Approximating the Price Effects of Mergers: Numerical Evidence and an Empirical Application

Nathan H. Miller* Marc Remer* Conor Ryan* Gloria Sheu*

October 1, 2012

Abstract

The results of counterfactual simulations in industrial economics can be sensitive to demand curvature assumptions. We provide numerical evidence on the accuracy of first order approximation, a potentially more robust methodology developed theoretically in Jaffe and Weyl (2011) to predict the price effects of mergers. We find that approximation returns predictions that are on average 40% and 2.6% percent different than actual price changes when consumer behavior is governed by the logit demand system and the Almost Ideal Demand System (AIDS), respectively; the approximation is exact with linear demand and less accurate with log-linear demand. This level of precision is superior to merger simulation conducted with an incorrect assumption on demand curvature in most cases. We also provide an empirical application to demonstrate how price approximation can be applied given scanner data with sufficient variation.

Keywords: price approximation; merger simulation; upward pricing pressure
JEL classification: K21; L13; L41

*Economic Analysis Group, Antitrust Division, U.S. Department of Justice, 450 5th St. NW, Washington DC 20530. The views expressed herein are entirely those of the authors and should not be purported to reflect those of the U.S. Department of Justice. We thank Sonia Jaffe, Charles Taragin, Glen Weyl, Nathan Wilson and seminar participants at the U.S. Department of Justice for valuable comments.

1 Introduction

Horizontal mergers can diminish the incentives of the merging firms to compete, as each merging firm internalizes the impact of aggressive actions on the profits of the other. The literature on antitrust economics characterizes this effect as arising due to the creation of opportunity costs; each merging firm, when making a sale, forgoes with some probability a sale by the other merging firm (e.g., Farrell and Shapiro (2010a); Jaffe and Weyl (2011)). This interpretation is useful for antitrust policymakers because these opportunity costs can be measured with data on consumer substitution patterns and margins in the pre-merger equilibrium.[1] Building on this logic, Jaffe and Weyl (2011) provide general conditions under which the price effects of mergers can be calculated, to a first-order approximation, by multiplying these opportunity costs with an appropriate measure of cost pass-through.[2] This calculation, hereafter referred to as "approximation," is the subject of our research.

The logic of approximation extends to many counter-factual exercises of interest in industrial economics, outside strict antitrust analysis, in which (1) the "policy" perturbs firms' first order conditions; and (2) the first order conditions are sufficiently well behaved. Relative to most simulation techniques, approximation requires additional information on either cost pass-through or demand curvature in the neighborhood of the original equilibrium.[3] This information can obviate the need to specify the functional form of demand. The promise of approximation is that knowledge of cost pass-through or local demand curvature can enable more robust counter-factual predictions.

We make two primary contributions in this paper. First, we use numerical experiments to assess the accuracy of approximation in predicting the price effects of mergers. The experiments are valuable because the theoretical results of Jaffe and Weyl (2011) demonstrate

[1]Farrell and Shapiro (2010a) refer to the opportunity costs created by a merger as gross upward pricing pressure (UPP). The Horizontal Merger Guidelines of the U.S. Department of Justice and the Federal Trade Commission, as revised in 2010, endorse upward pricing pressure as informative of the likely competitive effects of mergers. See Horizontal Merger Guidelines ∮6.1:

> "The value of sales diverted to a product is equal to the number of units diverted to that product multiplied by the margin between price and incremental cost on that that product. In some cases, where sufficient information is available, the Agencies assess the value of diverted sales, which can serve as a diagnostic of the upward pricing pressure.... The Agencies rely much more on the value of diverted sales than on the level of the HHI for diagnosing unilateral price effects in markets with differentiated products."

[2]Froeb, Tschantz, and Werden (2005) derives a similar approximation for the specific case of Nash-Bertrand competition and constant marginal costs.

[3]The connection between cost pass-through and consumer demand is emphasized in the recent theoretical literature (e.g. Jaffe and Weyl (2011), Miller, Remer, and Sheu (2012), Weyl and Fabinger (2009)).

the precision of approximation only with upward pricing pressure that is arbitrarily small and with profit functions that are quadratic in price (e.g., with linear demand and constant marginal costs). Accuracy is theoretically ambiguous outside these special cases – and while it is reasonable to expect the accuracy of approximation to decrease with the magnitude of upward pricing pressure and with the importance of the higher order properties of demand, it is unclear how these factors interact and at what rate the precision degrades.

Fourth, we provide an empirical application to demonstrate how approximation can be applied given scanner data with sufficient price variation. The data employed characterize unit sales and average sales prices in a consumer products industry evaluated in the past by the Antitrust Division of the U.S. Department of Justice. We use standard econometric techniques to obtain a second-order approximation to the unknown demand surface in the range of the data, which we interpret as representing the neighborhood of the pre-merger equilibrium. The results allow us to infer the appropriate measure of cost pass-through and apply the approximation to evaluate the likely price effects of a hypothetical merger. This approach is in stark relief to more conventional demand estimation, which seeks to obtain the first derivatives of demand (i.e., the demand elasticities) based on functional form assumptions that restrict the second-order properties of demand.

The results of the numerical experiments indicate that approximation returns price predictions that are on average 40% and 2.6% percent different than the actual price effect when consumer behavior is governed by the logit demand system and the Almost Ideal Demand System (AIDS), respectively; approximation is exact with linear demand and less accurate with log-linear demand. In most cases, this level of precision is superior to merger simulation conducted with an incorrect assumption on demand curvature. The results also indicate that (i) "simple" approximation is conservative relative to approximation; (ii) accuracy often is retained when information on cost pass-through or second demand derivatives is observed partially; (iii) the accuracy of approximation generally improves when the actual price effects are modest.

This paper contributes a number of literatures within antitrust and industrial economics. On balance, the results indicate that approximation can be a useful complement to merger simulation when sufficient data area available, building on the extensive literature on the application of merger simulation to antitrust analysis (see Werden and Froeb (2008) for an overview). They also indicate that upward pricing pressure (UPP) analysis is more powerful than initially conceptualized (e.g., see Schmalensee (2009); Carlton (2010); Farrell and Shapiro (2010a)) because it can be combined with information on cost pass-through to predict price effects. The results have special relevance to several recent papers that compare *ex* *ante* predictions of merger simulation to direct *ex post* estimates of actual price effects (e.g., Nevo (2000); Peters (2006); Weinberg (2011); Weinberg and Hosken (2012)), in that they highlight the potential importance of demand curvature assumptions in creating discrepancies between merger simulations and realized price effects. Finally, while we have examined approximation within the specific context of antitrust economics, the implications of our research extend to the literature on empirical industrial economics more generally insofar as approximation provides an alternative to simulation in the examination of counter-factual scenarios.

The paper proceeds as follows. We first provide an overview of approximation in Section 2. There we derive the approximation and provide graphical intuition, discuss how one can obtain the appropriate measure of pass-through using information on pre-merger cost pass-through or the second-order properties of demand, compare approximation to merger simulation, and show that rearranging the firms' first order conditions leads to an alternative formulation of the approximation. In Section 3, we discuss the design of the numerical experiments and in Section 4 we provide the results. Finally, Section 5 develops the empirical application and Section 6 concludes.

2 Overview of Merger Approximation

2.1 Derivation and graphical illustration

We focus on models of Bertrand-Nash competition in which firms face well-behaved, twice-differentiable demand functions.[4] Each firm i produces some subset of the products available to consumers and sets prices to maximize short-run profits, taking as given the prices of its competitors. The profits of firm i have the expression

$$\pi_i = P_i^T Q_i(P) - C_i(Q_i(P)), \tag{1}$$

where P_i is a vector of firm i's prices, Q_i is a vector of firm i's sales, P is a vector containing the prices of every product, and C_i is the cost of firm i. The following first order conditions

[4] We focus on Bertrand-Nash competition for expositional and notational simplicity. The approximation generalizes to other strategic contexts, provided there is only a single strategic variable per product, including Cournot-Nash competition and competition with conjectured reactions. We refer the reader to Jaffe and Weyl (2011) for the more general notation.

characterize firm i's profit-maximizing prices:

$$f_i(P) \equiv - \left[\frac{\partial Q_i(P)}{\partial P_i}^T \right]^{-1} Q_i(P) - (P_i - MC_i) = 0, \tag{2}$$

where MC_i is a vector of firm i's marginal costs (i.e., $MC_i = \partial C_i / \partial Q_i$). While first order conditions can be manipulated to yield various expressions, each of which characterizes the same profit-maximizing prices, the selected formulation is increasingly popular among antitrust theorists for reasons that we make apparent shortly.

Mergers change the pricing incentives of the merging firms, causing each firm to internalize the effect that a change in its price has on the sales of its merging partner. This change in incentives is reflected in a new set of first-order conditions. Considering a merger between firms j and k, the post-merger first order conditions are

$$h_i(P) \equiv f_i(P) + g_i(P) = 0 \quad \forall i, \tag{3}$$

where

$$g_j(P) = \underbrace{- \left(\frac{\partial Q_j(P)^T}{\partial P_j} \right)^{-1} \left(\frac{\partial Q_k(P)^T}{\partial P_j} \right)}_{\text{Matrix of Diversion from } j \text{ to } k} \underbrace{(P_k - MC_k^{\text{post}})}_{\text{Markup of } k} - \underbrace{(MC_j - MC_j^{\text{post}})}_{\text{Cost Efficiencies}} \tag{4}$$

and MC^{post} denotes the post-merger marginal costs of production. The form of $g_k(P)$ is analogous and $g_i(P) = 0$ for $i \neq j, k$. The diversion matrix in equation 4 represents the fractions of sales lost by firm j's products that shift to firm k's products due to an increase in firm j's prices. When multiplied by the vector of firm k's markups this yields the value of diverted sales; the higher are the value of diverted sales, the greater incentive a firm has to raise price following a merger. These incentives are counterbalanced by any marginal cost efficiencies created by the merger. Farrell and Shapiro (2010a) refer to $g_j(P^0)$ and $g_k(P^0)$ as the net *upward pricing pressure* created by the merger.[5]

It is natural to conceptualize upward pricing pressure as capturing an opportunity cost of sales because each merging firm, when making a sale, forgoes with some probability a sale by the other merging firm (e.g., Weyl and Fabinger (2009); Farrell and Shapiro (2010a); Farrell and Shapiro (2010b); Kominers and Shapiro (2010); Jaffe and Weyl (2011)). Indeed, the opportunity cost created by a merger equals net upward pricing pressure less marginal

[5]See Willig (2011) for an upward pricing pressure measure that accommodates quality efficiencies.

cost efficiencies. This interpretation is supported by the formulation of the first order conditions in equations 2 and 3 because both upward pricing pressure and marginal costs enter quasi-linearly with a coefficient of one so that upward pricing pressure has the same effect on the first order conditions as a marginal cost perturbation of the same magnitude.

