

# Patent Thickets, Trolls and Unproductive Entrepreneurship

John L. Turner<sup>†</sup>

September 2012

## Abstract

I introduce and analyze an equilibrium model of invention, patenting and infringement under monopolistic competition. Profitable use of inventions requires adaptation to complementary technologies. With patents, a thicket of conflicting rights emerges and costly infringements occur. This taxes invention and lowers welfare. When an inventor may be a “troll”—patent without inventing—the rate of invention falls further. Intuitively, some trolls would invent if it were impossible to be a troll. More technology is patented with trolls, so the thicket grows and welfare falls. Being a troll is unprofitable unless a critical mass of inventions, made by other firms, exists.

**JEL Classification:** *K2, L2, O3.*

**Keywords:** litigation, patents, thickets, trolls.

---

<sup>†</sup>Department of Economics, Terry College of Business, University of Georgia, Athens, GA 30602. Tel 706-542-3682, Email [jltturner@uga.edu](mailto:jltturner@uga.edu). I thank Jim Bessen, Josh Lerner, Mike Meurer, seminar participants at the 2012 ALEA Meetings, the 2012 EEA-ESEM Meetings, the University of Georgia and the University of Tennessee for their very useful comments. Research support from the Terry-Sanford research program is gratefully acknowledged.

## 1. Introduction

Patents grant their owners (“patentees”) the right to exclude others from using their inventions. When multiple patents cover complementary components of a given technology, a common contemporary phenomenon (Heller and Eisenberg 1998), patentees can exclude each other from using the technology. Hence, producers often must navigate a “thicket” of conflicting rights just to use their own inventions.<sup>1</sup> In principle, this could be accomplished with little cost through bargaining or pooling (Lerner and Tirole 2004). In practice, however, navigating the thicket is quite costly. Producers frequently infringe other patents inadvertently (Cotropia and Lemley 2008),<sup>2</sup> because it is difficult to identify related patents and because patent boundaries are difficult to determine (Bessen and Meurer 2008). Inventors and producers also face potential litigation from firms that do not practice their patents and keep them in relative obscurity (Reitzig, Henkel and Heath 2007). Resolving such disputes requires significant resources.<sup>3</sup>

To capture and study these regularities, I introduce and analyze an equilibrium model of invention, patenting and infringement under monopolistic competition. In the model, profitable use of inventions requires adaptation to complementary technologies, which yields inadvertent infringements. I establish the existence of an equilibrium patent thicket and analyze both welfare and the effect of trolls. Equilibrium welfare emerges as a quadratic function of the levels of invention and patenting. Hence, the main analysis of the welfare effects of thickets and trolls is accessible using standard techniques.

Patents harm welfare in two ways. First, patents create additional disputes whose (unavoidable) costs are strict social losses. Second, since inventors bear these costs, patents distort the equilibrium level of invention. Intuitively, each patent exerts a negative externality. The marginal patented invention generates zero expected private profit, but increases the size of the patent thicket. This leads to costly disputes and yields a “tragedy of the

---

<sup>1</sup>This is not a new phenomenon. For example, Hayter (1947) argues that farmers faced a thicket of patents on farm equipment during the 1870s and 1880s, and radio patents in the 1920s arguably formed a thicket (Sabety 2005). Lemley and Shapiro (2007) cite several other examples.

<sup>2</sup>Cotropia and Lemley estimate that copying is alleged in patent cases only about 11% of the time.

<sup>3</sup>Using an event-study approach, Bessen and Meurer (2008, Table 6.2) estimate publicly-owned alleged infringers suffered a median cost of \$2.9 million per patent dispute in the US over 1984-99.

commons.”<sup>4</sup> Costly disputes both lower the overall returns to invention and drive a wedge between the social and private value of invention. This effect lowers the equilibrium rate of invention below the first-best.

The patent fee affects the rate of patenting non-monotonically. When the fee is very low, all inventions are patented. As a result, the expected profit from invention is effectively decreasing in the fee. For a sufficiently high fee, however, the expected profit from patenting is negative if all inventions are patented.

Intuitively, patents are parasitic in the model. Invention and production lead to inadvertent patent infringement, and a patent allows its owner to capitalize on such infringement by suing for damages. With a sufficiently high fee, the rate of invention would be driven so low (were all inventions patented) that the expected patent damages are not sufficient to compensate the patentee for the fee.

Stated differently, patentees that rely on inadvertent infringements for profit face a “critical mass problem.” To get patenting in equilibrium, the rate of invention must *increase* to the critical-mass level (so those who do patent get sufficiently rewarded to pay patent fees) but the rate of patenting must correspondingly *decrease* (so inventors do not get unduly taxed). Equilibrium consists of a fraction, of inventions patented, that decreases as the patent fee increases. The (critical-mass) rate of invention increases with the fee, while the rate of patenting decreases. Eventually, for an even higher fee, the rate of patenting falls to zero and the rate of invention achieves the first best.

The social costs of patents are more severe when it is possible and economically feasible for inventors to patent without inventing something that may be immediately reduced to practice. I define a “troll” as a firm engaging in precisely this behavior.<sup>5</sup> Trolls patent

---

<sup>4</sup>Hardin (1968) argued that when a resource is jointly owned, as is the case with many natural resources such as oceans and parks, there is a tendency of populations to overuse the resource. Hardin termed this general phenomenon the “tragedy of the commons.” Heller and Eisenberg (1998) coined the term “tragedy of the anticommons” as a sort of mirror image. When governments give too many people the right to exclude others from using a resource, there is a tendency of populations to underuse the resource. Both phenomena may emerge in the type of model used here.

<sup>5</sup>Merges (2009, p. 1583), for example, refers to a troll as an entity that does not engage in technological invention yet participates in the secondary market for patent rights. He distinguishes the “legitimate” secondary market, where inventors sell patent rights to firms that commercialize them, from the “more questionable market for the settlement of lawsuits involving weak, outdated or irrelevant patents.” Defined this way, trolls in the extreme buy worthless patents from inventors, then create economic harm by enabling

technology in the hope that other firms will infringe that technology in making products.<sup>6</sup> An extreme example of this behavior is “submarine patenting” (Gallini 2002), where patentees delay a patent’s issue until others make products using the patented technology. Reforms of the patent term under the Agreement on Trade Related Aspects of Intellectual Property Rights (TRIPS) in 1994 curb the practice of delaying patent issue, but do not prevent firms from patenting in speculation that some other firm will infringe.<sup>7</sup>

In the model, trolls patent purely as a means of rent seeking. When there is at least a critical mass of inventions, firms facing relatively high invention costs prefer to avoid those costs, surrender the direct gains from producing, and rely exclusively on earning damages from inadvertent infringements. Since some of these firms would be willing to invent if being a troll is not an option, the equilibrium rate of invention with trolls is lower. As a result, the troll option both allocates inventive talent away from invention and directly increases the amount of (complementary) technology covered by patents. Hence, trolls lead to both “unproductive entrepreneurship,” in the sense of Baumol (1990),<sup>8</sup> and higher per-invention dispute costs. Welfare is lower for two reasons distinct from standard deadweight losses of monopoly. In contrast to firms that invent, firms cannot earn positive profits as trolls without a critical mass of inventions.

My results predict that patent thickets are likely to be a problem in industries with differentiated final goods made using numerous components that have significant technological overlaps with other products’ components. This description matches the current environment of the modern information technology industry. As Ramirez and Kimmel (2011, p. 5) write: “Those in the IT sector voice the greatest concern, noting that merely identifying the large number of patents that could be relevant to complicated, multi-component products

---

markets where the things being exchanged have no social value. Viewed this way, patent enforcement by trolls is analogous to blackmail. Alternatively, I focus on the problem created when an agent can patent technology without spending the necessary resources to invent something that can be readily reduced to practice.

<sup>6</sup>Thus, as in Merges (2009), they do not reduce inventions to practice and they sue for damages in my model. In contrast to Merges (2009), trolls do not buy patents from other inventors in my model.

<sup>7</sup>Specifically, TRIPS changed the end of the patent term from 17 years after the patent issues to 20 years after the initial patent application is made. It took effect June 8, 1995.

<sup>8</sup>Baumol (1990) and the closely-related and independent work of Murphy, Schliefer and Vishny (1991) both recognize that innovation is “productive” entrepreneurship and discuss the potential usefulness of patents in stimulating such activities. They do not formally model patenting or discuss troll behavior, however.

<b>NBER Technology Class</b>	<b>Non-NPE Cases</b>	<b>NPE Cases</b>
Chemical	11.5%	0.7%
Computers & Communications	21.9%	77.1%
Drugs & Medical	25.7%	0.9%
Electrical & Electronic	11.5%	10.7%
Mechanical	12.1%	6.4%
Other	17.3%	4.1%
N	7,699	869

Table 1: *US Patent Lawsuits, At Least One Public Firm, 1990-2009.*

is often cost-prohibitive.”<sup>9</sup> In contrast, when the frequency of inadvertent infringement is lower, my model predicts that thickets will cause less of a change in rates of invention and patenting and that their presence will harm welfare less. The model also predicts that input complementarity drives both the thicket problem and troll behavior.

Interestingly, the litigation behavior of non-practicing entities (NPEs) supports these predictions.<sup>10</sup> Perhaps most notably, the technology distribution of patents in lawsuits filed by NPEs is wildly different from the technology distribution in other patent lawsuits. Consider Table 1, which matches 1990-2009 data on public-firm patent lawsuits from Derwent Litalert with data for the same time period from Patent Freedom [used by Bessen, Ford and Meurer (2011)] and classifies these cases according to the NBER technology classes of Hall, Jaffe and Trajtenberg (2001).<sup>11</sup> Among non-NPE cases, the most highly represented technology class is Drugs & Medical, at about 26%.<sup>12</sup> Among NPE lawsuits, however, less than 1% of lawsuits involve patents from this category. This suggests that the level of invention in this industry is below the level needed to induce widespread troll behavior.<sup>13</sup> On the other hand,

<sup>9</sup>See also Department of Justice and Federal Trade Commission (2011, pp. 81-82) for further discussion of the “patent notice” problem.

