is a (finite or infinite) union of closed intervals, i.e., $\mathcal{A}_k \equiv \cup_{r=1}^{R} [l_k^r, h_k^r]$, where $l \leq l_k^r < h_k^r \leq h_k$ (R can be infinite). Note that the status quo M_0 is always "approved."

Given a realized set of feasible mergers $\mathfrak{F}$ and the antitrust authority's approval set $\mathcal{A}$, the set of feasible mergers that would be approved if proposed is given by $\mathfrak{F} \cap \mathcal{A}$. A bargaining process amongst the firms determines which feasible merger is actually proposed. Note that this bargaining problem involves externalities as firms' payoffs depend on the identity of the acquirer. There are various ways in which one could model this situation. For now, suppose the bargaining process takes the form of an offer game, as in Segal (1999), where the target (firm 0) makes public take-it-or-leave-it offers. In Segal (1999), the principal's offers consist of a profile of trades $x = (x_1, ..., x_K)$ with x_k the trade of agent k. Here, $x_k \in \{0,1\}$, where $x_k = 1$ if the target proposes a merger with firm k. Specifically, suppose firm 0 can make a take-it-or-leave-it offer t_k to a single firm k of its choosing, where k is such that $M_k \in (\mathfrak{F} \cap \mathcal{A})$. If the offer is accepted by firm k, then merger M_k is proposed to the antitrust authority, who will approve it since $M_k \in (\mathfrak{F} \cap \mathcal{A})$, and firm k acquires the target in return for the transfer payment t_k. If the offer is rejected, or if no offer is made, then no merger is proposed and no payments are made.

Let

$$\Delta\Pi(M_k) \equiv \pi_k(M_k) - \left[\pi_0^0 + \pi_k^0\right],\ k \geq 1,$$

denote the change in the bilateral profit to the merging parties, firms 0 and k, induced by merger M_k. Given the realized set of feasible and approvable mergers, $\mathfrak{F} \cap \mathcal{A}$, the proposed merger in the equilibrium of the offer game is $M^*(\mathfrak{F}, \mathcal{A})$, where

$$M^*(\mathfrak{F}, \mathcal{A}) \equiv \begin{cases} \tilde{M}(\mathfrak{F}, \mathcal{A}) & \text{if } \Delta\Pi\left(\tilde{M}(\mathfrak{F}, \mathcal{A})\right) > 0 \\ M_0 & \text{otherwise,} \end{cases}$$

and

$$\tilde{M}(\mathfrak{F}, \mathcal{A}) \equiv \arg \max_{M_k \in (\mathfrak{F} \cap \mathcal{A})} \Delta\Pi(M_k).$$

That is, the proposed merger M_k is the one that maximizes the induced change in the bilateral profit to firms 0 and k, provided that change is positive; otherwise, no merger is proposed.

In line with legal standards in the U.S. and many other countries, we assume that the antitrust authority acts in the consumers' interests. That is, the antitrust authority selects the approval set $\mathcal{A}$ that maximizes expected consumer surplus given that firms' proposal rule is $M^*(\cdot)$:

$$\max_{\mathcal{A}} E_{\mathfrak{F}}\left[\Delta CS\left(M^*(\mathfrak{F}, \mathcal{A})\right)\right],$$

where the expectation is taken with respect to the set of feasible mergers, $\mathfrak{F}$. (We discuss aggregate surplus maximization in Section 4.)

We are interested in studying how the optimal approval set depends on the pre-merger characteristics of the alternative mergers. For this reason, we assume that the potential acquirers differ in terms of their pre-merger marginal costs. Without loss of generality, let $\mathcal{K} \equiv \{1, ..., K\}$ and re-label firms 1 through K in decreasing order of their pre-merger marginal costs: $c_1 > c_2 > ... > c_K$. Thus, in the pre-merger equilibrium, firm $k \in \mathcal{K}$ produces more than firm $j \in \mathcal{K}$, and has a larger market share, if $k > j$. We will say that merger M_k is *larger* than merger M_j if $k > j$ as the combined pre-merger market share of firms 0 and k is larger than that of firms 0 and j.

3 Optimal Merger Policy

We now investigate the form of the antitrust authority's optimal policy when the bargaining process amongst firms takes the form of the offer game. Given a realized set of feasible mergers $\mathfrak{F}$ and an approval set $\mathcal{A}$, this bargaining process results in the merger $M^*(\mathfrak{F},\mathcal{A})$, as discussed in the previous section. We begin with some preliminary observations before turning to our main result.

3.1 Preliminaries

As firms produce a homogeneous good, a merger M_k raises [reduces] consumer surplus if and only if it raises [reduces] aggregate output Q. The following lemma summarizes some useful properties of a *CS-neutral* merger M_k, i.e., a merger that leaves consumer surplus unchanged, $\Delta CS(M_k) = 0$.

Lemma 1. *Suppose merger M_k is CS-neutral. Then*

1. *the merger causes no changes in the output of any nonmerging firm $i \notin \{0, k\}$ nor in the joint output of the merging firms 0 and k;*

2. *the merged firm's margin at the pre- and post-merger price $P(Q^\circ)$ equals the sum of the merging firms' pre-merger margins:*

$$P(Q^\circ) - \overline{c}_k = [P(Q^\circ) - c_0] + [P(Q^\circ) - c_k]\,; \tag{1}$$

3. *the merger is profitable for the merging firms, $\Delta\Pi(M_k) > 0$;*

4. *the merger increases aggregate profit, $\sum_{i\in\mathcal{N}\setminus\{0\}} \pi_i(M_k) > \sum_{i\in\mathcal{N}} \pi_i^0$.*

Proof. See Nocke and Whinston (2008) for a proof of parts (1)-(3). For part (4), note that the merger raises the joint profit of the merging firms 0 and k by part (3) and it leaves the profit of any nonmerging firm unchanged (as neither price nor their output changes). □

Rewriting equation (1), merger M_k is CS-neutral if the post-merger marginal cost satisfies

$$\overline{c}_k = \widehat{c}(Q^0) \equiv c_k - \left[P(Q^0) - c_0\right]. \tag{2}$$

An implication of (2), emphasized by Farrell and Shapiro (1990), is that a CS-neutral merger must involve a reduction in marginal cost below the marginal cost level of the more efficient merger partner: i.e., M_k can be CS-neutral only if $\overline{c}_k < \min\{c_0, c_k\}$.

As the following lemma shows, reducing the merged firm's marginal cost $\overline{c}_k$ not only increases consumer surplus but also the merged firm's profit:

Lemma 2. *Conditional on merger M_k being implemented, a reduction in the post-merger marginal cost $\overline{c}_k$ causes:*

1. *aggregate output $Q(M_k)$ and, therefore, consumer surplus $CS(M_k)$ to increase;*

2. *the induced change in the merging firms' bilateral profit, $\Delta\Pi(M_k)$, to rise.*

Proof. To see part (1), sum up the $N-1$ first-order conditions after merger M_k to obtain

$$(N-1)P(Q(M_k)) - \sum_{i\in\mathcal{N}\setminus\{0,k\}} c_i - \overline{c}_k + Q(M_k)P'(Q(M_k)) = 0.$$

Applying the implicit function theorem yields

$$\frac{dQ(M_k)}{d\overline{c}_k} = \left[NP'(Q(M_k)) + Q(M_k)P''(Q(M_k))\right]^{-1} < 0,$$

where the inequality follows from Assumption 1.

To see part (2), rewrite firm i's first-order condition to obtain

$$q_i(M_k) = -\frac{[P(Q(M_k)) - c_i]}{P'(Q(M_k)).}$$

As the RHS is decreasing in $Q(M_k)$, this implies that $dq_i(M_k)/d\bar{c}_k > 0$. Next, take the derivative of the merged firm's profit with respect to its post-merger marginal cost:

$$\frac{d}{d\bar{c}_k}\left[P(Q(M_k)) - \bar{c}_k\right] q_k(M_k) = -q_k(M_k) + P'(Q(M_k)) \sum_{i \in \mathcal{N} \setminus \{0,k\}} \frac{dq_i(M_k)}{d\bar{c}_k}.$$

But this expression is strictly negative. □

Part (1) of the lemma implies that merger M_k is CS-increasing [i.e., $\Delta CS(M_k) > 0$] if $\bar{c}_k < \widehat{c}(Q^0)$ and CS-decreasing [i.e., $\Delta CS(M_k) < 0$] if $\bar{c}_k > \widehat{c}(Q^0)$.

To make the antitrust authority's problem interesting, and avoid certain degenerate cases we will henceforth assume the following:

Assumption 2. *For all $k \in \mathcal{K}$, the probability that the merger M_k is CS-increasing is positive but less than one: $\Delta CS(k, h_k) < 0 < \Delta CS(k, l)$.*

The following lemma gives a key result that indicates that there is a systematic bias in the proposal incentives of firms, relative to the interests of consumers, in favor of larger mergers:

Lemma 3. *Suppose two mergers, M_j and M_k, with $k > j \geq 1$, induce the same non-negative change in consumer surplus, $\Delta CS(M_j) = \Delta CS(M_k) \geq 0$. Then the larger merger M_k induces a greater increase in the bilateral profit of the merger partners: $\Delta\Pi(M_k) > \Delta\Pi(M_j) > 0$.*

Proof. Suppose otherwise that $\Delta\Pi(M_k) \leq \Delta\Pi(M_j)$, i.e.,

$$\pi_k(M_k) - \pi_j(M_j) \leq \pi_k^0 - \pi_j^0. \tag{3}$$

Using the first-order conditions of profit maximization, the term on the r.h.s. of equation (3) can be re-written as

$$\begin{aligned} \pi_k^0 - \pi_j^0 &= \left[P(Q^0) - c_k\right] q_k^0 - \left[P(Q^0) - c_j\right] q_j^0 \\ &= \frac{\left[P(Q^0) - c_k\right]^2 - \left[P(Q^0) - c_j\right]^2}{-P'(Q^0)} \\ &= \left[\left(\frac{P(Q^0) - c_j}{-P'(Q^0)}\right) + \left(\frac{P(Q^0) - c_k}{-P'(Q^0)}\right)\right] [c_j - c_k] \\ &= \left[q_j^0 + q_k^0\right] [c_j - c_k]. \end{aligned}$$

As both mergers induce the same aggregate output (i.e., $Q(M_j) = Q(M_k)$), the term on the l.h.s. of equation (3) can similarly be re-written as

$$\pi_k(M_k) - \pi_j(M_j) = [q_j(M_j) + q_k(M_k)] [\bar{c}_j - \bar{c}_k].$$

Next, we claim that

$$[\bar{c}_j - \bar{c}_k] = [c_j - c_k].$$

To see this, let $\overline{Q} \equiv Q(M_j) = Q(M_k)$ denote the level of aggregate output after either merger. Summing up the N first-order conditions of profit maximization after merger M_l, $l = j, k$, we obtain

$$NP(\overline{Q}) - \left(\sum_{i \geq 1, i \neq l} c_i + \overline{c}_l \right) + \overline{Q}P'(\overline{Q}) = 0.$$

It follows that $c_i + \overline{c}_l$, $i, l = j, k$, $i \neq l$, is the same under either merger, proving the claim.

Combining these observations, we can re-write equation (3) as

$$[q_j(M_j) + q_k(M_k)] \leq \left[q_j^0 + q_k^0 \right].$$

Now, as merger M_l, $l = j, k$, is CS-nondecreasing by assumption, the merger induces a weak increase in the joint output of the merger partners and a weak decrease in the output of any other firm $i \neq 0, l$. That is,

$$q_l(M_l) \geq q_0^0 + q_l^0 > q_l^0 \geq q_l(M_r), \; l, r = j, k, \; l \neq r,$$

implying that

$$[q_j(M_j) + q_k(M_k)] > \left[q_j^0 + q_k^0 \right],$$

and thus resulting in a contradiction. Hence, equation (3) cannot hold. □

Assumptions 1-2 and Lemmas 2-3 imply that the possible mergers can be represented as shown in Figure 1 (where there are four possible mergers; i.e., $K = 4$). In the figure, the change in the merging firms' profit, $\Delta\Pi$, is measured on the horizontal axis and the change in consumer surplus, ΔCS, is measured on the vertical axis. The CS-increasing mergers therefore are those lying above the horizontal axis. The bilateral profit and consumer surplus changes induced by a merger between firms 0 and $k \geq 1$, $(\Delta\Pi(M_k), \Delta CS(M_k))$, fall somewhere on the curve labeled "M_k." (The figure shows only the parts of these curves for which the bilateral profit change $\Delta\Pi$ is nonnegative.) Since by Lemma 1 a CS-neutral merger is profitable for the merger partners, each curve crosses the horizontal axis to the right of the vertical axis. By Lemma 2, the curve for each merger M_k, $k \geq 1$, is upward sloping everywhere above the horizontal axis. By Lemma 3, above the horizontal axis the curves for larger mergers lie everywhere to the right of those for smaller mergers.

A useful corollary of Lemma 3, which can easily be seen in Figure 1, is the following:

Corollary 1. *If two CS-nondecreasing mergers M_j and M_k with $k > j \geq 1$ have $\Delta\Pi(M_k) \leq \Delta\Pi(M_j)$, then $\Delta CS(M_k) < \Delta CS(M_j)$.*

Proof. Suppose instead that $\Delta CS(M_k) \geq \Delta CS(M_j)$. Then there exists a $\overline{c}'_k > \overline{c}_k$ such that $\Delta CS(k, \overline{c}'_k) = \Delta CS(M_j)$. But this implies (using Lemma 2 for the first inequality and Lemma 3 for the second) that $\Delta\Pi(M_k) > \Delta\Pi(k, \overline{c}'_k) > \Delta\Pi(M_j)$, a contradiction. □

3.2 Optimal Merger Policy

We can now turn to the optimal policy of the antitrust authority. Recall that the antitrust authority can without loss restrict itself to approval sets in which the set of acceptable cost levels for a merger between firm 0 and each firm k, $\mathcal{A}_k \subseteq [l, h_k]$, is a union of closed intervals.

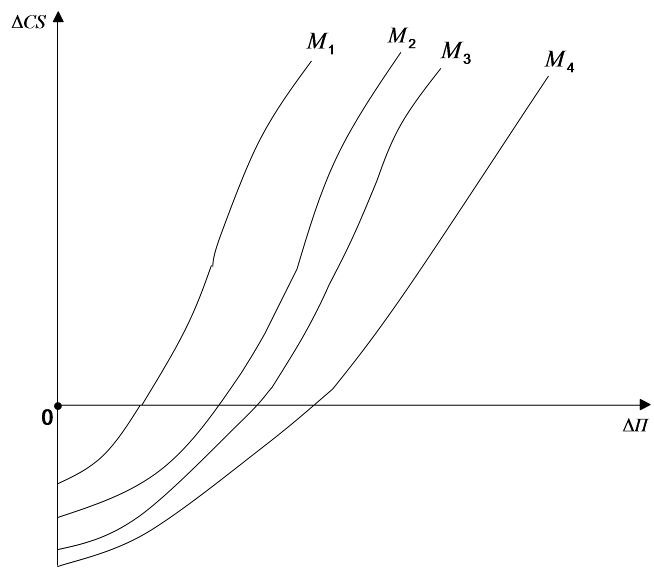

Figure 1: The curves depict the relationship between consumer surplus effect and bilateral profit effect of the various mergers, where each point on a curve corresponds to a different realization of post-merger marginal cost for that merger.

Throughout we restrict attention to such policies.[1] Let $\overline{a}_k \equiv \max\{\overline{c}_k | \overline{c}_k \in \mathcal{A}_k\}$ denote the largest allowable post-merger cost level for a merger (i.e., the "marginal merger") between firms 0 and k. Also let $\underline{\Delta CS}_k \equiv \Delta CS(k, \overline{a}_k)$ and $\underline{\Delta \Pi}_k \equiv \Delta\Pi(k, \overline{a}_k)$ denote the changes in consumer surplus and bilateral profit levels, respectively, induced by that marginal merger. These are the lowest levels of consumer surplus and bilateral profit in any allowable merger between firms 0 and k.