Mergers can affect equilibrium prices as firms pass through to consumers the net upward pricing pressure. The insight of Jaffe and Weyl (2011) is that these price effects can be approximated using only information local to the pre-merger equilibrium:

Theorem 1 (Jaffe and Weyl 2011). *Let P^0 be the pre-merger equilibrium price vector. If the functions f(P), g(P) and h(P) characterize the pre-merger first order conditions, the net upward pricing pressure created by the merger, and the post-merger first order conditions, respectively, so that $\frac{\partial h(P)}{\partial P} = \frac{\partial f(P)}{\partial P} + \frac{\partial g(P)}{\partial P}$ and $h(P^0) = g(P^0)$, and if $h(P)$ is invertible, then the price changes due to a merger, to a first approximation, are given by the vector*

$$\Delta P = -\left(\frac{\partial h(P)}{\partial P}\right)^{-1}\Bigg|_{P=P^0} h(P^0).$$

Here the vector $h(P^0)$ is equivalent to net upward pricing pressure because $f(P^0) = 0$ by definition. The matrix $-\left(\partial h(P)/\partial P\right)^{-1}|_{P=P_0}$ is the opposite inverse Jacobian of $h(P)$, evaluated at pre-merger prices, and captures how net upward pricing pressure is transmitted to consumers. Jaffe and Weyl (2011) refer to this matrix as *merger pass-through.* Consistent with the interpretation of upward pricing pressure as an opportunity cost, merger pass-through is related closely to the cost pass-through rates that arise in the pre-merger equilibrium. We explore this connection more deeply in Section 2.2.

To build intuition, we represent a simplified version of the approximation graphically.[6] Figure 1 plots a hypothetical function $h_i(P_i; P^0_{-i})$ for the single-product firm i, holding the prices of other products fixed at pre-merger equilibrium levels. Thus, the intersection of $h_i(P_i; P^0_{-i})$ with the horizontal axis provides the optimal price of firm i given that other prices remain unchanged from the pre-merger equilibrium.[7] The dashed line is the tangent to $h_i(P_i; P^0_{-i})$ at the pre-merger price. The post-merger price of firm i can be approximated

[6] We impose that $\partial h(P)/\partial P$ is diagonal solely for the purpose of the graphical demonstration. The restriction implies that prices are unaffected by the costs of other products so that, for instance, there is no strategic complementarity or substitutability as defined by Bulow, Geanakoplos, and Klemperer (1985). Economic theory dictates that the Jacobian of $h(P)$ is never actually diagonal. Even in the case of log-linear demand, where there is no strategic complementarity or substitutability among the prices of competitors, the off-diagonal terms of $\partial h(P)/\partial P$ are non-zero for the products of the merging firms.

[7] To be explicit, this optimum does not characterize the post-merger price of firm i because the point

by projecting this tangent to its point of intersection with the horizontal axis, which is equivalent to applying a single step of Newton's method. In this example, the convexity of $h_i(P_i; P^0_{-i})$ leads the approximation to understate the optimal price of the product given other prices at pre-merger levels. The convexity or concavity of the $h_i(P_i; P^0_{-i})$ depends on the higher-order properties of demand and, in general, the approximation could understate or overstate the profit-maximizing post-merger prices.

[Figure 1 about here.]

Theorem 1 implies that approximation is precise when upward pricing pressure is arbitrarily small and also with profit functions that are quadratic in price (e.g., with linear demand and constant marginal costs). Outside of these special cases, the precision of approximation is theoretically ambiguous. While the accuracy of the approximation may be expected to decrease with the magnitude of upward pricing pressure and with the curvature in $h(P)$, it is unclear how these factors interact and at what rate the precision degrades. The numerical experiments that we conduct are designed to evaluate the accuracy of approximation in such settings.

2.2 Obtaining merger pass-through

First order approximation requires knowledge of merger pass-through which, as can be ascertained from equations 2-4, depends on the first and second derivatives of demand.[8] The informational demands of approximation therefore exceed those of merger simulation, which requires knowledge only of first derivatives. In this section, we discuss how knowledge of merger pass-through can be obtained. We encourage the reader to keep in mind that the results of our numerical experiments suggest that approximation often retains precision when knowledge of merger pass-through is imperfect.

One approach to obtaining the requisite demand derivatives is to estimate them from data. The translog demand model of Christensen, Jorgenson, and Lau (1975) and the almost ideal demand system (AIDS) of Deaton and Muellbauer (1980) each have somewhat flexible second order properties and, given sufficient data, could be estimated. Alternatively, models with fully flexible first and second order properties could be used. Along these lines, in our

of intersection shifts as other prices re-equilibrate. Whether the post-merger price is higher or lower than this optimum depends on whether prices are strategic complements or substitutes. Approximation explicitly adjusts for the strategic complementarity and substitutability of prices through the off-diagonal terms of $\partial h(P)/\partial P$. This adjustment is not present in the simplified version represented graphically.

[8] We defer the derivation of merger pass-through to Appendix A.

empirical application we use scanner data to estimate a system of equations that provides second-order approximations to demand in the neighborhood of pre-merger equilibrium. We derive the first and second demand derivatives from the regression coefficients and apply approximation to evaluate a hypothetical merger.[9] The estimation of demand systems with flexible second order properties typically requires data with unusually rich price variation and is not feasible for many applications.

An alternative approach is to infer the second derivatives of demand from pre-merger cost pass-through and knowledge of the first derivatives of demand. Cost pass-through has been estimated in the academic literature (e.g., Besanko, Dube, and Gupta (2005)) and in conjunction with antitrust litigation (e.g., Ashenfelter, Ashmore, Baker, and McKernan (1998)). The first derivatives of demand are estimated routinely in the academic literature based on demand systems with flexible first order properties, such as the random coefficients logit model of Berry, Levinsohn, and Pakes (1995).

The key to this alternative approach is that cost pass-through is tightly linked to demand curvature. Following Jaffe and Weyl (2011), this connection can be derived from the first order conditions of equation 2. Consider the imposition of a per-unit tax on each product, which serves to perturb marginal costs, and denote the vector of taxes t. Since marginal costs enter quasi-linearly into the first order conditions of each firm with a coefficient of one, the post-tax pre-merger first order conditions can be written

$$f(P) + t = 0.$$

Differentiating with respect to t obtains

$$\frac{\partial P}{\partial t}\frac{\partial f(P)}{\partial P} + I = 0,$$

and algebraic manipulations then yield the pre-merger cost pass-through matrix:

$$\rho^{\text{pre}} \equiv \frac{\partial P}{\partial t} = -\left(\frac{\partial f(P)}{\partial P}\right)^{-1}. \tag{5}$$

The Jacobian of $f(P)$ depends on the first and second derivatives of demand, as is clear from equation 2.[10] Provided that the first derivatives are known, numerical optimization can be

[9] See Section 5 for details.

[10] Equation 5 clarifies the link between pre-merger cost pass-through and merger pass-through: the former depends on the Jacobian of the $f(P)$ while the latter depends on the Jacobian of $h(P)$; evaluated at pre-merger prices in both cases.

used to select second derivatives that rationalize pre-merger cost pass-through, i.e. second derivatives that minimize the "distance" between the elements in the implied opposite inverse Jacobian of $f(P)$ and the elements in the observed pre-merger cost pass-through matrix.[11] These second derivatives can then be used, in conjunction with the first derivatives, to calculate merger pass-through.

Some additional assumptions are necessary. Since the matrices that appear in equation (5) are of dimensionality $N \times N$, where N is the number of products, the relationship between pre-merger cost pass-through and the Jacobian of $f(P)$ provides N^2 equations with which to identify unknown second derivatives. An assumption that demand satisfies Slutsky symmetry is sufficient for identification in the special case of a merger among single product duopolists.[12] In other cases, second derivatives of the form $\frac{\partial^2 Q_i}{\partial P_j \partial P_k}$, for $i \neq j$, $i \neq k$ and $j \neq k$, are not identified from equation (5) even with Slutsky symmetry. These second derivatives are plausibly small, however, and it may be reasonable to normalize them to zero. Alternatively, Jaffe and Weyl (2011) suggest the following "horizontality" assumption on demand:

$$Q_i(P) = \psi\left(P_i + \sum_{j \neq i} \mu_j(P_j)\right), \tag{6}$$

for some $\psi : \mathbb{R} \to \mathbb{R}$ and $\mu : \mathbb{R} \to \mathbb{R}$, which is sufficient for full identification. The needed second derivatives then take the form

$$\frac{\partial^2 Q_i}{\partial P_j \partial P_k} = \frac{\partial^2 Q_i}{\partial^2 P_i} \frac{\frac{\partial Q_i}{\partial P_j} \frac{\partial Q_i}{\partial P_k}}{\left(\frac{\partial Q_i}{\partial P_i}\right)^2}. \tag{7}$$

The numerical experiments that we conduct explore how these identifying assumptions affect the accuracy of the approximation in a variety of economic environments.

2.3 Simple approximation

[To be completed]

[11] In our numerical experiments, we select the second derivatives to minimize the sum of squared deviations.

[12] Slutsky symmetry implies $\frac{\partial Q_i}{\partial P_j} = \frac{\partial Q_j}{\partial P_i}$ and it follows that:

$$\frac{\partial^2 Q_i}{\partial^2 P_j} = \frac{\partial}{\partial P_j} \frac{\partial Q_i}{\partial P_j} = \frac{\partial}{\partial P_j} \frac{\partial Q_j}{\partial P_i} = \frac{\partial^2 Q_j}{\partial P_j \partial P_i}.$$

2.4 Comparison to merger simulation

We find it instructive to compare approximation to merger simulation, which is employed routinely by researchers and antitrust authorities to predict the price effects of mergers (e.g., Nevo (2000); Werden and Froeb (2008)). Merger simulation begins with the selection of a functional form for the demand system. The structural parameters then are estimated to bring the implied first derivatives of demand close to those implied by the data.[13,14] With the specified demand system and appropriate structural parameters, post-merger prices can be calculated as the P^* that solves

$$h(P^*) \equiv f(P^*) + g(P^*) = 0, \tag{8}$$

where the functions f, g, and h are as defined in Section 2.1. This step often, but not necessarily, entails numerical optimization.[15]

In merger simulation models, the functional form of demand determines how elasticities change as prices move away from the pre-merger equilibrium and, as a result, the predicted price effects can be sensitive to functional form assumptions (e.g., Shapiro (1996); Crooke, Froeb, Tschantz, and Werden (1999)). Absent efficiencies, merger simulations based on demand systems with little or no curvature (e.g., linear demand) produce smaller predicted price increases than simulations based on demand systems that are more convex (e.g., log-linear demand).[16] It is worth pointing out that demand estimation typically is employed to recover the first derivatives of demand within the range of the data while the second derivatives are dictated by the assumed functional form. In our experience, the functional

[13] The estimation of demand elasticities is operationally equivalent to the estimation of first derivatives. We frame our discussion of merger simulation in the context of first derivatives so as to facilitate the comparison to approximation.

[14] The structural parameters instead can be calibrated with evidence on price-cost margins and consumer substitution patterns gleaned from surveys, marketing studies, or other documentary evidence. Demand calibration is less commonly employed by academic researchers because it often requires access to confidential information. However, firms have a strong incentive to understand their costs and consumer substitution patterns, and the resulting documentation often becomes available to economists employed by the Antitrust Division and the Federal Trade Commission under the Hart-Scott-Rodino Act.

[15] The post-merger prices can be computed as the vector $\widetilde{P}$ that satisfies $\frac{1}{N} \parallel h(\widetilde{P}) \parallel < \delta$. Analytical solutions are available for the case of linear demand and constant marginal costs.