<sup>10</sup>Patent Freedom defines an NPE as a company that does not “practice its invention in products or service, or otherwise derive a substantial portion of their revenues from the sale of products and services in the marketplace. Instead, NPEs seek to derive the majority of their income from the enforcement of patent rights (Bessen, Meurer and Ford 2011). This closely matches the troll behavior in my model.

<sup>11</sup>The data are the subset of all Derwent cases that include at least one public firm. The technology class reflects the “main patent” recorded by Derwent. 460 Patent Freedom cases could not be reliably matched to a Derwent case. See Bessen, Meurer, Turner and Williams (2012) for more detail. Note the NPE cases do not match those from Bessen, Ford and Meurer (2011) exactly because we did not have 2010 Derwent data.

<sup>12</sup>Every case in the Patent Freedom data is classified as an NPE case.

<sup>13</sup>Pharmaceutical products are often based primarily on chemical compounds with physical characteristics

more than 77% of NPE cases involve Computers & Communications patents, compared to about 22% in non-NPE cases. Here, troll behavior appears to be flourishing.<sup>14</sup>

This monopolistic competition model shares a key qualitative feature with Melitz (2003) that firms have heterogeneous levels of productivity, but is otherwise far simpler. Indeed, it is set up specifically to exemplify patent thickets, and the harm they are capable of causing, in the cleanest way. There is a continuum of goods, but utility is specified so that there are no consumption complementarities across goods and firms may perfectly price discriminate. In addition, invention and production occur in vertically integrated firms. These assumptions equate private and social returns to invention (notwithstanding patents), eliminate deadweight losses due to market power and preclude hold-up problems that would face inventors not capable of producing final goods in-house. In contrast to Melitz (2003), firms are equally productive at making final goods but differ in their cost of inventing production processes.

The earliest related work considers the thicket problem in a partial equilibrium environment. In an influential paper, Shapiro (2001) notes that when a producer must license multiple patents to develop a given technology, he faces the same problem as a producer who must buy multiple perfect-complement inputs controlled by separate monopolists. In this classic “economic theory of complements” (Cournot 1838), the production costs get marked up multiple times over marginal costs, generating more deadweight losses than if the inputs were all controlled by a single monopolist. Companies may therefore gain by forming “patent pools” to eliminate the multiple-markup problem. Lerner and Tirole (2004) show that efficient pools typically increase welfare when patents are complements (as in the case discussed by Shapiro) and decrease welfare when patents are substitutes.<sup>15</sup>

The two most closely related papers are the analysis of patent thickets by Ayres and that are relatively easy to distinguish from other compounds. Litigation more often centers on direct copying (Cotropia and Lemley 2008) because the US Food and Drug Administration (FDA) requires generic entrants to both imitate directly a pioneer-branded-company’s compound (to avoid going through clinical trials), and to prove a pioneer-branded-company’s compound patents invalid (if they have not yet expired).

<sup>14</sup>The highly controversial *NTP, Inc. v. Research in Motion, Ltd.* (E.D. Va. 2003) case arose in this industry. It eventually concluded with a 2006 settlement, where the maker of the Blackberry (RIM) paid \$612.5 million in license fees to NTP, a patent “holding company” (i.e. troll), rather than discontinue selling the Blackberry. See Shapiro (2010) for further discussion.

<sup>15</sup>In the pure substitutes case, there is no thicket problem and a patent pool operates much like a price-fixing cartel.

Parchomovsky (2007) and the analysis of patent trolls by Reitzig, Henkel and Heath (2007). Ayres and Parchomovsky (2007) analyze a stylized model of patent thickets using a traditional marginal-social-cost, marginal-social-benefit framework, and assess the relative value of using price-based regulation (i.e., patent fees) versus quantity-based regulation (i.e., quotas) under uncertainty. As in my paper, their model yields an equilibrium thicket and a well-defined notion of an “optimal” rate of patenting. However, their model does not microfound the sources of marginal benefits and costs, nor does it consider the impact of trolls. It would be very interesting to revisit their “fees vs. quotas” question in an extension.

In contrast, Reitzig, Henkel and Heath (2007) study the behavior of an inventor and patent trolls in a sequential-move game. In their model, as in mine, inventors may be unable to avoid infringing. Thus, certain firms (“trolls”) may benefit by trying to be infringed, so they can sue for damages. Using this model, Reitzig, Henkel and Heath (2007) argue that court rules for assigning damages benefit trolls by refusing to consider the costs of inventing around patents. Their model, oriented toward highlighting legal inequities, includes far more detail about an inventor’s attempts to determine whether patents block entry and about inventing around them. In contrast, however, it treats rates of patenting (i.e., thickets) as exogenous, does not allow firms to choose whether to be inventors or trolls, and does not formalize conditions for when it is feasible to be a troll.<sup>16</sup>

This paper also contributes to the literature discussing the appropriate nature of patent enforcement. Numerous economists have analyzed optimal patent length (Nordhaus 1969), patent scope (Klemperer 1990; Gilbert and Shapiro 1990), patent damages (Ayres and Klemperer 1999; Schankerman and Scotchmer 2001; Anton and Yao 2007; Choi 2009; Henry and Turner 2010), etc., and numerous legal scholars have analyzed optimal interpretation and application of aspects of patent law, such as the doctrine of equivalents (Merges and Nelson 1990), standards for non-obviousness (Rhodes 1991) and the computation of damages (Werden, Froeb and Beavers 1999). In contrast to my work, these analyses have typically focused on how optimal patent policy trades off the incentives to invent versus the deadweight losses due to the grant of market power that is necessary to generate those incentives.

---

<sup>16</sup>Reitzig, Henkel and Schneider (2010) also argue that trolls are endemic to markets for technology.

## 2. The Model

There are two categories of risk-neutral agents: a representative consumer and firms. The representative consumer has preferences over a numeraire good and all varieties of a continuum of goods indexed by  $\omega$  in  $[0,1]$ . Specifically, it has reservation value  $V$  for a single unit of all varieties on the continuum. Firms also fall on the same continuum. Invention and production are vertically integrated within the same firm. To make final-good  $\omega$ , firm  $\omega$  must use technology  $\omega$  to invent production process  $\omega$ . There is also a mass of old, free technology.

If it makes an invention, I say a firm is *active*. To make a final good, an active firm must also adapt other technology to *complete* its own invention. Completion, whose direct cost is normalized to zero, requires complementary input technology. With probability  $1 - \alpha$ , where  $\alpha \in (0, 1)$ , complementary technology comes from the old, free technology. With probability  $\alpha$ , complementary input technology is a random uniform draw from the continuum of new technologies. Complementarity is perfect—it is impossible to produce without the adaptations. Moreover, the producer can not determine whether the complementary technology it uses is old or new until after it has produced and sold. Intuitively, I assume that each firm can neither control nor predict how its effort to use its invention to make final goods will use other technology. I interpret  $\alpha$  as the *level of input complementarity*.

Given demand, each active firm with a completed invention can perfectly price discriminate, charging price  $V$  for its good and earning revenue  $V$ . Hence, a firm's revenue from production is  $V$  if it has completed its invention and 0 otherwise. This specification has the virtue of shutting down beneficial effects of product diversity on utility, inappropriability problems due to benefits that spill over to consumers, and monopoly deadweight losses.

Firm  $\omega$  may invent at exogenous cost  $b\omega$ , with  $b > V$ . Denote the fraction of firms that invent as  $\omega_I^*$  and call this the *rate of invention*. Note that because the size of the continuum of technologies is normalized to 1, the rate and *level of invention* coincide. The *first-best* rate of invention obtains if and only if all firms for whom revenue matches or exceeds direct invention cost ( $V > \omega b$ ) invent, and other firms do not. Hence, define  $\omega_{FB}^* = \frac{V}{b}$ .

Firm  $\omega$  may also obtain a patent for technology  $\omega$  for fee  $F$ . Patent fees are rebated back



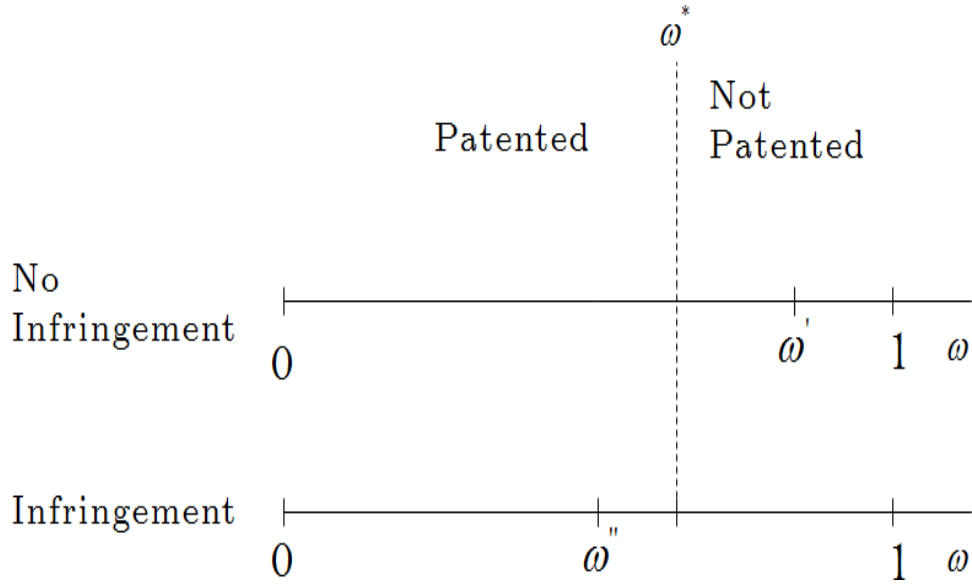


Figure 1: *Complementary Technology and Infringement*

to the representative consumer in lump sum. If a firm's adaptation of technology results in infringement of patented technology, then the patentee has the choice to initiate a dispute. Define the *level of patenting*,  $\omega_p^*$ , as the fraction of points on the  $[0, 1]$  continuum such that a given firm expects the technology to be patented. The *ex ante* likelihood that use of a given piece of new complementary technology creates a dispute equals  $\omega_p^*$ .