We now state our main result:

Proposition 1. *Any optimal approval policy $\mathcal{A}$ approves the smallest merger if and only if it is CS-nondecreasing, approves only mergers $k \in \mathcal{K}^+ \equiv \{1, ..., \widehat{K}\}$ with positive probability ($\widehat{K}$ may equal K) and satisfies $0 = \underline{\Delta CS}_1 < \underline{\Delta CS}_2 < ... < \underline{\Delta CS}_{\widehat{K}}$ for all $k \leq \widehat{K}$. That is, the lowest level of consumer surplus change that is acceptable to the antitrust authority equals zero for the smallest merger M_1, is strictly positive for every other merger M_k with $k > 1$, and is monotonically increasing in the size of the merger, while the largest merger(s) may never be approved.*

Proof. The proof proceeds in a number of steps.

Step 1. We observe first that an optimal policy does not approve CS-decreasing mergers. To see this, suppose the approval set $\mathcal{A}$ includes CS-decreasing mergers, and consider the set $\mathcal{A}^+ \subseteq \mathcal{A}$ that removes any mergers in $\mathcal{A}$ that reduce consumer surplus. Figure 2 depicts such a pair of approval sets, each containing the points shown with heavy trace. Since this change only matters when the bilateral profit-maximizing merger $M^*(\mathfrak{F},\mathcal{A})$ under set $\mathcal{A}$ is no longer approved under $\mathcal{A}^+$, the change in expected consumer surplus from this change in the approval policy equals $\Pr(M^*(\mathfrak{F},\mathcal{A}) \in \mathcal{A} \backslash \mathcal{A}^+)$, the probability of this event happening, times the conditional expectation

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F},\mathcal{A}^+)) - \Delta CS(M^*(\mathfrak{F},\mathcal{A}))|M^*(\mathfrak{F},\mathcal{A}) \in \mathcal{A} \backslash \mathcal{A}^+].$$

Since $\Delta CS(M^*(\mathfrak{F},\mathcal{A}^+))$ is necessarily nonnegative by construction of $\mathcal{A}^+$, and $\Delta CS(M^*(\mathfrak{F},\mathcal{A}))$ is strictly negative whenever $M^*(\mathfrak{F},\mathcal{A}) \in \mathcal{A} \backslash \mathcal{A}^+$, this change is strictly positive.

Step 2. Next, any smallest merger M_1 that is CS-nondecreasing must be approved. To see this, suppose that the approval set is $\mathcal{A}$ but that $\mathcal{A} \subset \mathcal{A}' \equiv (\mathcal{A} \cup \{(1, \overline{c}_1) : \Delta CS(1, \overline{c}_1) \geq 0\})$. Figure 3 depicts two such sets, $\mathcal{A}$ and $\mathcal{A}'$. Because a change from $\mathcal{A}'$ to $\mathcal{A}$ matters only when the bilateral profit-maximizing merger $M^*(\mathfrak{F},\mathcal{A}')$ under $\mathcal{A}'$ is no longer approved under $\mathcal{A}$, the change in expected consumer surplus by using $\mathcal{A}'$ rather than $\mathcal{A}$ equals $\Pr(M^*(\mathfrak{F},\mathcal{A}') \in \mathcal{A}' \backslash \mathcal{A})$ times

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F},\mathcal{A}')) - \Delta CS(M^*(\mathfrak{F},\mathcal{A}))|M^*(\mathfrak{F},\mathcal{A}') \in \mathcal{A}' \backslash \mathcal{A}]. \tag{4}$$

By Corollary 1 and the fact that $\mathcal{A}' \backslash \mathcal{A}$ contains only smallest mergers (between firms 0 and 1), whenever $M^*(\mathfrak{F},\mathcal{A}') \in \mathcal{A}' \backslash \mathcal{A}$ [which implies $\Delta\Pi(M^*(\mathfrak{F},\mathcal{A}')) > \Delta\Pi(M^*(\mathfrak{F},\mathcal{A}))$] we have $\Delta CS(M^*(\mathfrak{F},\mathcal{A}')) > \Delta CS(M^*(\mathfrak{F},\mathcal{A}))$, so (4) is strictly positive. This can be seen in Figure 3. This implies in particular that $\underline{\Delta CS}_1 = 0$.

Step 3. Next, let $\mathcal{K}^+$ denote those acquirers with $k \neq 1$ for whom the probability of having a merger $M_k \in \mathcal{A}$ is strictly positive. We claim that, in any optimal policy, $\underline{\Delta CS}_k > 0$ for all

[1] Thus, when we state that any optimal policy must have a particular form, we mean any optimal interval policy of this sort. There are other optimal policies that add or subtract in addition some measure zero sets of mergers, since these have no effect on expected consumer surplus.

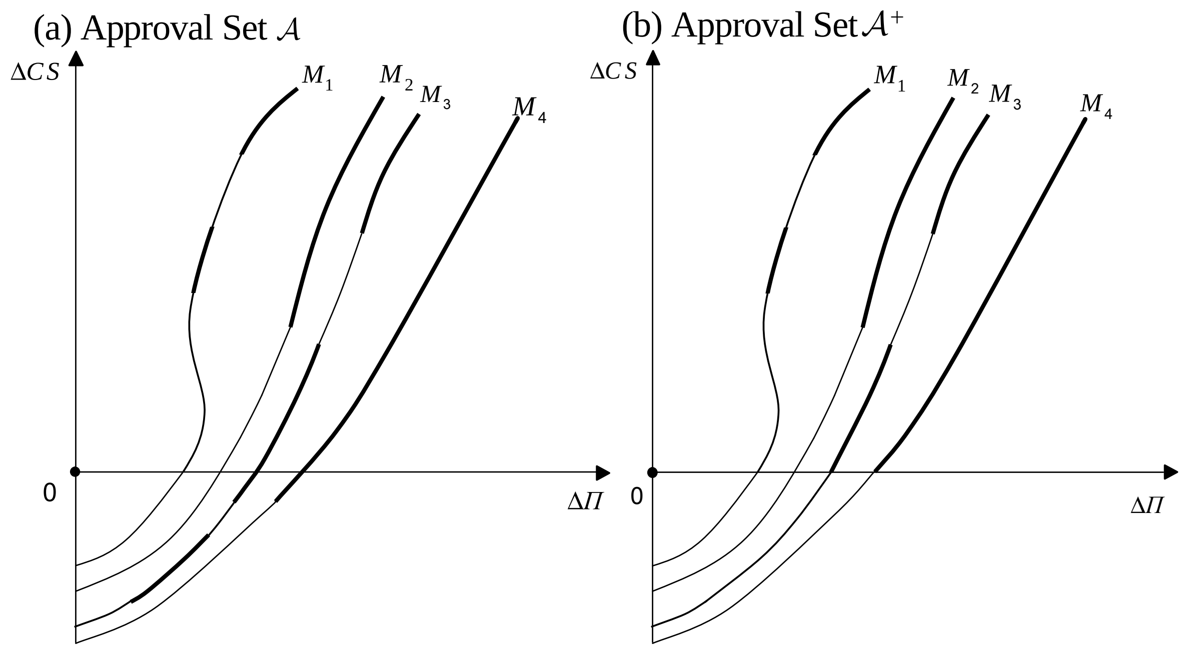

Figure 2: Changing the approval set $\mathcal{A}$ by blocking all mergers that induce a reduction in consumer surplus, resulting in approval set $\mathcal{A}^+$, raises expected consumer surplus.

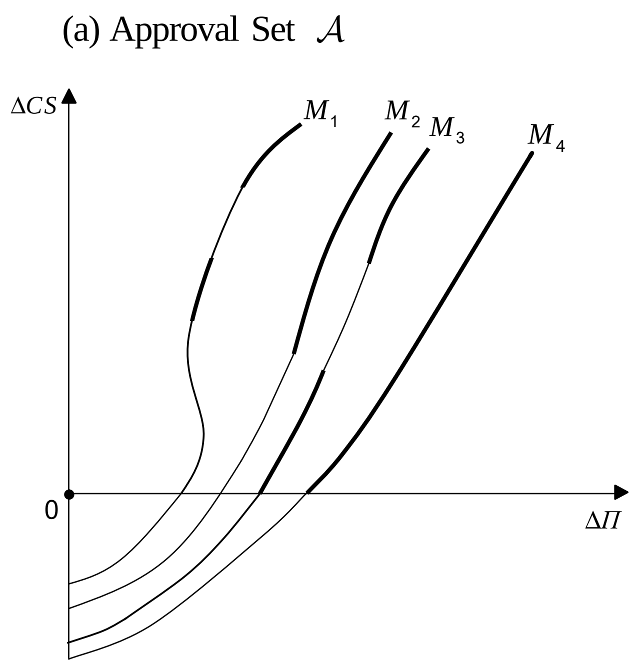

Figure 3: Changing the approval set $\mathcal{A}$ by approving the smallest merger M_1 whenever it does not reduce consumer surplus, resulting in approval set $\mathcal{A}^{+}$, raises expected consumer surplus.

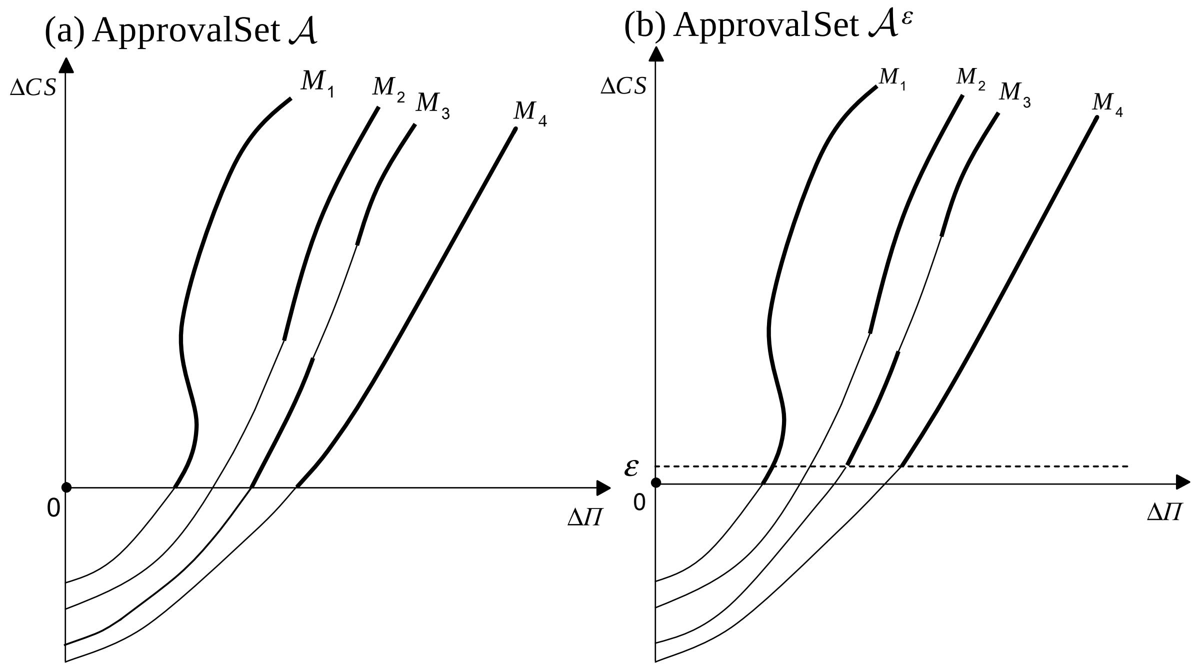

Figure 4: Changing the approval set $\mathcal{A}$ by blocking all those mergers other than the smallest that raise consumer surplus by less than ε, resulting in approval set $\mathcal{A}^\varepsilon$, raises expected consumer surplus for ε sufficiently small.

$k \in \mathcal{K}^+$. To see this, consider switching from the policy $\mathcal{A}$ to $\mathcal{A}^\varepsilon \equiv \{M_k \in \mathcal{A} : k \in \mathcal{K}^+$ and $\Delta CS(M_k) > \varepsilon\}$ where $\varepsilon > 0$, as shown in Figure 4. The change in expected consumer surplus equals $\Pr(M^*(\mathfrak{F},\mathcal{A}) \in \mathcal{A}\backslash\mathcal{A}^\varepsilon)$ times

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F},\mathcal{A}^\varepsilon)) - \Delta CS(M^*(\mathfrak{F},\mathcal{A}))|M^*(\mathfrak{F},\mathcal{A}) \in \mathcal{A}\backslash\mathcal{A}^\varepsilon].$$

Now, as $\varepsilon \to 0$, this conditional expectation approaches

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F},\mathcal{A}^\varepsilon))|M^*(\mathfrak{F},\mathcal{A}) \in \mathcal{A}\backslash\mathcal{A}^\varepsilon],$$

which is strictly positive given steps 1 and 2.

Step 4. Next, we claim that in any optimal policy, for all $k \in \mathcal{K}^+$, $\underline{\Delta CS}_k$ must equal the expected change in consumer surplus from the next-most-profitable merger (i.e., from the merger with the second-highest bilateral profit change) $M^*(\mathfrak{F}\backslash(k,\overline{a}_k),\mathcal{A})$, conditional on merger $M_k = (k, \overline{a}_k)$ being the most profitable merger in $\mathfrak{F} \cap \mathcal{A}$. Defining the expected change in consumer surplus from the next-most-profitable merger $M^*(\mathfrak{F} \backslash M_k, \mathcal{A})$, conditional on merger $M_k = (k, \overline{c}_k)$ being the most profitable merger in $\mathfrak{F} \cap \mathcal{A}$, to be

$$\begin{aligned} E_k^{\mathcal{A}}(\overline{c}_k) &\equiv E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F} \backslash M_k, \mathcal{A})) | M_k = (k, \overline{c}_k) \text{ and } M_k = M^*(\mathfrak{F}, \mathcal{A})] && (5) \\ &= E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F} \backslash M_k, \mathcal{A})) | M_k = (k, \overline{c}_k) \text{ and } \Delta\Pi(M^*(\mathfrak{F} \backslash M_k, \mathcal{A})) \leq \Delta\Pi(M_k)], && (6) \end{aligned}$$

this means that

$$\underline{\Delta CS}_k = E_k^{\mathcal{A}}(\overline{a}_k). \tag{7}$$

In Figure 5 the possible locations of the next-most-profitable merger when the most profitable merger is $M_2 = (2, \overline{a}_2)$ are shown as a shaded set. The quantity $E_2^{\mathcal{A}}(\overline{a}_2)$ is the expectation of the change in consumer surplus for the merger that has the largest change in bilateral profit among mergers other than M_2, conditional on all of these other mergers lying in the shaded region of the figure.