[16] Shapiro (1996) considers a merger between two single-product firms with identical margins (m) and diversion ratios (d) and shows that $\Delta p/p = md/2(1-d)$ if demand is linear and $\Delta p/p = md/(1-m-d)$ if demand is log-linear. For any m and d, the predicted price effects with log-linear demand are more than double those with linear demand. See also Crooke, Froeb, Tschantz, and Werden (1999), which conducts numerical experiments and documents that for a given set of pre-merger elasticities, a log-linear demand specification yields substantially greater price increases than logit or AIDS specifications, which in turn yield greater price increases than a linear specification.

form of demand is rarely selected with demand curvature in mind.[17]

Approximation differs from merger simulation primarily in how the second derivatives of demand are treated, or equivalently, in how demand elasticities are projected to change as prices move away from the pre-merger equilibrium. Whereas merger simulation employs an assumption on the second derivatives, imposed via the functional form of demand, approximation utilizes knowledge of either cost pass-through or the second-order properties of demand around the pre-merger equilibrium. The promise of approximation is that such knowledge, when available, can enable researchers and practitioners to forecast more robustly the price effects of mergers.

2.5 Alternative first order conditions

The first order conditions used in approximation are obtained by differentiating the profit function expressed in equation 1 with respect to price and then pre-multiplying by the opposite inverse of $\partial Q_i/\partial P_i$. The pre-multiplication has conceptual advantages insofar as it facilitates the interpretation of upward pricing pressure as an opportunity cost, and it is innocuous for merger simulation because the prices that characterize the post-merger equilibrium are unchanged. However, the technique of approximation requires the evaluation of $h(P)$ at pre-merger prices rather than at post-merger prices. Transformations such as the pre-multiplication employed by Jaffe and Weyl (2011) can affect the level, slope, and curvature of $h(P)$ away from post-merger prices. Consequently, the way that the first order conditions are expressed can have implications for the accuracy of approximation.

Froeb, Tschantz, and Werden (2005) proposes that first order approximations to merger price effects can obtained based on first order conditions constructed in the usual manner by taking the derivative of the profit function with respect to price:

$$f_i^{alt}(P) \equiv Q_i(P) + \left(\frac{\partial Q_i(P)}{\partial P_i}^T\right)(P_i - MC_i) = 0. \qquad (9)$$

[17] The trade-off between the tractability of estimation and the reasonableness of implied consumer behavior typically receives greater weight. For instance, the almost ideal demand system (AIDS) model of Deaton and Muellbauer (1980) allows for flexible substitution patterns but suffers from the curse of dimensionality as N^2 price coefficients must be estimated (N being the number of products). By contrast, the logit demand system has only a single price coefficient but restricts substitution patterns. The random coefficients logit model of Berry, Levinsohn, and Pakes (1995) is widely used in the academic literature because it does not suffer from the curse of dimensionality while allowing for flexible substitution patterns.

Considering a merger between firms j and k, the post-merger first order conditions are

$$h_i^{alt}(P) \equiv f_i^{alt}(P) + g_i^{alt}(P) = 0, \tag{10}$$

where

$$g_j^{alt}(P) = \left(\frac{\partial Q_k(P)^T}{\partial P_j}\right)(P_k - MC_k^{\text{post}}) + \left(\frac{\partial Q_j(P)^T}{\partial P_j}\right)(MC_j - MC_j^{\text{post}}). \tag{11}$$

Theorem 1 can be modified to apply to these alternative formulations. We see no reason, *a priori*, to expect approximation to perform better or worse than this alternative, and in practice both methods require the same set of primitives. We evaluate the performance of both formulations in our numerical experiments.

It worth noting that marginal costs do not enter these alternative first order conditions quasi-linearly, and therefore the interpretation of $g_j^{alt}(P^0)$ and $h_j^{alt}(P^0)$ as opportunity costs is less straight-forward than in approximation.[18] Further, while the alternative merger pass-through matrix $-(\partial h^{alt}(P)/\partial P)^{-1}|_{P=P^0}$ retains its interpretation as a measure of how $g_j^{alt}(P)$ is transmitted to consumers through prices, its connection to pre-merger cost pass-through rates is tenuous because the Jacobian of $f^{alt}(P)$ does not yield pre-merger cost pass-through, as does the Jacobian of $f(P)$ by equation (5).

3 Design of the Numerical Experiments

3.1 Conceptual overview

We use numerical experiments to evaluate the accuracy of approximation across a range of economic environments. In each experiment, we first posit the demand and cost functions that fully characterize the market, and treat these as the "truth." We then simulate a merger between two firms in the market. This obtains the true price effect of the merger and provides a baseline against which to measure the accuracy of approximation. Our use of merger simulation is distinguished from most practical applications, which are conducted with imperfect information about the economic environment and thus yield imperfect price

[18]The emphasis of the antitrust literature on upward pricing pressure, as expressed in equation (4), rather than on $g_j^{alt}(P)$, stems from the fact that upward pricing pressure can be calculated with diversion ratios whereas $g_j^{alt}(P)$ requires knowledge of demand elasticities. This advantage of upward pricing pressure does not extend to the calculation of approximate price effects, which requires knowledge of these elasticities regardless of how the first order conditions are expressed.

predictions. In our experiments, by contrast, merger simulation is conducted with perfect knowledge of demand and supply conditions and provides the true price effects.

To support approximation, we derive the first and second derivatives of demand that arise in pre-merger equilibrium based on the posited demand and cost functions. We also calculate the pre-merger cost pass-through from these elements, following equation 5. We then compare approximation to the merger simulation results and assess the accuracy of approximation. In practical applications, demand derivatives and cost pass-through likely would be estimated from data or inferred from documentary evidence. Our approach provides a clean assessment of the accuracy of approximation because it links the demand derivatives and cost pass-through that arise in pre-merger equilibrium to the underlying demand system used to conduct merger simulation.

3.2 Data generating process

In each experiment, we consider an industry with three single-product firms and evaluate merger between the first two firms. We begin with prices, quantities and the first firm's margin, which is sufficient information to calibrate a logit demand model. Then, using the implied logit elasticities, we calibrate five additional demand systems: the AIDS, linear demand, log-linear demand and two forms of mixed logit demand. Customer substitution among the products is imposed as proportional to share for the AIDs, linear demand and log-linear demand in the pre-merger equilibrium; the property is maintained away from the pre-merger equilibrium only for logit demand.[19] The mixed logit experiments allow us to approximate more flexible consumer substitution patterns.

We turn now to the details of the data generating process. We randomly draw the market shares of firms 1 and 2 from a uniform distribution with support between 5% and 65%. So as not to exceed the size of the market, the share of firm 2 also faces the upper bound of one minus the first firm's share, and firm 3 receives the remaining market share. The margin of firm 1 is drawn from a uniform distribution with support between 10% and 90%. We assume all prices are one, the market size is 100, and marginal costs are constant.

Calibration starts with the logit demand system, which takes the form

$$S_i = \frac{e^{(\eta_i - P_i)/\tau}}{\sum_k e^{(\eta_k - P_k)/\tau}}, \tag{12}$$

[19] Consumer substitution that is proportional to share is a well-recognized property of logit demand (e.g., Nevo (2001)). An implication of our calibration strategy is that the degree of substitutability between the merging firms' products is greater when their shares are greater.

where S_i is the share of firm i (i.e. $S_i = Q_i/M$ for market size M). The unknowns include the J product-specific terms (η_i) and a single scaling/price coefficient (τ). The system is under-defined, which we account for by normalizing the η value for the last product to one. The implied elasticities evaluated at pre-merger equilibrium are

$$\epsilon_{jk} = \begin{cases} -(1-S_j)/\tau & \text{if } j = k \\ S_k/\tau & \text{if } j \neq k \end{cases}, \tag{13}$$

keeping in mind that prices are equal to one. We obtain the margins of firms 2 and 3 based on these elasticities and the first order conditions.[20]

We use these elasticities and margins to calibrate the linear demand system, the AIDS, and the log-linear demand system. The linear demand system takes the form

$$Q_i = \alpha_i + \sum_j \beta_{ij} P_j. \tag{14}$$

where α represents the product-specific intercepts, β represents the price coefficients, and Q is quantity. The log-linear demand system takes the form

$$\ln(Q_i) = \delta_i + \sum_j \epsilon_{ij} \ln P_j, \tag{15}$$

where δ represents the product-specific intercepts and ϵ is as defined in equation (13). The AIDS of Deaton and Muellbauer (1980) takes the form

$$W_i = \psi_i + \sum_j \phi_{ij} \log P_j + \theta_i \log(X/P^*), \tag{16}$$

where W_i is an expenditure share (i.e., $W_i = P_i Q_i / \sum_k P_k Q_k$), X is the total expenditure and P^* is a price index given by

$$\log(P^*) = \psi_0 + \sum_k \psi_k \log(P_k) + \frac{1}{2} \sum_k \sum_l \phi_{kl} \log(P_k) \log(P_l).$$

We focus on the special case of $\theta_i = 0$, consistent with common practice in antitrust applications (e.g, Epstein and Rubinfeld (1999)). The restriction is equivalent to imposing an

[20] We discard draws that yield margins for firms 2 and 3 that exceed unity. This tends to occur when firm 1 is assigned a margin near the upper limit of 0.80 with a small share (the latter implies a small relative margin).

income elasticity of one. While the log-linear and linear demand systems require all the margins, the restricted AIDS model only requires two. Thus, the margins for the AIDS model are slightly different than those of the previous three models.

We also generate results for the mixed (or "random coefficients") logit demand system that is popular in empirical industrial economics research. We focus on a specific case in which market shares take the form

$$S_i = \int \frac{e^{(\eta_i - (1+\sigma\nu)P_i)/\tau}}{\sum_k e^{(\eta_k - (1+\sigma\nu)P_k)/\tau}} \partial F(\nu),$$

where $F(\nu)$ is a distribution that we assume to be normal with mean zero and variance one. We select τ based on the already calibrated standard logit model. We select two values of σ for investigation: $\sigma = 1/(2\tau)$, which implies that roughly 95% of consumers have downward-sloping demand, and $\sigma = 1/(4\tau)$, which is selected as a halfway point to the standard logit model. We then we take 1,000 draws from the distribution of ν and calibrate the product specific intercepts to match the observed market shares. As in the standard logit model, we normalize the intercept of the third product to one. The results generated for this particular specification of the mixed logit model may not generalize to other specification employed in the empirical literature that feature different or multiple distributions of consumer tastes. We nonetheless consider the exercise to have value, insofar as it shows how the accuracy of approximation can change based on the true underlying preferences of consumers.[21]

3.3 Summary statistics

Table 1 provides summary statistics on the randomly-generated industries. As shown, the average market share and margin the first firm are 40% and 30%, respectively. Substantial variation exists in each. For instance, the fifth and ninety-fifth market share percentiles are 16% and 62%. The market shares and margins of the second firm are somewhat smaller, due to the mathematical restriction that the second firm's share can never exceed one minus the first firm's share. The average margins of the two firms are 0.46 and 0.42, respectively, which corresponds to average own-price demand elasticities of demand. Given the market

[21]Three of the posited demand systems can exhibit idiosyncracies in merger simulation. First, merger simulations with linear demand can predict price increases large enough to make quantities negative in the post-merger equilibrium. Second, post-merger equilibrium does not always exist with log-linear demand or mixed logit; this occurs when one or more of the cross-price elasticities is sufficiently large relative to the own-price elasticities. While the first idiosyncracy does not arise in our data generating process, the second arises with somewhat greater frequency. When a randomly-generated industry produces a log-linear or mixed logit merger simulation that does not converge we retain the industry for the other demand systems.

shares, the average implied diversion ratio from firm 1 to firm 2 is 50% in the pre-merger equilibrium and the corresponding diversion ratio from firm 2 to firm 1 is 57%.