Figure 1 illustrates the simple case where all firms with  $\omega \leq \omega_I^*$  both invent and patent, so that  $\omega_I^* = \omega_P^* \equiv \omega^*$ . The continuum of technologies is represented on the horizontal axis. All technology such that  $\omega \leq \omega^*$  is patented, while technology  $\omega > \omega^*$  is free for use in adaptation by all producers. In the top panel, the complementary technology is  $\omega'$ . Since  $\omega' > \omega^*$ , complementary technology is not patented, so there is no infringement. In the bottom panel, complementary technology  $\omega'' \in [0, \omega^*]$ , so there is infringement.

Patent litigation is a simple sequential-move game. If infringement occurs, the patentee decides whether to pay cost  $\frac{C_D}{2} \in (0, V)$  to initiate a suit. Defending a suit carries unavoidable cost  $\frac{C_D}{2}$ .<sup>17</sup> Patents subject to infringement disputes are enforced perfectly. The

<sup>17</sup>Implicitly, this assumes that the defendant is always better off spending  $\frac{C_D}{2}$  to mount some defense rather than capitulating to the patentee's initial demands. For simplicity, I do not model that part of litigation. Note that, in practice, costs are not always symmetric. Provided both the patentees and defendants credibly try to improve their own outcomes by spending resources to contest disputes, however, my welfare results depend only on the total dispute cost. These costs are motivated by costs of discovery and coordination of

infringer pays the patentee royalty damages of  $rV$ , where  $r \in (0, 1)$  is the *rate of patent enforcement*. Hence, the patentee gains  $rV - \frac{C_D}{2}$  while the defendant loses  $rV + \frac{C_D}{2}$ .<sup>18</sup> It is credible to initiate a suit if and only if  $r \geq \frac{C_D}{2V} \equiv r_C$ .<sup>19</sup>

The timing is as follows. First, the level of demand, the costs and complementarities of production technology, the costs of disputes, patent fees and the rate of patent enforcement are given exogenously.<sup>20</sup> Second, firms choose whether to invent and whether to patent. Third, active producers complete their inventions, produce and sell. Fourth, patentees learn which patents are infringed and choose whether to initiate litigation. Last, litigation concludes and final payouts are realized.

For firm  $\omega$ , define the expected profit from inventing as  $\pi_I(\omega, \omega_P^*, r)$ . The firm pays cost  $b\omega$  to invent. Upon completing the invention and selling output, it achieves revenue  $V$ . In completing its invention, it uses a new technology with probability  $\alpha$ . Fraction  $\omega_P^*$  of new technologies are patented. If sued for infringement, it pays expected damages  $rV$  and dispute cost  $\frac{C_D}{2}$ . The firm is sued if and only if litigation is credible. Hence, we have

$$\pi_I(\omega, \omega_P^*, r) = \begin{cases} V - b\omega & \text{if } r < r_C \\ V - b\omega - \alpha\omega_P^* \left( rV + \frac{C_D}{2} \right) & \text{if } r \geq r_C. \end{cases} \quad (1)$$

Define the expected profit from patenting as  $\pi_P(\omega_I^*, r, F)$ . By patenting, the firm pays the patent fee  $F$  and earns expected damages for itself. The total quantity of infringements is  $\alpha\omega_I^*\omega_P^*$ , so it expects  $\alpha\omega_I^*$  for itself. If infringed, the patentee sues and earn an expected profit of  $rV - \frac{C_D}{2}$ , if and only if litigation is credible. Hence, we have

$$\pi_P(\omega_I^*, r, F) = \begin{cases} -F & \text{if } r < r_C \\ \alpha\omega_I^* \left( rV - \frac{C_D}{2} \right) - F & \text{if } r \geq r_C. \end{cases} \quad (2)$$

Define the *critical mass* of invention as the minimum level of invention such that the expected

---

legal strategy.

<sup>18</sup>With risk-neutral parties, imperfect enforcement would introduce only cosmetic differences to the analysis. It is entirely appropriate to think of  $rV$  as an expected payment from the defendant to the patentee and  $C_D$  as the expected total costs of a suit, including suits litigated to trial as well as those settled out of court after some initial costs are paid.

<sup>19</sup>Our assumptions on parameters guarantee that  $r_C \in (0, 1)$ .

<sup>20</sup>A planner wishing to intervene would do so at this stage, choosing patent fees and/or the rate of enforcement conditional on the other exogenous parameters.

profit from patenting is non-negative,

$$\underline{\omega}_I^* = \frac{F}{\alpha \left( rV - \frac{C_D}{2} \right)}.$$

Finally, define  $\Pi(\omega, \omega_I^*, \omega_P^*, r, F) = \pi_I(\omega, \omega_P^*, r) + \pi_P(\omega_I^*, r, F)$  to be the total expected profit from inventing and patenting.

I analyze no-troll (NT) and troll (T) versions of the model. In the no-troll model, a firm must invent to be eligible to get a patent protecting its technology. In the troll model, all technology is eligible for patent protection, so any firm  $\omega$  may patent technology  $\omega$  regardless of whether it invents. Denote the equilibrium levels of invention and patenting as  $\{\omega_I^*(NT), \omega_P^*(NT)\}$  for the no-troll case, and  $\{\omega_I^*(T), \omega_P^*(T)\}$  for the troll case. Define the *rate of patenting*  $\rho$  as the fraction of technology patented among technology that is eligible for patent protection. In the no-troll case, the equilibrium rate of patenting  $\rho(NT) = \frac{\omega_P^*(NT)}{\omega_I^*(NT)}$ , while in the troll case,  $\rho(T) = \omega_P^*(T)$ .

### 3. Analysis Without Trolls

If it is not possible to patent without inventing, then expected profits from patenting may influence the decision to invent. It is therefore simplest to work through the set of decisions recursively. Start by letting all decisions to invent be given, and consider the decision to patent conditional on the rate of invention  $\omega_I^*$ .

Since firm  $\omega$ 's expected profit from patenting does not depend on  $\omega$ , each inventing firm faces the same decision. If the rate of invention is above critical mass ( $\omega_I^* > \underline{\omega}_I^*$ ), then the expected profit from patenting is strictly positive and all inventing firms patent. If the rate of invention is below critical mass, the expected profit from patenting is strictly negative and no inventing firms patent. If the rate of invention is exactly at critical mass, the expected profit from patenting is identically zero, and an inventing firm is indifferent between patenting and not patenting. I assume that firms may patent or may not patent in this case.

Now consider the decision to invent. Since firms are atomistic, each treats the overall rate of invention and patenting as given. For firm  $\omega$ , if the expected profit from patenting

is negative, it will choose to invent (and not patent) if and only if its expected profit from inventing is non-negative [ $\pi_I(\omega, \omega_P^*, r) \geq 0$ ]. If the expected profit from patenting is positive, however, then firm  $\omega$  will choose to invent and patent if and only if the *total* expected profit is non-negative [ $\Pi(\omega, \omega_I^*, \omega_P^*, r, F) \geq 0$ ]. Notably, there may be firms whose expected profit from invention is negative but whose expected profit from patenting more than makes up for that loss, so that it is better to invent and patent than to do neither. If the expected profit from patenting is identically zero, then firm  $\omega$  will invent if and only if  $\pi_I(\omega, \omega_P^*, r, F) \geq 0$ .<sup>21</sup>

The following chart highlights how the decisions to invent and patent are interdependent.

$\pi_I(\omega, \omega_P^*, r)$	$\pi_P(\omega_I^*, r, F)$	Optimal Decision
+ or 0	+	Invent and Patent
+ or 0	0	Invent and Sometimes Patent
+ or 0	-	Invent and Not Patent
-	+	Invent and Patent iff $\Pi(\omega, \omega_I^*, \omega_P^*, r, F) \geq 0$
-	0	Not Invent and Not Patent
-	-	Not Invent and Not Patent

### 3.1. Equilibrium

Equilibrium consists of rates of invention and patenting  $\{\omega_I^*(NT), \omega_P^*(NT)\}$  such that all firms make optimal inventing and patenting decisions, taking the rates of invention and patenting as given. Firms with low values of  $\omega$  have higher expected profits and are more likely to invent. Therefore, for equilibria where the expected profit from patenting is strictly positive, equilibrium consists of a cutoff firm  $\omega = \omega_I^*(NT)$  such that

$$\begin{aligned}
\Pi_I(\omega, \omega_I^*(NT), \omega_P^*(NT), r, F) &> 0 && \text{for all } \omega < \omega_I^*(NT) \\
\Pi_I(\omega_I^*(NT), \omega_I^*(NT), \omega_P^*(NT), r, F) &= 0 && \\
\Pi_I(\omega, \omega_I^*(NT), \omega_P^*(NT), r, F) &< 0 && \text{for all } \omega > \omega_I^*(NT),
\end{aligned} \tag{3}$$

and where the level of patenting  $\omega_P^*(NT) = \omega_I^*(NT)$ . Thus, all firms with  $\omega \leq \omega_I^*(NT)$  invent, those with  $\omega > \omega_I^*(NT)$  do not invent, and every inventing firm patents.

<sup>21</sup>For technical convenience, I assume that invention is certain if the expected profit from invention (or invention and patenting in the no-troll model) is non-negative, while patenting is certain only if the expected profit from patenting is strictly positive.

On the other hand, in equilibria where the expected profit from patenting is negative or zero, equilibrium is a cutoff firm  $\omega = \omega_I^*(NT)$  and a level of patenting  $\omega_P^*(NT)$  such that

$$\begin{aligned}\pi_I(\omega, \omega_P^*(NT), r) &> 0 && \text{for all } \omega < \omega_I^*(NT) \\ \pi_I(\omega_I^*(NT), \omega_P^*(NT), r) &= 0 \\ \pi_I(\omega, \omega_P^*(NT), r) &< 0 && \text{for all } \omega > \omega_I^*(NT).\end{aligned}\tag{4}$$

and

$$\omega_P^*(NT) \in \begin{cases} [0, \omega_I^*(NT)] & \text{if } \pi_P(\omega_I^*(NT), r, F) = 0 \\ 0 & \text{if } \pi_P(\omega_I^*(NT), r, F) < 0. \end{cases}$$

If  $\pi_P(\omega_I^*(NT), r, F) = 0$ , then equilibrium invention equals critical mass,  $\omega_I^*(NT) = \underline{\omega}_I^*$ , and  $\omega_P^*(NT) \in [0, \omega_I^*(NT)]$  solves  $\pi_I(\underline{\omega}_I^*, \omega_P^*(NT), r) = 0$ .