To see that (7) must hold for all $k \in \mathcal{K}^+$, suppose first that $\underline{\Delta CS}_{k'} > E_{k'}^{\mathcal{A}}(\overline{a}_{k'})$ for some $k' \in \mathcal{K}^+$ and consider the alternative approval set $\mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon}$ where

$$\mathcal{A}_{k'}^{\varepsilon} \equiv \{M_k : M_k = (k', \overline{c}_{k'}) \text{ with } \overline{c}_{k'} \in (\overline{a}_{k'}, \overline{a}_{k'} + \epsilon)\}.$$

For any $\varepsilon > 0$, the change in expected consumer surplus from changing from $\mathcal{A}$ to $\mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon}$ equals $\Pr(M^*(\mathfrak{F}, \mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon}) \in \mathcal{A}_{k'}^{\varepsilon})$ times

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F}, \mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon})) - \Delta CS(M^*(\mathfrak{F}, \mathcal{A})) | M^*(\mathfrak{F}, \mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon}) \in \mathcal{A}_{k'}^{\varepsilon}]. \tag{8}$$

This conditional expectation can be rewritten as

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F}, \mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon})) - E_{k'}^{\mathcal{A}}(\overline{c}_{k'}) | M^*(\mathfrak{F}, \mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon}) \in \mathcal{A}_{k'}^{\varepsilon}], \tag{9}$$

where $\overline{c}_{k'}$ is the realized cost level in the bilateral profit-maximizing merger $M^*(\mathfrak{F}, \mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon})$, which is a merger of firms 0 and k' when the conditioning statement is satisfied. By continuity of $\Delta CS(k', \overline{c}_{k'})$ and $E_k^{\mathcal{A}}(\overline{c}_{k'})$ in $\overline{c}_{k'}$, there exists an $\overline{\varepsilon} > 0$ such that $\Delta CS(M_{k'}) > E_k^{\mathcal{A}}(\overline{c}_{k'})$ for all $M_{k'} \in \mathcal{A}_{k'}^{\varepsilon}$ provided $\varepsilon \in (0, \overline{\varepsilon}]$. For all such ε, the conditional expectation (9) is strictly positive so this change in the approval set would strictly increase expected consumer surplus. A similar argument applies if $\underline{\Delta CS}_{k'} < E_{k'}^{\mathcal{A}}(\overline{a}_{k'})$.

Step 5. Next, we argue that for all $j < k$ such that $j, k \in \mathcal{K}^+$ it must be that $\underline{\Delta\Pi}_j \leq \underline{\Delta\Pi}_k$; that is, the bilateral profit change in the marginal merger by acquirer j must be no greater than the bilateral profit change in the marginal merger by any larger acquirer k. Figure 6(a) shows a situation that violates this condition, where the marginal merger by acquirer 3 causes a smaller bilateral profit change $\underline{\Delta\Pi}_3$, than the marginal merger by the smaller acquirer 2, $\underline{\Delta\Pi}_2$.

For $j \in \mathcal{K}^+$, let $k' \equiv \arg\min_{k \in \mathcal{K}^+, k > j} \underline{\Delta\Pi}_k$ and suppose that $\underline{\Delta\Pi}_{k'} < \underline{\Delta\Pi}_j$. We know from the previous step that $\underline{\Delta CS}_{k'} = E_{k'}^{\mathcal{A}}(\overline{a}_{k'})$. Let $\overline{c}_j'$ be the post-merger cost level satisfying $\Delta\Pi(j, \overline{c}_j') = \underline{\Delta\Pi}_{k'}$ and consider a change in the approval set from $\mathcal{A}$ to $\mathcal{A} \cup \overline{\mathcal{A}}_j^{\varepsilon}$ where

$$\overline{\mathcal{A}}_j^{\varepsilon} \equiv \{M_j : M_j = (j, \overline{c}_j) \text{ with } \overline{c}_j \in \left(\overline{c}_j', \overline{c}_j' + \epsilon\right)\}.$$

The set $\overline{\mathcal{A}}_j^{\varepsilon}$ is shown in Figure 6(b). The change in expected consumer surplus from this change in the approval set equals $\Pr(M^*(\mathfrak{F}, \mathcal{A} \cup \overline{\mathcal{A}}_j^{\varepsilon}) \in \overline{\mathcal{A}}_j^{\varepsilon})$ times

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F}, \mathcal{A} \cup \overline{\mathcal{A}}_j^{\varepsilon})) - E_j^{\mathcal{A}}(\overline{c}_j) | M^*(\mathfrak{F}, \mathcal{A} \cup \overline{\mathcal{A}}_j^{\varepsilon}) \in \overline{\mathcal{A}}_j^{\varepsilon}], \tag{10}$$

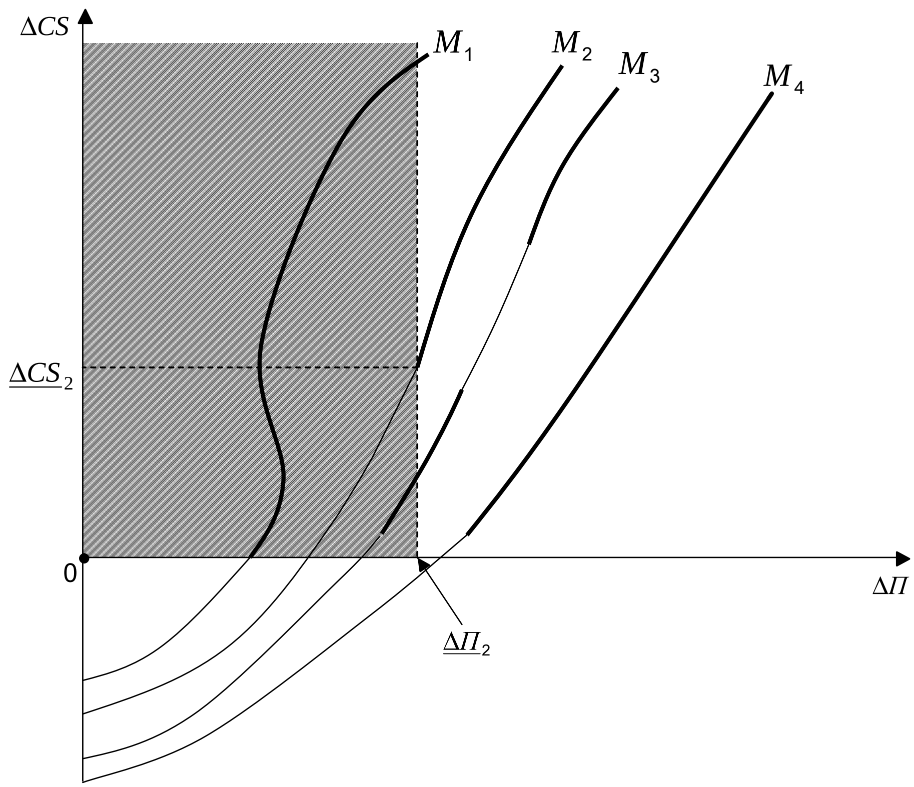

Figure 5: The optimal approval policy is such that the increase in consumer surplus induced by the worst allowable merger M_k is equal to the expected consumer surplus change from the next-most profitable merger, conditional on the marginal merger being the most profitable merger in the set of feasible and allowable mergers.

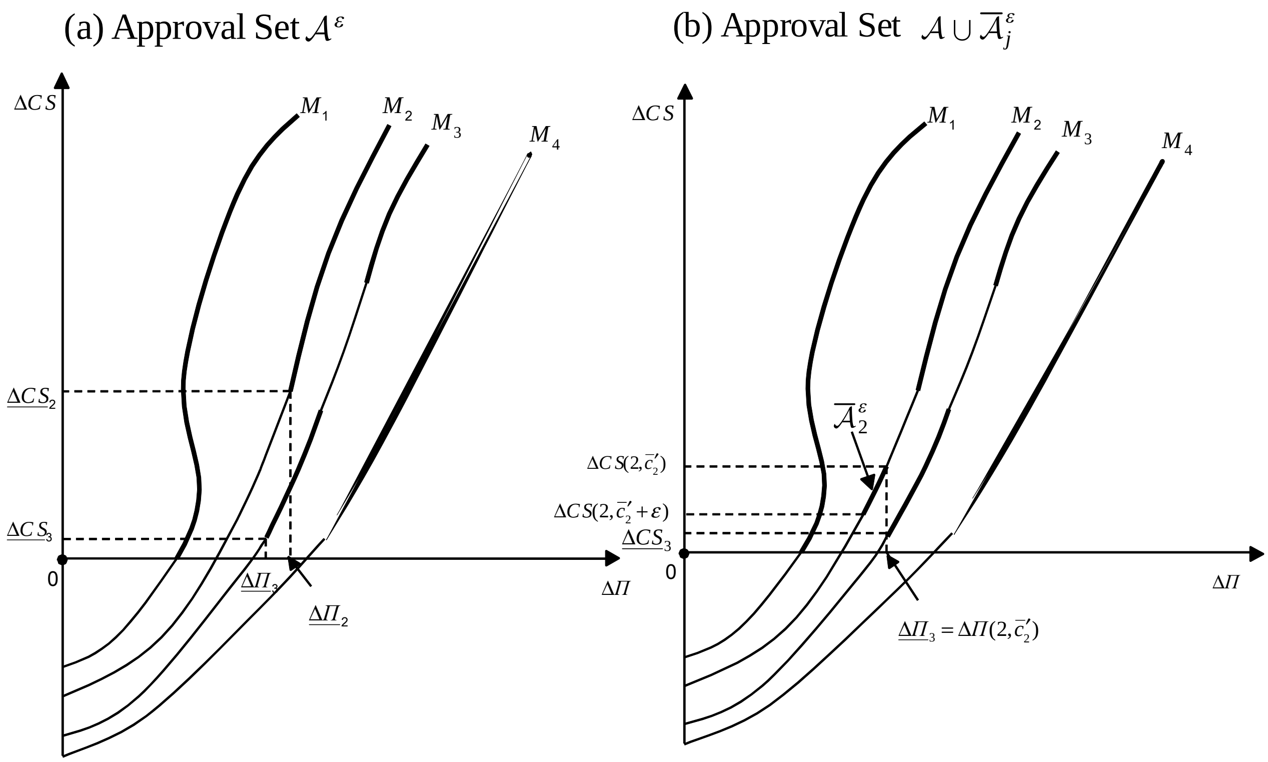

Figure 6: The optimal proval policy is such that the profit increase induced by the worst allowable merger M_j, is no greater than that by the worst allowable larger merger M_k, $k > j$, i.e., $\underline{\Delta\Pi}_j \leq \underline{\Delta\Pi}_k$. Panel (a), where $\underline{\Delta\Pi}_2 > \underline{\Delta\Pi}_3$, shows a violation of that property. Panel (b) illustrates that, in case of a violation, the antitrust authority can increase expected consumer surplus by approving some mergers M_2 whose induced profit change is just below $\underline{\Delta\Pi}_3$.

where $\overline{c}_j$ is the realized cost level in the aggregate profit-maximizing merger $M^*(\mathfrak{F},\mathcal{A}\cup\overline{\mathcal{A}}_j^{\varepsilon})$, which is a merger of firms 0 and j when the conditioning statement is satisfied. As $\varepsilon \to 0$, the expected change in (10) converges to

$$\begin{aligned} \Delta CS(j,\overline{c}_j') - E_j^{\mathcal{A}}(\overline{c}_j') &= \Delta CS(j,\overline{c}_j') - E_{k'}^{\mathcal{A}}(\overline{a}_{k'}) \\ &> \underline{\Delta CS}_{k'} - E_{k'}^{\mathcal{A}}(\overline{a}_{k'}) \\ &= 0, \end{aligned}$$

where the inequality follows from Corollary 1 since $\Delta\Pi(j,\overline{c}_j') = \underline{\Delta\Pi}_{k'}$.

Step 6. We next argue that $\underline{\Delta CS}_j < \underline{\Delta CS}_k$ for all $j,k \in \mathcal{K}^+$ with $j < k$. Suppose otherwise; i.e., for some $j,h \in \mathcal{K}^+$ with $h > j$ we have $\underline{\Delta CS}_j \geq \underline{\Delta CS}_h$. Define $k = \arg\min\left\{h \in \mathcal{K}^+ : h > j \text{ and } \underline{\Delta CS}_j \geq \underline{\Delta CS}_h\right\}$. Figure 7 depicts such a situation where $j = 2$ and $k = 3$.

By Step 4, we must have $E_j^{\mathcal{A}}(\overline{a}_j) = \underline{\Delta CS}_j \geq \underline{\Delta CS}_k = E_k^{\mathcal{A}}(\overline{a}_k)$. But recalling (6), $E_k^{\mathcal{A}}(\overline{a}_k)$ can be written as a weighted average of two conditional expectations:

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F}\backslash M_k,\mathcal{A}))|M_k = (k,\overline{c}_k),\ M_k = M^*(\mathfrak{F},\mathcal{A}), \text{ and } \Delta\Pi(M^*(\mathfrak{F}\backslash M_k,\mathcal{A})) < \underline{\Delta\Pi}_j] \quad (11)$$

and

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F}\backslash M_k,\mathcal{A}))|M_k = (k,\overline{c}_k),\ M_k = M^*(\mathfrak{F},\mathcal{A}), \text{ and } \Delta\Pi(M^*(\mathfrak{F}\backslash M_k,\mathcal{A})) \in [\underline{\Delta\Pi}_j, \underline{\Delta\Pi}_k]]. \quad (12)$$

Expectation (11) conditions on the event that the next-most-profitable merger other than $(k,\overline{a}_k)$ induces a bilateral profit change less than $\underline{\Delta\Pi}_j$, the bilateral profit change of merger $(j,\overline{a}_j)$. Since no merger in $\mathcal{A}$ by either acquirer k or j can have such a profit level (since $\underline{\Delta\Pi}_k > \underline{\Delta\Pi}_j$ by Step 5), the expectation (11) must exactly equal $E_j^{\mathcal{A}}(\overline{a}_j)$. Now consider the expectation (12). If $\Delta\Pi(M^*(\mathfrak{F}\backslash M_k,\mathcal{A})) \in [\underline{\Delta\Pi}_j, \underline{\Delta\Pi}_k)$, it could be that (i) $M^*(\mathfrak{F}\backslash M_k,\mathcal{A}) = (j,\overline{c}_j)$ for some $\overline{c}_j \leq \overline{a}_j$, or (ii) $M^*(\mathfrak{F}\backslash M_r,\mathcal{A}) = (r,\overline{c}_r)$ for some $r < j$, or (iii) $M^*(\mathfrak{F}\backslash M_r,\mathcal{A}) = (r,\overline{c}_r)$ for some $r > j$ and $r < k$. Now, in case (i) it is immediate that $\Delta CS(M^*(\mathfrak{F}\backslash M_k,\mathcal{A}) \geq \underline{CS}_j$, with strict inequality whenever $\overline{c}_j = \overline{a}_j$. In case (ii), the fact that $\Delta\Pi(r,\overline{c}_r) \geq \underline{\Delta\Pi}_j$ implies by Corollary 1 that

$$\Delta CS(M^*(\mathfrak{F}\backslash M_k,\mathcal{A}) = \Delta CS(r,\overline{c}_r) > \underline{CS}_j = E_j^{\mathcal{A}}(\overline{a}_j). \quad (13)$$

In case (iii), (13) follows from the definition of k. Thus, expectation (12) must strictly exceed $E_j^{\mathcal{A}}(\overline{a}_j)$, which leads to a contradiction.

