Labor Market Structure and Offshoring

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Abstract

In this paper, I propose a two-country, two-sector model where a firm’s decision on offshoring depends on labor market rigidities, due to which, firms need to bear not only wages but also additional costs from the labor market. In this model, firms endogenously choose their organizational form considering their productivity level and fixed organizational costs. The labor market cost generated by search frictions plays a key role in changing the variable benefits of each choice, and thus works as a key determinant in the process of selecting the organizational form for offshoring. I found four different types of equilibria depending on relative levels of two labor market costs (domestic, foreign) and the price of the intermediate input. In all equilibria, a relative rise in the domestic labor market cost increases the share of offshoring firms, while decreasing domestic integration. Furthermore, an economy with offshoring has a higher welfare level and a lower unemployment rate than autarky.

1 Introduction

Rapidly increasing international disintegration in production has gained attention in many studies. When a firm decides to move part of its production process or tasks abroad, it has two choices. It can forge an arm’s-length relationship with other intermediate goods suppliers; or it can opt for vertical integration by setting up affiliates overseas. The former is called ‘Outsourcing’ and the latter, ‘Foreign Direct Investment (FDI)’.

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As some firms choose to outsource tasks while others choose FDI, a question naturally arises about the key determinants in each decision. Although many studies have attempted to answer this question, the labor market structure has received little attention, as most studies in the literature have assumed a frictionless labor market. Considering the fact that saving variable costs is the fundamental motivation of firms choosing to offshore, models with a frictionless labor market may be missing an important factor that affects a firm’s decision.

In this paper, I consider the labor market structure to be a key determinant in a firm’s decision to offshore. In the model I propose, two sectors exist, in one of which the labor market is under search and matching frictions. Specifically, the labor market structure is based on Helpman and Itskhoki (2010), where homogeneous workers search firms for jobs, while firms are heterogeneous across their skill levels. In this setting, it is costly for a firm to hire or fire workers, and thus firms consider not only the wage level, but also the labor market cost generated by the frictions. I show that a country with a relatively high labor market cost will have more firms choosing to offshore; and firms with a high productivity level will choose offshoring, while less productive firms remain in the domestic market.

When I analyze the impact of allowing offshoring on the economy, I find that the equilibrium with offshoring gives a weakly higher welfare level. Firms facing a higher labor market cost in the domestic labor market in autarky try to reduce the production to keep the profit level unchanged. Allowing offshoring, however, offsets the decrease in the quantity level by structural changes in the economy, as more firms choose to offshore. In an analysis of the total hiring and unemployment rate, I find that even though the total hiring in autarky is higher, the unemployment rate is also higher. These seemingly contradictory results have originated from the fact that the unemployment rate depends on not only the total jobs available, but also the number of workers searching for jobs in the labor market. When offshoring is available, workers realize that the number of jobs decreases; and choose the other sector, where labor market frictions do not exist.

The proposed model lies in an intersection of two strands of literatures. One of them focuses on the influence of the labor market structure on issues related to the international trade. Since the seminal work of Davidson, Martin, and Matusz (1999), there have been many studies on this topic and now there exists well-developed literature on search-induced unemployment in different trade environments. Among this literature, two studies from Mitra and Ranjan (2010, 2012) are closely related to the model. In both studies, the labor market is under search and matching frictions. Mitra and
Ranjan (2010) analyze the impact of offshoring on the unemployment rate. Interestingly, they find the impact of offshoring on the unemployment rate depends on inter-sectoral labor mobility. Under imperfect inter-sectoral labor mobility, unemployment may increase in the offshoring sector, while it may decrease in the non-offshoring sector. When the economy is under perfect inter-sectoral labor mobility, however, the economy-wide unemployment decreases unambiguously. Mitra and Ranjan (2012) analyze the role of offshoring in a model with fair wage consideration. In their model, workers are heterogeneous in terms of skill level, while firms are homogeneous. When the fair wage constraint is binding and a firm hires two types of labor, a distortion can arise in the production process. Mitra and Ranjan (2012) show that fairness consideration can motivate offshoring, as it could be a way of resolving the distortion. But neither study distinguishes FDI from Outsourcing as they do not focus on the firms’ choice between the two.

Davidson, Matusz, and Shevchenko (2008) also introduce search and matching frictions into the labor market and analyze the impact of Outsourcing in high-tech jobs on low-skilled workers’ wages. In their model, there exist two types of labor, high-skilled and low-skilled, and firms endogenously choose their technology level. Thus, there exists heterogeneity on both sides of the labor market. Interestingly, Davidson, Matusz, and Shevchenko (2008) find that under certain conditions, Outsourcing can increase the wage level of low-skilled labor in the long run. In the long run equilibrium, there will be more entry of firms and if the new firms select the low-level technology, it will become easier for low-skilled labor to find employment and their wage level can increase. In the short run, however, both types of labor are worse off, as high-skilled labor now faces fewer job opportunities and lowered wages, while low-skilled labor encounters greater competition for jobs.

Although many papers in the literature on trade issues related to labor market rigidities use novel models, several recent papers have attempted to introduce search-induced unemployment into standard trade models.\footnote{See Cuñat and Melitz (2010, 2012), Davidson et al. (2012), and Felbermayr et al. (2011).} In particular, Helpman and Itskoki (2010) introduce Diamond-Mortensen-Pissarides-type search and matching frictions into the labor market. They use a two-sector model where one sector is monopolistically competitive. Firms become heterogeneous when they enter the market, and some choose to export, while others decide whether to exit or remain in the domestic market. Helpman and Itskoki (2010) show that labor market flexibility can be a
source of comparative advantage in trade, i.e., a country with lowered labor market inefficiencies can get welfare improvement, while its trade partner becomes worse off.

Another strand of literature this paper relies on tries to answer the question of why some firms choose to outsource, while others choose FDI. In some studies, incomplete contracts play a central role. Antras (2003) focuses on the incomplete contract between a firm and a supplier. With Outsourcing, a firm can have efficiency gains from having a specialized supplier. However, due to incomplete contracts, hold-up problems occur, and both sides tend to underinvest compared to the optimal level. Similarly, Grossman and Helpman (2002, 2003) have modeled an economy where firms are placed in a trade-off between extra governance costs in FDI and the incomplete contract problem in Outsourcing.

In other studies, the heterogeneity of firms’ productivity level induces different sorting across firms. In Antras and Helpman (2004), firms are heterogeneous in terms of productivity. In this model, they make two choices. In choosing location between North and South, they face a trade-off between the low fixed cost at home and low variable costs in a foreign country. On the other hand, in choosing between vertical integration and Outsourcing, firms face a trade-off between an ownership advantage in vertical integration and better incentives in Outsourcing. These trade-offs, together with the incomplete contract problem in Outsourcing, drive firms to choose certain organizational forms depending on their realized productivity levels. In Antras and Helpman’s (2004) model, the most productive firms choose FDI, while less productive firms choose Outsourcing.

While many studies focus on the role of incomplete contracts and productivity levels, Chen (2011) places informational asymmetry at the center of the analysis. In this model, firms face an adverse selection problem in choosing an intermediate goods supplier, while they face an inefficient monitoring problem when they choose FDI. This model is helpful in explaining why FDI is heavily concentrated in capital-intensive industries, in which monitoring costs are significantly lower than in other industries, which alleviates the inefficient monitoring problem in FDI.

In this paper, I extend Helpman and Itskhoki’s (2010) model. While following the basic setup of their model, I allow firms to have offshoring choices, which include both FDI and Outsourcing. Compared to previous studies, firms have a different source of trade-off in choosing offshoring. They can lower their production costs by selecting a foreign country with a low labor market cost, even though it would incur an additional fixed organizational cost. Thus, the labor market cost directly affects a firm’s decision to
offshore.

The remainder of this paper is organized as follows. In Section 2, I develop the model and in Section 3, I calculate the profit levels of the three different choices. In Section 4, I derive four different types of equilibria and analyze them in Section 5. Finally, in Section 6, I offer a summary of the analysis with concluding remarks. The proof of the main results are in Appendices A and B.

2 The Model

In this model, there exist two sectors. One sector produces a homogeneous good, while the other produces differentiated goods. In the homogeneous-good sector, there are no labor market frictions and all firms produce domestically. The price of the homogeneous good is normalized to one and it serves as a numeraire. In the differentiated-goods sector, search and matching frictions exist in the labor market. Firms in this sector can produce domestically by using domestic labor, but they can also choose to offshore by paying the fixed organizational cost of FDI or Outsourcing. As in many previous studies, I assume that the fixed organizational cost of FDI is higher than that of Outsourcing.

2.1 Preferences

A representative household gets utility from consuming \( q_0 \) homogeneous goods and a continuum of differentiated goods,

\[
Q = \left[ \int_{i \in I} q(i)^\beta di \right]^{\frac{1}{\beta}}, \quad 0 < \beta < 1,
\]

where \( q(i) \) denotes consumption of variety \( i \), \( I \) denotes the set of varieties, and \( \beta \) is a measure of the elasticity of substitution between varieties. The total utility from consuming them is defined as

\[
U = q_0 + \frac{1}{\varsigma}Q^\varsigma, \quad 0 < \varsigma < \beta.
\]

The restriction \( \varsigma < \beta \) implies that differentiated goods are better complements to each other than the homogeneous good.