Depending upon the rate of enforcement  $r$  and the patent fee  $F$ , equilibrium may take either form. If litigation is not credible ( $r < r_C$ ), or if it is just barely credible ( $r = r_C$ ) and the patent fee is positive, then the expected profit from patenting is negative regardless of  $\omega_I^*$ , so  $\omega_P^*(NT) = 0$  is the unique level of equilibrium patenting. In this case there is no patent thicket, and solving for the (unique) equilibrium invention according to (4) yields  $\omega_I^*(NT) = \frac{V}{b}$ , that is, equilibrium invention satisfies the first-best. In addition, conditional on the first-best rate of invention, the expected profit from patenting is negative for a sufficiently high patent fee even if litigation is credible. We see that  $\pi_P(\omega_{FB}^*, r, F) < 0$  if

$$F > \frac{\alpha V \left( rV - \frac{C_D}{2} \right)}{b} \equiv \bar{F}.\tag{5}$$

For such *prohibitive* values of  $F > \bar{F}$ ,  $\omega_P^*(NT) = 0$  and  $\omega_I^*(NT) = \omega_{FB}^*$  also form a unique equilibrium.

Let litigation be credible ( $r > r_C$ ) and let the patent fee be non-prohibitive. Start by considering the case where  $F$  is very close to zero. Because patents are cheap, the critical mass of inventions is low and firms patent all inventions. Setting  $\omega_I^* = \omega_P^*$  and solving (3), we find the equilibrium marginal firm,

$$\omega_I^*(NT) = \omega_P^*(NT) = \frac{V - F}{b + \alpha C_D}.\tag{6}$$

The expected profit from patenting is non-negative provided

$$F \leq \frac{\alpha V \left( rV - \frac{C_D}{2} \right)}{b + \alpha \left( rV + \frac{C_D}{2} \right)} \equiv \underline{F}.$$

Hence, the equilibrium levels of invention and patenting given by (6) holds for any  $F \leq \underline{F}$ .<sup>22</sup>

Clearly, the rate of invention in (6) is below the first-best.<sup>23</sup> With a low patent fee, the patent system generates a costly thicket—it is privately optimal to patent, but patenting generates costly disputes. The direct effect of the patent system is a tax of the expected total dispute costs,  $\alpha C_D \omega_I^*(NT)$ , per invention. The marginal firm earns zero additional surplus, while imposing a negative externality on other firms and producers by making their adaptation of complementary inputs more costly.

The rate of invention in (6) is decreasing in the patent fee. When patents are very cheap, the marginal firm generates a negative expected profit from invention that exactly offsets the positive expected profit from patenting  $[\pi_I(\omega_I^*(NT), \omega_P^*(NT), r) = -\pi_P(\omega_I^*(NT), r, F)]$ . Although the patent thicket lowers the rate of invention, patenting subsidizes invention at the margin. As  $F$  increases, that subsidy declines, reducing invention and patenting.<sup>24</sup>

As  $F$  increases past  $\underline{F}$ , the *critical-mass problem* emerges. Equilibrium follows (4) and is characterized by *higher* levels of invention but *lower* levels of patenting. Start with the intuition. If  $F < \underline{F}$ , then the critical-mass firm (located precisely at  $\omega = \underline{\omega}_I^*$ ) earns, in equilibrium, a strictly positive expected profit from inventing, equilibrium invention exceeds critical mass and patenting is strictly profitable for all inventing firms. If  $F \in (\underline{F}, \bar{F}]$ , however, the critical-mass firm earns a negative expected profit from inventing unless  $\omega_P^*(NT) < \underline{\omega}_I^*$ , that is, unless some inventions are not patented (so that the firm suffers fewer inadvertent infringements). The only way this can happen is for the expected profit from patenting to be zero, so equilibrium invention equals critical mass,  $\omega_I^*(NT) = \underline{\omega}_I^*$ .

---

<sup>22</sup>The uniqueness of the equilibrium follows from the fact that  $\Pi(\omega, \omega_I^*, \omega_P^*, r, F)$  is monotone decreasing in  $\omega_I^*(NT)$ , so it may equal zero at most once.

<sup>23</sup>Our assumptions on parameter ranges guarantee that, with free patents ( $F = 0$ ), the equilibrium  $\omega_I^*(NT) < 1$  for all model specifications in this paper.

<sup>24</sup>The rate of enforcement,  $r$ , does not affect the rate of invention for low  $F < \underline{F}$ . Firms suffer from a higher enforcement rate (facing higher expected damages when sued) and benefit from a higher enforcement rate (expecting higher damages when they sue for infringement), and the effects exactly offset each other.

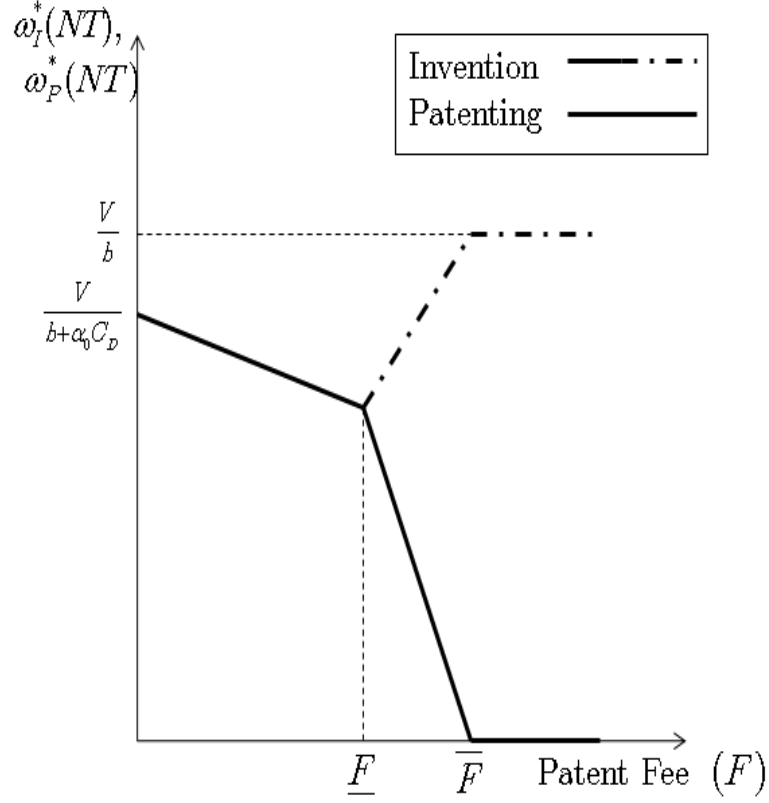


Figure 2: *Equilibrium Invention and Patenting, Credible Litigation, No Trolls*

To find the equilibrium rate and level of patenting, set  $\omega_P^* = \rho\omega_I^*$  and rewrite the firm's expected profit from inventing as

$$\pi_I(\omega, \omega_P^*, r) = V - b\omega - \alpha\rho\omega_I^* \left( rV + \frac{C_D}{2} \right).$$

Solving for the cutoff firm  $\omega_I^*(NT)$ , according to (4), we have

$$\omega_I^*(NT) = \frac{V}{b + \alpha\rho \left( rV + \frac{C_D}{2} \right)}.$$

Setting  $\omega_I^*(NT) = \underline{\omega}_I^*$  and solving, we find the equilibrium rate of patenting

$$\rho(NT) = \frac{\alpha V \left( rV - \frac{C_D}{2} \right) - Fb}{F \left[ \alpha \left( rV + \frac{C_D}{2} \right) \right]}, \quad (7)$$

and the level of patenting,

$$\omega_P^*(NT) = \rho(NT)\underline{\omega}_I^* = \frac{\alpha V \left( rV - \frac{C_D}{2} \right) - Fb}{\alpha^2 \left( rV - \frac{C_D}{2} \right) \left( rV + \frac{C_D}{2} \right)}.$$

Figure 2 illustrates how equilibrium levels of invention and patenting change with  $F$ .<sup>25</sup> Invention and patenting diverge at  $F = \underline{F}$ , and invention is a non-monotonic function of  $F$ .<sup>26</sup> Critical-mass invention is represented by the upward-sloping dashed line. With a prohibitive patent fee, the first-best level of invention obtains as an equilibrium, without any patenting, because this level of invention is below critical mass.

### 3.2. Welfare

Equilibrium welfare is

$$W(NT) = \int_0^{\omega_I^*(NT)} \pi_I(\omega, \omega_P^*(NT), r) d\omega + \omega_P^*(NT) [\pi_P(\omega_I^*(NT), r, F) + F] d\omega,$$

where  $F$  is added back in because all patent fees are rebated back to the representative consumer. This simplifies to a quadratic function of the level of invention and a linear function of the level of patenting,

$$W(NT) = V\omega_I^*(NT) - \frac{b}{2}\omega_I^*(NT)^2 - \alpha C_D \omega_I^*(NT)\omega_P^*(NT).$$

Clearly, this is decreasing in the level of patenting, all else equal. More patents yield more disputes. First-best invention  $\omega_{FB}^*$  maximizes this function for the case  $\omega_P^* = 0$ .

Welfare is never a monotonic function of invention, however, and is non-monotonic in patenting for the interesting case  $F < \underline{F}$ , where  $\omega_I^*(NT) = \omega_P^*(NT)$ . In particular,

$$W(NT) = V\omega_I^*(NT) - \left( \frac{b}{2} + \alpha C_D \right) \omega_I^*(NT)^2.$$

---

<sup>25</sup>For intermediate  $F \in (\underline{F}, \bar{F}]$ , where the rate of invention is at critical mass, an increase in  $r$  raises the expected profit from patenting, which lowers the critical mass rate of invention.

<sup>26</sup>One unfortunate feature of the model is that the equilibrium for  $F \in (\underline{F}, \bar{F}]$  is not unique. Namely, any combination of firms such that the total equals  $\rho(NT)$  forms an equilibrium. Thus, we cannot say if the patents are obtained by relatively productive (low- $\omega$ ) firms or not.