Step 7. Finally, we argue that $\mathcal{K}^+ = \{1,...,\widehat{K}\}$ for some $\widehat{K}$. To establish this fact, we show that if $k \notin \mathcal{K}^+$, then $k+1 \notin \mathcal{K}^+$. We first observe that $\Delta CS(k,l) > \Delta CS(k+1,l)$, which follows because the profile of firms' costs following merger (k,l) are lower than following merger $(k+1,l)$ (the post-merger industry cost profile differs only for firms k and $k+1$, which have costs of l and c_{k+1} with the first merger and c_k and l with the second). Thus, if $k+1 \in \mathcal{K}^+$, then $\Delta CS(k+1,\overline{a}_{k+1}) < \Delta CS(k,l)$. But, an argument like that in Step 6 [using the fact that, by an argument like that in Step 4, $\Delta CS(k,l) \leq E_k^{\mathcal{A}}(\overline{a}_k)$] shows that $\Delta CS(k,l) < E_{k+1}^{\mathcal{A}}(\overline{a}_{k+1})$, so that $\Delta CS(k+1,\overline{a}_{k+1}) < E_{k+1}^{\mathcal{A}}(\overline{a}_{k+1})$, contradicting the conclusion of Step 4. □

We have shown that there is a misalignment between firms' proposal incentives and the interests of the antitrust authority: firms tend to have an incentive to propose a merger that

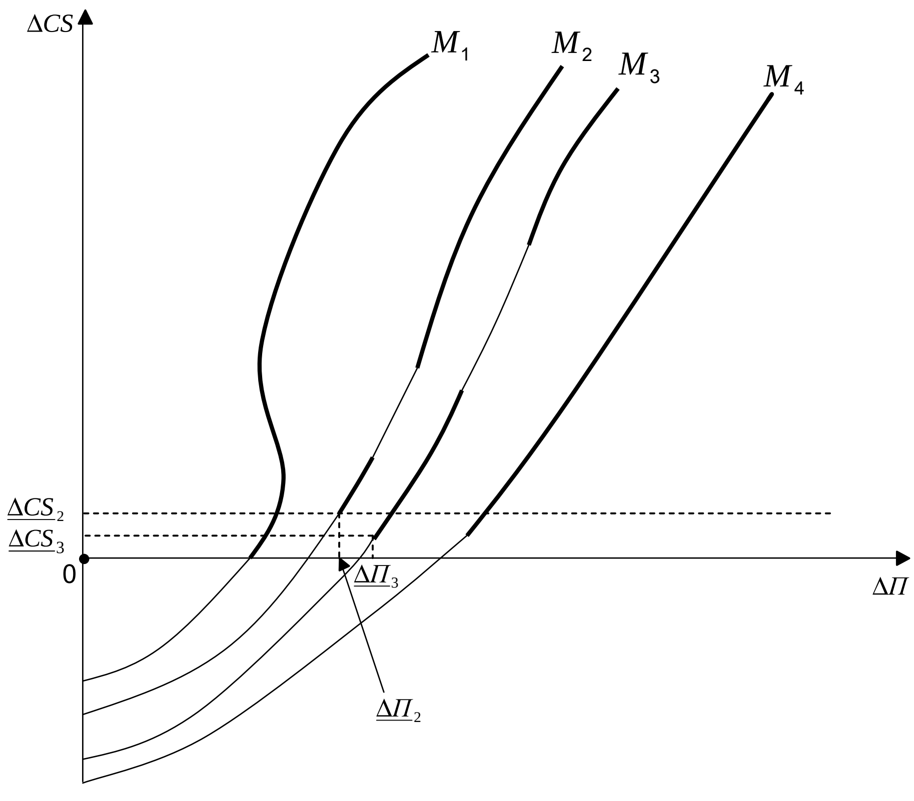

Figure 7: The optimal approval set is such that the consumer surplus increase induced by the worst allowable merger M_j, is less than that by the worst allowable larger merger M_k, $k > j$, i.e., $\underline{\Delta CS}_j < \underline{\Delta CS}_k$. In the figure, $\underline{\Delta CS}_2 > \underline{\Delta CS}_3$, which is a violation of that property.

is larger (in terms of the pre-merger size of the merger partner) than the one that would maximize consumer surplus. To compensate for this intrinsic bias in firms' proposal incentives, the antitrust authority should optimally adopt a higher minimum CS-standard the larger is the proposed merger.

Remark 1. *Does the optimal policy have a cut-off structure so that $\mathcal{A}_k = [l, \overline{a}_k]$? The answer is no, as the following example illustrates. (For simplicity, the example considers the case where, contrary to the assumption of the model, one of the mergers has a finite support of post-merger marginal costs. But the same insight would obtain if we perturbed the example and assumed that the support is continuous with no atoms.)*

Suppose that there are two possible mergers, M_1 and M_2 . The smaller merger, M_1, is always feasible. Its post-merger marginal cost is either $\overline{c}_1 = l$ or $\overline{c}_1 = h_1$, where the probability on the latter is 0.9. The corresponding changes in consumer surplus and bilateral profit are given by $(\Delta CS(1,l), \Delta\Pi(1,l)) = (5,5)$ and $(\Delta CS(1,h_1), \Delta\Pi(1,h_1)) = (1,1)$. The unconditional expected increase in consumer surplus from approving M_1 is thus equal to 4.6. The post-merger marginal cost of the larger merger, M_2, has a continuous support $[l, h_2]$ with no atoms, satisfying $\Delta CS(2,h_2) < 1$ and $5 < \Delta CS(2,l)$. It is straightforward to verify that the optimal approval policy $\mathcal{A}^*$ is such that $\mathcal{A}_1 = \{l, h_1\}$ and $\mathcal{A}_2 = [l, c_2'] \cup [c_2'', \overline{a}_2]$, where c_2' and $c_2'' > c_2'$ are implicitly defined by $\Delta CS(2, c_2') = 4.6$ and $\Delta CS(2, c_2'') = 4$. This situation is illustrated in Figure 8. To see why the optimal approval policy for M_2 does not have a cut-off structure, note that for any post-merger marginal cost $\overline{c}_2 \in (c_2', c_2'')$, the induced change in consumer surplus is less than 5 (which is the induced change in consumer surplus of the best realization of M_1). But, if approved, the firms would propose the larger merger even if the realized M_1 is better for consumers as, for $\overline{c}_2 \in (c_2', c_2'')$, $\Delta\Pi(2, \overline{c}_2) > 5 = \Delta\Pi(1, l)$. The optimal policy corrects for this bias in firms' proposal policies by not approving M_2 whenever $\overline{c}_2 \in (c_2', c_2'')$.

4 Other Bargaining Processes

In our analysis so far, we have focused on the case where the bargaining process between firms is given by the offer game, resulting in the proposal of the merger that maximizes the change in the bilateral profit of the merger partners in the realized set of feasible and approvable mergers. In this section, we explore two alternative bargaining processes. First, we consider the benchmark case of efficient bargaining. Second, we consider the case where there is (efficient) bargaining only between a subset of firms (including all of those firms that are involved in potential mergers). We show that, in both cases, the main result continues to hold: the optimal approval policy has the property that the minimum CS-standard is increasing in the size of the proposed merger.

4.1 Efficient Bargaining

Suppose the outcome of the bargaining processes is efficient for the firms in the industry in the sense that it maximizes aggregate profit. That is, we assume that, from the realized set of feasible and approvable mergers, $\mathfrak{F} \cap \mathcal{A}$, firms choose to propose merger

$$M^*(\mathfrak{F}, \mathcal{A}) \equiv \arg \max_{M_k \in (\mathfrak{F} \cap \mathcal{A})} \Delta\Pi(M_k),$$

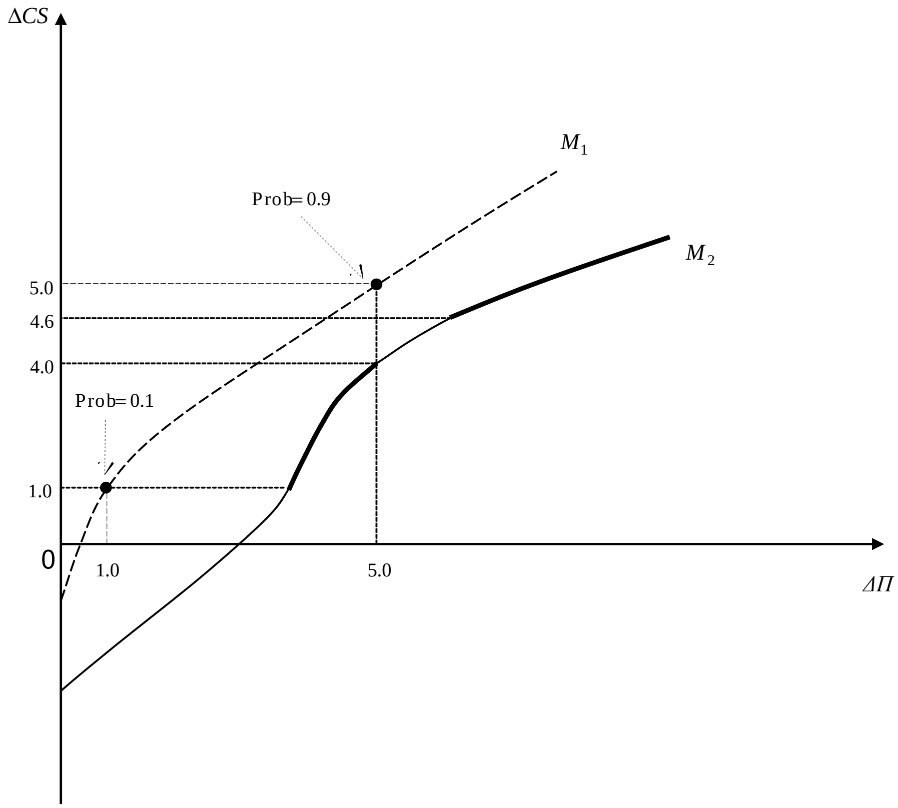

Figure 8: The figure depicts an example where the optimal approval set does not have a cutoff structure.

where $\Delta\Pi(M_k)$ now denotes the change in aggregate profit induced by merger M_k,

$$\Delta\Pi(M_k) \equiv \sum_{i\in\mathcal{N}\setminus\{0\}} \pi_i(M_k) - \sum_{i\in\mathcal{N}} \pi_i^0.$$

There are several bargaining processes which could lead to aggregate profit maximization:

1. Multilateral "Coasian bargaining" under complete information amongst all firms would lead to an efficient (aggregate-profit maximizing) outcome.

2. Suppose the auctioneer (here, firm 0) conducts a "menu auction" in which each firm $i \geq 1$ submits a nonnegative bid $b_i(M_k) \geq 0$ for each merger $M_k \in (\mathfrak{F} \cap \mathcal{A})$ with $k \geq 1$. Firm 0 then selects the merger that maximizes its profit, where the profit from selecting merger M_k is given by the sum of all bids for that merger, $\sum_{i\in\mathcal{N}\setminus\{0\}} b_i(M_k)$, and the profit from selecting the null merger M_0 is $\pi_0(M_0)$. Bernheim and Whinston (1996) show that there is an efficient equilibrium which, in this setting, implements the merger that maximizes aggregate profit.

3. Suppose the target (firm 0) can commit to any sales mechanism. Jehiel, Moldovanu and Stacchetti (1996) show that one such optimal mechanism has the following structure: The target proposes to implement merger $M_k \in (\mathfrak{F} \cap \mathcal{A})$ and requires payment $\pi_i(M_k) - \pi_i(\underline{M}_i)$ from each firm $i \geq 1$, where $\underline{M}_i \in (\mathfrak{F} \cap \mathcal{A})$ is the merger in set $(\mathfrak{F} \cap \mathcal{A}) \setminus M_i$ that minimizes firm i's profit. If a firm i does not accept participation in the mechanism when all other firms do, then the principal commits to proposing merger $\underline{M}_i$ to the antitrust authority [who will then approve it since $\underline{M}_i \in (\mathfrak{F} \cap \mathcal{A})$].[2] Jehiel, Moldovanu and Stacchetti show that there exists an equilibrium in which all firms participate in the mechanism. Given the set of feasible and approvable mergers, $\mathfrak{F} \cap \mathcal{A}$, the resulting outcome maximizes aggregate profit; that is, merger $M^*(\mathfrak{F}, \mathcal{A})$ is proposed.[3]

We claim that Proposition 1 carries over to this bargaining process: the optimal approval policy $\mathcal{A}$ is such that the minimum CS-standard is zero for the smallest merger and increasing in the size of the proposed merger, $0 = \underline{\Delta CS}_1 < \underline{\Delta CS}_2 < \cdots < \underline{\Delta CS}_{\widehat{K}}$, where $\widehat{K}$ is the largest merger that is approved with positive probability. The key steps in the argument are the following. First, note that Lemma 1 states that a CS-neutral merger M_k, $k \geq 1$, raises not only the bilateral profit of the merger partners but also aggregate profit, $\Delta\Pi(M_k) > 0$. Second, part (2) of Lemma 2 does not extend to the case of aggregate profit without imposing some condition. We therefore *assume* that a reduction in post-merger marginal cost increases aggregate profit if the merger is CS-nondecreasing, and then discuss when this condition does indeed hold true.

Assumption 3. *If merger M_k, $k \geq 1$, is CS-nondecreasing [i.e., $\bar{c}_k \leq \widehat{c}(Q^0)$], then reducing its post-merger marginal cost $\bar{c}_k$ increases the aggregate profit $\Pi \equiv \sum_{i\in\mathcal{N}\setminus\{0\}} \pi_i(M_k)$.*[4]

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[2] That is, similar to Bernheim and Whinston's (1996) menu auction, firms make payments even for mergers that they are not a party to.

[3] To see this, note that the target's program can be written as:

$$\max_{M_k\in(\mathfrak{F}\cap\mathcal{A})} \Pi(M_k) - \sum_{i\in\mathcal{N}} \pi_i(\underline{M}_i).$$

But this is equivalent to $\max_{M_k\in(\mathfrak{F}\cap\mathcal{A})} \Pi(M_k)$.

[4] In fact, this assumption is stronger than necessary. What we actually require is that it holds for $k \geq 2$.

As we now show, this assumption must hold for merger M_k if whenever it is CS-nondecreasing we have $\overline{c}_k \leq \min_{l \neq k} c_l$; i.e., the merged firm has the lowest marginal cost. Since this would always be true were the firms in set $\mathcal{N}\backslash\{0\}$ to have identical initial marginal costs, it clearly holds provided their initial marginal costs are sufficiently close. To see why Assumption 3 holds in this case, note that summing up the post-merger first-order conditions for profit maximization yields

$$\Pi = \sum_{i \in \mathcal{N}\backslash\{0\}} \left[P(Q) - c_i\right] q_i = \left|Q^2 P'(Q)\right| H, \tag{14}$$

where $H \equiv \sum_{i \in \mathcal{N}\backslash\{0\}} (s_i)^2$ is the post-merger industry Herfindahl Index. Assumption 1 ensures that the first term, $\left|Q^2 P'(Q)\right|$, is increasing in Q. By part (1) of Lemma 2, a reduction in post-merger marginal cost leads to a larger Q, so that a sufficient condition for the claim to hold is that reducing the merged firm's marginal cost induces an increase in H. But this is indeed the case if the merged firm has lower costs, and hence a larger market share, than any of its (unmerged) rivals, since then a further reduction in its marginal cost increases its share and lowers the shares of all of its rivals, increasing H (see Lemma 5 in the Appendix).

Third, the systematic misalignment of interests between firms and the antitrust authority, as stated in Lemma 3, is also present when bargaining is efficient:

Lemma 4. *Suppose two mergers, M_j and M_k, with $j < k$, induce the same non-negative change in consumer surplus, $\Delta CS(M_j) = \Delta CS(M_k) \geq 0$. Then, the larger merger M_k induces a greater increase in aggregate profit: $\Delta\Pi(M_k) > \Delta\Pi(M_j) > 0$.*

Proof. From the discussion above, the post-merger aggregate profit is given by (14). As both mergers induce the same level of consumer surplus (and thus the same Q), the first term on the right-hand side of (14) is the same for both mergers. It thus suffices to show that the larger merger M_k induces a larger value of H than the smaller merger M_j.