It is well known that CES preferences yield the following constant elasticity demands:

\[
q(i) = Q \left( \frac{p(i)}{P} \right)^{-\frac{1}{1-\beta}}.
\]
And the price index for $Q$ is defined as

$$P = \left[ \int p(i)^{-\frac{\beta}{1-\beta}} di \right]^{-\frac{1-\beta}{\beta}}. $$

With total spending $E$, the representative household maximizes its utility by choosing

$$Q = P^{-\frac{1}{1-\xi}}; \quad (3)$$
$$q_0 = E - P^{-\frac{1}{1-\xi}}. \quad (4)$$

### 2.2 Technology

In the homogeneous-good sector, firms make one unit of good by using one unit of labor. As the market is competitive, the wage is equal to the price of the homogeneous good, one.

Following Melitz (2003), the market in the differentiated-goods sector is monopolistically competitive, and each firm needs to pay the entry cost, $f_e$, to enter the market. After paying the entry cost, firms draw their productivity level, $\theta$, from a known common distribution. The production function of a firm with $\theta$ is given by

$$q(\theta) = \theta h, \quad (5)$$

where $h$ is the number of workers the firm hires. Firms can choose to offshore by hiring foreign labor (FDI) or buying intermediate inputs from intermediate-goods suppliers (Outsourcing) to substitute $h$. In order to produce, firms also have to pay a fixed production cost, $f_d$.

Using Equation (2) and Equation (5), we can calculate the price and revenue of a firm with productivity level $\theta$ as a function of $Q$ and $h$:

$$p(\theta) = (\theta h)^{-(1-\beta)} Q^{-(\beta-\xi)};$$
$$R(\theta) = (\theta h)^{\beta} Q^{-(\beta-\xi)}. \quad (6)$$

### 2.3 The Labor Market

In this economy, there is a continuum of identical households of measure one. As each household has $L$ units of workers, the total labor endowment of this economy is $L$. Workers can choose to work either in the homogeneous-good sector or in the differentiated-goods sector. A household allocates its labor into two sectors. Out of $L$ workers, it allocates $N$ to the differentiated-good sector and $L - N$ to the homogeneous-good sector. As the wage in the
homogeneous-good sector is equal to one, the average wage from working in
the differentiated-goods sector should be one.

In the differentiated-goods sector, labor market frictions exist, and as a
result of which, firms face a labor market cost, \( b \), whenever it hires a worker.
In this setting, firms consider not only the wage level, but also the labor
market cost when they make a decision on the size of their labor.

The labor market cost can be decomposed into hiring and firing costs.
When firms hire workers, they have to pay costs in opening vacancies. As
it is not possible to have immediate matchings with potential workers, an
inefficiency in the matching process also incurs costs.

When a vacancy is filled, firms have to fire a fraction of workers they
hired, as they are assumed to realize whether workers are suitable or not for
the jobs they are matched, once they are hired. Thus, firms need to hire
more than an optimal number of employees, and they have to bear other
costs related to the firing process.

Following the approach that is proposed in Helpman and Itskhoki (2010),
I assume that the hiring cost \( b_h \) is a function of labor market tightness,

\[
x = \frac{H}{(1 - \sigma)N},
\]

where \( H \) is the total hiring in the differentiated-goods sector and \( \sigma \) is the
fraction of job openings which need to be fired. As firms anticipate to fire a
fraction \( \sigma \) of total matches, they hire \( \frac{H}{1 - \sigma} \) to have \( H \) workers.

Hiring costs in this economy are defined as

\[
b_h = ax^\delta, \quad a > 1 \text{ and } \delta > 0,
\]

where \( a \) represents frictions in the labor market during the hiring process.
Higher costs of opening a vacancy or lower efficiency of matching technology
will give us a higher \( a \).

When firms fire a worker, they bear firing costs, \( \psi \). Under the assumption
that I made regarding the firing process, the total labor market cost becomes

\[
b = \frac{1}{1 - \sigma} (b_h + \sigma \psi)
\]

and the economy-wide unemployment rate is defined as

\[
u = \frac{N - H}{L}.
\]

Following Stole and Zwieble (1996a, 1996b), firms engage in a generalized
Nash Bargaining procedure over the revenue they create with matches. For
simplicity, I assume equal bargaining power for a firm and a worker. As a result, the equilibrium wage as a function of employment is the solution of the following equation:

$$\frac{\partial}{\partial h} (R(h) - w(h) h) = w(h), \quad (8)$$

where $R(h)$ is the revenue and $h$ is the number of workers. As an additional worker affects the overall wage level, Equation (8) yields a differential equation of $w$. With zero outside option for a worker,\(^2\) the bargaining procedure makes the marginal gains from the additional worker equal to the marginal gains to the worker.

3 Choices of Firms

3.1 Domestic Integration (DI)

Firms in the differentiated-goods sector must pay a fixed cost, $f_d$, regardless of their organizational choices when they decide not to exit. This fixed cost may include costs associated with headquarter services, such as accounting, finance operations, and R&D.

By using Equation (8), we can calculate equilibrium wages, $w(\theta)$, as a function of employment:

$$\frac{\partial}{\partial h} [R(\theta) - w(\theta) h] = w(\theta). \quad (9)$$

By solving this, we get

$$w(\theta) = \frac{\beta}{1 + \beta} \frac{R(\theta)}{h}. \quad (10)$$

Thus, a firm loses $\frac{\beta}{1 + \beta} R(\theta)$ after the wage bargaining and faces

$$\max \frac{1}{1 + \beta} R(\theta) - bh - f_d. \quad (11)$$

And the optimal level of hiring becomes

$$h_d^*(\theta) = \left[ \frac{\beta}{b(1 + \beta)} \right]^{\frac{1}{1-\beta}} Q^{\frac{\beta + \gamma}{1-\beta}} \theta^{\frac{\beta}{1-\beta}}. \quad (12)$$

\(^2\)Once a worker enters the differentiated-goods sector, s/he cannot go back to the homogeneous-good sector.
By plugging this into the wage equation, we get the wage and profit level from choosing DI:3

\[ w(\theta) = b, \quad \forall \theta, \]

\[ \pi^*_d(\theta) = (1 - \beta) \left( \frac{1}{1 + \beta} \right)^{\frac{1}{1+\beta}} \left( \frac{\beta}{b} \right)^{\frac{\beta}{1+\beta}} Q^{-\frac{\beta}{1+\beta}} \theta^{\frac{1}{1+\beta}} - f_d \]

\[ \equiv A_d \Theta - f_d, \tag{13} \]

where \( A_d = (1 - \beta) \left( \frac{1}{1+\beta} \right)^{\frac{1}{1+\beta}} \left( \frac{\beta}{b} \right)^{\frac{\beta}{1+\beta}} Q^{-\frac{\beta}{1+\beta}} \) and \( \Theta = \theta^{\frac{\beta}{1+\beta}} \) is a different measure of productivity.

### 3.2 FDI

Instead of hiring domestic labor, a firm can hire foreign labor by engaging in FDI. Similar to the domestic labor market, there also exist labor market frictions in the foreign labor market, and firms have to bear \( b_f \) whenever they hire workers.4

I assume that a firm engaging in FDI faces the same bargaining procedure as in Equation (9),

\[ \frac{\partial}{\partial h} [R(\theta) - w_f(\theta) h] = w_f(\theta), \]

and the resulting wage level becomes

\[ w_f(\theta) = \frac{\beta}{1 + \beta} \frac{R(\theta)}{h}. \]

Therefore, the result of the wage bargaining is the same: a firm pays \( \frac{\beta}{1 + \beta} R(\theta) \) for the total wage. A firm solves the same problem as in DI with the foreign labor market cost, \( b_f \), but with an additional fixed organizational cost of FDI, \( f_f \), which is assumed to be greater than that of Outsourcing, \( f_u \). Thus, a firm faces

\[ \max \frac{1}{1 + \beta} R(\theta) - b_f h - f_d - f_f \]

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3 In more general settings where the bargaining power of two parties are not equal, the wage level is proportional to the labor market cost. Specifically, with a relative bargaining power of firms \( \mu \), we get \( w(\theta) = b/\mu \).