[Table 1 about here.]

In the numerical experiments, we focus on the precision of approximation for price changes that do not exceed 50%, in order to provide more clarity over a reasonable range. As shown, the average simulated price changes are 20%, 21%, 21% and 29% for logit demand, the AIDS, linear demand and log-linear demand, respectively, conditional on the restriction.[22] In these calculations we focus exclusively on the price effects for the first firm; the accuracy of approximation is not materially different for the other price changes. The random draws cover the range well. For instance, with logit demand the fifth and ninety-fifth percentiles are 4% and 44%, respectively. When we further restrict the sample to include only "small" price changes under 10%, the average simulated price change is 6% for each of the four posited demand systems.

3.4 Research objectives

We develop three main sets of numerical results. The first pertains to the accuracy of approximation when complete information is available either on pre-merger cost pass-through or on the second derivatives of demand in the neighborhood of pre-merger equilibrium. For each combination of draws and each demand system, we calculate approximation three ways: based on the second derivatives of demand, based on pre-merger cost through with the horizontality assumption, and based on pre-merger cost pass-through setting derivatives of the form $\partial^2 Q_i/\partial P_j \partial P_k$ equal to zero.

The second set of results pertains to simple approximation.

The third set of results pertains to the accuracy of approximation when the second derivatives of demand are unknown and only incomplete information is available on the pre-merger cost pass-through. These results may prove valuable to practitioners presented with

[22] It is often argued that merger simulations based on linear demand systems return smaller price effects than merger simulations based on logit demand, for a given set of pre-merger margins and diversion (e.g., see Crooke, Froeb, Tschantz, and Werden (1999)). In actuality, this depends on the relative strength of two countervailing influences. First, own-price elasticities increase in magnitude more quickly with linear demand than with logit demand; this leads to larger price increases with logit demand, all else equal. Second, the diversion ratios between the merging firms decrease with prices with logit demand (since substitution is proportional to share) but are constant with linear demand; this is the practical result of the inconsistency between linear and logit demand discussed in Jaffe and Weyl (2010) and it leads to larger price increases with linear demand, all else equal. In our experiments, we find that the first consideration tends to dominate for smaller price increases but that the second dominates for larger price increases.

data that are insufficiently rich to identify the full pass-through matrix. We consider two scenarios in which some of elements of the cost pass-through are known:

- Cost pass-through is available only for the merger firms.[23] To implement approximation, we impute the own-cost pass-through rate of non-merging firm using the mean of the own-cost pass-through rates of the merging firms and impute cross-cost pass-through rates involving the non-merging firms using the mean of the cross-cost pass-through rates of the merging firms.

- Only own-cost pass-through is available, i.e., the off-diagonal elements of the cost pass-through matrix are unknown. To implement approximation, we treat the cross-cost pass-through terms as equaling zero.

We also consider two scenarios in which only industry cost pass-through rates are available. Industry pass-through captures the effects of a cost shock common to all firms; from a mathematical standpoint, the industry pass-through can be calculated by summing across the rows of the cost pass-through matrix. We implement approximation two ways:

- We calculate the cost pass-through matrix that would arise given linear demand, given the the first derivatives of demand, and then scale the matrix to reproduce industry cost pass-through. We refer to this as the "adjusted-linear" method.[24]

- We set the own-cost pass-through rates equal to the industry pass-through rates and set the cross-cross pass-through rates to zero. This treatment is consistent with log-linear demand and we refer to it at the "log-linear" method.

Finally, we provide three extensions that evaluate approximation under our specification of mixed logit demand, examine the accuracy of approximation for small price changes, and analyze the use of the alternative first order conditions of Section 2.5.

[23]In practice, this scenario could arise when an antitrust authority has superior ability to compel document and data productions from merging firms than from non-merging firms.

[24]To obtain obtain the cost pass-through matrix, we first calculate $\partial f(P)/\partial P$ based on the equation in Appendix A, making use of the known first derivatives and presumption that the second derivatives equal zero, and then invert following equation 5. See also Miller, Remer, and Sheu (2012), which provides an expression of $\partial f(P)/\partial P$ that is specific to linear demand.

4 Results of Numerical Experiments

4.1 Accuracy with complete information

4.1.1 Prediction error

Table 2 summarizes the absolute prediction error of approximation that arises when complete information is available for either pre-merger cost pass-through or the second derivatives of demand in the neighborhood of pre-merger equilibrium. We define absolute prediction error as the absolute value of the difference between approximation and the true price increase. Thus, absolute error indicates the precision of approximation but not whether price predictions are overstated or understated. The table provides separate statistics for each of the posited demand systems. Observations are included in the sample only when the true price effect does not exceed 50 percent in order to provide more clarity over a reasonable range. We calculate approximations alternately based on full knowledge of the second-order properties of demand in neighborhood of pre-merger equilibrium ("Known Second Derivatives"); based on full knowledge of pre-merger cost pass-through and the horizontality assumption ("PTRs with Horizontality"); and based on full knowledge of pre-merger cost pass-through and the assumption that derivatives of the form $\partial^2 Q_i/\partial P_j \partial P_k$ equal zero ("PTRs with Zeros").

[Table 2 about here.]

The mean absolute prediction error (MAPE) that arises with logit demand ranges from 0.082 to 0.084. This indicates that approximation yields price predictions that are, on average, 8.2 to 8.4 percentage points different than the true price effect. Since the average true price effect with logit demand is 0.20, approximation is on average 41%-42% from the true effect. We explore below how that level of accuracy compares to merger simulation conducted with potentially incorrect assumptions on demand curvature. The MAPE that arises with the AIDS ranges from 0.8 to 2.6 percentage points. The average true price effect with the AIDS is 0.21 so, in our sample, approximation is on average 4.7%-15.3% from the true effect.

There is no prediction error when demand is linear. This follows from the theoretical result that approximation is exact with profit functions that are quadratic in price, as they are with linear demand and constant marginal costs. The MAPE that arises with log-linear demand and known second derivatives (in the neighborhood of pre-merger equilibrium) is 1.07. This level of prediction error is attributable to the influence of numerous "outliers" with prediction error well above two (e.g., the maximum prediction error is 66). These outliers appear to be a characteristic of the approximation, rather than a statistical quirk, in that informational setting. The MAPE that arises when the approximation is based on cost pass-through rates is 0.193 and, given the average true price effect with log-linear demand of 0.27, approximation is on average 71.5% from the true effect. The approximation does not seem to provide consistently accurate predictions under the extreme curvature of the log-linear demand system.

Figure 2 provides scatter-plots of approximation against the true price effects for logit demand, the AIDS and log-linear demand. The case of linear demand is omitted because approximation is exact in that setting. Printed on each scatter-plot is the 45-degree line; dots that appear above the line represent instances in which approximation over-predicts the true price effect while dots under the line represent under-predictions. The figure clarifies the relative accuracy of the approximation across demand systems and shows how using cost pass-through rather than direct knowledge of the second-order properties of demand (in the neighborhood of pre-merger equilibrium) does little to adversely affect accuracy. Also notable is that approximation systematically over-predicts price increases when the true underlying demand system is logit. This pattern is strongest when approximation is calculated with known second derivatives and more attenuated when approximation is calculated with cost pass-through.

[Figure 2 about here.]

4.1.2 Relative accuracy of approximation and merger simulation

Table 3 tabulates the frequency with which approximation outperforms merger simulation (in the top panel) and provides the MAPEs that arise with approximation and merger simulation (in the bottom panel). Approximation is calculated assuming full knowledge of the second demand derivatives in the neighborhood of pre-merger equilibrium. Merger simulation is conducting alternately assuming logit demand, the AIDS, and linear demand.[25] We compare approximation to each of these merger simulations when the true underlying demand system is alternately logit, the AIDS, linear and log-linear. Given the design of the experiments, merger simulation returns the true price effect only when the demand curvature assumption is correct. For example, linear demand merger simulation returns the true price effect when the true underlying demand system is linear but not when it is logit.

[Table 3 about here.]

[25] We exclude log-linear merger simulations because the merger simulations often do not identify any post-merger equilibrium when the true underlying demand system is logit, the AIDS or linear.

When the true underlying demand system is logit, the approximation is more accurate than AIDS simulation in 79.1 percent of the industries considered and more accurate than linear simulation in 90.3 percent of the industries considered. When true demand is the AIDS, the approximation is more accurate than merger simulations based on logit demand and linear demand, in 94.8 percent and 87.4 percent of the industries considered, respectively. The approximation always outperforms misspecified merger simulation when true demand is linear because approximation is exact in that setting. When true demand is log-linear, approximation outperforms merger simulation based on logit demand, the AIDS, and linear demand in about half the considered industries. In most cases, approximation generates smaller MAPEs than misspecified merger simulation. Together, these comparisons showcase the potential usefulness of approximation in generating robust predictions when uncertainty exists regarding the true underlying demand schedule.

4.2 Accuracy of simple approximation

[To be completed]

4.3 Accuracy with incomplete information

Table 4 summarizes the absolute prediction error that arises when the second derivatives of demand are unknown and when only incomplete information is available on the pre-merger cost pass-through. Four informational scenarios are considered: pre-merger cost pass-through that is available only for own costs, i.e., the off-diagonal elements are unknown ("Own Cost PTRs"); pre-merger cost pass-through that is available only for the merging firms ("Merging Firms' PTRs"); industry cost pass-through that is apportioned using the adjusted-linear method ("Industry PTRs – Adj.-Linear"); and industry cost pass-through that is apportioned using the log-linear method ("Industry PTRs – Log-Linear"). Details on these scenarios are provided in Section 3.4.

[Table 4 about here.]

When own cost pass-through is known but cross pass-through is unknown, and one proceeds as if cross pass-through is zero, the MAPEs that arise are larger, at 9.6, 9.4, 12.8, and 19.3 percentage points for logit demand, the AIDS, linear demand and log-linear demand, respectively. These larger errors are due to a systematic under-prediction of price increases caused by the failure to account for prices being strategic complements. The exception is log-linear demand, where accuracy is retained because prices are neither strategic complements nor strategic substitutes so cross pass-through is zero in actuality. We conclude that when somewhat conservative predictions of price increases are sufficient, as they may be many antitrust enforcement actions, approximation retains its value when information on cross pass-through is unavailable.

When instead cost pass-through is observed for the merging firms but not the non-merging firm, the MAPEs that arise are 1.9, 1.8, 2.5 and 19.3 percentage points with logit demand the AIDS, linear demand and log-linear demand, respectively. Thus, approximation generally retains its accuracy in this informational setting. (Accuracy is identical for log-linear demand because prices are neither strategic complements nor strategic substitutes.) The result could have value to antitrust authorities in the U.S. because, under the Hart-Scott-Rodino Act, merging firms under investigation have legal obligations to provide substantial information that may not be available from non-merging firms.

We evaluate two approaches to dealing with situations in which information is available on industry pass-through rates but not the individual elements of the cost pass-through matrix. The first apportions industry pass-through as if demand were linear while the second apportions industry pass-through as if demand were log-linear. As shown, the relative performance of these two approaches depends on the true underlying demand system – the log-linear method is more accurate for logit demand but the linear method is more accurate for the other demand systems. Since the underlying demand system would be unknown in practical applications, the numerical results to not provide clear guidance on the most appropriate treatment of industry pass-through. It is notable, however, that the MAPEs that arise with both methods often are smaller than those of misspecified merger simulation (reported in Table 3).