Now, as both mergers induce the same Q, Assumption 1 implies that the output of any firm not involved in M_j or M_k is the same under both mergers. Hence,

$$s_k(M_k) + s_j(M_k) = s_k(M_j) + s_j(M_j). \tag{15}$$

Next, recall that a CS-nondecreasing merger increases the share of the merging firms and reduces the share of all nonmerging firms. Thus, we have $s_k(M_k) \geq s_k + s_0 > s_k(M_j)$ and $s_j(M_j) \geq s_j + s_0 > s_j(M_k)$. In addition, since total output is the same after both mergers and $c_k < c_j$, we also have $s_j(M_k) < s_k(M_j)$. By (15), this in turn implies that $s_k(M_k) > s_j(M_j)$. Hence, the distribution of market shares after the larger merger M_k is a sum-preserving spread of those after the smaller merger M_j:

$$s_k(M_k) > \max\{s_j(M_j), s_k(M_j)\} \geq \min\{s_j(M_j), s_k(M_j)\} > s_j(M_k). \tag{16}$$

By Lemma 5 in the Appendix, H is therefore larger after M_k than after M_j. $\square$

The final step consists in noting that all of the steps in the proof of Proposition 1 continue to hold if we replace the change in bilateral profit by the change in aggregate profit.

4.2 Efficient Bargaining Between a Subset of Firms

Suppose that the outcome of the bargaining process maximizes the joint profit of only a subset of firms, $\mathcal{L}$, that includes the target and all of the acquirers, i.e., $(\{0\} \cup \mathcal{K}) \subseteq \mathcal{L} \subset \mathcal{N}$. That is, the proposal rule is

$$M^* (\mathfrak{F}, \mathcal{A}) \equiv \arg \max_{M_k \in (\mathfrak{F} \cap \mathcal{A})} \Delta\Pi(M_k),$$

where $\Delta\Pi(M_k)$ now denotes the induced change in the joint profit of the firms in set $\mathcal{L}$, $\Delta\Pi(M_k) \equiv \sum_{i \in \mathcal{L} \setminus \{0\}} \pi_i(M_k) - \sum_{i \in \mathcal{L}} \pi_i^0$.

Under the same conditions as in the case of efficient bargaining, our main result – Proposition 1 – carries over to this bargaining process. The key argument is the following: If any CS-nondecreasing merger or any reduction in a merged firm's marginal cost induces an increase in aggregate profit, then it also induces an increase in the joint profit of the firms in set $\mathcal{L}$. To see this, note that both a CS-nondecreasing merger and a reduction in a firm's post-merger marginal cost weakly reduce the profit of any other firm, including the firm(s) not in set $\mathcal{L}$. This observations has several implications. First, it means that part (4) of Lemma 1 continues to hold if we replace aggregate profit by the joint profit of the firms in set $\mathcal{L}$. Second, it also means that Assumption 3 implies that a reduction in the post-merger marginal cost $\bar{c}_k$ raises the joint profit of the firms in set $\mathcal{L}$ for any CS-nondecreasing merger. Third, a similar type of argument implies that Lemma 4 continues to hold if we replace the induced change in aggregate profit by the induced change in the joint profit of the firms in $\mathcal{L}$. To see this, recall that both mergers in the statement of the lemma, M_j and M_k, induce (by assumption) the same change in consumer surplus. Hence, the profit of any firm $i \neq j, k$ is the same under both mergers. Finally, it is straightforward to see that all of the steps in the proof of Proposition 1 carry over as well.

5 Extensions

In this section, we consider four extensions of our baseline model. First, we consider the case of price competition with differentiated products. Second, we study the optimal merger approval policy when the antitrust authority cares not only about consumer surplus but also about producer surplus. Third, we extend the model by allowing for synergies in fixed costs. Fourth, we consider a simple situation where there is no single "pivotal" firm that is part of every potential merger.

5.1 Differentiated Products

In our analysis we have assumed that firms produce a homogeneous good and compete in a Cournot fashion. Restricting attention to the case of efficient bargaining between firms, we now show that our main insights carry over to the case where firms produce differentiated goods (with consumers having a CES or multinomial logit demand system) and compete in prices. Specifically, we assume that the initial market structure is such that every firm produces one differentiated good at marginal cost. If a merger is proposed and approved, then the merged firm produces the two products of its merger partners at the same post-merger marginal cost.

CES Demand. In the CES model, the utility function of the representative consumer is given by

$$U = \left(\sum_{i=0}^{N} X_i^{\rho} \right)^{1/\rho} Z^{\alpha},$$

where $\rho \in (0,1)$ and $\alpha > 0$ are parameters, X_i is consumption of differentiated good i, and Z is consumption of the numeraire. Utility maximization implies that the representative consumer spends a constant fraction $1/(1+\alpha)$ of his income Y on the $N+1$ differentiated goods (and the remainder on the numeraire). Using the normalization $Y/(1+\alpha) \equiv 1$, the resulting demand for differentiated good i is

$$X_i = \frac{p_i^{-\lambda-1}}{\sum_{j=0}^{N} p_j^{-\lambda}},$$

where p_i is the price of good i, and $\lambda \equiv \rho/(1-\rho)$. The consumer's indirect utility can be written as

$$V = (1+\alpha)\ln Y + \frac{1}{\lambda} \ln \left(\sum_{j=0}^{N} p_j^{-\lambda} \right). \tag{17}$$

We assume that firms compete in prices.

Multinomial Logit Demand. In the multinomial logit model, expected demand for product i is given by

$$X_i = \frac{\exp\left(\frac{a-p_i}{\mu}\right)}{\sum_{j=0}^{N} \exp\left(\frac{a-p_j}{\mu}\right)},$$

where $a > 0$ and $\mu > 0$ are parameters, and p_j the price of product j. Letting Y denote income, the indirect utility of the representative consumer can be written as

$$V = Y + \mu \ln \left[\sum_{j=0}^{N} \exp\left(\frac{a-p_j}{\mu} \right) \right]. \tag{18}$$

Again, we assume that firms compete in prices.

The CES and multinomial logit models share important features with the Cournot model. In particular, all of these models can be written as "aggregative games." That is, the profit a firm obtains from its plant or product i can be written as

$$\pi(\psi_i, c_i; \Psi),$$

where $\psi_i \geq 0$ is the firm's strategic variable, c_i (constant) marginal cost, and $\Psi \equiv \sum_j \psi_j$ an aggregator summarizing the "aggregate outcome." (If a merged firm runs two plants or produces two products at the same marginal cost $\overline{c}_k$ and chooses the same value ψ_k of its strategic variable for both of its plants or products, then its total profit is $2\pi(\psi_k, \overline{c}_k; \Psi)$.) Further, consumer surplus is an increasing function of the aggregator, and does not depend on its composition, so that it can be written as $V(\Psi)$. In the Cournot model, ψ_i is output q_i and Ψ aggregate output Q, so that profit can be written as $\pi(\psi_i, c_i; \Psi) = [P(\Psi) - c_i]\psi_i$ and consumer surplus as $V(\Psi) = \int_0^{\Psi} [P(x) - P(\Psi)]\, dx$. In the CES model, we have $\psi_i = p_i^{-\lambda}$ and $\Psi = \sum_j p_j^{-\lambda}$, so that profit from product i can be written as

$$\pi(\psi_i, c_i; \Psi) = [\psi_i^{-1/\lambda} - c_i] \frac{\psi_i^{(\lambda+1)/\lambda}}{\Psi}.$$

From the indirect utility (17), it follows that consumer surplus is an increasing function of Ψ. Finally, in the multinomial model, we have $\psi_i = \exp((a - p_i)/\mu)$ and $\Psi = \sum_j \exp((a - p_j)/\mu)$, so that profit from product i can be written as

$$\pi(\psi_i, c_i; \Psi) = [a - \mu \ln \psi_i - c_i] \frac{\psi_i}{\Psi}.$$

From the indirect utility (18), it follows that consumer surplus is an increasing function of Ψ.

In the Appendix, we show that the equilibrium profit functions of these three models share some important properties. Using this common structure, we show in the Appendix that if merger M_k is CS-neutral, then it raises the joint profit of the merging firms as well as aggregate profit. Moreover, a reduction in post-merger marginal cost increases the merged firm's profit and, provided pre-merger differences between firms are not too large, aggregate profit. Moreover, if any two mergers M_j and M_k, $k > j$, induce the same nonnegative change in consumer surplus, then the larger merger M_k induces a greater increase in aggregate profit than the smaller merger M_j. In sum, in the two differentiated goods models, the merger curves have the same features in $(\Delta CS, \Delta\Pi)$-space as in the Cournot model. Our main result, Proposition 1, therefore carries over as well.

5.2 Alternative Welfare Standard

In our baseline model, we have assumed that the antitrust authority seeks to maximize consumer surplus. While this is in line with the legal standard in the U.S. and many other countries, it might seem unsatisfactory that the antitrust authority completely ignores any effect of its policy on producer surplus. We now show that our main result extends to the case where the antitrust antitrust authority seeks to maximize any convex combination of consumer surplus and aggregate surplus. For brevity, we consider only the case of efficient bargaining between firms; but the same result would hold if the bargaining process between firms is given by the offer game.

Specifically, suppose the antitrust authority's welfare criterion is $W \equiv CS + \lambda\Pi$, where $\lambda \in [0, 1]$. When $\lambda = 1$, welfare W thus amounts to aggregate surplus. Let

$$\Delta W(M_k) \equiv \Delta CS(M_k) + \lambda\Delta\Pi(M_k)$$

denote the change in welfare induced by approving merger M_k. We will say that merger M_k is W-increasing [W-decreasing] if $\Delta W(M_k) > 0$ [$\Delta W(M_k) < 0$], and W-nondecreasing [W-nonincreasing] if $\Delta W(M_k) \geq 0$ [$\Delta W(M_k) \leq 0$].

Since a W-increasing merger may be CS-decreasing, we require a slightly stronger version of Assumption 3:

Assumption 3' *If merger M_k for $k \geq 2$ is W-nondecreasing, then reducing its post-merger marginal cost $\bar{c}_k$ increases the aggregate profit Π. Moreover, for any W-nondecreasing merger M_k, $k \in K$, $\bar{c}_k < \min\{c_0, c_k\}$ [i.e., the merger involves synergies].*

To understand when Assumption 3' must hold, consider the extreme case where all firms have the same pre-merger marginal cost c. Then, for merger M_k to be W-nondecreasing, it must involve synergies in that $\overline{c}_k < c$.[5] Hence, if M_k is W-nondecreasing, the merged firm is the firm with the lowest marginal cost post merger. Reducing the merged firm's marginal cost $\overline{c}_k$ induces an increase in aggregate output Q, thereby raising $|Q^2 P'(Q)|$, and a further increase in the Herfindahl index H. From equation (14), a lower level of post-merger marginal cost $\overline{c}_k$ thus results in a greater level of aggregate profit Π. By continuity of consumer and producer surplus in marginal costs, it follows that $\Delta W(M_k) \geq 0$ implies that $\overline{c}_k < \min\{c_0, c_k\}$, and that Π is decreasing in $\overline{c}_k$, if pre-merger marginal cost differences are sufficiently small.

We also impose the following analog of Assumption 2:

Assumption 2' *For all $k \in \mathcal{K}$, the probability that the merger M_k is W-increasing is positive but less than one: $\Delta W(k, h_k) < 0 < \Delta W(k, l)$.*

Assumption 3' allows us to obtain a slightly stronger version of Lemma 4:

Lemma 4' *Suppose two W-nondecreasing mergers, M_j and M_k, with $k > j \geq 1$, induce the same change in consumer surplus, $\Delta CS(M_j) = \Delta CS(M_k)$. Then the larger merger M_k induces a greater increase in aggregate profit: $\Delta\Pi(M_k) > \Delta\Pi(M_j) > 0$.*

Proof. The proof proceeds exactly as that of Lemma 4, except that the inequalities $s_k(M_k) > s_k(M_j)$ and $s_j(M_j) > s_j(M_k)$ in equation (16) now hold since any W-nondecreasing merger involves synergies, $\overline{c}_k < c_k$ and $\overline{c}_j < c_j$, by Assumption 3' (and since $Q(M_k) = Q(M_j)$ as both mergers induce the same CS-level by assumption). $\square$

Figure 9 depicts the "merger curves" in $(\Delta\Pi, \Delta CS)$-space. The dotted lines are isowelfare curves, each with slope $-\lambda$; the hatched line is the isowelfare curve corresponding to no welfare change, $\Delta W = 0$. Lemma 4' states that, above the line $\Delta W = 0$, the curve corresponding to a larger merger lies everywhere to the right of that corresponding to a smaller merger. The figure also illustrates another result. That result is the analog of Corollary 1 and shows that there is a systematic misalignment between the proposal incentives of firms and the objectives of the antitrust authority:

Corollary 1' *If two W-nondecreasing mergers M_j and M_k with $k > j \geq 1$ have $\Delta\Pi(M_k) \leq \Delta\Pi(M_j)$, then $\Delta W(M_k) < \Delta W(M_j)$.*

Proof. Suppose instead that $\Delta W(M_k) \geq \Delta W(M_j)$. As $\Delta\Pi(M_k) \leq \Delta\Pi(M_j)$ by assumption, this implies that $\Delta CS(M_k) \geq \Delta CS(M_j)$. Then there exists a $\overline{c}'_k > \overline{c}_k$ such that $\Delta CS(k, \overline{c}'_k) = \Delta CS(M_j)$. But this implies (using Assumption 3' for the first inequality and Lemma 4' for the second) that $\Delta\Pi(M_k) > \Delta\Pi(k, \overline{c}'_k) > \Delta\Pi(M_j)$, a contradiction. $\square$

Figure 10 depicts the merger curves in $(\Delta\Pi, \Delta W)$-space. Note that each merger curve has a positive horizontal intercept: since a CS-nondecreasing merger raises aggregate profit, a W-neutral merger must be CS-decreasing and therefore increase aggregate profit. Moreover, each

[5] To see this, suppose otherwise that $\overline{c}_k \geq c$. We can decompose the induced change in market structure into two steps: (i) a move from N to $N-1$ firms, each with marginal cost c, and (ii) an increase in the marginal cost of one firm from c to $\overline{c}_k \geq c$. Step (i) induces a reduction in aggregate output but does not affect average production costs, and so reduces W. Step (ii) weakly reduces aggregate output and weakly increases average costs in the industry, and so weakly reduces W.

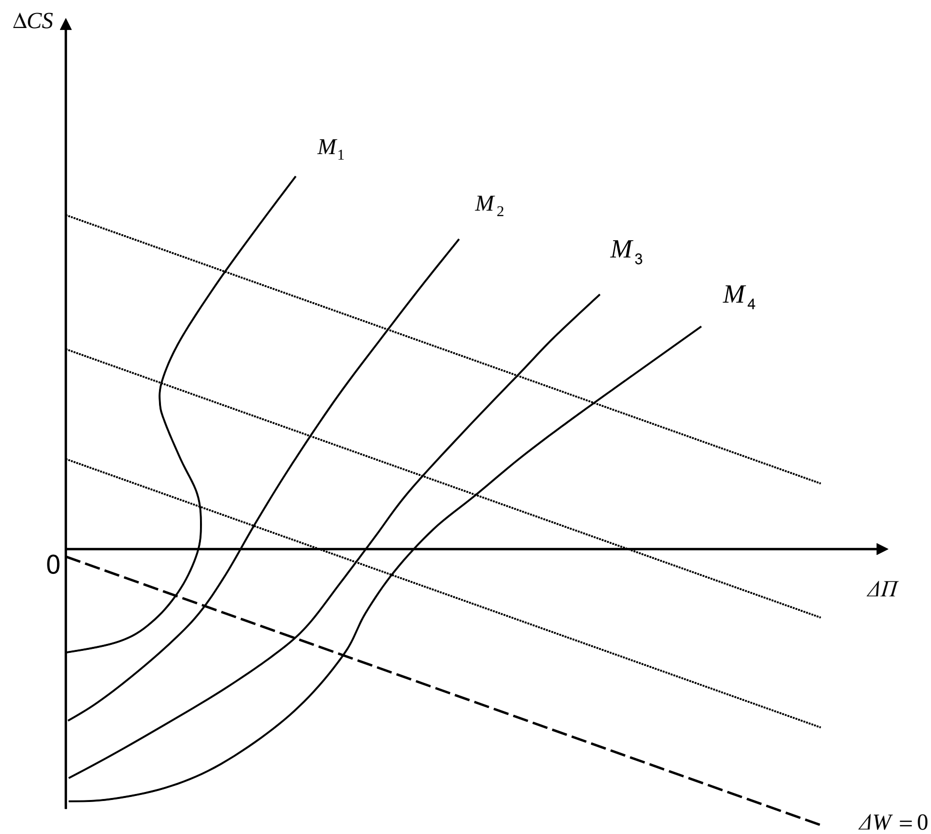

Figure 9: The merger curves in $(\Delta\Pi, \Delta CS)$-space. The downward-sloping lines are the iso-welfare curves.