4 Same as in the domestic labor market, \( b_f \) is generated from search and matching frictions in the foreign labor market. For example, we can think of a labor market structure in a foreign country to be the same as a domestic labor market with different parameters, 

\[ b_f = \frac{1}{1-\sigma_f} (\alpha_f x_f^\gamma + \sigma_f \psi_f). \]
and by solving this, we can derive the optimal hiring level,

\[ h^*_f(\theta) = \left[ \frac{\beta}{b_f (1 + \beta)} \right]^{1/\beta} Q^{-{\beta-\varsigma \over 1-\beta} \theta^{1-\beta}}. \]

With the optimal hiring level, we can derive the following wage and profit level from choosing FDI:

\[ w_f(\theta) = b_f, \quad \forall \theta, \]

\[ \pi^*_f(\theta) = (1 - \beta) \left( \frac{1}{1 + \beta} \right)^{1/\beta} \left( \frac{\beta}{b_f} \right)^{1/\beta} Q^{-{\beta-\varsigma \over 1-\beta} \theta^{1-\beta}} - f_d - f_f \]

\[ \equiv A_f \Theta - f_d - f_f. \quad (14) \]

### 3.3 Outsourcing

Now suppose that a firm can buy intermediate goods from foreign suppliers to substitute for labor.\(^5\) The intermediate goods market is assumed to be perfectly competitive, and suppliers in the foreign country are assumed to have relative strength in dealing with the foreign labor market compared to firms from the home country, as they have more information about the labor market structure. Thus, they face a lower labor market cost compared to firms engaging in FDI, i.e.,

\[ b_u < b_f, \]

where \( b_u \) denotes the labor market cost that suppliers face.

With the competitive price of an intermediate good, \( p'_u \), suppliers also has to bargain with theirs labor forces,

\[ \frac{\partial}{\partial h} \left[ p'_u h - w_u h \right] = w_u, \]

and by solving this equation, we get

\[ w_u = \frac{1}{2} p'_u. \]

Thus, after the wage bargaining, a supplier maximizes the following problem:

\[ \max \frac{1}{2} p'_u h - b_u h. \]

\(^5\)For simplicity, domestic Outsourcing is assumed to be dominated by domestic integration. Under an assumption that domestic firms are exposed to the same information about the domestic labor market, this is satisfied with the condition, \( p^*/b = (1 + \beta)^{1/\beta} \), where \( p^* \) denotes the unit price of a domestic intermediate good.
And as the market is perfectly competitive, the price of an intermediate good should be at a level where the supplier gets zero profit, i.e.,

\[ p_u' = 2b_u. \]

By choosing Outsourcing, a firm can keep all of its revenue, as it does not have to deal with the labor market anymore. However, it is assumed that it needs to pay the adjustment cost, \( c \), for every intermediate good to make it fit into its production process. Thus, the unit price of the intermediate good, \( p_u \), becomes \( p_u' + c \). So the problem that a firm with Outsourcing faces is

\[ \max R(\theta) - p_u h - f_d - f_u \]

and the resulting level of hiring and profit becomes

\[
\begin{align*}
    h_u^*(\theta) &= \left( \frac{\beta}{p_u} \right)^{\frac{1}{1-\beta}} Q^{-\frac{\beta-\varsigma}{1-\beta}} \frac{\beta}{1-\beta} \\
    \pi_u^*(\theta) &= (1 - \beta) \left( \frac{\beta}{p_u} \right)^{\frac{\beta}{1-\beta}} Q^{-\frac{\beta-\varsigma}{1-\beta}} \frac{\beta}{1-\beta} - f_d - f_u \\
    &\equiv A_u \Theta - f_d - f_u.
\end{align*}
\]

\[ (15) \]

4 Equilibrium

As firms are different in terms of productivity and three choices come with different levels of fixed costs, we can observe different organizational types across firms in equilibrium. With three choices, we could have seven different cases. Although it is theoretically possible to have equilibrium with only FDI or only Outsourcing, I consider them as extreme cases and will discuss four other equilibria, which include at least Domestic Integration. From here on, I refer to these equilibria as Type 1, Type 2, Type 3, or Type 4 equilibrium: Type 1 has only DI and Type 2 has DI and Outsourcing. Type 3 is with DI and FDI and Type 4 includes all three choices.

4.1 Two Possible Paths

As a thought experiment, we now consider what would happen if the home labor market cost starts to increase from a very low level. When it is very low compared to \( p_u \) and \( b_f \), firms located at home have no incentive to engage in offshoring. Thus, we would have Type 1 equilibrium, where only DI exists. As \( b \) increases, while \( p_u \) and \( b_f \) remain fixed, the benefits from choosing
offshoring increase. When the gains from offshoring are sufficient to cover its fixed organizational cost, firms would change their organizational form into Outsourcing or FDI.

At this point, it is unclear which type of offshoring would be selected in the new equilibrium without further analysis. But that choice depends on the relative values of $p_u$ and $b_f$. If the price of the intermediate good is relatively low compared to the foreign labor market cost, it is reasonable to expect that the equilibrium will change to Type 2 because Outsourcing involves lower fixed organizational cost. If that is not the case, the equilibrium will change to Type 3. This naturally gives rise to the following questions: What would be the possible equilibrium paths as $b$ changes? And what are the conditions for each path? It turns out that two different paths exist in this model, depending on certain parameter values.

The first path is from Type 1 to Type 2. This happens when the gains from FDI are not sufficient to cover the additional fixed cost, $f_f - f_u$. In this path, as $b$ increases, firms choose Outsourcing and then it becomes impossible to have FDI in equilibrium. This is because the relative variable benefits from having Outsourcing over FDI do not rely on the domestic labor market cost. If we derive the relative ratio between two coefficients, we get

$$\frac{A_f}{A_u} = (1 + \beta)^{-\frac{1}{1-\beta}} \left( \frac{p_u}{b_f} \right)^{\frac{\beta}{1-\beta}},$$

which is independent of $b$. This means that the relative gains from having Outsourcing over FDI do not change as $b$ increases. If $A_f/A_u$ is smaller or equal to one, the gains from FDI would never become larger than that of Outsourcing due to higher fixed organizational cost, and the equilibrium would stay in Type 2 as $b$ increases.

Instead, if $A_f/A_u$ is greater than one, we would have another path which includes FDI. In this path, FDI comes in first; and later we can see both choices in the equilibrium. In other words, the equilibrium changes from Type 1 to Type 3 and then moves on to Type 4. As $b$ increases, FDI becomes more attractive for the most productive firms, while other firms stick to DI due to their inability to cover the fixed organizational cost of FDI. When $b$ further increases, some firms begin to change to Outsourcing as the relative variable benefits from DI further shrink. In this equilibrium, firms with a high-productivity level choose FDI and firms with a modest-productivity level choose Outsourcing, while firms with a low-productivity level stay with DI. And as the condition of two paths cannot be satisfied at the same time, we can derive the following results.
Lemma 4.1. In any path, Type 2 equilibrium cannot coexist with Type 3 or Type 4 equilibrium.

Proof. This is clear if we compare the equilibrium conditions of each type. To have Type 2 equilibrium, we need the condition, 
\[(1 + \beta)^{-\frac{1}{1-\beta}} \left( \frac{p_u}{b_f} \right)^{\frac{\beta}{1-\beta}} \leq 1.\]
But to have Type 3 or Type 4 equilibrium, we must have the condition,
\[(1 + \beta)^{-\frac{1}{1-\beta}} \left( \frac{p_u}{b_f} \right)^{\frac{\beta}{1-\beta}} > 1.\]
These two conditions cannot be satisfied at the same time. \qed

Proposition 4.1. If the domestic labor market cost keeps increasing from a very low level, while keeping \(p_u\) and \(b_f\) fixed, the equilibrium would change as

1) Type 1 \(\rightarrow\) Type 2, if \((1 + \beta)^{-\frac{1}{1-\beta}} \left( \frac{p_u}{b_f} \right)^{\frac{\beta}{1-\beta}} \leq 1\)

2) Type 1 \(\rightarrow\) Type 3 \(\rightarrow\) Type 4, if \((1 + \beta)^{-\frac{1}{1-\beta}} \left( \frac{p_u}{b_f} \right)^{\frac{\beta}{1-\beta}} > 1\)

4.2 Equilibrium Conditions

To characterize the equilibrium in this model, let’s find the equilibrium conditions. For each type of the equilibrium, there exists a zero profit cutoff productivity, \(\Theta_d\), and (possibly) other productivity cutoffs that are defined by the intersections of different profit equations:

\[
\begin{align*}
A_d \Theta_d &= f_d, \quad \text{(16a)} \\
A_d \Theta_i - f_d &= A_i \Theta_i - f_d - f_i, \quad i \in \{u, f\}, \quad \text{(16b)} \\
A_f \Theta_f - f_d - f_f &= A_u \Theta_u - f_d - f_u. \quad \text{(16c)}
\end{align*}
\]

Equation (16a) set the zero-profit cutoff and Equation (16b) pin down the productivity level which gives us the same profit from DI and FDI (or Outsourcing). Equation (16c) only appears in Type 4 equilibrium, where profit equations of FDI and Outsourcing intersect each other.

As firms draw their productivity level after they pay the fixed entry cost, the free entry condition equalizes the expected profit to the fixed entry cost:

\[f_e = E[\pi(\Theta)].\]

The total output in the differentiated-goods sector is derived as

\[Q = M^{\frac{1}{\beta}} \left[ \int q(\Theta)^{\beta} dG(\Theta) \right]^{\frac{1}{\beta}},\]

\(^6\)In Appendix A, I list the sufficient conditions for each type.
where $M$ is the measure of firms in the sector and $q(\Theta)$ is the output of a firm with productivity level $\Theta$.