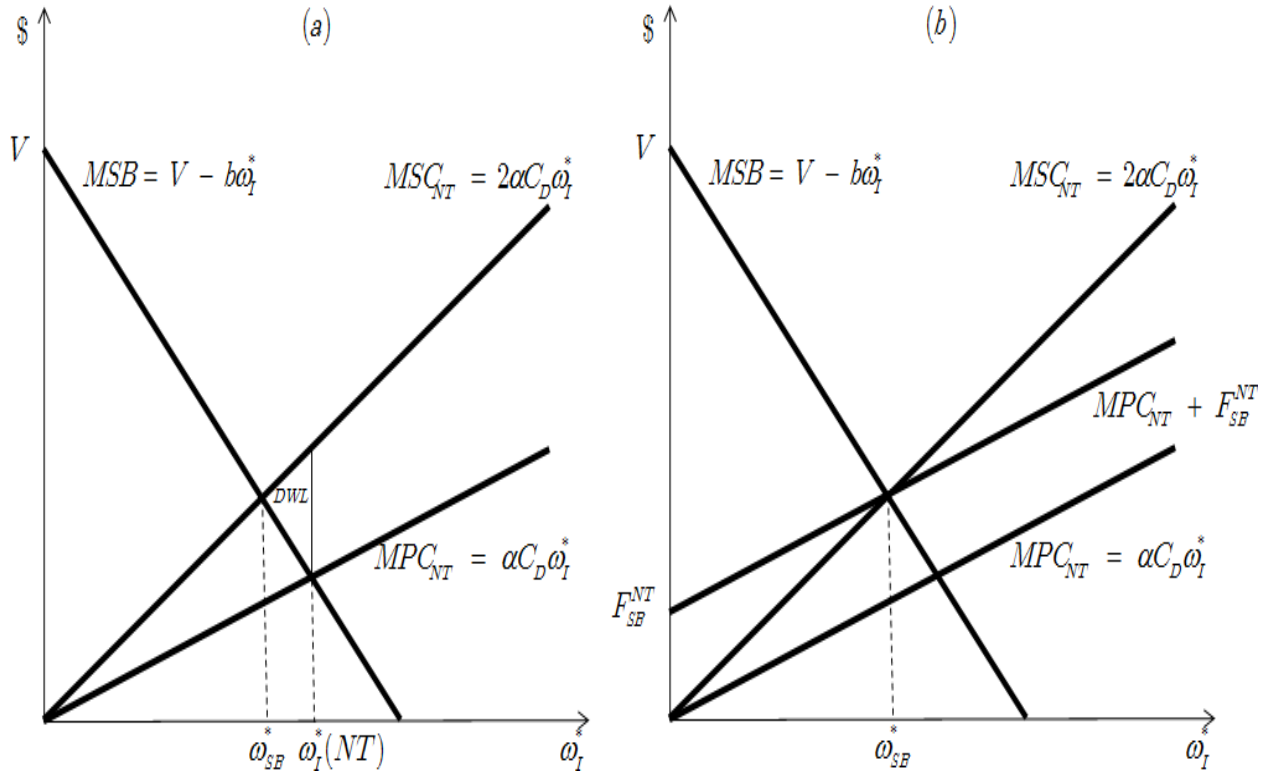


Figure 3: *Equilibrium and Welfare,  $F < \underline{E}$ , No Trolls*

Here, first-best welfare is infeasible. This function achieves a maximum at

$$\omega_{SB}^* = \frac{V}{b + 2\alpha C_D},$$

where the  $SB$  subscript denotes the *second-best* level of invention. Conditional on equilibrium patenting  $\rho(NT) = 1$ , this is the optimal level of invention (if feasible). This level of invention is achieved with patent fee  $F_{SB}^{NT} = \alpha C_D \omega_{SB}^*$  and is feasible if  $F_{SB}^{NT} < \underline{E}$ . It balances the direct gains from new inventions versus the additional dispute costs they yield. The patent fee  $F_{SB}^{NT}$  is Pigouvian, equaling the expected externality of an additional patent. This yields a local, but not a global, maximum. Welfare is strictly higher for any prohibitive patent fee, which yields first-best invention and zero patenting.

Figures 3(a)-(b) illustrate the externality problem created by patent thickets for the case where all inventions are patented,  $F < \underline{E}$ . The level of invention and patenting ( $\omega_I^* = \omega_P^*$ ) is shown on the x-axis. Notwithstanding disputes, the marginal social benefit of an invention is

$MSB = V - b\omega_I^*$ . The marginal social cost is  $MSC_{NT} = 2\alpha C_D$ . Intuitively, each additional patented invention adds an average of  $2\alpha$  inadvertent infringements, because the invention (when completed) infringes with probability  $\alpha$  and the patent gets infringed with probability  $\alpha$ . Each of these costs  $C_D$ . Damage payments net out to zero, so they do not show up in these formulas. The marginal private cost, however, is  $MPC_{NT} = \alpha C_D$ , because an inventing firm only internalizes half of the expected dispute costs created by an additional invention and patent.

Panel (a) shows the equilibrium level of invention for the case of free patents  $F = 0$ . Because the thicket exerts a negative externality, equilibrium invention  $\omega_I^*(NT)$  is too high and yields a standard dead-weight loss. Panel (b) illustrates equilibrium with the Pigouvian fixed fee  $F_{SB}^{NT}$ . This raises an inventing firm's cost by the size of the externality and achieves the second best level of invention.

### 3.3. Comparative Statics

Consider first the effect of a change in the level of input complementarity. An increase in  $\alpha$  increases the incidence of inadvertent infringements. For low  $F$ , when all inventions are patented, the primary effect of higher input complementarity is to increase the frequency of costly disputes, which tax invention and lower the equilibrium rate of invention. For higher  $F$ , higher input complementarity makes the expected profit from patenting higher, which lowers the critical mass of inventions needed to support patents and increases  $\underline{F}$ , the maximum patent fee at which all inventions are patented. The level of input complementarity is irrelevant when  $F$  is prohibitive, as there are no patents. We have the following proposition.

**Proposition 1.** *In the no-troll model, the equilibrium rate of invention is decreasing in the level of input complementarity, and is strictly decreasing whenever there is some patenting in equilibrium.*

Figure 4 illustrates the effects of an increase in input complementarity from  $\alpha_0$  to  $\alpha_1$ . The function  $\omega_I^*(NT) = \frac{V-F}{b+\alpha C_D}$ , which gives equilibrium inventing for  $F < \underline{F}$ , shifts lower. The cutoff  $\underline{F}$  shifts out from  $\underline{F}_0$  to  $\underline{F}_1$ . The function for critical mass invention,  $\underline{\omega}_I^* = \frac{F}{\alpha\left(rV - \frac{C_D}{2}\right)}$ ,

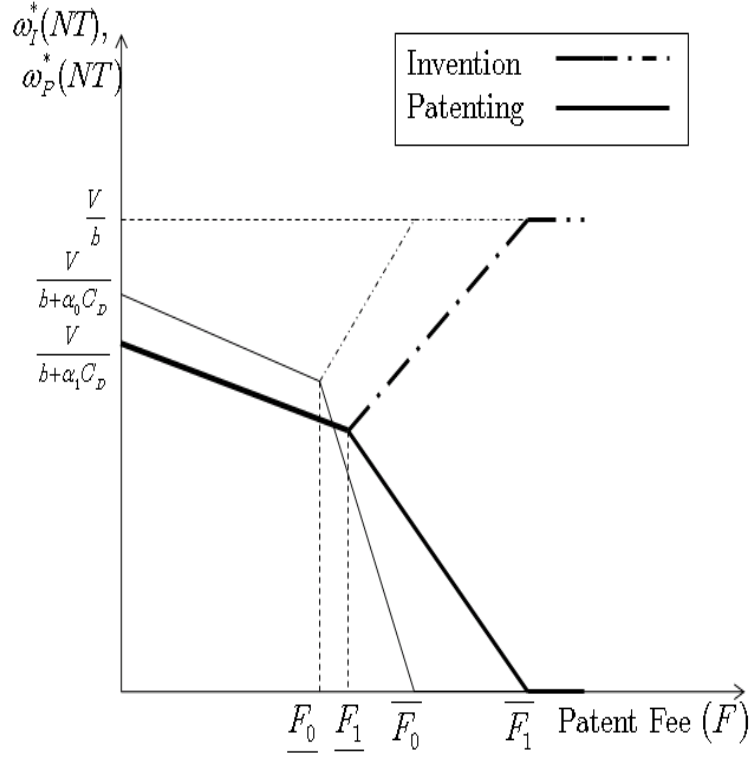


Figure 4: *The Effects of an Increase in the Level of Input Complementarity*

shifts lower. The cutoff  $\bar{F}$  shifts out from  $\bar{F}_0$  to  $\bar{F}_1$ .

A change in  $\alpha$  lowers the level of patenting  $\omega_P^*(NT)$  for low  $F$ , naturally, but the rate of patenting  $\rho(NT)$  remains at 1. For  $F \in (\underline{F}, \bar{F}]$ , increasing  $\alpha$  has two effects on the rate of patenting. First, as  $\alpha$  increases, the expected profit from patenting rises. This lowers the critical mass of inventions, which lowers the direct inventing costs,  $b\underline{\omega}_I^*$ , of the critical-mass firm. That firm can tolerate a higher level of patenting (notwithstanding the higher  $\alpha$ ), forcing the rate of patenting upwards. The second effect is that as  $\alpha$  rises, the per-invention cost from inadvertent infringement,  $\alpha \left( rV + \frac{C_D}{2} \right)$ , rises. This reduces the critical-mass firm's tolerance for patents, pushing the rate of patenting down. However, the total cost from infringement for the critical-mass firm is

$$\rho \underline{\omega}_I^* \alpha \left( rV + \frac{C_D}{2} \right) = \rho F \frac{rV + \frac{C_D}{2}}{rV - \frac{C_D}{2}},$$

which is *independent* of  $\alpha$ . Since  $\alpha$  lowers the direct inventing costs to this firm,  $\rho$  must rise to set this firm's expected profit from invention to zero. Hence, the first effect dominates

and we have the following result.