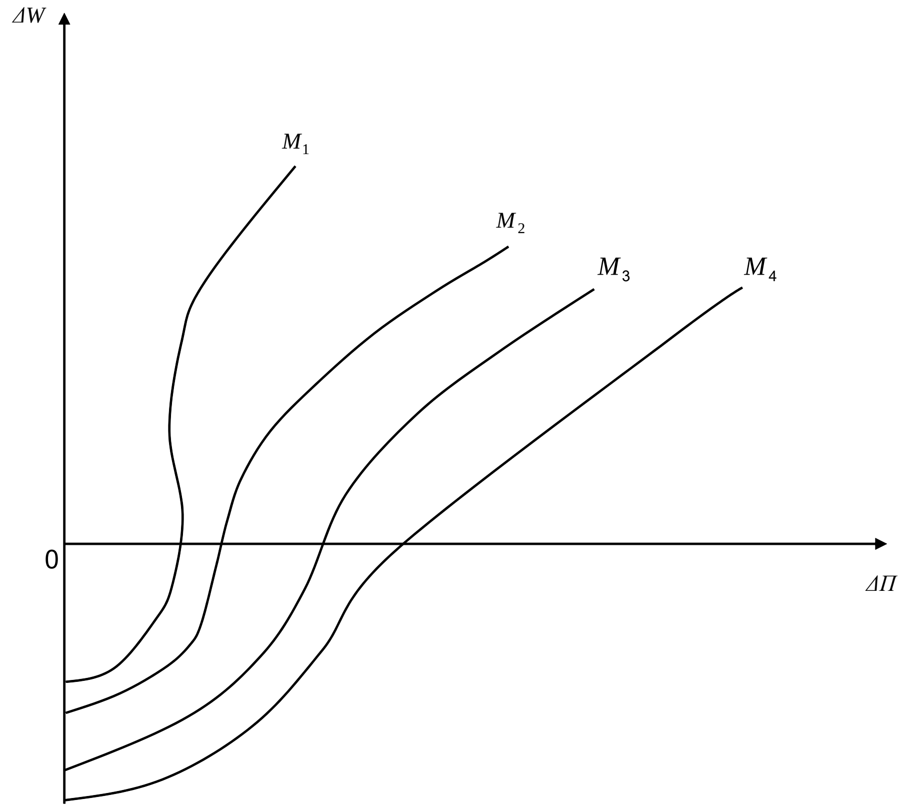

Figure 10: The merger curves in $(\Delta\Pi, \Delta W)$-space.

curve is upward-sloping in the positive orthant (except possibly for the curve corresponding to M_1). Finally, in the positive orthant, the curve of a larger merger lies everywhere to the right of that of a smaller merger.

Let $\underline{\Delta W}_k \equiv \Delta W(k, \overline{a}_k)$ denote the welfare level of the "marginal merger," i.e., the lowest welfare level in any allowable merger between firms 0 and k. The following proposition shows that our main result (Proposition 1) extends to the case where the antitrust authority maximizes an arbitrary convex combination of consumer surplus and aggregate surplus:

Proposition 1' *Any optimal approval policy A approves the smallest merger if and only if it is W-nondecreasing, and satisfies $0 = \underline{\Delta W}_1 < \underline{\Delta W}_j < \underline{\Delta W}_k$ for all $j, k \in \mathcal{K}^+$, $1 < j < k$, where $\mathcal{K}^+ \subseteq \mathcal{K}$ is the set of mergers that is approved with positive probability. Moreover, if $j \notin \mathcal{K}^+$ and $k \in \mathcal{K}^+$, $j < k$, then $\Delta W(j, l) < \underline{\Delta W}_k$. That is, the lowest level of welfare change that is acceptable to the antitrust authority equals zero for the smallest merger*

M_1, is strictly positive for every other merger M_k with $k > 1$, and is monotonically increasing in the size of the merger.

Proof. The proof proceeds in seven steps. Steps 1 through 6 are as in the proof of Proposition 1 but with the welfare criterion replacing the consumer surplus criterion. Step 7 does not carry over as we cannot guarantee that $\Delta W(k, l) > \Delta W(k + 1, l)$. But the same type of argument can be used to show that if $j \notin \mathcal{K}^+$ and $k \in \mathcal{K}^+$, $j < k$, then $\Delta W(j, l) < \underline{\Delta W}_k$. □

5.3 Synergies in Fixed Costs

So far, we have assumed that firms have constant returns, implying that all merger-specific efficiencies involve marginal cost savings. We now consider the case where firms have to incur a fixed cost, a part of which may be saved by merging, and show that our main result carries over to this setting.

Let f_i denote the fixed cost of firm i.[6] A feasible merger M_k is described by $M_k = (k, \bar{c}_k, \overline{f}_k)$, where $\overline{f}_k \in [\overline{f}_k^l, \overline{f}_k^h]$ is the realization of its post-merger fixed cost. The merger induces a fixed cost saving if $f_0 + f_k - \overline{f}_k \equiv \phi_k > 0$. Graphically, a fixed cost saving shifts the merger curve in a parallel fashion (by the amount of the saving) to the right in $(\Delta\Pi, \Delta CS)$-space. When a feasible merger is proposed, the antitrust authority can observe all aspects of that merger, including the induced fixed cost saving. The antitrust authority's approval set is now described by $\mathcal{A} \equiv \left\{M_k : \left(\bar{c}_k, \overline{f}_k\right) \in \mathcal{A}_k\right\} \cup M_0$, where $\mathcal{A}_k \subseteq [l, h_k] \times [\overline{f}_k^l, \overline{f}_k^h]$. Without loss of generality, we restrict attention to approval sets that are *regular* in the sense that every $\mathcal{A}_k$ is the closure of its interior, i.e., $\mathcal{A}_k = \mathrm{cl}\,(\mathrm{int}\,(\mathcal{A}_k))$. Let $\bar{a}_k(\overline{f}_k) \equiv \max\{\bar{c}_k | (\bar{c}_k, \overline{f}_k) \in \mathcal{A}_k\}$ denote the largest allowable post-merger marginal cost level for a merger between firms 0 and k, conditional on the realized post-merger fixed cost being equal to $\overline{f}_k$. Let $\underline{\Delta CS}_k(\overline{f}_k) \equiv \underline{\Delta CS}_k(k, \bar{a}_k(\overline{f}_k), \overline{f}_k)$ and $\underline{\Delta\Pi}_k(\overline{f}_k) \equiv \underline{\Delta\Pi}_k(k, \bar{a}_k(\overline{f}_k), \overline{f}_k)$ denote the changes in consumer surplus and bilateral profits, respectively, induced by the "marginal merger" between firms 0 and k, and let $\underline{\Delta CS}_k \equiv \min_{\overline{f}_k \in [\overline{f}_k^l, \overline{f}_k^h]} \underline{\Delta CS}_k(\overline{f}_k)$ and $\underline{\Delta\Pi}_k \equiv \min_{\overline{f}_k \in [\overline{f}_k^l, \overline{f}_k^h]} \underline{\Delta\Pi}_k(\overline{f}_k)$.

An immediate observation is the following. Suppose fixed cost savings are nonnegative and perfectly correlated across mergers so that $\phi_k = \phi \geq 0$ for every feasible merger $M_k \in \mathfrak{F}$. Then, the optimal approval set is constant in ϕ in the sense that $(\bar{c}_k, f_0 + f_k - \phi) \in \mathcal{A}_k$ implies $\left(\bar{c}_k, f_0 + f_k - \phi'\right) \in \mathcal{A}_k$, from which follows that $\underline{\Delta CS}_k(\overline{f}_k) = \underline{\Delta CS}_k$ for all $\overline{f}_k$ and k. Moreover, as before, the optimal policy is characterized by Proposition 1. To see this, note that the expected CS-maximizing antitrust authority cares about fixed cost savings only insofar as they affect firms' merger proposals. But if fixed cost savings are perfectly correlated and nonnegative, the profit ranking of mergers is unaffected by the actual fixed cost realization and all CS-nondecreasing mergers remain profitable.

Suppose now that the realized fixed cost saving of merger M_k is nonnegative, $\phi_k \geq 0$, and can be decomposed as follows:

$$\phi_k = \phi + \eta_k,$$

where $\phi \in [\phi^l, \phi^h]$ is the (random or deterministic) component that is common across all feasible mergers (as above) and $\eta_k \in [\eta_k^l, \eta_k^h]$ is the component idiosyncratic to merger M_k. We assume that the idiosyncratic shocks are conditionally independent across mergers, have full support

[6] Firm i can *not* avoid paying this fixed cost by exiting the industry.

and no mass points. Assume also that when merger M_k is proposed, the antitrust authority can observe ϕ and η_k separately (and condition the approval set on both components separately).[7] Using the same arguments as above, it is straightforward to show that the optimal approval set is constant in ϕ. For notational simplicity, we will from now on assume that there is no common component, $\phi \equiv 0$, so that $\overline{f}_k = f_0 + f_k - \eta_k$.

Graphically, the possibility of fixed cost savings implies that the merger curves in $(\Delta\Pi, \Delta CS)$-space are "broad bands" rather than curves, with each point in the band of merger M_k corresponding to a different realization of $(\overline{c}_k, \overline{f}_k)$, and with the horizontal width of the band given by $\left|f_k^h - f_k^l\right|$ at any $\Delta CS(M_k)$. We assume that $\left|f_k^h - f_k^l\right|$ is sufficiently small so that the bands of the different mergers are non-overlapping in the positive orthant. From Lemma 3 it follows that if any two mergers M_j and M_k, $j < k$, induce the same nonnegative change in consumer surplus, $\Delta CS(M_j) = \Delta CS(M_k) \geq 0$, then the larger merger is more profitable, independently of the realized fixed cost savings. As fixed cost savings are nonnegative by assumption, the conclusion of Lemma 1 – that a CS-neutral merger is profitable – continues to hold.

Our main result, Proposition 1, carries over to this setting:

Proposition 2. *In the model with fixed cost savings, any optimal approval policy $\mathcal{A}$ approves the smallest merger if and only if it is CS-nondecreasing, approves only mergers $k \in \mathcal{K}^+ \equiv \{1, ..., \widehat{K}\}$ with positive probability ($\widehat{K}$ may equal K) and satisfies $0 = \underline{\Delta CS}_1 < \underline{\Delta CS}_2 < ... < \underline{\Delta CS}_{\widehat{K}}$ for all $k \leq \widehat{K}$.*

Proof. Steps 1-3 proceed along the same lines as those in the proof of Proposition 1.

Step 4. As in the absence of fixed cost savings, any optimal policy has the property that, for all $k \in \mathcal{K}^+$, $\underline{\Delta CS}_k(\overline{f}_k)$ is equal to the expected change in consumer surplus from the next-most profitable merger $M^*(\mathfrak{F}\backslash(k, \overline{c}_k, \overline{f}_k), \mathcal{A})$, conditional on merger $M_k = (k, \overline{c}_k, \overline{f}_k)$ maximizing the change in the merging firms' bilateral profit in $\mathfrak{F} \cap \mathcal{A}$. That is,

$$\begin{aligned}\underline{\Delta CS}_k(\overline{f}_k) &= E_k^{\mathcal{A}}(\overline{a}_k(\overline{f}_k), \overline{f}_k) \\ &\equiv E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F}\backslash M_k, \mathcal{A}))|M_k = (k, \overline{a}_k(\overline{f}_k), \overline{f}_k) \text{ and } \Delta\Pi(M^*(\mathfrak{F}\backslash M_k, \mathcal{A})) \leq \Delta\Pi(M_k)].\end{aligned}$$

To see that this equation must hold for all $k \in \mathcal{K}^+$, suppose first that $\underline{\Delta CS}_{k'}(\overline{f}'_{k'}) > E_{k'}^{\mathcal{A}}(\overline{a}_{k'}(\overline{f}'_{k'}), \overline{f}'_{k'})$ for some firm $k' \in \mathcal{K}^+$ and fixed cost realization $\overline{f}'_{k'}$, and consider the alternative approval set $\mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon}$, where

$$\mathcal{A}_{k'}^{\varepsilon} \equiv \left\{M_k : M_k = (k', \overline{c}_{k'}, \overline{f}_{k'}) \text{ with } \overline{c}_{k'} \in \left(\overline{a}_{k'}(\overline{f}'_{k'}), \overline{a}_{k'}(\overline{f}'_{k'}) + \varepsilon\right) \text{ and } \overline{f}_{k'} \in \left(\overline{f}'_{k'} - \varepsilon, \overline{f}'_{k'} + \varepsilon\right)\right\}.$$

Using the same type of argument as in the proof of Proposition 1, it is straightforward to show that, for $\varepsilon > 0$ small enough, the change in expected consumer surplus from changing the approval set from $\mathcal{A}$ to $\mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon}$ is strictly positive. A similar logic can be used to show that we cannot have $\underline{\Delta CS}_{k'}(\overline{f}'_{k'}) < E_{k'}^{\mathcal{A}}(\overline{a}_{k'}(\overline{f}'_{k'}), \overline{f}'_{k'})$.

Step 5. Let $\mathcal{M}_j^{CS} \equiv \{M_j : \Delta CS(M_j) = \underline{\Delta CS}_j \text{ and } M_j \in \mathcal{A}_j\}$ denote the set of marginal mergers M_j that induce a change in consumer surplus of $\underline{\Delta CS}_j$, and let $M_j^{CS} \in \mathcal{M}_j^{CS}$ denote the most profitable amongst these mergers, i.e., $\Delta\Pi(M_j^{CS}) \geq \Delta\Pi(M'_j)$ for all $M'_j \in \mathcal{M}_j^{CS}$. An

[7] That is, a feasible merger M_k is described by $M_k = (k, \overline{c}_k, \phi, \overline{f}_k)$, and the approval set by $\mathcal{A} \equiv \left\{M_k : \left(\overline{c}_k, \phi, \overline{f}_k\right) \in \mathcal{A}_k\right\} \cup M_0$, where $\mathcal{A}_k \subseteq [l, h_k] \times [\phi^l, \phi^h] \times [\overline{f}_k^l, \overline{f}_k^h]$.

optimal approval set must have the property that, for all $j < k$ such that $j, k \in \mathcal{K}^+$, we have $\Delta\Pi\left(M_j^{CS}\right) \leq \underline{\Delta\Pi}_k$. The argument proceeds in two parts.