In the labor market, each household divides its labor endowment into two sectors, and this process equalizes expected wages in both sectors, i.e.,

$$x(1 - \sigma)\bar{w} = 1.$$ 

The LHS simply indicates an expected wage from choosing the differentiated-goods sector. A worker expects to receive an average wage, $\bar{w}$, when s/he is hired, $x$, and is not fired, $1 - \sigma$. And the RHS is the wage level that a worker could get from the homogeneous-good sector.

Total hiring in the differentiated-goods sector is defined as

$$H = M \times E[h(\Theta)]$$

and the average wage in the sector can be calculated by

$$\bar{w} = \frac{M \times E[w(\Theta)h(\Theta)]}{H}.$$ 

Without an additional assumption on the distribution of $\Theta$, these conditions will implicitly determine $M$, $N$, $H$, $Q$, and cutoff productivity levels.

### 4.3 Equilibrium with DI, Outsourcing, and FDI (Type 4)

In this section, I will characterize equilibrium conditions and firms’ decision under Type 4 equilibrium. As the equilibrium conditions depend on the distribution of $\Theta$, I assume that $\Theta$ follows the Pareto distribution with shape parameter $\alpha$ and minimum value $\Theta_m$ for the rest of the discussion. I list the equilibrium conditions of the other three types in Appendix A.

To have Type 4 equilibrium, we should have sufficiently low foreign labor market costs so that firms can cover the high fixed organizational cost in FDI. The unit price of an intermediate good should also be low enough to cover the fixed organizational cost of Outsourcing, but it should not be too low, as it would drive FDI out of the equilibrium.

**Lemma 4.2.** Sufficient conditions to have Type 4 equilibrium are

1) $(1 + \beta)^{\frac{1}{1 - \beta}} \left( \frac{b}{p_u} \right)^{\frac{1}{1 - \beta}} > 1$ \quad $1 + \beta^{-\frac{1}{1 - \beta}} \left( \frac{p_u}{b_f} \right)^{\frac{1}{1 - \beta}} < 1$,

2) $(1 + \beta)^{\frac{1}{1 - \beta}} \left( \frac{b}{p_u} \right)^{\frac{1}{1 - \beta}} - 1 < \frac{f_d}{f_u}$ \quad $1 + \beta^{-\frac{1}{1 - \beta}} \left( \frac{p_u}{b_f} \right)^{\frac{1}{1 - \beta}} < \left( \frac{f_u - f_d}{f_u} \right) \left[ (1 + \beta)^{\frac{1}{1 - \beta}} \left( \frac{b}{p_u} \right)^{\frac{1}{1 - \beta}} - 1 \right]$.
Proof. As \( f_f > f_u > 0 \), we should have \( A_f > A_u > A_d \) to have Type 4 equilibrium. The first condition comes from \( A_d < A_u < A_f \). The second condition is derived from \( \Theta_d < \Theta_u < \Theta_f \), where \( \Theta_d, \Theta_u, \) and \( \Theta_f \) are defined by

\[
\begin{align*}
A_d \Theta_d - f_d & \equiv 0 \\
A_u \Theta_u - f_u & \equiv A_d \Theta_u \\
A_f \Theta_f - f_f & \equiv A_u \Theta_f - f_u.
\end{align*}
\]

The first condition of this Lemma states that the variable benefits from FDI should be greatest, while the second condition restricts the variable gains from FDI, so that we can have both Outsourcing and DI in the equilibrium. If conditions in Lemma 4.2 are satisfied, we can find the following decision rule for firms in Type 4 equilibrium.

**Proposition 4.2.** A firm with \( \Theta \) will

\[
\begin{align*}
\text{exit} & \quad \text{if } \Theta < \Theta_d \\
\text{choose DI} & \quad \text{if } \Theta_d < \Theta < \Theta_u \\
\text{choose Outsourcing} & \quad \text{if } \Theta_u < \Theta < \Theta_f \\
\text{choose FDI} & \quad \text{if } \Theta_f < \Theta.
\end{align*}
\]

This proposition tells us that a firm makes a decision over its structure by comparing its realized productivity level with three productivity cutoffs. It is interesting to note that this is similar to the segregation result of the headquarter intensive sector in Antras and Helpman (2004). Although both papers share the same assumption on the relative sizes of fixed organizational costs, the fundamental source of the result is different. In Antras and Helpman (2004), a trade-off between the ownership advantages in FDI and better incentive in outsourcing, and the incomplete contracts in outsourcing drive the results. In this model, however, the labor market condition, along with the price of intermediate goods, is the basis of the result as they determine three productivity cutoffs.

Using the equilibrium conditions that we discussed in the previous section, we can get analytical solutions of \( \Theta_d, \Theta_u, \) and \( \Theta_f \). Before proceeding further, it will be helpful to express \( \Theta_u \) and \( \Theta_f \) in terms of \( \Theta_d \) by using three cutoff conditions:
\[ \Theta_u = \frac{f_u}{f_d} \left[ (1 + \beta) \left( \frac{b}{p_u} \right)^{\frac{\beta}{1-\beta}} - 1 \right] -1 \Theta_d \equiv k_1 \Theta_d, \quad (17) \]

\[ \Theta_f = \frac{f_f - f_u}{f_d} \left[ \left( \frac{b}{b_f} \right)^{\frac{\beta}{1-\beta}} - (1 + \beta) \left( \frac{b}{p_u} \right)^{\frac{\beta}{1-\beta}} \right] -1 \Theta_d \equiv k_2 \Theta_d, \quad (18) \]

With these, we can express three profit equations in terms of three productivity cutoffs:

\[ \pi_d(\Theta) = f_d \Theta \Theta_d - f_d, \]

\[ \pi_u(\Theta) = (f_u + k_1 f_d) \frac{\Theta}{k_1 \Theta_d} - f_d - f_u, \]

\[ \pi_f(\Theta) = \left[ k_2 f_d + \left( \frac{k_2 - k_1}{k_1} \right) f_u + f_f \right] \frac{\Theta}{k_2 \Theta_d} - f_d - f_f. \]

If we plug these into the free entry condition and apply the Pareto distribution assumption, we get

\[ f_e = \int_{\Theta_d}^{\Theta_u} \left[ f_d \frac{\Theta}{\Theta_d} - f_d \right] dG(\Theta) + \int_{\Theta_u}^{\Theta_f} \left[ (f_u + k_1 f_d) \frac{\Theta}{k_1 \Theta_d} - f_d - f_u \right] dG(\Theta) + \int_{\Theta_f}^{\infty} \left[ \left( k_2 f_d + \left( \frac{k_2 - k_1}{k_1} \right) f_u + f_f \right) \frac{\Theta}{k_2 \Theta_d} - f_d - f_f \right] dG(\Theta) \quad (19) \]

\[ = \frac{(k_1 k_2)^{-\alpha} [(f_f - f_u) k_1^\alpha + (f_u + f_d k_1^\alpha) k_2^\alpha]}{\alpha - 1} \left( \frac{\Theta_m}{\Theta_d} \right)^\alpha, \]

where the last expression, which relates \( \Theta_d \) in terms of \( k_1 \) and \( k_2 \), is decreasing in \( \Theta_d \).

The labor market condition and the average wage condition are calculated as

\[ x(1 - \sigma)\bar{w} = 1 \quad (20) \]

and as

\[ \bar{w} = \frac{M}{H} \int_{\Theta_d}^{\Theta_u} w(\Theta) h_d(\Theta) dG(\Theta) = \frac{b M}{H} \int_{\Theta_d}^{\Theta_u} h_d(\Theta) dG(\Theta). \quad (21) \]

The second equality of the average wage condition holds because all domestic labor receives the same wage, \( b \).
The total hiring, \( H \), and the total output in the differentiated-goods sector, \( Q \), are defined as

\[
H = M \int_{\Theta_d}^{\Theta_u} h_d(\Theta) dG(\Theta) \tag{22}
\]

and

\[
Q = M^{\frac{1}{\beta}} \left[ \int_{\Theta_d}^{\Theta_u} q_d(\Theta)^\beta dG(\Theta) + \int_{\Theta_u}^{\Theta_f} q_u(\Theta)^\beta dG(\Theta) + \int_{\Theta_f}^{\infty} q_f(\Theta)^\beta dG(\Theta) \right]^{\frac{1}{\beta}}, \tag{23}
\]

where

\[
q_d(\Theta) = \Theta^{1-\beta} h_d, \\
q_u(\Theta) = \Theta^{1-\beta} h_u, \\
q_f(\Theta) = \Theta^{1-\beta} h_f.
\]

Combining (20), (21), and (22), we get

\[
\frac{H}{N} = \frac{1}{b}. \tag{24}
\]

Using this result and Equation (7), we get the following equation that determines the labor market cost:

\[
b = \frac{1}{1-\sigma} \left( ax^\delta + \sigma \psi \right) = \frac{1}{1-\sigma} \left\{ a \left[ \frac{1}{b(1-\sigma)} \right]^\delta + \sigma \psi \right\}. \tag{25}
\]

Note that we can calculate \( b \) from Equation (25) as a function of labor market parameters. In other words, \( b \) does not depend on other fixed costs, nor the distribution of productivity. This result is consistent with the result of the closed economy model in Helpman and Itskhoki (2010).\(^7\)

As the labor market cost is determined solely by exogenous labor market parameters, we can calculate \( \Theta_d \) from Equation (19). As the right hand side of Equation (19) is decreasing in \( \Theta_d \), under the conditions of Proposition 4.2, there exists \( \Theta_d \), which is unique at a given level of \( b \).