Figure 3 provides scatter-plots of approximation against the true price effects for logit demand and the AIDS, for each of the partial information scenarios considered. Again, dots that appear above the 45-degree line represent instances in which approximation over-predicts the price effect while dots under the line represent under-predictions.

[Figure 3 about here.]

4.4 Extensions

4.4.1 Accuracy with mixed logit demand

Figure 4 provides scatter-plots of approximation against the true price effects for logit and mixed logit demand systems. The approximation is calculated based on full knowledge of the second-order properties of demand in neighborhood of pre-merger equilibrium. Two particular mixed logit models are considered, as developed in Section 3.2, based on two different price parameters: $\sigma = 1/(4\tau)$ and $\sigma = 1/(2\tau)$, the latter of which moves farther from the standard logit demand model. Once the underlying demand system is characterized by a mixed logit, rather than the standard logit, approximation does not always over-predict the true price increases. Average accuracy is relatively unchanged – the average MAPEs are 0.086 and 0.110 for $\sigma = 1/(4\tau)$ and $\sigma = 1/(2\tau)$, respectively, are similar to the average MAPE of 0.084 that arises with standard logit demand. The results are useful because they demonstrate that some properties of approximation under logit demand can change for mixed logit demand. They do not necessarily inform how these changes likely would play out for the many different mixed logit specification that have been used in the literature.

[Figure 4 about here.]

4.4.2 Accuracy with small price changes

Jaffe and Weyl (2011) prove that approximation is exact for arbitrarily small price changes. In this section, we restrict attention to those randomly-drawn industries that generate merger price increases under 10 percent. The results help identify whether, in practice, the accuracy of the approximation is improved for small price changes. The value of accuracy arguably is greater in these settings because whether antitrust enforcement action is warranted may be more uncertain. Table 5 provides the MAPEs of approximation that arise when the true price effects are under 10 percent. For logit demand, the MAPEs reported indicate that the approximation is on average 17%-26% from the true effect, given the average true price of effect of 6.1 percentage points and depending on the precise method with approximation is conducted. The range is 3.6%-9.9% for AIDS and with log-linear it is 14%, setting aside the case of known second derivative which is again driven by outliers. Thus, under each demand system, average accuracy is improved relative to the full sample of randomly drawn industries (see Section 4.1.1). We conclude that approximation likely has enhanced usefulness when the counter-factual exercise when the perturbation to pre-merger equilibrium is less pronounced.

[Table 5 about here.]

4.4.3 Accuracy with alternative first order conditions

We now develop the accuracy of approximation based on the alternative first order conditions of Section 2.5. As already stated, transformations can affect the level, slope and curvature of $h(P)$ and therefore the performance of approximation. Table 6 provide the MAPEs of approximation conducted with both the baseline and alternative first order conditions. With known second derivatives, approximation with the baseline first order conditions is relatively more accurate for logit demand but relatively less accurate for the AIDS. This reflects, in both instances, the unexpected result that the alternative first order conditions systematically generate smaller price increases.[26] Since approximation with the baseline first order conditions overstates price increases for logit demand but not (much) for the AIDS, the leads approximation with the alternative first order conditions to be accurate for logit demand and less accurate for the AIDS. A similar, though less pronounced, pattern characterizes the results with known cost pass-through rates and the horizontality assumption. Overall, the results indication that neither method of approximation dominates the other in terms of accuracy, and we conclude that in most applications it would be appropriate to examine the results of both.

[Table 6 about here.]

5 Empirical Application

In this section, we demonstrate how the first and second derivatives of demand can be estimated and subsequently used as inputs into the J-W approximation. The data employed characterize unit sales and average sales prices in a consumer products industry evaluated in the past by the Antitrust Division. Weekly observations on four popular brands are available for more than 40 cities over roughly a four year period.[27] Our objective is to obtain a second-order approximation to the unknown demand surface over the range of the data, which we interpret as representing the neighborhood of pre-merger equilibrium. To that end, we specify the following demand system:

$$Q_i = \alpha_i + \sum_j \beta_{ij} P_j + \sum_j \sum_{k \leq j} \gamma_{ijk} P_j P_k + \epsilon_i, \tag{17}$$

where we have suppressed city and week subscripts on Q, P, and ϵ. This system of equations is sufficiently flexible to approximate any arbitrary set of first and second-order demand

[26]Approximation with the alternative first order conditions generate smaller price increases in 99.5% of the logit demand industries and 100% of the AIDS industries. This also holds true for log-linear demand, where the alternative first order conditions generate smaller price increases in 93.3% of the randomly-drawn industries.

[27]We take a number of steps to to preserve the confidentiality of the data and are unable to identify the products or provide meaningful summary statistics.

properties, as there is one parameter for each derivative of interest:

$$\frac{\partial Q_i}{\partial P_j} = \beta_{ij} + 2\gamma_{ijj} P_j + \sum_{k \neq j} \gamma_{ijk} P_k \tag{18}$$

and

$$\frac{\partial^2 Q_i}{\partial P_j \partial P_k} = \begin{cases} 2\gamma_{ijj} & \text{if } j = k \\ \gamma_{ijk} & \text{if } j \neq k \end{cases} . \tag{19}$$

Absent constraints on the parameters, equation 17 provides a second-order approximation to any model of consumer behavior.[28] This approach is in contrast to the standard practice of estimating only the first derivatives of demand (i.e., the demand elasticities) using a functional form assumption that dictates the second order properties of demand.

The estimation of demand systems with flexible second-order properties, such as the system specified by equation 17, requires rich price variation in order to identify the parameters of interest; such price variation is present in the scanner data we employ. To illustrate, Figure 5 provides a scatter-plot of the weekly average sales price and unit sales for one product in a representative city.[29] The data are suggestive of a demand curve that is downward sloping and convex. Similar empirical relationships are present in most of the product-city combinations in the data.

[Figure 5 about here.]

We use OLS to estimate the system of equations and incorporate product, time and city fixed effects to control for perceived product quality, temporal fluctuations in demand, and geographic heterogeneity in consumer preferences, respectively.[30] The error term represents demand shocks that are specific to particular product-city-time combinations; the estimated regression coefficients are unbiased provided that prices are uncorrelated with these shocks.

[28] Fully non-parametric estimation is not necessary to support approximation, which requires information on only the first and second derivatives of demand. Nonetheless, nonparametric estimation places fewer restrictions on the data than the selected specification, which imposes that the second derivatives are constant within the range of the data. This restriction likely is meaningful when the data have broad support, well beyond an epsilon-ball around the pre-merger equilibrium. In such instances, estimation of the selected specification provides some measure of the average second derivatives. Nonparametric estimation may be preferable when the data have broad support and the empirical variation is sufficiently rich. Such estimation is beyond the scope of this paper.

[29] To protect the confidentiality of the data, a small number of outliers have been omitted and both average sales price and unit sales have been scaled by an unspecified constant and perturbed additively by a uniformly distributed random variable.

[30] We use the share of unit sales as the dependent variable rather than total unit sales. Absent this transformation, equation 17 would imply that a price change of a given size would have the same effect on total unit sales in each city, regardless of size or propensity-to-buy of the city's population.

Such as assumption would be warranted, for example, if prices are set before demand is realized in the market (e.g., see Hausman, Leonard, and Zona (1994); Weinberg and Hosken (2012)). Otherwise estimation plausibly could proceed with 2SLS, using prices in other cities/weeks as instruments, under the appropriate conditions.

Table 7 provides the demand elasticities and cost pass-through rates that are implied by the OLS regression coefficients.[31] The own-price elasticities of -3.89, -1.50, -1.56, and -2.25 imply margins for the four products of 25%, 67%, 64% and 44%, respectively. All of the cross-price elasticities are positive, consistent with consumer substitution between the products in response to price fluctuations. The own-cost pass-through rates well exceed 50% and therefore are consistent with convex demand schedules.[32] The cross-cost pass-through rates are positive, with one exception, consistent with prices being strategic complements in the sense of Bulow, Geanakoplos, and Klemperer (1985).

[Table 7 about here.]

Table 8 reports the results of approximation for a hypothetical merger of the first two products. When calculated using the baseline first order conditions and the estimated demand derivatives the predicted price changes are 36.5%, 41.1%, 27.3%, and 21.1% for the four products, respectively. Also shown are permutations based on different first order conditions and different information sets (demand derivatives versus cost pass-through) and the results of simple approximation. The advantage of these price predictions relative to merger simulation is that they make use of the estimated second-order properties of demand rather than imposing these properties through a functional form assumption – that is, they more fully allow the variation that is present in the data to inform the counter-factual predictions. While the estimation of demand systems with flexible second-order properties requires data with rich variation in prices, it is feasible that such data will become increasingly available to researchers and practitioners as firms collect, store and utilize data more intelligently and efficiently.

[Table 8 about here.]

[31] We make use of equation 5 to convert the regression coefficients into cost pass-through.

[32] The implied convexity does not approach that of a log-linear demand system. In that system, the own-cost pass-through rate equals $e/(1+e)$, where e is the own-price elasticity of demand. The own-cost pass-through rates that would arise with log-linear demand, given our elasticity estimates, are 1.31, 3.00, 2.70, and 1.85, respectively, for the four products examined.

6 Discussion

We interpret our results as indicating that approximation can be a useful complement to merger simulation when sufficient data are available. Whether these complementarities are likely to be recognized by the antitrust community is unclear to us. Certainly the approximation has advantages. It provides a methodology that, in appropriate settings, can be more robust and data driven than merger simulation. Furthermore, approximation can be explained on an intuitive level simply as the product of upward pricing pressure and the appropriate measure of cost pass-through. We see the downside, relative to merger simulation, as relating primarily to economists' ability to discern cost pass-through or local demand curvature in the course of merger investigations. There is also uncertainty as to whether the derived theoretical relationship between local demand curvature and cost pass-through extends to real-world settings, or whether firms more typically apply rules-of-thumb to guide pass-through behavior. We hope that our work proves helpful to the antitrust community in identifying and evaluating these tradeoffs.

Our work also has implications for industrial economics research. In particular, one standard methodology employs model-based simulations to evaluate counter-factual scenarios that are outside the range of the available data. The structural parameters of the models typically are estimated to bring the implied first derivatives of demand close to those implied by the data. Our work highlights the importance of the *second* derivatives in driving the outcomes of simulations. Further, the numerical results we develop indicate the potential value of approximation as an alternative methodology that is applicable to some of the counter-factual scenarios of interest in industrial economics. Our results also could motivate econometric research into how to best to obtain second-order approximations to the unknown demand surface, using non-parametric regression or other techniques. The value of such research likely is enhanced by the fact that researchers increasingly have access to data with rich variation that could be exploited in estimation.

Several topics surrounding approximation remain unexplored. We provide a partial list of potential research questions here with future work in mind. First, under what theoretical conditions does approximation overstate and understate the price effects of mergers? Our numerical results indicate that approximation overstates price increases when true underlying demand schedule is logit but this relationship is ambiguous when the underlying demand schedule is instead almost ideal, log-linear or mixed logit. Research that discerns how the specific theoretical properties of these demand systems affect the performance of approximation would have value. Second, what are the most accurate ways to translate information that may be available to researchers (e.g., industry pass-through) into the cost pass-through or demand curvature information required for approximation? We have proposed a number of possibilities but have not addressed the question systematically. Finally, how accurate is approximation under different equilibrium concepts? We have focused solely on Nash-Bertrand competition but both upward pricing pressure and first order approximation are generalizable and can accommodate, for example, equilibria based on Nash-Cournot competition and consistent conjectures.