Part (i). For all $j < k$ such that $j, k \in \mathcal{K}^+$, we must have $\underline{\Delta\Pi}_j \leq \underline{\Delta\Pi}_k$. The argument is along the lines of Step 5 in the proof of Proposition 1. For $j \in \mathcal{K}^+$, let $k' \equiv \arg\min_{k\in\mathcal{K}^+,k>j} \underline{\Delta\Pi}_k$ and suppose that $\underline{\Delta\Pi}_{k'} < \underline{\Delta\Pi}_j$. Let $M_{k'}^{\Pi} = (k', \overline{a}_{k'}(\overline{f}_{k'}^{\Pi}), \overline{f}_{k'}^{\Pi})$ denote the marginal merger $M_{k'}$ that induces the bilateral profit change $\underline{\Delta\Pi}_{k'}$, i.e., $\Delta\Pi\left(M_{k'}^{\Pi}\right) = \underline{\Delta\Pi}_{k'}$. By Step 4, $M_{k'}^{\Pi}$ is uniquely defined, and $\underline{\Delta CS}_{k'}\left(\overline{f}_{k'}^{\Pi}\right) = E_{k'}^{\mathcal{A}}\left(\overline{a}_{k'}\left(\overline{f}_{k'}^{\Pi}\right), \overline{f}_{k'}^{\Pi}\right)$. Let $\left(\overline{c}_j', \overline{f}_j'\right)$ be a pair of post-merger marginal and fixed cost levels satisfying $\Delta\Pi(j, \overline{c}_j', \overline{f}_j') = \underline{\Delta\Pi}_{k'}$ and consider a change in the approval set from $\mathcal{A}$ to $\mathcal{A}\cup\overline{\mathcal{A}}_j^{\varepsilon}$, where

$$\overline{\mathcal{A}}_j^{\varepsilon} \equiv \{M_j : \Delta\Pi(M_j) \in [\underline{\Delta\Pi}_{k'} - \varepsilon, \underline{\Delta\Pi}_{k'}]\},$$

and $\varepsilon > 0$. The change in expected consumer surplus from this change in the approval set equals $\Pr(M^*(\mathfrak{F},\mathcal{A}\cup\overline{\mathcal{A}}_j^{\varepsilon}) \in\overline{\mathcal{A}}_j^{\varepsilon})$ times

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F},\mathcal{A}\cup\overline{\mathcal{A}}_j^{\varepsilon})) - E_j^{\mathcal{A}}(\overline{c}_j, \overline{f}_j)|M^*(\mathfrak{F},\mathcal{A}\cup\overline{\mathcal{A}}_j^{\varepsilon}) \in\overline{\mathcal{A}}_j^{\varepsilon}], \tag{19}$$

where $(\overline{c}_j, \overline{f}_j)$ is the pair of realized cost levels in the most profitable merger $M^*(\mathfrak{F},\mathcal{A}\cup\overline{\mathcal{A}}_j^{\varepsilon})$, which is a merger of firms 0 and j when the conditioning statement is satisfied. As $\varepsilon \to 0$, the expected change in (19) converges to

$$\begin{aligned}\Delta CS(j, \overline{c}_j', \overline{f}_j') - E_j^{\mathcal{A}}(\overline{c}_j', \overline{f}_j') &= \Delta CS(j, \overline{c}_j', \overline{f}_j') - E_{k'}^{\mathcal{A}}\left(\overline{a}_{k'}\left(\overline{f}_{k'}^{\Pi}\right), \overline{f}_{k'}^{\Pi}\right) \\ &> 0,\end{aligned}$$

where the inequality follows since $j < k'$ and $\Delta\Pi(j, \overline{c}_j', \overline{f}_j') = \Delta\Pi\left(M_{k'}^{\Pi}\right)$.

Part (ii). For all $j < k$ such that $j, k \in \mathcal{K}^+$, we must have $\Delta\Pi\left(M_j^{CS}\right) \leq \underline{\Delta\Pi}_k$. For $j \in \mathcal{K}^+$, let $k' \equiv \arg\min_{k\in\mathcal{K}^+,k>j} \underline{\Delta\Pi}_k$, and suppose that $\underline{\Delta\Pi}_j \leq \underline{\Delta\Pi}_{k'} < \Delta\Pi\left(M_j^{CS}\right)$. (By part (i), we cannot have $\underline{\Delta\Pi}_j > \underline{\Delta\Pi}_{k'}$.) Let $\left(\overline{a}_j\left(\overline{f}_j'\right), \overline{f}_j'\right)$ be such that $\Delta\Pi\left(j, \overline{a}_j\left(\overline{f}_j'\right), \overline{f}_j'\right) = \underline{\Delta\Pi}_{k'}$, and consider a change from $\mathcal{A}$ to $\mathcal{A}\backslash\widetilde{\mathcal{A}}_{k'}^{\varepsilon}$, where

$$\widetilde{\mathcal{A}}_{k'}^{\varepsilon} \equiv \{M_{k'} : \Delta\Pi(M_{k'}) \in [\underline{\Delta\Pi}_{k'}, \underline{\Delta\Pi}_{k'} + \varepsilon] \text{ and } M_{k'} \in \mathcal{A}_{k'}\},$$

and $\varepsilon > 0$. The induced change in expected consumer surplus is $\Pr\left(M^*(\mathfrak{F},\mathcal{A}) \in \widetilde{\mathcal{A}}_{k'}^{\varepsilon}\right)$ times

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F},\mathcal{A}\backslash\widetilde{\mathcal{A}}_{k'}^{\varepsilon})) - E_{k'}^{\mathcal{A}}(\overline{c}_{k'}, \overline{f}_{k'})|M^*(\mathfrak{F},\mathcal{A}) \in\widetilde{\mathcal{A}}_{k'}^{\varepsilon}], \tag{20}$$

where $(\overline{c}_{k'}, \overline{f}_{k'})$ is the pair of realized cost levels in the most profitable merger $M^*(\mathfrak{F},\mathcal{A})$, which is a merger of firms 0 and k' when the conditioning statement is satisfied. As $\varepsilon \to 0$, the expected change in (20) converges to

$$\begin{aligned}\Delta CS\left(j, \overline{a}_j\left(\overline{f}_j'\right), \overline{f}_j'\right) - E_{k'}^{\mathcal{A}}(\overline{c}_{k'}, \overline{f}_{k'}) &= \underline{\Delta CS}_j\left(\overline{f}_j'\right) - \Delta CS\left(M_{k'}^{\Pi}\right) \\ &> 0,\end{aligned}$$

where $M_{k'}^{\Pi}$ is defined as in part (i), and where the inequality follows since $j < k'$ and $\Delta\Pi\left(j, \overline{a}_j\left(\overline{f}_j'\right), \overline{f}_j'\right) = \underline{\Delta\Pi}_{k'}$.

Step 5. For all $j, k \in \mathcal{K}^+$, $j < k$, we must have $\underline{\Delta CS}_j < \underline{\Delta CS}_k$. Suppose otherwise so that for some $j, h \in \mathcal{K}^+$, $h > j$, we have $\underline{\Delta CS}_j \geq \underline{\Delta CS}_h$. Let $k \equiv \arg\min\{h \in \mathcal{K}^+ : h > j$

and $\underline{\Delta CS}_j \geq \underline{\Delta CS}_h\}$. By Step 4, we must have $E_j^{\mathcal{A}}(\overline{a}_j(\overline{f}_j^{CS}), \overline{f}_j^{CS}) = \underline{\Delta CS}_j \geq \underline{\Delta CS}_k = E_k^{\mathcal{A}}(\overline{a}_k(\overline{f}_k^{CS}), \overline{f}_k^{CS})$. Now, $E_k^{\mathcal{A}}(\overline{a}_k(\overline{f}_k^{CS}), \overline{f}_k^{CS})$ can be written as a weighted average of two conditional expectations:

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathfrak{F}\backslash M_k,\mathcal{A}))|M_k = (k, \overline{c}_k^{CS}, \overline{f}_k^{CS}),\ M_k = M^*(\mathfrak{F},\mathcal{A}), \text{ and } \Delta\Pi(M^*(\mathfrak{F}\backslash M_k,\mathcal{A})) < \underline{\Delta\Pi}_j] \tag{21}$$

and

$$E_{\mathfrak{F}}\left[\Delta CS(M^*(\mathfrak{F}\backslash M_k,\mathcal{A}))|M_k = (k, \overline{c}_k^{CS}, \overline{f}_k^{CS}),\ M_k = M^*(\mathfrak{F},\mathcal{A}),\right.$$
$$\left.\text{and } \Delta\Pi(M^*(\mathfrak{F}\backslash M_k,\mathcal{A})) \in [\underline{\Delta\Pi}_j, \Delta\Pi(M_k)]\right]. \tag{22}$$

Using the same arguments as in the proof of Proposition 1, we obtain that the term in (21) is equal to $\Delta CS(M_j^{\Pi})$, which weakly exceeds $\underline{\Delta CS}_j$ by definition, and that the second term strictly exceeds $E_j^{\mathcal{A}}(\overline{a}_j(\overline{f}_j^{CS}), \overline{f}_j^{CS}) = \underline{\Delta CS}_j$, which leads to a contradiction.

Step 7. The argument proceeds proceeds along the same lines as that in the proof of Proposition 1. □

5.4 No Single Pivotal Firm

So far, we have assumed that there is a single target (and therefore a single 'pivotal player'), firm 0, that is part of every potential merger. We now show that our main result continues to hold in the simplest possible setting where there is no single target but, as before, all mergers are mutually exclusive, and there is efficient bargaining between firms. Specifically, we assume that there are three potential mergers, a merger between firms 1 and 2, a merger between firms 1 and 3, and a merger between firms 2 and 3. The merger between firms i and $j > i$ is denoted $M_{ij} \equiv (\{i, j\}, \overline{c}_{ij})$, where $\overline{c}_{ij}$ is the corresponding post-merger marginal cost, which (conditional on the merger being feasible, $\phi_{ij} = 1$) is drawn from distribution G_{ij} with support $[l, h_{ij}]$.

Note that any two of these three potential mergers have in common exactly one merger partner. As $c_1 > c_2 > c_3$, this implies that we can order the three mergers by the combined pre-merger market shares of their merger partners: M_{23} is larger than M_{13}, which in turn is larger than M_{12}. With this ordering of merger size, our previous analysis carries over to this setting. In particular, any optimal policy approves the smallest merger M_{12} if and only if it is CS-nondecreasing, satisfies $\underline{CS}(M_{ij}) > 0$ if merger M_{ij} is approved with positive probability, and $\underline{CS}(M_{13}) < \underline{CS}(M_{23})$ if both M_{13} and M_{23} are approved with positive probability. Moreover, if the largest merger M_{23} is approved with positive probability, then so is M_{13}.

6 Appendix

6.1 Proofs

Lemma 5. *Consider the function $H(s_1, ..., s_N) = \sum_n (s_n)^2$ and two vectors $s' = (s'_1, ..., s'_N)$ and $s'' = (s''_1, ..., s''_N)$ having $\sum_{n=1}^N s'_n = \sum_{n=1}^N s''_n$. If for some r, (i) $s'_r \geq s'_j$ for all $j \neq r$, (ii) $s''_r > s'_r$, and (iii) $s''_j \leq s'_j$ for all $j \neq r$, then $H(s'') > H(s')$.*

Proof. Without loss of generality, take $r = 1$ and define $\Delta_n \equiv s'_n - s''_n$ for $n > 1$. Observe that $\Delta_n \geq 0$ for all $n > 1$ and $\Delta_n > 0$ for some $n > 1$. Define as well the vectors $s^n \equiv (s'_1 + \sum_{t=2}^n \Delta_t, s'_2 - \Delta_2, ..., s'_n - \Delta_n, s'_{n+1}, ..., s'_N)$ for $n > 1$ and $s^1 \equiv s'$. Note that $s^N = s''$. Then

$$H(s'') - H(s') = \sum_{n=1}^{N-1} [H(s^{n+1}) - H(s^n)].$$

Now letting $\overline{s}_1^1 \equiv s'_1$ and $\overline{s}_1^n \equiv s'_1 + \sum_{t=2}^n \Delta_t \geq s'_1$ for all $n > 1$, each term in this sum is nonnegative,

$$\begin{aligned} H(s^{n+1}) - H(s^n) &= (\overline{s}_1^n + \Delta_{n+1})^2 + (s'_n - \Delta_{n+1})^2 - (\overline{s}_1^n)^2 - (s'_n)^2 \\ &= 2\Delta_{n+1}(\overline{s}_1^n - s'_n) + 2(\Delta_{n+1})^2 \geq 0, \end{aligned}$$

and strictly positive if $\Delta_{n+1} > 0$. Since $\Delta_{n+1} > 0$ for some $n \geq 1$, the result follows. □

6.2 Notes on the Aggregative Game Approach

Assumptions. Suppose an unmerged firm i's profit can be written as

$$\pi(\psi_i, c_i; \Psi),$$

where $\psi_i \geq 0$ is firm i's strategic variable, c_i the firm's constant marginal cost, and $\Psi \equiv \sum_j \psi_j$ an aggregator summarizing the "aggregate outcome." The firm's cumulative best response, $r(c_i; \Psi) \equiv \arg\max_{\psi_i} \pi(\psi_i, c_i; \psi_i + \sum_{j \neq i} \psi_j)$ is assumed to be decreasing in its marginal cost c_i. Similarly, a merged firm k's profit is given by $2\pi(\psi_i, \overline{c}_i; \Psi)$, and its cumulative best response, $\overline{r}(\overline{c}_k; \Psi) \equiv \arg\max_{\psi_k} 2\pi(\psi_k, \overline{c}_k; 2\psi_k + \sum_{j \neq 0,k} \psi_j)$, is decreasing in $\overline{c}_k$. Consumer surplus, denoted $V(\Psi)$, is an increasing function of the aggregator and does not depend on the composition of the aggregator.

Suppose that there exists a unique stable equilibrium. Let $\psi_i(M_k)$ denote firm i's equilibrium action under market structure M_k, and $\Psi(M_k) \equiv \sum_j \psi_j(M_k)$. Further, suppose that firm i's equilibrium profit can be written as

$$\begin{aligned} g(\psi_i(M_k); \Psi(M_k)) &\equiv \max_{\psi_i} \pi(\psi_i, c_i; \Psi(M_k)) \text{ if firm } i \text{ is unmerged;} \\ g(2\psi_i(M_k); \Psi(M_k)) &\equiv \max_{\psi_i} 2\pi(\psi_i, \overline{c}_i; \Psi(M_k)) \text{ if firm } i = k \text{ is merged.} \end{aligned}$$

The equilibrium profit function g has the following properties: (i) $g(0; \Psi) = 0$; (ii) for $0 \leq \psi_i \leq \Psi$, $g(\psi_i; \Psi)$ is strictly increasing and strictly convex in ψ_i. We assume that a reduction in post-merger marginal cost $\overline{c}_k$ leads to (a) an increase in $\psi_k(M_k)$ and in the aggregate outcome $\Psi(M_k)$; (b) an increase in $\psi_k(M_k)/\Psi(M_k)$ and a decrease in $\psi_j(M_k)/\Psi(M_k)$, $j \neq 0, k$; and (c) an increase in the merged firm's equilibrium profit $g(2\psi_k(M_k), \Psi(M_k))$ and a reduction in any other firm i's equilibrium profit $g(\psi_i(M_k); \Psi(M_k))$.

Our assumptions hold for several textbook models of competition.

Example 1 (Cournot)**.** In the homogeneous goods Cournot model with constant marginal costs, let ψ_i denote the output of plant i. All unmerged firms can be thought of as single-plant firms, whereas a merged firm can be thought of as running two plants at the same marginal cost (producing the same output at both plants). We impose the same assumptions on demand as in the main text. The profit maximization problem of a single-plant firm i with marginal cost c_i can be written as

$$\max_{\psi_i} \left[P(\psi_i + \sum_{j \neq i} \psi_j) - c_i \right] \psi_i.$$

From the first-order condition of profit maximization, $P(\Psi) - c_i + \psi_i P'(\Psi) = 0$, we can write the equilibrium profit under merger M_k as

$$g(\psi_i(M_k); \Psi(M_k)) = - \left[\psi_i(M_k)\right]^2 P'(\Psi(M_k)).$$

The profit maximization problem of a merged firm k with marginal cost $\overline{c}_k$ (and two plants) can be written as

$$\max_{\psi_k} \left[P(2\psi_k + \sum_{j \neq 0,k} \psi_j) - \overline{c}_k \right] 2\psi_k.$$

From the first-order condition of profit maximization, $P(\Psi) - \overline{c}_k + 2\psi_k P'(\Psi) = 0$, so that we can write the merged firm's equilibrium profit under merger M_k as

$$g(2\psi_k(M_k); \Psi(M_k)) = - \left[2\psi_k(M_k)\right]^2 P'(\Psi(M_k)).$$

It can easily be verified that g has all of the required properties (it takes the value of zero if its first argument is zero and is increasing and convex in its first argument). and that a reduction in post-merger marginal cost $\overline{c}_k$ has the posited effects. (The other assumptions were shown to hold in the main text.)