\(^7\)Note that this model focuses on not a horizontal FDI, but a vertical FDI. As discussed in Antras and Yeaple (2013), it is useful to assume zero transportation costs to shut down the horizontal incentive in FDI. Thus, if I introduce exports into this model, it will create unnecessary complications, while keeping the main results the same, as it would only increase the market size that firms face.
Then, we can calculate the total output in the differentiated-goods sector using the zero profit condition:

\[
1 - \beta \left[ \frac{\beta}{b(1 + \beta)} \right]^{\frac{\beta}{1+\beta}} Q^{-\frac{\beta-\zeta}{1-\beta}} \Theta_d = f_d.
\]

With \( \Theta_d \) and \( Q \), we can solve for \( M \), \( N \), and \( H \) using (22), (23), and (24), while \( \Theta_u \) and \( \Theta_f \) can be solved from (17) and (18).

5 Analysis

5.1 FDI vs. Outsourcing

In this section, I attempt to analyze the trade-off between FDI and Outsourcing. To do this, let me list three different profit equations in three different choices:

\[
\begin{align*}
\pi_d(\Theta) &= A_d \Theta - f_d, \\
\pi_f(\Theta) &= A_f \Theta - f_d - f_f, \\
\pi_u(\Theta) &= A_u \Theta - f_d - f_u
\end{align*}
\]

where

\[
\begin{align*}
A_d &= (1 - \beta) \left( \frac{1}{1 + \beta} \right)^{\frac{1}{1-\beta}} \left( \frac{\beta}{b} \right)^{\frac{\beta}{1-\beta}} Q^{-\frac{\beta-\zeta}{1-\beta}}, \\
A_f &= (1 - \beta) \left( \frac{1}{1 + \beta} \right)^{\frac{1}{1-\beta}} \left( \frac{\beta}{b_f} \right)^{\frac{\beta}{1-\beta}} Q^{-\frac{\beta-\zeta}{1-\beta}}, \\
A_u &= (1 - \beta) \left( \frac{\beta}{p_u} \right)^{\frac{\beta}{1-\beta}} Q^{-\frac{\beta-\zeta}{1-\beta}}.
\end{align*}
\]

By comparing three different expressions, we can see that all three choices have a simple linear relationship with the productivity level and the three coefficients of \( \Theta \) take similar forms. If we calculate ratios between them, we get

\[
\frac{A_u}{A_d} = (1 + \beta)^{\frac{1}{1-\beta}} \left( \frac{b}{p_u} \right)^{\frac{\beta}{1-\beta}}, \quad \frac{A_f}{A_d} = \left( \frac{b}{b_f} \right)^{\frac{\beta}{1-\beta}}, \quad \text{and} \quad \frac{A_f}{A_u} = (1 + \beta)^{-\frac{1}{1-\beta}} \left( \frac{p_u}{b_f} \right)^{\frac{\beta}{1-\beta}}.
\]

From here, we can see that the relative levels of \( p_u \) and \( b_f \) along with \( b \) determine the relative variable benefits of each choice.

As \( \frac{A_f}{A_u} = (1 + \beta)^{-\frac{1}{1-\beta}} \left( \frac{p_u}{b_f} \right)^{\frac{\beta}{1-\beta}} = (1 + \beta)^{-\frac{1}{1-\beta}} \left( \frac{2b_u+c}{b_f} \right)^{\frac{\beta}{1-\beta}} \), \( \beta \) and \( c \) affect the relative attractiveness of two choices at a given level of \( b_u \) and \( b_f \). The
term with $\beta$ originates from the fact that in Outsourcing, firms do not have to bargain over their revenue with their labor, as they can replace workers with intermediate goods. Thus, we can interpret this term as an additional benefit of choosing Outsourcing over FDI.

**Proposition 5.1.** By choosing Outsourcing, a firm can avoid the labor market and this gives a secondary benefit to the firm, which in turn increases as workers have more shares in the wage bargaining process.

**Proof.** The term $(1 + \beta)^{\frac{1}{1-\beta}}$, which can be interpreted as a secondary relative benefit of choosing Outsourcing over FDI, is an increasing function of $\beta$. It has a value of one when $\beta$ is zero, and it goes to infinity as $\beta$ goes to one. As the worker’s share in the wage bargaining, $\frac{\beta}{1+\beta}$, increases in $\beta$, the secondary benefit of having Outsourcing increases in the workers’ share.

The intuition of this proposition is quite simple. The larger the share that workers receive in wage bargaining, the more the gains that firms have as a result of avoiding it.

The other parameter that affects the trade off between two choices is $c$. If we have very low $c$, Outsourcing will dominate FDI as it also has lower organizational cost. On the contrary, if $c$ is too high, Outsourcing would not be chosen by any firms, as the variable benefits of choosing it would be lower than FDI and DI.

To be more specific, let’s rewrite the first sufficient condition of Type 4 equilibrium:

$$b_f (1 + \beta)^{\frac{1}{\beta}} - 2b_u < c < b (1 + \beta)^{\frac{1}{\beta}} - 2b_u.$$  

This condition tells us that, in Type 4 equilibrium, $c$ has to be low enough to make Outsourcing a profitable option for some firms ($c < b (1 + \beta)^{\frac{1}{\beta}} - 2b_u$), while it should not be too low, as it would drive FDI out of the equilibrium ($b_f (1 + \beta)^{\frac{1}{\beta}} - 2b_u < c$). If $c$ is larger than $b (1 + \beta)^{\frac{1}{\beta}} - 2b_u$, we will have Type 3 equilibrium and if $c$ is smaller than $b_f (1 + \beta)^{\frac{1}{\beta}} - 2b_u$, it will become Type 2 equilibrium.

We can interpret $c$ as a generality of a skill that a firm uses. If a firm uses unique technology, it will be very costly for it to buy an intermediate good and adjust it to fit into its production process. In this case, the firm would be better-off by choosing integration, instead of an arm’s-length relationship. On the contrary, if the technology is a general one, it will be profitable to choose Outsourcing. We can also think of $c$ as a fixed cost of producing an intermediate input as in Antras (2003); but in my model firms have to bear all of the cost as suppliers get zero profit.
5.2 Effect of the Labor Market Cost on Firms’ Decision

One benefit of using the proposed model is that we can derive analytic solutions of $\Theta_d$, $\Theta_u$, and $\Theta_f$ in all types of equilibria. With these, we can analyze the effect of the domestic labor market cost on individual firms’ decision.

Equilibrium values of the zero profit cutoff in all types are summarized in Table 5.1. Note that unlike all the other equilibria, in Type 1, $\Theta_d$ does not depend on the domestic labor market cost. This means that as long as $b$ lies in a range which supports Type 1 equilibrium, changes in $b$ do not change the zero profit cutoff. It turns out that changes in $b$ are completely offset by changes in the quantity index in the differentiated-goods sector, $Q$, so that the profit level of an individual firm remains the same. For three other types of equilibria, we can do comparative statics to determine the effect of changes in $b$ on $\Theta_d$ and it turns out that $\Theta_d$ decreases in $b$ in all of them.

<table>
<thead>
<tr>
<th>Type</th>
<th>$\Theta_d$</th>
<th>$\Theta_u$</th>
<th>$\Theta_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1</td>
<td>$\left[\frac{f_u}{f_u(a-1)}\right]^{1/\alpha} \Theta_m - k_1 \Theta_d$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Type 2</td>
<td>$\left[\frac{f_d+k_1^u f_u}{f_u(a-1)}\right]^{1/\alpha} \Theta_m$</td>
<td>$k_1 \Theta_d$</td>
<td>-</td>
</tr>
<tr>
<td>Type 3</td>
<td>$\left[\frac{f_d+k_1^u f_u+k_2^u (f_f-f_u)}{f_u(a-1)}\right]^{1/\alpha} \Theta_m$</td>
<td>-</td>
<td>$k_3 \Theta_d$</td>
</tr>
<tr>
<td>Type 4</td>
<td>$\left[\frac{f_d+k_1^u f_u+k_2^u (f_f-f_u)}{f_u(a-1)}\right]^{1/\alpha} \Theta_m$</td>
<td>$k_1 \Theta_d$</td>
<td>$k_2 \Theta_d$</td>
</tr>
</tbody>
</table>

Table 5.1: Cutoffs

To understand this result, let’s see the equilibrium conditions of Type 1. Once we derive $\Theta_d$ as in Table 5.1, we use the zero profit cutoff condition,

$$\pi_d(\Theta_d) = (1 - \beta) \left( \frac{1}{1 + \beta} \right) \theta^\beta \left( \frac{\beta}{b} \right) \bar{\sigma} \theta^{\frac{\beta}{1+\beta}} Q^{-\frac{\beta-\bar{\sigma}}{1-\beta}} \theta_d - f_d = 0,$$  \hspace{1cm} (27)

to get the equilibrium level of $Q$. From here, we can see that in Type 1 equilibrium, the production index should be adjusted to cancel out the changes in $b$, as $\Theta_d$ is unaffected by that. This means that if firms in Type 1 face a higher domestic labor market cost, they reduce production quantities

$$k_1 = \frac{f_u}{f_d} \left[ (1 + \beta) \frac{\theta^\beta}{\theta^\beta} - 1 \right]^{-1}, k_2 = \frac{f_u-f_u}{f_d} \left[ \left( \frac{\theta^\beta}{\theta^\beta} - (1 + \beta) \frac{\theta^\beta}{\theta^\beta} \right) \frac{\theta^\beta}{\theta^\beta} \right]^{-1},$$

and

$$k_3 = \frac{f_f}{f_d} \left[ \left( \frac{\theta^\beta}{\theta^\beta} - 1 \right) \frac{\theta^\beta}{\theta^\beta} \right]^{-1}. $$
to make the profit level the same. And the reduced $Q$ makes the profit curves of Outsourcing and FDI steeper, as firms with either of them now enjoy higher domestic prices.