References

Ashenfelter, O., D. Ashmore, J. B. Baker, and S.-M. McKernan (1998). Identifying the firm-specific cost pass-through rate. *FTC Working Paper*.

Berry, S., J. Levinsohn, and A. Pakes (1995, July). Automobile prices in market equilibrium. *Econometrica 63*(4), 847–890.

Besanko, D., J.-P. Dube, and S. Gupta (2005, Winter). Own-brand and cross-brand retail pass-through. *Marketing Science 1*(1), 123–137.

Bulow, J. I., J. D. Geanakoplos, and P. D. Klemperer (1985). Multimarket oligopoly: Strategic substitutes and complements. *Journal of Political Economy 93*(3), pp. 488–511.

Carlton, D. W. (2010). Revising the horizontal merger guidelines. *Journal of Competition, Law, and Economics*.

Christensen, L., D. Jorgenson, and L. Lau (1975, June). Transcendental logarithmic utility functions. *American Economic Review 65*, 367–383.

Crooke, P., L. Froeb, S. Tschantz, and G. J. Werden (1999). The effects of assumed demand form on simulated post-merger equilibria. *Review of Industrial Organization 15*, 205–217.

Deaton, A. and J. Muellbauer (1980). An almost ideal demand system. *The American Economic Review 70*(3), pp. 312–326.

Epstein, R. J. and D. L. Rubinfeld (1999). Merger simulation: A simplified approach with new applications. *Antitrust Law Journal 69*, 883–919.

Farrell, J. and C. Shapiro (2010a). Antitrust evaluation of horizontal mergers: An economic alternative to market definition. *B.E. Journal of Theoretical Economics: Policies and Perspectives 10*(1).

Farrell, J. and C. Shapiro (2010b). Recapture, pass-through, and market definition. *Antitrust Law Journal 76*(3), 585 – 604.

Froeb, L., S. Tschantz, and G. J. Werden (2005). Pass through rates and the price effects of mergers. *International Journal of Industrial Organization 23*, 703–715.

Hausman, J., G. K. Leonard, and J. D. Zona (1994). Competitive analysis with differentiated products. *Annales D'Economie et de Statistique 34*(1), 159–180.

Jaffe, S. and E. G. Weyl (2010). Linear demand systems are inconsistent with discrete choicew. *B. E. Journal of Theoretical Economics (Advances) 10*(1).

Jaffe, S. and E. G. Weyl (2011). The first order approach to merger analysis.

Kominers, S. and C. Shapiro (2010). Second-order critical loss analysis.

Miller, N. H., M. Remer, and G. Sheu (2012). Using cost pass-through to calibrate demand.

Nevo, A. (2000). Mergers with differentiated products: The case of the ready-to-eat cereal industry. *The RAND Journal of Economics 31*(3), pp. 395–421.

Nevo, A. (2001). Measuring market power in the ready-to-eat cereal industry. *Econometrica 69*(2), pp. 307–342.

Peters, C. (2006, October). Evaluating the performance of merger simulation: Evidence from the u.s. airline industry. *Journal of Law & Economics 49*(2), 627–49.

Schmalensee, R. (2009). Should new merger guidelines give upp market definition? *CPI Antitrust Chronicle 12*(1).

Shapiro, C. (1996, Spring). Mergers with differentiated products. *Antitrust 10*(2), 23–30.

Weinberg, M. C. (2011). More evidence on the performance of merger simulations. *American Economic Review (Papers and Proceedings) 101*((3)), 5155.

Weinberg, M. C. and D. Hosken (2012). Evidence on the accuracy of merger simulations.

Werden, G. J. and L. M. Froeb (2008). *Handbook of Antitrust Economics*, Chapter Unilateral Competitive Effects of Horizontal Mergers, pp. 43–104. MIT Press.

Weyl, E. G. and M. Fabinger (2009, October). Pass-through as an economic tool.

Willig, R. (2011). Unilateral competitive effects of mergers: Upward pricing pressure, product quality, and other extensions. *Review of Industrial Organization 39*(2), 19–38.

Appendix

A Merger Pass-Through Defined

In this appendix, we provide an expression for the Jacobian of $h(P)$, which can be used to construct merger pass-through as defined by Jaffe and Weyl (2011). Using the definition $h(P) \equiv f(P) + g(P)$, we have

$$\frac{\partial h(P)}{\partial P} = \frac{\partial f(P)}{\partial P} + \frac{\partial g(P)}{\partial P}. \tag{20}$$

The Jacobian of $f(P)$ can be written as:

$$\frac{\partial f(P)}{\partial P} = \begin{bmatrix} \frac{\partial f_1(P)}{\partial p_1} & \cdots & \frac{\partial f_1(P)}{\partial p_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_J(P)}{\partial p_1} & \cdots & \frac{\partial f_J(P)}{\partial p_N} \end{bmatrix}, \tag{21}$$

where N is the total number of products and J is the number of firms. The vector P includes all prices; we use lower case to refer to the prices of individual products, so that p_n represents the price of product n. In the case that product n is sold by firm i,

$$\frac{\partial f_i(P)}{\partial p_n} = - \begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{bmatrix} + \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1} \left[\frac{\partial^2 Q_i}{\partial P_i \partial p_n}^T\right] \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1} Q_i - \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1} \left[\frac{\partial Q_i}{\partial p_n}\right], \tag{22}$$

where Q_i and P_i are vectors representing the quantities and prices respectively of the products owned by firm i, and the initial vector of constants has a 1 in the firm-specific index of the product n. For example, if product 5 is the third product of firm 2, then the 1 will be in the 3^{rd} index position when calculating $\partial f_2(P)/\partial p_5$. If product n is not sold by firm i, the vector of constants is $\vec{0}$, and thus

$$\frac{\partial f_i(P)}{\partial p_n} = \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1} \left[\frac{\partial^2 Q_i}{\partial P_i \partial p_n}^T\right] \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1} Q_i - \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1} \left[\frac{\partial Q_i}{\partial p_n}\right]. \tag{23}$$

The matrix $\partial g(P)/\partial P$ can be decomposed in a similar manner:

$$\frac{\partial g(P)}{\partial P} = \begin{bmatrix} \frac{\partial g_1(P)}{\partial p_1} & \cdots & \frac{\partial g_1(P)}{\partial p_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_K(P)}{\partial p_1} & \cdots & \frac{\partial g_K(P)}{\partial p_N} \\ \\ 0 & \cdots & 0 \\ \downarrow & & \downarrow \end{bmatrix}, \tag{24}$$

where N is the number of products and K is the number of merging firms. Notice that $\partial g(P)/\partial P$ includes zeros for non-merging firms, because the merger does not create opportunity costs for these firms. In the case that product n is sold by a firm merging with firm i (this does not include firm i itself),

$$\begin{aligned} \frac{\partial g_i(P)}{\partial p_n} &= -\left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1}\left[\frac{\partial Q_j}{\partial P_i}^T\right]\begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{bmatrix} \\ &+ \left(\left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1}\left[\frac{\partial^2 Q_i}{\partial P_i \partial p_n}^T\right]\left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1}\left[\frac{\partial Q_j}{\partial P_i}^T\right] - \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1}\left[\frac{\partial^2 Q_j}{\partial P_i \partial p_n}^T\right]\right)(P_j - C_j), \end{aligned} \tag{25}$$

where Q_j, P_j, and C_j are vectors of the quantities, prices, and marginal costs respectively of products sold by firms merging with firm i, and the vector of 1s and 0s has a 1 in the merging firm's firm-specific index of the product n. For example, if product 5 is the third product of firm 2, and firm 2 is merging with firm 1, then the 1 will be in the 3^{rd} index position when calculating $\partial g_1(P)/\partial p_5$. It is an important distinction that – supposing there are more than two merging parties – the index j refers to all of the merging parties' products, excluding firm i's products. If product n is not sold by any firm merging with firm i (including a product sold by firm i),

$$\frac{\partial g_i(P)}{\partial p_n} = \left(\left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1}\left[\frac{\partial^2 Q_i}{\partial P_i \partial p_n}^T\right]\left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1}\left[\frac{\partial Q_j}{\partial P_i}^T\right] - \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1}\left[\frac{\partial^2 Q_j}{\partial P_i \partial p_n}^T\right]\right)(P_j - C_j). \tag{26}$$