**Proposition 2.** *In the no-troll model, the equilibrium rate of patenting is increasing in the level of input complementarity, and is strictly increasing whenever the equilibrium rate of patenting is interior [ $\rho(NT) \in (0, 1)$ ].*

Clearly, when the patent fee is prohibitive, there is no patenting and a small increase in  $\alpha$  has no effect on the rate or level of patenting. Note that the effect of  $\alpha$  on the level of patenting is ambiguous. In Figure 4, for a patent fee just above  $\underline{F}_0$ , the increase in  $\alpha$  from  $\alpha_0$  to  $\alpha_1$  causes the level of patenting to fall.

An increase in the dispute cost,  $C_D$ , decreases the rate of invention when  $F$  is low, as it increases the implicit tax firms face. It increases the critical-mass rate of invention, however, as an increase in  $C_D$  lowers the expected profit from patenting by lowering the return to suing for infringement. For the same reason, an increase in the dispute cost lowers the rate of patenting when  $F \in (\underline{F}, \bar{F}]$ .<sup>27</sup> The dispute cost is irrelevant when  $F$  is prohibitive.

An increase in the value of output,  $V$ , increases invention when  $F$  is low and when  $F$  is prohibitive, but lowers the critical-mass rate of invention. Intuitively, when  $V$  increases (for  $F \in (\underline{F}, \bar{F}]$ ), the expected profit from suing for infringement, and thus the expected profit from patenting, increases. This lowers the critical mass rate of invention but raises the rate of patenting. When  $F$  is prohibitive, the rate of invention achieves the first-best level, which increases with  $V$ .

An increase in the cost of invention,  $b$ , lowers invention when  $F$  is low and when  $F$  is prohibitive. It does not affect the critical-mass rate of invention, as the returns to patenting do not depend on the invention cost. The rate of patenting does fall with  $b$  (for  $F \in (\underline{F}, \bar{F}]$ ), however, as  $b$  increases the level of direct invention costs to the cutoff inventing firm.

---

<sup>27</sup>See the appendix for calculations of derivatives of  $\rho(NT)$ .

## 4. Analysis With Trolls

Now consider a slightly different version of the model where a firm may choose to patent without inventing. Here, profits from patenting do not affect the decision to invent, or vice-versa. As a result, decisions never depend on the total expected profit  $\Pi(\omega, \omega_I^*, \omega_P^*, r, F)$ , and firms allocate themselves differently (noted in **bold**) than in the model without trolls.

$\pi_I(\omega, \omega_P^*, r)$	$\pi_P(\omega_I^*, r, F)$	Optimal Decision
+ or 0	+	Invent and Patent
+ or 0	0	Invent and Sometimes Patent
+ or 0	–	Invent and Not Patent
–	+	<b>Not Invent and Patent (Troll)</b>
–	0	<b>Not Invent and Sometimes Patent (Troll)</b>
–	–	Not Invent and Not Patent

### 4.1. Equilibrium

Equilibrium is a rate of invention  $\omega_I^*(T)$  and patenting  $\omega_P^*(T)$  such that

$$\begin{aligned}
 \pi_I(\omega, \omega_P^*(T), r) &> 0 && \text{for all } \omega < \omega_I^*(T) \\
 \pi_I(\omega_I^*(T), \omega_P^*(T), r) &= 0 \\
 \pi_I(\omega, \omega_P^*(T), r) &< 0 && \text{for all } \omega > \omega_I^*(T),
 \end{aligned} \tag{8}$$

and

$$\omega_P^*(T) \in \begin{cases} 1 & \text{if } \pi_P(\omega_I^*(T), r, F) > 0 \\ [0, 1] & \text{if } \pi_P(\omega_I^*(T), r, F) = 0 \\ 0 & \text{if } \pi_P(\omega_I^*(T), r, F) < 0. \end{cases}$$

In the special case where  $\pi_P(\omega_I^*(T), r, F) = 0$ , then equilibrium invention equals critical mass and  $\omega_P^*(T) \in [0, 1]$  solves  $\pi_I(\omega_I^*, \omega_P^*(T), r) = 0$ .

As in the model without trolls, if litigation is not credible, then there is no patenting and invention achieves the first-best. The same is true if the patent fee is sufficiently high. Since there is no patenting, there are no trolls. Hence, the troll option does not adversely affect invention, patenting or welfare if  $r \leq r_C$  or the patent fee is prohibitive.<sup>28</sup>

---

<sup>28</sup>Note that if  $r = r_C$ , then  $\bar{F} = 0$ .

In contrast, consider the case where  $r > r_C$  and  $F$  is close to zero, so that patents are so cheap that inventing firms patent all inventions. In contrast to the case without trolls, all remaining technology is also patented in equilibrium, so that  $\omega_P^*(T) = 1$ . The expected profit from inventing and patenting is non-negative [ $\Pi(\omega, \omega_I^*(T), 1, r, F) \geq 0$ ] if

$$\omega \leq \frac{V - F - \alpha \left( rV + \frac{C_D}{2} \right)}{b - \alpha \left( rV - \frac{C_D}{2} \right)} \equiv \hat{\omega}_I(T),$$

but this does not determine equilibrium invention. Instead, the expected profit from being a troll is higher if  $\pi_I(\omega, 1, r) < 0$ , that is, for

$$\omega > \frac{V - \alpha \left( rV + \frac{C_D}{2} \right)}{b} \equiv \omega_I^*(T). \quad (9)$$

This latter cutoff is the equilibrium rate of invention. Firms with  $\omega \leq \omega_I^*(T)$  invent and those with  $\omega > \omega_I^*(T)$  become trolls. Note that  $\omega_I^*(T)$  does not depend on  $F$ , because the decisions to invent and patent are separate. The rate of invention is decreasing in  $r$  (instead of being independent of  $r$  as in the no-troll case), because inventing firms expect to pay damages for inadvertent infringement, but do not factor expected damages received (from owning a patent) into the decision to invent. With this rate of invention, the expected profit from patenting is non-negative provided

$$F \leq \frac{\alpha \left( rV - \frac{C_D}{2} \right) \left[ V - \alpha \left( rV + \frac{C_D}{2} \right) \right]}{b} \equiv \underline{F}^T.$$

Hence,  $\omega_I^*(T)$  according to (9) and  $\omega_P^*(T) = 1$  forms a unique equilibrium for  $F \leq \underline{F}^T$ .

The measure of firms  $\omega \in (\hat{\omega}_I(T), \omega_I^*(T)]$  finds invention, in and of itself, unprofitable. As a result, they choose to be trolls in equilibrium. The effect of unproductive entrepreneurship is a drop in the level of invention equaling

$$\omega_I^*(NT) - \omega_I^*(T) = \frac{V - F}{b + \alpha C_D} - \frac{V - \alpha \left( rV + \frac{C_D}{2} \right)}{b}.$$

Because firms with  $\omega \in (\omega_I^*(NT), \omega_I^*(T)]$  are inefficiently allocated out of productive inventive activity and into troll behavior, the economy loses  $V - b\omega - \alpha C_D \omega_I^*(NT) > 0$  on each invention

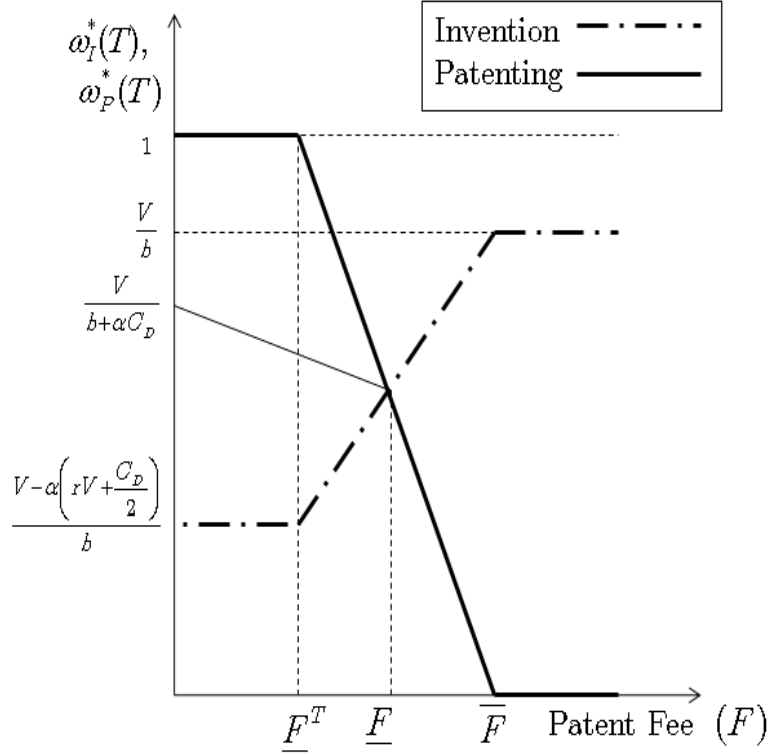


Figure 5: *Equilibrium Invention and Patenting, Credible Litigation, With Trolls*

not completed. Since all technology is patented, the measure of firms  $\omega \in (\omega_I^*(NT), 1]$  is inefficiently allocated out of doing nothing and into troll behavior.

If  $F > \underline{F}^T$ , there is a critical-mass problem. Patenting is unprofitable if the rate of invention is  $\omega_I^*(T) = \frac{V - \alpha \left( rV + \frac{C_D}{2} \right)}{b}$ . As  $F$  rises past  $\underline{F}^T$ , equilibrium invention equals critical mass,

$$\omega_I^*(T) = \frac{F}{\alpha \left( rV - \frac{C_D}{2} \right)},$$

and the level of patenting equals

$$\omega_P^*(T) = \frac{\alpha V \left( rV - \frac{C_D}{2} \right) - Fb}{\alpha^2 \left( rV - \frac{C_D}{2} \right) \left( rV + \frac{C_D}{2} \right)}.$$

These levels hold for all  $F \in [\underline{F}^T, \bar{F}]$ ,<sup>29</sup> and the effect of unproductive entrepreneurship is a

<sup>29</sup>Again, equilibrium is not unique because the identity of those who patent is not unique.

drop in the level of invention equaling

$$\omega_I^*(NT) - \omega_I^*(T) = \frac{V - F}{b + \alpha C_D} - \frac{F}{\alpha \left( rV - \frac{C_D}{2} \right)}.$$

Equilibrium invention and patenting, as a function of the patent fee, are shown in Figure 5.<sup>30</sup> For comparison, equilibrium invention and patenting are also shown for the case without trolls. We have the following proposition.