When the profit curve of FDI or Outsourcing becomes steeper than DI, firms with a high productivity level start to choose offshoring (Type 2 or Type 3). Now, firms that choose offshoring are not affected by the changes in the domestic labor market cost as they do not hire domestic labor. So the changes in $Q$ cannot fully absorb all of the effects of the changes in $b$, and the profit level of firms that choose DI start to decrease. This process in turn forces more firms to exit (increase in $\Theta_d$), and thus the average productivity level is increased.

And this process continues as $b$ increases. The profit level of DI decreases, and that of offshoring increases. Thus, in both Type 2 and Type 3 equilibria, more firms exit ($\Theta_d$ increases), while more firms choose offshoring. This result implies that $\Theta_u$ in Type 2 and $\Theta_f$ in Type 3 decreases in $b$.

Even in Type 4 equilibrium, $\Theta_d$ will increase as $b$ increases. Although it is difficult to predict the movement of two other cutoffs, it turns out that we can prove both $\Theta_u$ and $\Theta_f$ decreases in $b$. Intuitively, this result is driven by the following fact: the relative benefits of selecting FDI instead of Outsourcing do not depend on $b$. As we have seen in the Section 4.1, $\frac{A_f}{A_u}$ does not include any terms containing $b$. As $b$ increases, the relative benefits of choosing Outsourcing or FDI compared to DI increases; but its effect is not biased toward a certain choice, i.e., both $\Theta_u$ and $\Theta_f$ decrease. Above discussion can be summarized in the following proposition.

**Proposition 5.2.** As $b$ increases, while keeping $b_f$ and $p_u$ fixed,

1) $\Theta_d$ is not affected in Type 1 equilibrium,
2) $\Theta_d$ increases in Type 2, Type 3, and Type 4 equilibrium,
3) both $\Theta_u$ and $\Theta_f$ decrease,
4) average firm productivity in the differentiated-goods sector increases.

**Proof.** In Appendix B \[\square\]

### 5.3 Economic Implications of Offshoring

This model predicts how firms’ organizational decision is made across heterogeneous firms when three possible choices are given. To see the economic implications of having these options, let’s think about an autarky equilibrium, where offshoring is not allowed. In autarky, firms cannot choose offshoring, and thus the equilibrium will stay in Type 1. As discussed in the previous chapter, in Type 1, $\Theta_d$ remains at the same level and $Q$ decreases
when $b$ increases. Considering the indirect utility function of this economy, $V = E + \frac{1-\zeta}{\zeta}Q$, these results imply that the welfare level of the autarky equilibrium decreases.

To compare the welfare level in autarky with that of an economy where offshoring is allowed, let’s check the zero profit cutoff condition in both cases. In both economies, $Q$ is defined by Equation (27):

$$Q_A^{-\frac{\beta-\zeta}{1-\beta}} = \frac{1 + \beta}{1 - \beta} \left[ \frac{b (1 + \beta)}{\beta} \right]^{\frac{\beta}{1-\beta}} \frac{f_d}{\Theta_{d,A}},$$

$$Q_O^{-\frac{\beta-\zeta}{1-\beta}} = \frac{1 + \beta}{1 - \beta} \left[ \frac{b (1 + \beta)}{\beta} \right]^{\frac{\beta}{1-\beta}} \frac{f_d}{\Theta_{d,O}},$$

where subscript $A$ denotes the autarky equilibrium and $O$ denotes an equilibrium with offshoring. By dividing one equation with the other, we get

$$\left( \frac{Q_O}{Q_A} \right)^{-\frac{\beta-\zeta}{1-\beta}} = \frac{\Theta_{d,A}}{\Theta_{d,O}}.$$  \hfill (28)

By Proposition 5.2, when $b$ is sufficiently large, the right hand side of Equation (28) is smaller than one, and thus $Q_O$ is greater than $Q_A$ (they will be the same if $b$ lies in a region that support Type 1 equilibrium). Simply taking a derivative with respect to $b$ in Equation (28), we can prove that $\frac{Q_O}{Q_A}$ increases in $b$.

These are summarized in the following proposition.

**Proposition 5.3.** The welfare level of an economy with offshoring is

1) higher than that in autarky

2) and the difference between the welfare levels increases in $b$.

Now, let’s discuss the hiring level of domestic workers and the unemployment rate in this model. By using the same trick of Equation (28) with Equation (12), we get

$$\frac{h_O(\Theta)}{h_A(\Theta)} = \left( \frac{Q_O}{Q_A} \right)^{-\frac{\beta-\zeta}{1-\beta}}.$$  \hfill (29)

As $\frac{Q_O}{Q_A} > 1$, we can see that the hiring level of individual firms in autarky is higher than that of the offshoring equilibrium.

Moreover, under the Pareto assumption that I made, the number of firms that choose DI is smaller in the offshoring equilibrium. Thus, we can conclude that the total hiring, $H$, is smaller in an economy with offshoring (of

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\textsuperscript{9}It can also be shown that $Q$ in all types of equilibria decreases in $b$.\hfill 22
course, if the economy stays in Type 1 equilibrium even though offshoring is allowed, \( Q \) and \( H \) will be the same).

The economy-wide unemployment rate is defined as

\[
    u = \frac{(N - H)}{L} = (b - 1) \frac{H}{L},
\]

where the second equality holds by Equation (24). From here, we can see that the unemployment rate equals zero when \( b = 1 \). When \( b \) goes to infinity, the term \( (1 - \frac{1}{b}) \) will get closer to one, but as \( H \) approaches zero, the unemployment rate will also go to zero. Thus, as \( b \) increases, we would expect a bell-shaped unemployment rate curve.

The bell-shaped unemployment rate curve is driven by the labor movement across two sectors. As \( b \) increases, more workers enter the differentiated-goods sector, as higher \( b \) means a higher wage level. However, as the total hiring, \( H \), decreases at the same time, workers realize that the probability of getting a job in the differentiated-goods sector becomes lower and they choose the other sector. Even though this process hinders workers from entering the differentiated-goods sector in both the offshoring equilibrium and the autarky equilibrium, it is much slower in autarky. In the offshoring equilibrium, increased labor market inefficiencies make firms choose offshoring, and thus adjustment in \( H \) is more dramatic than in autarky. With lower \( H \), more workers choose the homogeneous sector, and thus the unemployment rate in the offshoring equilibrium becomes lower than that in autarky. Analytically, this is an obvious result, as \( H \) in autarky is higher than that in the offshoring equilibrium, and Equation (30) tells us, at a given level of \( b \), higher \( H \) leads to a higher unemployment rate.

**Proposition 5.4.** In an economy with offshoring, the total hiring in the differentiated-goods sector and the unemployment rate are lower than those in autarky.

For illustrative purposes, I simulate the model with parameter values that support the second path (Type 1→Type 2→Type4).\(^{10}\) From Figure 5.3, we can see that the total hiring in the differentiated-goods sector and the economy-wide unemployment rate are lower in the offshoring equilibrium. We can also find that the total output is higher, and the zero profit cutoff is increasing in \( b \) in the offshoring equilibrium. All of these results are fundamentally driven by the fact that offshoring allows the economy more

\(^{10}\) I used \( b_f = 1.2, p_u = 3, f_d = 5, f_u = 10, f_f = 15, f_e = 4, \Theta_m = 1, \alpha = 1.5, \zeta = 0.2, \beta = 0.6 \) for this example.
options to react to the increased inefficiencies in the labor market. By choosing offshoring, firms can prevent the quantity from dropping too much, and this is the main source of the welfare gains in this economy.

Figure 1: Autarky vs. Offshoring

6 Conclusion

In this paper, I studied an economy with two sectors where the labor market in one is under search and matching frictions. By focusing on four types of equilibria, I find that the welfare level of the economy decreases as the labor market inefficiency increases. Increased labor market inefficiency makes more firms choose offshoring, in which foreign labor is used and more firms exit. As firms with a high-productivity level choose to offshore and firms with a low-productivity level exit, the average productivity level increases. Since fewer firms hire domestic labor, total hiring is reduced and total output decreases in the differentiated-goods sector. As the increased wage level and the decreased total hiring cause the economy-wide unemployment rate to move in different directions, it shows a bell-shaped curve.