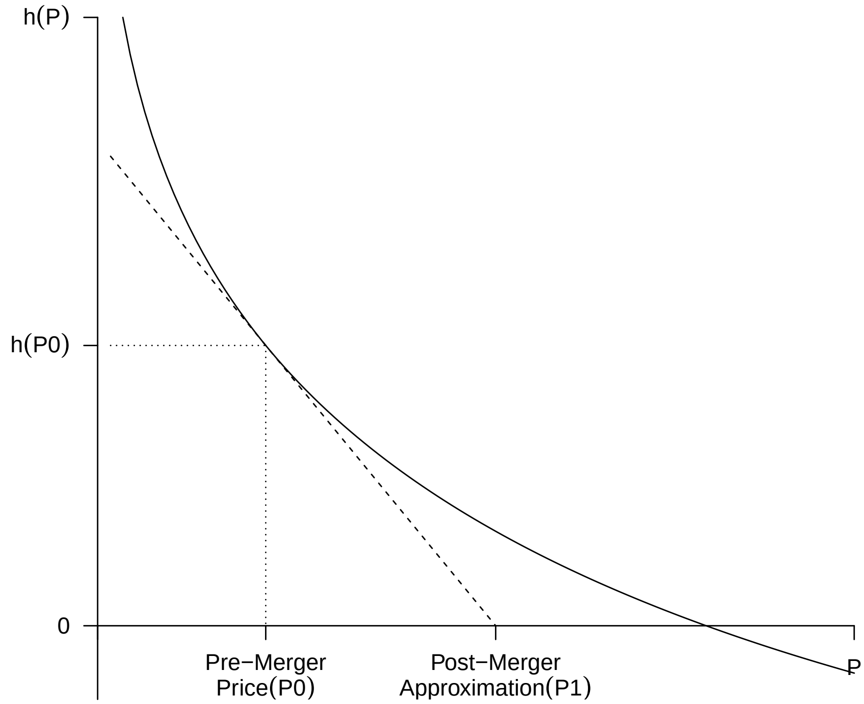

Figure 1: Simplified Version of Approximation

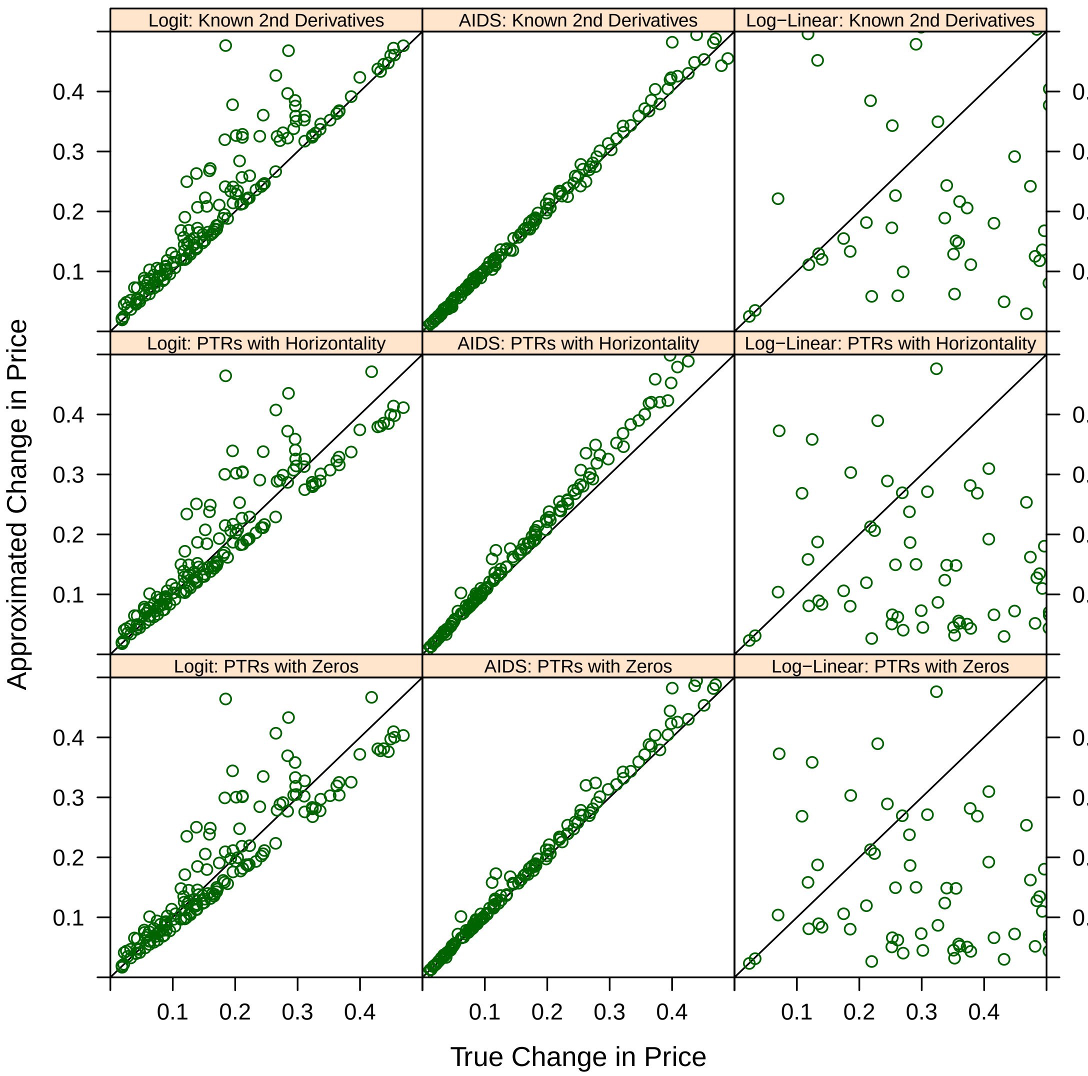

Figure 2: Prediction Error with Complete Information.

Notes: The figure provides scatter-plots of approximation against the true price effect for logit demand, the AIDS and log-linear demand. The case of linear demand is omitted because approximation is exact in that setting. Approximations are calculated alternately based on full knowledge of the second-order properties of demand in neighborhood of pre-merger equilibrium ("Known 2nd Derivatives"); based on full knowledge of pre-merger cost pass-through and the horizontality assumption ("PTRs with Horizontality"); and based on full knowledge of pre-merger cost pass-through and the assumption that derivatives of the form $\partial^2 Q_i/\partial P_j \partial P_k$ equal zero ("PTRs with Zeros").

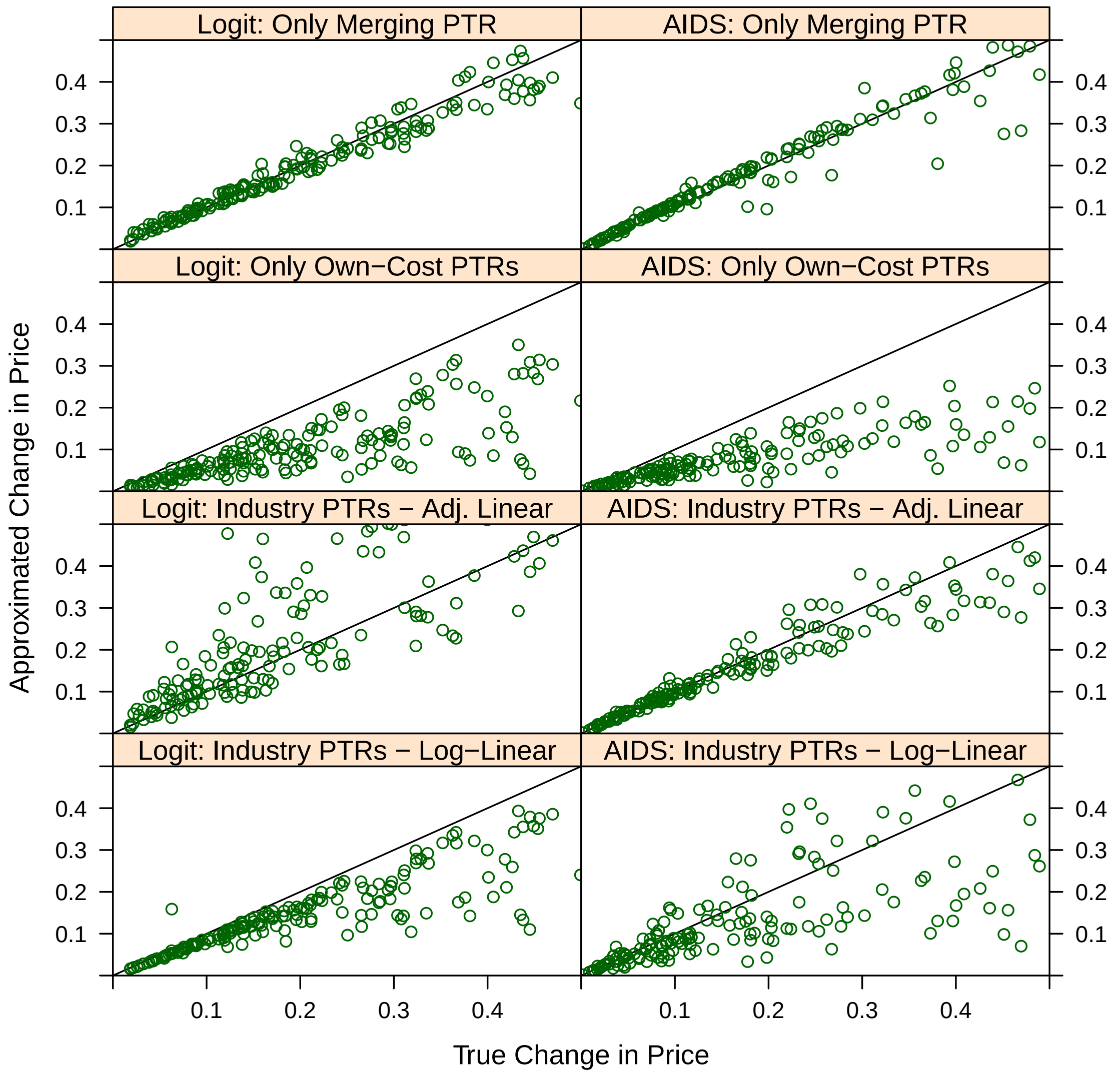

Figure 3: Prediction Error with Incomplete Information

Notes: The figure provides scatter-plots of approximation against the true price effect for logit demand and the AIDS. Five informational scenarios are considered: pre-merger cost pass-through that is available only for the merging firms ("Only Merging Firm PTRs"); pre-merger cost pass-through that is available only for own costs, i.e., the off-diagonal elements are unknown ("Only Own-Cross PTRs"); industry cost pass-through that is apportioned using the adjusted-linear method ("Industry PTRs – Adj.-Linear"); and industry cost pass-through that is apportioned using the log-linear method ("Industry PTRs – Log-Linear").

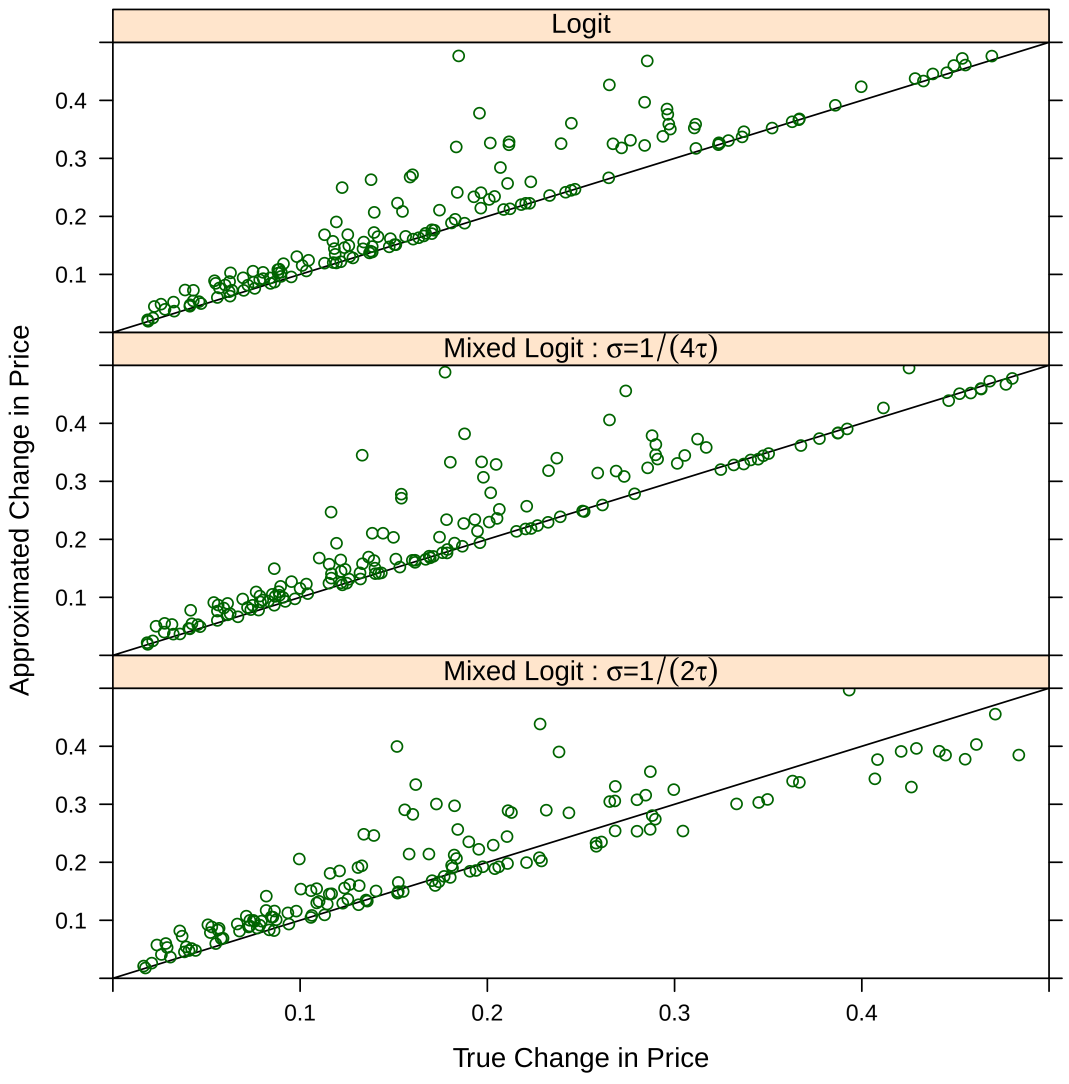

Figure 4: Prediction Error with Complete Information – Logit and Mixed Logit Demand

Notes: The figure provides scatter-plots of approximation against the true price effect for logit and mixed logit demand. The approximation is calculated based on full knowledge of the second-order properties of demand in neighborhood of pre-merger equilibrium.