**Proposition 3.** *If all inventions are patented in equilibrium in the no-troll model, then the introduction of trolls lowers the equilibrium level of invention and raises the equilibrium level of patenting. If some inventions are not patented in equilibrium in the no-troll model, then the introduction of trolls has no effect on equilibrium invention or patenting.*

#### 4.2. Welfare

With trolls, welfare is

$$W(T) = V\omega_I^*(T) - \frac{b}{2}\omega_I^*(T)^2 - \alpha C_D \omega_I^*(T)\omega_P^*(T).$$

I compare this to welfare without trolls,  $W(NT)$ . The intuition is easiest to see for the case  $F < \underline{F}$ , where  $\omega_I^*(NT) = \omega_P^*(NT)$ .

Recall that we can write  $MSB = V - b\omega_I^*$ . With trolls, the rate of patenting is higher,  $\omega_P^*(NT) = 1$ . As a result, the total costs of disputes for a given level of invention,  $\alpha C_D \omega_I^*$ , are higher than in the no-troll case. Marginal social cost is a constant,  $MSC_T = \alpha C_D$ , and the “second best”  $\omega_{SB}^*(T) = \frac{V - \alpha C_D}{b}$  is different. Intuitively, because all technology is patented, each invention adds  $\alpha$  additional disputes, at cost  $C_D$  each. In contrast to the no-troll case, the marginal private cost  $MPC_T = \alpha \left( rV + \frac{C_D}{2} \right)$  is higher than marginal social cost.<sup>31</sup> Here, the decision to invent is separate from the decision to patent, so an inventing firm does not factor profits it receives from inadvertent infringements into its decision to invent.

<sup>30</sup>Note that it is always true that  $\underline{F}^T \leq \underline{F}$ .

<sup>31</sup>Note that  $r \geq r_C$ , so  $rV + \frac{C_D}{2} > C_D$ .



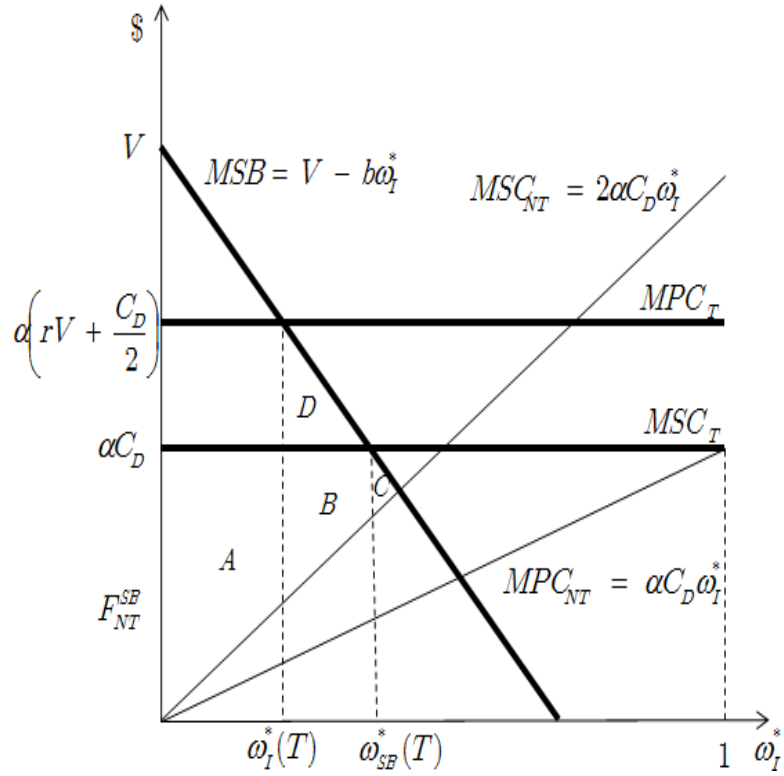


Figure 6: *Equilibrium and Welfare,  $F < \underline{F}^T$ , Trolls*

Figure 6 illustrates the welfare losses brought on by troll behavior. Relative to the second-best in the no-troll case, welfare is reduced by the sum of areas  $A$ ,  $B$  and  $C$  due to the increased costs (per invention) due to the higher number of patents.<sup>32</sup> Welfare is further reduced by area  $D$  due to unproductive entrepreneurship that lowers the equilibrium level of invention.<sup>33</sup>

For sufficiently high  $F$ , of course, invention is above critical mass, so  $\omega_P^*$  does not equal 1 and welfare may be the same as in the no-troll case. The following result summarizes the welfare consequences of trolls.

**Proposition 4.** *Equilibrium welfare is at least as low in the troll model as in the no-troll model. Welfare is strictly lower in the troll model for all levels of the patent fee such that all inventions are patented in the no-troll model.*

<sup>32</sup>For each piece of technology that is invented both with and without trolls,  $\omega \in [0, \omega_T^*(T)]$ , dispute costs rise by  $\alpha C_D[1 - \omega_P^*(NT)]$ .

<sup>33</sup>Note that  $\omega_{SB}^*(T)$  can be either higher or lower than  $\omega_{SB}^*$ . It is easier to illustrate welfare losses in a diagram for the case  $\omega_{SB}^*(T) > \omega_{SB}^*$ , which holds for sufficiently high  $b > 2(V - \alpha C_D)$ .

### 4.3. Comparative Statics

Just as in the case without trolls, the rate of invention is decreasing in the level of input complementarity. For  $F < \underline{F}^T$ , the rate of invention exceeds critical mass and falls with  $\alpha$  at rate  $-\frac{(rV + \frac{C_D}{2})}{b}$ . For  $F \in (\underline{F}^T, \overline{F}]$ , the rate of invention is at critical mass,  $\underline{\omega}_I^*$ , shown earlier to be decreasing in  $\alpha$ .

**Proposition 5.** *In the troll model, the equilibrium rate of invention is decreasing in the level of input complementarity, and is strictly decreasing whenever there is some patenting in equilibrium.*

For the rate of patenting, things change just a bit. Recalling the discussion just prior to Proposition 2, there remain two effects of an increase in  $\alpha$ . The invention cost of the critical-mass firm,  $b\underline{\omega}_I^*$ , falls. However, the total cost from infringement it faces,

$$\rho\alpha \left( rV + \frac{C_D}{2} \right),$$

increases with  $\alpha$ . The reason is that the level of patenting is no longer proportional to  $\underline{\omega}_I^*$  and therefore is not inversely proportional to  $\alpha$ . When  $\alpha$  is very high, the cost increase from the latter effect may dominate,<sup>34</sup> so that the rate of patenting falls. We have the following sufficient condition.

**Proposition 6.** *For sufficiently small  $\alpha < \frac{1}{4}$  in the troll model, the equilibrium rate and level of patenting are increasing in the level of input complementarity and are strictly increasing whenever the rate of patenting is interior [ $\rho(T) \in (0, 1)$ ] in equilibrium.*

Note that this same condition applies to the *level* of patenting in the no-troll model. Also, while the rate of patenting is the same as in the no-troll model, the identity of patentees changes. Here, some patentees may be trolls. Although the functions for invention and patenting differ for low  $F < \underline{F}$ , they are identical for higher  $F$ . Comparative statics are

---

<sup>34</sup>Which effect dominates depends on other parameters, and there is no alpha such that the latter effect always dominates. For example, it is obvious that  $\overline{F}$  is increasing in  $\alpha$ , so that the first effect always dominates for  $F$  very close to  $\overline{F}$ .

qualitatively the same, for other parameters, as in the no-troll model.<sup>35</sup>

## 5. Discussion

An important feature of this model is that it is possible to identify optimal levels of invention and patenting. Clearly, such levels will differ in different industrial organization circumstances and may differ in dynamic environments. As a result, the model lends itself to several natural extensions.

The most natural of these would consider the case where invention and production are not vertically integrated. If inventors specialize in invention but do not produce, they must license their inventions to final-goods producers to earn returns on their research efforts and expenditures. Lamoreaux and Sokoloff (2001) argue that the US patent system facilitated such specialization in the 19th Century, while Gans and Stern (2003) argue that such an “Ideas Factory” environment emerges naturally in cases where technology is excludable via intellectual property rights and where technology reinforces complementary assets held by incumbent firms. In a companion paper (Turner 2012), I study this case.<sup>36</sup> When inventors must patent and license their invention to earn returns, patents are positive for welfare—indeed, they are necessary for *any* equilibrium invention.<sup>37</sup>

In this version of the model, there are three notable differences. First, I require royalties to be “reasonable.” Specifically, United States courts set royalties to equal those from a hypothetical, pre-infringement, negotiation conducted between patentee and licensee. Since, in my model, bargains occur in equilibrium, I require royalties to equal those paid in those bargains.<sup>38</sup> Notably, this restricts possible equilibrium royalties—if set too high, a producer

---

<sup>35</sup>The exception is that, for  $F \leq \underline{F}^T$ ,  $\omega_P^*(T) = 1$  is unaffected by these parameters.

<sup>36</sup>Somewhat in contrast to Gans and Stern (2003), the complementary assets in my model are held by producers who do not invent.

<sup>37</sup>Khan and Sokoloff (1993) note that prior to 1836, inventors typically had to commercially develop their inventions, which constrained their ability to specialize in invention. In 1836, US patent law reform replaced a registration system with a rigorous examination system, significantly lowering legal uncertainty surrounding the novelty and scope of patented inventions. After 1836, inventors were more free to specialize in invention and markets for assigning inventions flourished (Lamoreaux and Sokoloff 2001).