I also analyzed the economic implications of offshoring by comparing the offshoring equilibrium with the autarky equilibrium. In the offshoring equilibrium, the welfare level is weakly higher and the economy-wide unemployment rate is weakly lower than those in the autarky equilibrium. This implies that even though offshoring reduces the size of total hiring, it does help the economy by lowering the unemployment rate and lifting the welfare level. It is interesting to note that the clear prediction of the unemployment rate is
driven by the assumption of a frictionless labor market in the homogeneous-good sector. With this assumption, the impact of the reduced total hiring in the differentiated-goods sector can be absorbed by the homogeneous-good sector. This is similar to Mitra and Ranjan’s (2010) finding that offshoring decreases the economy-wide unemployment rate when perfect inter-sectoral mobility is satisfied. In both papers, the impact of offshoring on the unemployment rate depends on whether or not the other sector can partly absorb the negative effects of offshoring on unemployment.

It may be interesting to extend the proposed model in this study by relaxing the assumption of a frictionless labor market in the homogeneous-good sector; but the main point of this model will still be the same: The difference in the labor market cost across countries is the key factor that determines a firm’s decision on offshoring. Only with the labor market frictions is this model able to produce the same segregation result that was found in previous studies. This result hints that the full employment assumption which was used in many previous studies could be too strong in analyzing the motivation of offshoring.

References


Appendix

A Equilibrium Conditions for Type 1, Type 2 and Type 3

A.1 Equilibrium with DI (Type 1)

Suppose that there only exist Domestic Integration in equilibrium. This happens when gains from offshoring are not enough to cover the fixed organizational costs.

Lemma A.1. The sufficient condition for Type 1 equilibrium are

\[(1 + \beta) \frac{1}{\beta} \left( \frac{b}{p_u} \right)^{\frac{\beta}{1 - \beta}} \leq 1 \quad \text{and} \quad \left( \frac{b}{b_f} \right)^{\frac{\beta}{1 - \beta}} \leq 1.\]

Proof. The first condition comes from \( A_u \leq A_d \) and the second condition comes from \( A_f \leq A_d \). As I assume \( f_f > f_u > 0 \), these two conditions ensure Type 1 equilibrium.

In Type 1 equilibrium, there only exist zero profit cutoff, which is defined by Equation (16a). Other equilibrium conditions for Type 1 will become as following:

- (free entry) \( f_e = \int_{\Theta_d}^\infty \pi_d(\Theta) dG(\Theta) \)
- (labor market) \( x(1 - \sigma)\bar{w} = 1 \)
- (average wage) \( \bar{w} = \frac{M}{H} \int_{\Theta_d}^\infty w(\Theta) h_d(\Theta) dG(\Theta) \)
- (total hiring) \( H = M \int_{\Theta_d}^\infty h_d(\Theta) dG(\Theta) \)
- (total output) \( Q = M^{\frac{1}{\beta}} \left[ \int_{\Theta_d}^\infty q_d(\Theta)^{\frac{1}{\beta}} dG(\Theta) \right]^{\frac{\beta}{\beta}} \)
A.2 Equilibrium with DI and Outsourcing (Type 2)

Type 2 equilibrium exists when the benefits of choosing Outsourcing dominate those of choosing FDI.

Lemma A.2. Sufficient conditions to have Type 2 equilibrium are

\[ 1) \quad (1 + \beta)^{\frac{1}{1-\beta}} \left( \frac{b}{pu} \right)^{\frac{\beta}{1-\beta}} > 1 \quad \& \quad (1 + \beta)^{-\frac{1}{1-\beta}} \left( \frac{pu}{b_f} \right)^{\frac{\beta}{1-\beta}} \leq 1, \]

\[ 2) \quad (1 + \beta)^{\frac{1}{1-\beta}} \left( \frac{b}{pu} \right)^{\frac{\beta}{1-\beta}} - 1 < \frac{fu}{fd}. \]

Proof. The first condition comes from $\Theta_d < \Theta_u \& \Theta_f \leq \Theta_u$. And the second condition comes from $\Theta_d < \Theta_f \& \Theta_f < \Theta_u$. \(\square\)

In Type 2, there exist two cutoffs, zero profit cutoff, $\Theta_d$, and Outsourcing cutoff, $\Theta_u$. Other equilibrium conditions will be modified as following:

(free entry) \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quarter
In Type 3, two cutoffs exist, zero profit cutoff, $\Theta_d$, and FDI cutoff, $\Theta_f$. Other equilibrium conditions will be modified as following:

(free entry) \[ f_e = \int_{\Theta_d}^{\Theta_f} \pi_d(\Theta)dG(\Theta) + \int_{\Theta_f}^{\infty} \pi_f(\Theta)dG(\Theta) \]

(labor market) \[ x(1 - \sigma)\bar{w} = 1 \]

(average wage) \[ \bar{w} = \frac{M}{h} \int_{\Theta_d}^{\Theta_u} w(\Theta)h_d(\Theta)dG(\Theta) + \int_{\Theta_u}^{\infty} w(\Theta)h_u(\Theta)dG(\Theta) \]

(total hiring) \[ H = M \left[ \int_{\Theta_d}^{\Theta_f} h_d(\Theta)dG(\Theta) + \int_{\Theta_f}^{\infty} h_f(\Theta)dG(\Theta) \right] \]

(total output) \[ Q = M^{1/\beta} \left[ \int_{\Theta_d}^{\Theta_f} q_d(\Theta)^{\beta}dG(\Theta) + \int_{\Theta_f}^{\infty} q_f(\Theta)^{\beta}dG(\Theta) \right]^{1/\beta} \]

B A proof of Proposition 5.2

B.1 Zero Profit Cutoff

Before proceeding further, let’s define

\[ R_1 = \frac{A_u}{A_d} = (1 + \beta)^{\frac{1}{\beta}} \left( \frac{b}{p_u} \right)^{\frac{1}{\beta}} \]

and

\[ R_2 = \frac{A_f}{A_d} = \left( \frac{b}{b_f} \right)^{\frac{1}{\beta}}. \]

One can easily show that $\frac{dR_1}{db} > 0$ and $\frac{dR_2}{db} > 0$. It is also convenient to define $k_1$, $k_2$, and $k_3$ as

\[ k_1 = \frac{f_u}{f_d} (R_1 - 1)^{-1}, \]

\[ k_2 = \frac{f_f - f_u}{f_d} (R_2 - R_1)^{-1}, \]

\[ k_3 = \frac{f_f}{f_d} (R_2 - 1)^{-1}. \]

For Type 2 equilibrium, $\Theta_d = \left[ \frac{f_d + k_1^{-\alpha} f_u}{f_e (\alpha - 1)} \right]^{1/\alpha} \Theta_m$. By taking a derivative with respect to $b$, we get

\[ \frac{d\Theta_d}{db} = \frac{d\Theta_d}{dk_1} \frac{dk_1}{db} = \left\{ - \left[ \frac{f_d + k_1^{-\alpha} f_u}{f_e (\alpha - 1)} \right]^{\frac{1}{\alpha}} \frac{f_u}{f_e (\alpha - 1)} k_1^{-\alpha - 1} \Theta_m \right\} \left\{ - \frac{f_u}{f_d (R_1)^2} \frac{dR_1}{db} \right\} > 0. \]
For Type 3 equilibrium, $\Theta_d = \left[ \frac{f_d + k_1^{-\alpha} f_f}{f_e (\alpha - 1)} \right]^{1/\alpha} \Theta_m$. By taking a derivative with respect to $b$, we get

$$\frac{d\Theta_d}{db} = \frac{d\Theta_d}{dk_3} \frac{dk_3}{db} = \left\{ - \left[ \frac{f_d + k_3^{-\alpha} f_u}{f_e (\alpha - 1)} \right]^{1/\alpha} \frac{f_u}{f_e (\alpha - 1)} k_3^{-\alpha-1} \Theta_m \right\} \left\{ - \frac{f_u}{f_d (R_2 - 1)^2} \frac{dR_2}{db} \right\} > 0.$$

For Type 4 equilibrium, $\Theta_d = \left[ \frac{f_d + k_1^{-\alpha} f_u + k_2^{-\alpha} (f_f - f_u)}{f_e (\alpha - 1)} \right]^{1/\alpha} \Theta_m$. As both $k_1$ and $k_2$ depend on $b$, we have to check the sign of

$$\frac{d\Theta_d}{db} = \frac{\partial \Theta_d}{\partial k_1} \frac{dk_1}{db} + \frac{\partial \Theta_d}{\partial k_2} \frac{dk_2}{db}.$$

where $\frac{dk_1}{db} = -\frac{f_u}{f_d (R_1 - 1)^2} \frac{dR_1}{db} < 0$. We can find signs of other three parts as

$$\frac{\partial \Theta_d}{\partial k_1} = - \left[ \frac{f_d + k_1^{-\alpha} f_u + k_2^{-\alpha} (f_f - f_u)}{f_e (\alpha - 1)} \right]^{1/\alpha-1} \Theta_m \frac{f_u}{f_e (\alpha - 1)} k_1^{-\alpha-1} < 0,$$

$$\frac{\partial \Theta_d}{\partial k_2} = - \left[ \frac{f_d + k_1^{-\alpha} f_u + k_2^{-\alpha} (f_f - f_u)}{f_e (\alpha - 1)} \right]^{1/\alpha-1} \Theta_m \frac{f_f - f_u}{f_e (\alpha - 1)} k_2^{-\alpha-1} < 0,$$

$$\frac{dk_2}{db} = - \frac{f_f - f_u}{f_d} \left( \frac{1}{R_2 - R_1} \right)^2 \left[ \frac{\beta (R_2 - R_1)}{(1 - \beta) b} \right] < 0.$$

The inequality of the last equation holds because, in Type 4, $R_2 = \frac{A_f}{A_d} > \frac{A_u}{A_d} = R_1$.