Example 2 (CES)**.** In the CES demand model with price competition, suppose that an unmerged firm produces a single good, while a merged firm produces two goods at the same marginal cost (thus optimally charging the same price for each). Consider first a single-product firm i. The profit maximization problem of a single-plant firm i with marginal cost c_i can be written as

$$\max_{\psi_i} [\psi_i^{-1/\lambda} - c_i] \frac{\psi_i^{(\lambda+1)/\lambda}}{\psi_i + \sum_{j \neq i} \psi_j}.$$

From the first-order condition of profit maximization,

$$-\Psi + \left[\psi_i^{-1/\lambda} - c_i\right] \psi_i^{(\lambda+1)/\lambda} \left\{ \frac{(\lambda+1)\Psi}{\psi_i} - \lambda \right\} = 0$$

we can write the firm's equilibrium profit under merger M_k as

$$g\left(\psi_i(M_k); \Psi(M_k)\right) \equiv \left\{ \frac{(\lambda+1)\Psi(M_k)}{\psi_i(M_k)} - \lambda \right\}^{-1}.$$

Consider now the merged firm k and suppose the firm produces two products at marginal cost $\overline{c}_k$. The profit maximization problem can be written as

$$\max_{\psi_k} 2[\psi_k^{-1/\lambda} - \overline{c}_k] \frac{\psi_k^{(\lambda+1)/\lambda}}{2\psi_k + \sum_{j \neq 0,k} \psi_j}.$$

(It can easily be verified that the firm optimally chooses the same value of ψ_k for each one of its two products.) From the first-order condition,

$$-\Psi + \left[\psi_k^{-1/\lambda} - \overline{c}_k\right] \psi_k^{(\lambda+1)/\lambda} \left\{ \frac{(\lambda+1)\Psi}{\psi_k} - 2\lambda \right\} = 0,$$

we obtain the merged firm's equilibrium profit under merger M_k:

$$g\left(2\psi_k(M_k);\Psi(M_k)\right) \equiv \left\{\frac{(\lambda+1)\Psi(M_k)}{2\psi_k(M_k)} - \lambda\right\}^{-1}.$$

It can easily be verified that our assumptions hold in the CES model. In particular, the equilibrium profit function g has all of the required properties (it takes the value of zero if its first argument is zero and is increasing and convex in its first argument). Consider a reduction in post-merger marginal cost $\overline{c}_k$. For a given level of Ψ, the merged firm wants to choose a higher value of ψ_k, and every other firm i wants to choose a higher level of ψ_i, as can be seen from the first-order conditions. In any stable equilibrium, the reduction in $\overline{c}_k$ thus induces a higher value of Ψ. Rewrite the first-order condition of an unmerged firm i:

$$-1 + \left[1 - c_i\psi_i^{1/\lambda}\right]\left\{(\lambda+1) - \lambda\frac{\psi_i}{\Psi}\right\} = 0.$$

As the induced increase in Ψ induces an increase in ψ_i (i.e., prices are strategic complements), the ratio ψ_i/Ψ must fall as otherwise the l.h.s. of the first-order condition would decrease. But as

$$\frac{2\psi_k}{\Psi} + \frac{\sum_{i\neq 0,k}\psi_i}{\Psi} = 1,$$

it follows that the same ratio for the merged firm, ψ_k/Ψ, must increase. From the expression for the equilibrium profits, we thus obtain that the profit of the merged firm, $g\left(2\psi_k(M_k);\Psi(M_k)\right)$, increases and that of any unmerged firm i, $g\left(\psi_i(M_k);\Psi(M_k)\right)$, decreases.

Example 3 (Multinomial Logit)**.** In the multinomial logit demand model with price competition, suppose that an unmerged firm produces a single good, while a merged firm produces two goods at the same marginal cost (thus optimally charging the same price for each). Consider first a single-product firm i. The profit maximization problem of a single-plant firm i with marginal cost c_i can be written as

$$\max_{\psi_i}\left[a - \mu\ln\psi_i - c_i\right]\frac{\psi_i}{\psi_i + \sum_{j\neq i}\psi_j}$$

From the first-order condition of profit maximization,

$$\left\{-\mu + a - \mu\ln\psi_i - c_i\right\}\Psi - \left[a - \mu\ln\psi_i - c_i\right]\psi_i = 0,$$

we obtain firm i's equilibrium profit under merger M_k:

$$g(\psi_i(M_k);\Psi(M_k)) = \mu\left\{\frac{\Psi(M_k)}{\psi_i(M_k)} - 1\right\}^{-1}.$$

Consider now the merged firm k and suppose the firm produces two products at marginal cost $\overline{c}_k$. The profit maximization problem can be written as

$$\max_{\psi_k} 2\left[a - \mu\ln\psi_k - \overline{c}_k\right]\frac{\psi_k}{2\psi_k + \sum_{j\neq 0,k}\psi_j}.$$

(It can easily be verified that the firm optimally chooses the same value of ψ_k for each one of its two products.) The merged firm's first-order condition of profit maximization,

$$\left\{-\mu + a - \mu\ln\psi_k - \overline{c}_k\right\}\Psi - 2\left[a - \mu\ln\psi_k - \overline{c}_k\right]\psi_k = 0,$$

can be rewritten to obtain firm k's equilibrium profit under merger M_k :

$$g(2\psi_k(M_k);\Psi(M_k)) = \mu\left\{\frac{\Psi(M_k)}{2\psi_k} - 1\right\}^{-1}.$$

It can easily be verified that our assumptions hold in the CES model. In particular, the equilibrium profit function g has all of the required properties (it takes the value of zero if its first argument is zero and is increasing and convex in its first argument). Consider a reduction in post-merger marginal cost $\bar{c}_k$. For a given level of Ψ, the merged firm wants to choose a higher value of ψ_k, and every other firm i wants to choose a higher level of ψ_i, as can be seen from the first-order conditions. In any stable equilibrium, the reduction in $\bar{c}_k$ thus induces a higher value of Ψ. Rewrite the first-order condition of an unmerged firm i as

$$\frac{-\mu + a - \mu\ln\psi_i - c_i}{a - \mu\ln\psi_i - c_i} = \frac{\psi_i}{\Psi}.$$

It can easily be checked that the l.h.s. of this equation is decreasing in ψ_i. As the induced increase in Ψ induces an increase in ψ_i (i.e., prices are strategic complements), the ratio ψ_i/Ψ must fall as otherwise the l.h.s. of the equation would decrease. But as

$$\frac{2\psi_k}{\Psi} + \frac{\sum_{i\neq 0,k}\psi_i}{\Psi} = 1,$$

it follows that the same ratio for the merged firm, ψ_k/Ψ, must increase. From the expression for the equilibrium profits, we thus obtain that the profit of the merged firm, $g\left(2\psi_k(M_k);\Psi(M_k)\right)$, increases and that of any unmerged firm i, $g\left(\psi_i(M_k);\Psi(M_k)\right)$, decreases.

Results. Let $\psi_i^0 \equiv \psi_i(M_0)$ and $\Psi^0 \equiv \Psi(M_0)$, and note that, as consumer surplus $V(\Psi)$ is strictly increasing in Ψ, merger M_k is CS-neutral if $\Psi(M_k) = \Psi^0$; it is CS-increasing if $\Psi(M_k) > \Psi^0$, and CS-decreasing if $\Psi(M_k) < \Psi^0$.

Lemma 6. *Merger M_k is CS-neutral if $2\psi_k(M_k) = \psi_0^0 + \psi_k^0$, CS-increasing if $2\psi_k(M_k) > \psi_0^0 + \psi_k^0$, and CS-decreasing if $2\psi_k(M_k) < \psi_0^0 + \psi_k^0$.*

Proof. Suppose merger M_k is CS-neutral. Then, $\Psi(M_k) = \Psi^0$. From the profit maximization problem of any firm i not involved in the merger, it follows that $\psi_i(M_k) = \psi_i^0$. Hence, we must have $2\psi_k(M_k) = \psi_0^0 + \psi_k^0$. The claim then follows from the observation that consumer surplus is increasing in Ψ and that the equilibrium is stable. □

Lemma 7. *If merger M_k is CS-neutral, it raises the joint profit of the merging firms as well as aggregate profit.*

Proof. It is immediate to see that the profit of any firm not involved in the merger remains unchanged as Ψ remains unchanged. It thus remains to show that

$$g(2\psi_k(M_k);\Psi(M_k)) > g\left(\psi_0^0;\Psi^0\right) + g\left(\psi_k^0;\Psi^0\right).$$

But as M_k is CS-neutral, we have $\Psi(M_k) = \Psi^0$ and $2\psi_k(M_k) = \psi_0^0 + \psi_k^0$. The above inequality can thus be rewritten as

$$g\left(\psi_0^0 + \psi_k^0;\Psi^0\right) > g\left(\psi_0^0;\Psi^0\right) + g\left(\psi_k^0;\Psi^0\right).$$

But this follows from the above-mentioned properties of the function g. □

As a reduction in post-merger marginal cost increases the merged firm's profit, any CS-nondecreasing merger is profitable. Let us assume that a reduction in post-merger marginal cost (of a CS-nondecreasing merger) also increases aggregate profit. In the Cournot model, we have seen that this assumption holds if pre-merger cost differences are not too large. This observation also holds in the CES and multinomial logit models:

Example 4 (CES)**.** In the CES model, if pre-merger marginal cost differences are not too large so that for any CS-nondecreasing merger M_k we have $2\psi_k(M_k) > \psi_i(M_k)$, then the reduction in post-merger marginal cost $\overline{c}_k$ increases aggregate profit. To see this, note that from the argument given in our exposition of the CES model above, the reduction in $\overline{c}_k$ induces a change from ψ_i/Ψ to $(\psi_i/\Psi - \Delta_i)$, $i \neq 0, k$, $\Delta_i > 0$, and from ψ_k/Ψ to $(\psi_k/\Psi + \sum_{i\neq 0,k} \Delta_i)$. It thus suffices to show that the joint profit of the merged firm k and any other firm i,

$$h_i(\Delta) \equiv \left\{ \frac{r_k + \Delta}{(\lambda+1) - \lambda(r_k + \Delta)} \right\} + \left\{ \frac{r_i - \Delta}{(\lambda+1) - \lambda(r_i - \Delta)} \right\},$$

where $\Delta \geq 0$, $r_i = \psi_i \Psi$ and $r_k \geq \psi_k/\Psi$, is increasing in Δ. But this holds as we have

$$h_i'(\Delta) \equiv \frac{\lambda+1}{[(\lambda+1) - \lambda(r_k+\Delta)]^2} + \frac{\lambda+1}{[(\lambda+1) - \lambda(r_i - \Delta)]^2} > 0,$$

where the inequality follows as $r_k > r_i$ by assumption.

Example 5 (Multinomial Logit)**.** In the multinomial logit model, if pre-merger marginal cost differences are not too large so that for any CS-nondecreasing merger M_k we have $2\psi_k(M_k) > \psi_i(M_k)$, then the reduction in post-merger marginal cost $\overline{c}_k$ increases aggregate profit. To see this, note that from the argument given in our exposition of the multinomial logit model above, the reduction in $\overline{c}_k$ induces a change from ψ_i/Ψ to $(\psi_i/\Psi - \Delta_i)$, $i \neq 0, k$, $\Delta_i > 0$, and from ψ_k/Ψ to $(\psi_k/\Psi + \sum_{i\neq 0,k} \Delta_i)$. It thus suffices to show that the joint profit of the merged firm k and any other firm i,

$$h_i(\Delta) \equiv \mu \left\{ \frac{r_k + \Delta}{1 - (r_k + \Delta)} \right\} + \mu \left\{ \frac{r_i - \Delta}{1 - (r_i - \Delta)} \right\},$$

where $\Delta \geq 0$, $r_i = \psi_i \Psi$ and $r_k \geq \psi_k/\Psi$, is increasing in Δ. But this holds as we have

$$h_i'(\Delta) \equiv \frac{\mu}{[1 - (r_k + \Delta)]^2} + \frac{\mu}{[1 - (r_i - \Delta)]^2} > 0,$$

where the inequality follows as $r_k > r_i$ by assumption.

We are now in the position to extend Lemma 4 to this larger class of models:

Lemma 8. *Suppose mergers M_j and M_k, $k > j$, induce the same nonnegative change in consumer surplus so that $\Psi(M_j) = \Psi(M_k) \geq \Psi^0$. Then, the larger merger M_k induces a greater increase in aggregate profit than the smaller merger M_j.*

Proof. As the aggregate outcome Ψ is the same under both mergers, the profit of each firm not participating in either merger is also the same under both mergers. We thus only need to show that

$$g(2\psi_k(M_k); \overline{\Psi}) + g(\psi_j(M_k); \overline{\Psi}) > g(2\psi_j(M_j); \overline{\Psi}) + g(\psi_k(M_j); \overline{\Psi}),$$

where $\overline{\Psi} \equiv \Psi(M_j) = \Psi(M_k)$ is the common aggregate outcome after each of the two alternative mergers. As $\Psi(M_j) = \Psi(M_k)$, we must have

$$2\psi_k(M_k) + \psi_j(M_k) = 2\psi_j(M_j) + \psi_k(M_j).$$

Now, as $c_j > c_k$ and as $\Psi(M_j) = \Psi(M_k)$, we obtain (from the assumption that a firm's cumulative best response is decreasing in its marginal cost) that

$$\psi_j(M_k) < \psi_k(M_j),$$

implying that

$$2\psi_k(M_k) > 2\psi_j(M_j).$$

Next, note that as a CS-nondecreasing merger increases the profit of the merging firms and reduces everyone else's profit, we have

$$\begin{aligned} g(2\psi_k(M_k), \Psi(M_k)) &> g(\psi_k^0, \Psi^0) \\ &\geq g(\psi_k(M_j), \Psi(M_j)). \end{aligned}$$

As $\Psi(M_k) = \Psi(M_j)$ and as g is strictly increasing in its first argument, this implies

$$2\psi_k(M_k) > \psi_k(M_j).$$

Using the same type of argument, we also have

$$2\psi_j(M_j) > \psi_j(M_k).$$

We have thus shown that

$$2\psi_k(M_k) > \max\left\{2\psi_j(M_j), \psi_k(M_j)\right\} \geq \min\left\{2\psi_j(M_j), \psi_k(M_j)\right\} > \psi_j(M_k).$$

But since $2\psi_k(M_k) + \psi_j(M_k) = 2\psi_j(M_j) + \psi_k(M_j)$ and since g is strictly convex, this implies that

$$g(2\psi_k(M_k); \overline{\Psi}) + g(\psi_j(M_k); \overline{\Psi}) > g(2\psi_j(M_j); \overline{\Psi}) + g(\psi_k(M_j); \overline{\Psi}).$$

$\square$

Finally, note that if $\left|\overline{\Psi} - \Psi^0\right|$ is sufficiently small, where $\overline{\Psi} \equiv \Psi(M_j) = \Psi(M_k) \geq \Psi^0$, then the lemma also implies that the larger merger M_k induces a larger increase in the bilateral profit change than the smaller merger M_j. (This follows from the fact that if both mergers are CS-neutral, then the induced bilateral profit change is equal to the induced aggregate profit change.)

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