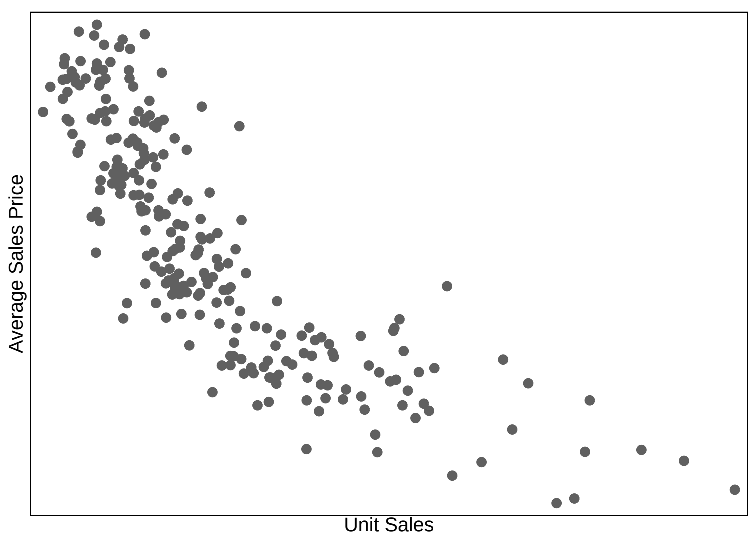

Figure 5: Prices and Unit Sales in a Representative City.

Notes: The figure provides a scatter-plot of the weekly average sales price and unit sales for one product in a representative city. To protect the confidentiality of the data, a small number of outliers have been omitted and both average sales price and unit sales have been scaled by an unspecified constant and perturbed additively by a uniformly distributed random variable.

Table 1: Summary Statistics

	Mean	St. Dev.	5th pctile	95th pctile
Characteristics of Firm 1				
Market share	0.37	0.15	0.10	0.58
Margin	0.46	0.17	0.22	0.75
Own-price elasticity	2.57	1.48	1.33	4.52
Characteristics of Firm 2				
Market share	0.31	0.15	0.08	0.54
Margin	0.44	0.21	0.17	0.87
Own-price elasticity	2.86	1.83	1.15	5.96
Consumer Substitution				
Diversion from 1 to 2	0.50	0.23	0.15	0.90
Diversion from 2 to 1	0.54	0.22	0.16	0.90
Merger Simulation Results Conditional on $\Delta P < 0.50$				
Logit demand	0.20	0.13	0.04	0.44
AIDS	0.17	0.13	0.02	0.43
Linear demand	0.20	0.13	0.04	0.46
Log-linear demand	0.27	0.13	0.06	0.47
Mixed Logit demand $(\frac{\alpha}{4})$	0.20	0.13	0.04	0.44
Mixed Logit demand $(\frac{\alpha}{2})$	0.19	0.12	0.03	0.44

Notes: Summary statistics are based on 300 randomly-drawn industries. The merger simulation results show changes in firm 1's price, conditional on that change being under 50%. With logit demand, 242 of the 300 randomly-drawn industries produce such a price change. With the AIDS, linear demand, and log-linear demand, 191, 190, and 45 industries produce such a price change, respectively.

Table 2: Absolute Prediction Error with Complete Information

	Logit Demand			AIDS		
	Mean	5th pctile	95th pctile	Mean	5th pctile	95th pctile
Known Second Derivatives	0.084	0.000	0.339	0.008	0.000	0.023
PTRs with Horizontality	0.082	0.001	0.319	0.026	0.001	0.081
PTRs with Zeros	0.083	0.002	0.320	0.013	0.000	0.046
	Linear Demand			Log-linear Demand		
	Mean	5th pctile	95th pctile	Mean	5th pctile	95th pctile
Known Second Derivatives	0	0	0	1.072	0.003	2.189
PTRs with Horizontality	0	0	0	0.193	0.006	0.395
PTRs with Zeros	0	0	0	0.193	0.006	0.395

Notes: The table provides summary statistics regarding the absolute prediction errors of approximation. Absolute prediction error is defined by the absolute value of the difference between approximation and the true price increase on product 1 arising due to a merger of products 1 and 2. Separate statistics are shown for logit demand, the AIDS, linear demand and log-linear demand. Observations are included only when the true price effect does not exceed 50 percent. The approximation is calculated alternately based on full knowledge of the second-order properties of demand in neighborhood of pre-merger equilibrium ("Known Second Derivatives"); based on full knowledge of pre-merger cost pass-through and the horizontality assumption ("PTRs with Horizontality"); and based on full knowledge of pre-merger cost pass-through and the assumption that derivatives of the form $\partial^2 Q_i/\partial P_j \partial P_k$ equal zero ("PTRs with Zeros").

Table 3: Approximation Versus Merger Simulation

Probability that Approximation Outperforms Simulation

	True Underlying Demand System			
	Logit	AIDS	Linear	Log-Linear
Logit Simulation	0%	94.8%	100%	50.3%
AIDS Simulation	79.1%	0%	100%	53.1%
Linear Simulation	90.3%	87.4%	.	49.7%

Mean Absolute Prediction Error

	True Underlying Demand System			
	Logit	AIDS	Linear	Log-Linear
Approximation	0.084	0.008	0	1.072
Logit Simulation	0	0.077	0.063	0.171
AIDS Simulation	0.181	0	0.050	0.186
Linear Simulation	0.266	0.152	0	0.164

Notes: The table provides (1) the frequency with which approximation outperforms merger simulations based on specific demand assumptions, and (2) the MAPE of approximation and these merger simulations. Separate statistics are shown for when true consumer preferences are characterized by logit demand, the AIDS, linear demand and log-linear demand.

Table 4: Absolute Prediction Errors with Incomplete Information

	Logit Demand			AIDS		
	Mean	5th pctile	95th pctile	Mean	5th pctile	95th pctile
Own Cost PTRs	0.096	0.013	0.260	0.094	0.007	0.288
Merging Firms' PTRs	0.019	0.001	0.058	0.018	0.002	0.071
Ind. PTRs – Adj.-Linear	0.266	0.003	1.107	0.025	0.001	0.106
Ind. PTRs – Log-Linear	0.052	0.005	0.169	0.067	0.001	0.246
	Linear Demand			**Log-linear Demand**		
	Mean	5th pctile	95th pctile	Mean	5th pctile	95th pctile
Own Cost PTRs	0.128	0.017	0.330	0.193	0.006	0.395
Merging Firms' PTRs	0.025	0.003	0.080	0.193	0.006	0.395
Ind. PTRs – Adj.-Linear	0	0	0	0.142	0.007	0.320
Ind. PTRs – Log-Linear	0.078	0.004	0.259	0.193	0.006	0.395

Notes: The table provides summary statistics regarding the absolute prediction errors of approximation. Separate statistics are shown for logit demand, the AIDS, linear demand and log-linear demand. Observations are included only when the true price effect does not exceed 50 percent. Four informational scenarios are considered: pre-merger cost pass-through that is available only for own costs, i.e., the off-diagonal elements are unknown ("Own Cost PTRs"); pre-merger cost pass-through that is available only for the merging firms ("Merging Firms' PTRs"); industry cost pass-through that is apportioned using the adjusted-linear method ("Ind. PTRs – Adj.-Linear"); and industry cost pass-through that is apportioned using the log-linear method ("Ind. PTRs – Log-Linear").

Table 5: Mean Absolute Prediction Error for Small Price Changes

	Logit	AIDS	Linear	Log-Linear
Known Second Derivatives	0.016	0.002	0	4.614
PTRs with Horizontality	0.010	0.005	0	0.091
PTRs with Zeros	0.010	0.004	0	0.091

Notes: The table provides the mean absolute prediction errors of approximation that arise when the true price effect does not exceed 10%. Separate statistics are shown for logit demand, the AIDS, linear demand and log-linear demand. Observations are included only when the true price effect does not exceed 50 percent. The approximation is calculated alternately based on full knowledge of the second-order properties of demand in neighborhood of pre-merger equilibrium ("Known Second Derivatives"); based on full knowledge of pre-merger cost pass-through and the horizontality assumption ("PTRs with Horizontality"); and based on full knowledge of pre-merger cost pass-through and the assumption that derivatives of the form $\partial^2 Q_i/\partial P_j \partial P_k$ equal zero ("PTRs with Zeros").

Table 6: Mean Absolute Prediction Error with Alternative FOCs

	Logit	AIDS	Linear	Log-Linear
Known Second Derivatives				
Baseline FOC	0.084	0.008	0	1.072
Alternative FOC	0.061	0.103	0	0.150
PTRs with Horizontality				
Baseline FOC	0.082	0.026	0	0.193
Alternative FOC	0.084	0.054	0	0.176

Notes: The table provides the mean absolute prediction errors that arise with both the baseline first order conditions and with the alternative first order conditions. Separate statistics are shown for logit demand, the AIDS, linear demand and log-linear demand. Observations are included only when the true price effect does not exceed 50 percent. The approximation is calculated alternately based on full knowledge of the second-order properties of demand in neighborhood of pre-merger equilibrium ("Known Second Derivatives") and based on full knowledge of pre-merger cost pass-through with the horizontality assumption("PTRs with Horizontality").

Table 7: Results from OLS Regressions

Panel A: Demand Elasticity Estimates

	Product 1	Product 2	Product 3	Product 4
Product 1	-4.22	0.19	0.09	0.55
Product 2	1.96	-1.50	0.35	1.78
Product 3	1.16	0.34	-1.59	0.65
Product 4	1.88	0.81	0.37	-2.17

Panel B: Cost Pass-Through Estimates

	Product 1	Product 2	Product 3	Product 4
Product 1	0.82	0.17	-0.07	0.31
Product 2	0.64	1.32	0.08	1.65
Product 3	1.14	0.52	2.54	3.75
Product 4	0.35	0.29	0.03	1.36

Notes: The elasticities and cost pass-through rates are inferred from OLS regression coefficients. In Panel A, the top number in the second column is the elasticity of demand for product 1 with respect to the price of product 2, and the remaining numbers are calculated accordingly. In Panel B, the top number in the second column is the pass-through rate of product 1 with respect to the costs of product 2, and again the remaining numbers are calculated accordingly.

Table 8: Approximation Results for Merger of Products 1 and 2

	Product 1	Product 2	Product 3	Product 4
Known 2nd Derivatives				
Baseline FOC	36.5%	41.1%	27.3%	21.1%
Alternative FOC	28.5%	31.0%	21.2%	16.2%
PTRs with Horizontality				
Baseline FOC	57.0%	51.1%	40.9%	29.7%
Alternative FOC	37.3%	32.8%	26.7%	19.3%
PTRs with Zeros				
Baseline FOC	41.1%	36.5%	29.4%	21.3%
Alternative FOC	29.8%	26.0%	21.2%	15.3%
Simple Approximation	26.0%	20.3%	18.2%	12.8%

Notes: Approximation is based on the estimated demand derivatives and either uses these derivatives directly ("Known 2nd Derivatives" or uses the implied cost pass-through rate matrix.