### B.2 Outsourcing and FDI cutoff

In Type 2 equilibrium, we have one additional cutoff,

$$\Theta_u = k_1 \Theta_d.$$

Taking a derivative with respect to $b$, we get $\frac{d\Theta_u}{db} = k_1 \frac{d\Theta_d}{db} + \Theta_d \frac{dk_1}{db}$. As the sign of $\frac{d\Theta_d}{db} (> 0)$ and $\frac{dk_1}{db} (< 0)$ are different, we have to solve for each term to see whether we could determine the sign of the whole equation.
Let’s define $B_2$ as $\frac{f_d + k_1^{-\alpha} f_u}{f_e(\alpha - 1)}$. Then, $\Theta_d = \left[ \frac{f_d + k_1^{-\alpha} f_u}{f_e(\alpha - 1)} \right]^{1/\alpha} \Theta_m \equiv B_2^{1/\alpha} \Theta_m$. Using this equation, we get

\[
\frac{d\Theta_u}{db} = \frac{dk_1}{db} \left( \Theta_d + \frac{d\Theta_d}{dk_1} k_1 \right) = \frac{dk_1}{db} \left[ B_2^{1/\alpha} \Theta_m - B_2^{1/\alpha - 1} \Theta_m \frac{f_u}{f_e(\alpha - 1)} k_1^{-\alpha} \right] = \frac{dk_1}{db} B_2^{1/\alpha - 1} \Theta_m \frac{f_d}{f_e(\alpha - 1)},
\]

which is negative as $\alpha$ assumed to be greater than 1 and $\frac{dk_3}{db} < 0$.

In Type 3 equilibrium, we have FDI cutoff,

\[ \Theta_f = k_3 \Theta_d. \]

Let’s denote $\Theta_d = B_3^{1/\alpha} \Theta_m$ by defining $B_3 \equiv \frac{f_d + k_1^{-\alpha} f_u}{f_e(\alpha - 1)}$. Then, we get

\[
\frac{d\Theta_f}{db} = \frac{dk_3}{db} \left( \Theta_d + \frac{d\Theta_d}{dk_3} k_3 \right) = \frac{dk_3}{db} \left[ B_3^{1/\alpha} \Theta_m - B_3^{1/\alpha - 1} \Theta_m \frac{f_u}{f_e(\alpha - 1)} k_1^{-\alpha} \right] = \frac{dk_3}{db} B_3^{1/\alpha - 1} \Theta_m \frac{f_d}{f_e(\alpha - 1)},
\]

which is negative as $\frac{dk_3}{db} < 0$.

Now, we have to check how two cutoffs react to changes in the labor market cost in Type 4 equilibrium. Offshoring and FDI cutoffs are defined as

\[ \Theta_u = k_1 \Theta_d, \quad \Theta_f = k_2 \Theta_d. \]

And by defining $B_4 \equiv \frac{f_d + k_1^{-\alpha} f_u + k_2^{-\alpha} (f_f - f_u)}{f_e(\alpha - 1)}$, we get

\[ \Theta_d = \left[ \frac{f_d + k_1^{-\alpha} f_u + k_2^{-\alpha} (f_f - f_u)}{f_e(\alpha - 1)} \right]^{1/\alpha} \Theta_m \equiv B_4^{1/\alpha} \Theta_m. \]
As both $k_1$ and $k_2$ depend on $b$, we have to check the sign of

$$\frac{d\Theta_u}{db} = \frac{d(k_1\Theta_d)}{db} = k_1 \frac{d\Theta_d}{db} + \Theta_d \frac{dk_1}{db}$$

(31)

to determine the sign of $\frac{d\Theta_u}{db}$. We can calculate each part as

$$\frac{dk_1}{db} = -\frac{\beta}{b(1-\beta)} \frac{f_u}{\bar{f}_d (R_1 - 1)^2},$$

$$\frac{dk_2}{db} = -\frac{\beta}{b(1-\beta)} \frac{f_f - f_u}{\bar{f}_d (R_2 - R_1)},$$

$$\frac{\partial \Theta_d}{\partial k_1} = -\Theta_m B_d^{1/2} \frac{f_u}{f_e (\alpha - 1)} k_1^{-\alpha - 1},$$

$$\frac{\partial \Theta_d}{\partial k_2} = -\Theta_m B_d^{1/2} \frac{f_f - f_u}{f_e (\alpha - 1)} k_2^{-\alpha - 1}.$$  

If we plug these four equations into the original equation, we get

$$\frac{d\Theta_d}{db} = B_d^{1/2} C \left[ \frac{R_1}{(R_1 - 1)^2 f_u^2 k_1^{-\alpha - 1} + (f_f - f_u)^2 R_2 - R_1 k_2^{-\alpha - 1}} \right]$$

where $C = \Theta_m \frac{1}{f_e f_d (\alpha - 1)} b^{1/2}$. If we plug this into (31), we get

$$\frac{d\Theta_u}{db} = B_d^{1/2} C \left[ \frac{R_1}{(R_1 - 1)^2 f_u^2 k_1^{-\alpha - 1} + (f_f - f_u)^2 R_2 - R_1 k_2^{-\alpha - 1}} \right]$$

$$- B_d^{1/2} C \left\{ \frac{R_1}{(R_1 - 1)^2 f_u [f_d + f_u k_1^{-\alpha} + (f_f - f_u) k_2^{-\alpha}]} \right\}$$

$$= B_d^{1/2} C \left[ \frac{(f_f - f_u)^2}{R_2 - R_1} k_1 k_2^{-\alpha - 1} - \frac{R_1}{(R_1 - 1)^2 f_u (f_f - f_u) k_2^{-\alpha} - R_1 f_d f_u}{(R_1 - 1)^2} \right].$$

Finally, if we plug in $k_1 = \frac{f_u}{f_d} (R_1 - 1)^{-1}$ and $k_2 = \frac{f_f - f_u}{f_d} (R_2 - R_1)^{-1}$ into the above equation, we get

$$\frac{d\Theta_u}{db} = B_d^{1/2} C \left[ -f_d^\alpha f_u (f_f - f_u)^{-\alpha + 1} \frac{(R_2 - R_1)^\alpha}{(R_1 - 1)^2} - \frac{R_1 f_d f_u}{(R_1 - 1)^2} \right]$$

which is negative as $R_2 > R_1$ and $f_f > f_u$.  

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Similarly, we can simplify \( \frac{d\Theta_f}{db} \) as

\[
\frac{d\Theta_f}{db} = \frac{d(k_2\Theta_d)}{db} = k_2 \frac{d\Theta_d}{db} + \Theta_d \frac{dk_2}{db}
\]

\[
= B_4^{\frac{1}{\alpha}-1} C \left[ \frac{R_1}{(R_1 - 1)^2} f_u^2 k_1^{-\alpha-1} k_2 + \frac{(f_f - f_u)^2}{R_2 - R_1} k_2^{-\alpha} \right]
\]

\[-B_4^{\frac{1}{\alpha}-1} C \left\{ \frac{f_f - f_u}{(R_2 - R_1)} \left[ f_d + k_1^{-\alpha} f_u + k_2^{-\alpha} (f_f - f_u) \right] \right\}
\]

\[
= B_4^{\frac{1}{\alpha}-1} C \left[ \frac{R_1}{(R_1 - 1)^2} f_u^2 k_1^{-\alpha-1} k_2 - \frac{f_u (f_f - f_u)}{(R_2 - R_1)} k_1^{-\alpha} - \frac{f_d (f_f - f_u)}{(R_2 - R_1)} \right].
\]

If we plug in \( k_1 \) and \( k_2 \) into the above equation, we get

\[
\frac{d\Theta_f}{db} = B_4^{\frac{1}{\alpha}-1} C f_u \left( \frac{f_f - f_u}{R_2 - R_1} \right) \left[ \left( \frac{f_u}{f_d} \frac{1}{R_1 - 1} \right)^{1-\alpha} - 1 \right].
\]

As \( \alpha > 1 \) and \( \frac{f_u}{f_d} \frac{1}{R_1 - 1} \) is greater than one by the Lemma 4.2, \( \left( \frac{f_u}{f_d} \frac{1}{R_1 - 1} \right)^{1-\alpha} < 1 \), and thus \( \Theta_f \) is also decreasing in \( b \).