<sup>38</sup>See Schankerman and Scotchmer (2001) and Henry and Turner (2010) for alternative models of reasonable royalties.

is not willing to pay. Because royalties cannot be too high, some of the returns to invention inevitably spill over to the producer. Second, inventing firms that patent no longer face a critical-mass problem, while trolls do face such a problem. This introduces a tradeoff where patents are helpful in stimulating invention, but an improperly-managed thicket taxes invention, especially if the thicket includes trolls. Third, because it is impossible to earn any returns to invention absent patents, the first-best level of invention is infeasible. Instead, optimal invention satisfies the second-best level. Because some of the returns to invention spill over to producers, it is sometimes impossible to achieve this level of invention as an equilibrium, even if trolls are prohibited. When troll behavior is possible, the problem is significantly worse. Now, to achieve the second-best level of welfare, it is necessary to achieve second-best invention and to preclude trolls. To preclude trolls requires a sufficiently high patent fee, however, and (I show) such a high fee necessarily lowers the rate of invention below the second best. Hence, the mere possibility of troll behavior prevents the optimal rate of invention.

Another natural and important extension would generalize the direct cost to trolls of troll behavior. In the model analyzed in this paper, I adopt the very strong assumption that the cost of patenting is the same (fixed amount) for any technology and is independent of whether an invention is made. This has the distinct advantage of highlighting the effect of trolls, who patent everything when the patent fee is sufficiently small. In practice, the cost of developing ideas to the point of convincing the patent authority of the ideas' novelty, utility and non-obviousness (though short of the point of reducing an invention to use) may differ across technologies. In a model specified this way, trolls would not typically patent everything. Rather, they would focus on those ideas that are cheapest to develop relative to the returns they expect to earn from inadvertent infringements. Potentially, this would alter the welfare consequences in important ways. Crucially, as long as the cost of patenting without inventing is lower than the cost of patenting and inventing, the threat of unproductive entrepreneurship remains. Moreover, the key insight that trolls face a critical-mass problem also remains as long as patent fees are strictly positive.

It would also be interesting to consider the more general case where consumers have a preference for a variety of goods and firms cannot price discriminate (Spence 1976; Dixit

and Stiglitz 1977) and where firms may have heterogeneous technology in producing final goods (Melitz 2003). Spence (1976) shows that free entry in such models leads to “too few” product varieties. Patent thickets, which both lower the rate of invention and increase its costliness, exacerbate this problem when firms are vertically integrated but may not worsen welfare in models where patents overcome hold-up problems facing small inventors.

Perhaps the most useful extension would be to embed my monopolistically competitive framework in an infinite-horizon model, to study the effects of patent thickets on economic growth. Baumol (1990) and Murphy, Schliefer and Vishny (1991) argue that unproductive entrepreneurship is a key impediment to growth. To study this more formally, one could adapt my model to an endogenous growth setting. Romer (1990) embraces the idea that product variety at the intermediate input level drives growth.<sup>39</sup> My model may be useful for determining how patent policy affects growth through its affect on the creation of such inputs.

My result that trolls hurt not only producers but welfare generally offer an important argument for policy changes. With trolls, the United States patent system is failing to live up to its constitutional mandate “to promote the progress of science and the useful arts” (Article I, Section 8), because the presence of trolls reduces the level of invention. What can practically be done? Among developed countries, patent law typically requires an applicant to prove it has invented something novel, useful and nonobvious to get a patent.<sup>40</sup> It does not require an applicant to prove that it spent resources to achieve its invention nor does it require the applicant to demonstrate a means of developing the invention to the point of commercial viability. Perhaps patent policy should include such a requirement. It is precisely patenting without invention that causes problems, because the thicket increases while the stock of viable inventions shrinks. Trolls do not reduce technology to practice yet close it off to use by complementary technologies. Patents that are written speculating about some future use should be prevented entirely.

If it is impossible to prevent patenting without invention through changes to the screening

---

<sup>39</sup>An alternative approach (Aghion and Howitt 1992) which is also influential is to treat innovation as yielding sequential quality improvements. This would be an interesting way to model invention in a monopolistically competitive framework.

<sup>40</sup>For the United States, this is the precise set of requirements.

of applications, an alternative approach is to raise patent fees. In cases where the applicant cannot demonstrate the ability to develop the invention to the point of commercial viability, it is more likely the patent is parasitic. Renewal fees (currently paid in years 4,8 and 12 in the United States) should be raised in such cases. I look forward to further progress in the area.

## Appendix

**Proof of Proposition 1.** If either  $r < r_C$ ,  $r = r_C$  and  $F > 0$ , or  $F > \frac{\alpha V \left( rV - \frac{c_D}{2} \right)}{b}$ , then there is no patenting in equilibrium [ $\omega_P^*(NT) = 0$ ]. We then have  $\omega_I^*(NT) = \frac{V}{b}$ . The rate of invention is unaffected by (and therefore weakly decreasing in)  $\alpha$ . Let  $r > r_C$ . Then, for  $F \leq \underline{F}$ , we have  $\rho(NT) = 1$ , and  $\omega_I^*(NT) = \omega_P^*(NT) = \frac{V-F}{b+\alpha C_D}$  is strictly decreasing in  $\alpha$ . For  $F \in (\underline{F}, \bar{F})$ , we have from (7) that  $\rho(NT) > 0$  and  $\omega_I^*(NT) = \underline{\omega}_I^* = \frac{F}{\alpha \left( rV - \frac{c_D}{2} \right)}$ , which is also strictly decreasing in  $\alpha$ . Hence, if  $r > r_C$  and  $F < \bar{F}$ , then the rate of invention is strictly decreasing in  $\alpha$ . It is also true that the rate of patenting is positive,  $\rho > 0$ , if and only if  $r > r_C$  and  $F < \bar{F}$ . **QED**

**Proof of Proposition 2.** If either  $r < r_C$ ,  $r = r_C$  and  $F > 0$ , or  $F \geq \frac{\alpha V \left( rV - \frac{c_D}{2} \right)}{b}$ , then there is no patenting in equilibrium. In that case,  $\rho(NT) = \omega_P^*(NT) = 0$  is unaffected by (and is therefore weakly decreasing in)  $\alpha$ . Let  $r > r_C$ . Then, for  $F \leq \underline{F}$ , we have  $\rho(NT) = 1$ , which is unaffected by (and is therefore weakly decreasing in)  $\alpha$ . For  $F \in (\underline{F}, \bar{F})$ , we have  $\rho(NT) = \frac{\alpha V \left( rV - \frac{c_D}{2} \right) - Fb}{F \left[ \alpha \left( rV + \frac{c_D}{2} \right) \right]} \in (0, 1)$ . Differentiating, we find  $\frac{d\rho(NT)}{d\alpha} = \frac{bF^2 \left( rV + \frac{c_D}{2} \right)}{\left[ \alpha F \left( rV - \frac{c_D}{2} \right) \right]^2} > 0$ . **QED**

**Proof of Proposition 3.** I have shown that  $\rho = 1$  in the no-troll model if and only if  $F \leq \underline{F}$ . For  $F \in [0, \underline{F}^T]$ , simple algebra shows that  $\omega_I^*(T) - \omega_I^*(NT) = \frac{V - \alpha \left( rV + \frac{c_D}{2} \right)}{b} - \frac{V-F}{b+\alpha C_D} < 0$ . For  $F \in (\underline{F}^T, \underline{F}]$ , simple algebra shows that  $\omega_I^*(T) - \omega_I^*(NT) = \frac{F}{\alpha \left( rV - \frac{c_D}{2} \right)} - \frac{V-F}{b+\alpha C_D} < 0$ . If  $F > \underline{F}$ , then  $\rho(NT) < 1$  (some inventions are not patented) when trolling is impossible. If  $F \in (\underline{F}, \bar{F}]$ , then  $\omega_I^*(T) = \omega_I^*(NT) = \underline{\omega}_I^*$ , so that the level of invention is unchanged with trolls. If  $F > \bar{F}$ , then  $\omega_I^*(T) = \omega_I^*(NT) = \frac{V}{b}$ , so that the level of invention is unchanged with trolls. **QED**

**Proof of Proposition 4.** For  $F \geq \underline{F}$ ,  $\omega_I^*(T) = \omega_I^*(NT)$  and  $\omega_P^*(T) = \omega_P^*(NT)$ , so it is

obvious that  $W(T) = W(NT)$  for such cases. Consider the case  $F < \underline{F}$ . We can write

$$\begin{aligned} W(NT) &= \int_0^{\omega_I^*(NT)} [V - b\omega] d\omega - \omega_I^*(NT)\omega_P^*(NT)\alpha C_D \\ W(T) &= \int_0^{\omega_I^*(T)} [V - b\omega] d\omega - \omega_I^*(T)\omega_P^*(T)\alpha C_D. \end{aligned}$$

Since  $\omega_P^*(NT) = \omega_I^*(NT)$ , we then have

$$\begin{aligned} W(NT) - W(T) &= \int_{\omega_I^*(T)}^{\omega_I^*(NT)} [V - b\omega] d\omega + \alpha C_D \{-\omega_I^*(NT)^2 + \omega_I^*(T)\omega_P^*(T)\} \\ &= \int_{\omega_I^*(T)}^{\omega_I^*(NT)} [V - b\omega] d\omega + \alpha C_D \{-\omega_I^*(NT)\omega_I^*(T) \\ &\quad - \omega_I^*(NT)(\omega_I^*(NT) - \omega_I^*(T)) + \omega_I^*(T)\omega_P^*(T)\} \\ &= \int_{\omega_I^*(T)}^{\omega_I^*(NT)} [V - b\omega - \omega_I^*(NT)\alpha C_D] d\omega + \alpha C_D \omega_I^*(T) \{\omega_P^*(T) - \omega_I^*(NT)\}. \end{aligned}$$

Recall from (3) that  $\omega_I^*(NT)$  satisfies  $V - b\omega_I^*(NT) - \omega_I^*(NT)\alpha C_D = F$ . Hence, the first term is strictly positive because  $\omega_I^*(NT) > \omega_I^*(T)$  and  $V - b\omega - \omega_I^*(NT)\alpha C_D > F \geq 0$  for all  $\omega \in [\omega_I^*(T), \omega_I^*(NT)]$ . The second term is strictly positive because (from Proposition 3)  $\omega_P^*(T) > \omega_I^*(NT)$  for any  $F < \underline{F}$ . Hence  $W(NT) - W(T) > 0$ . **QED**

**Proof of Proposition 5.** If either  $r < r_C$ ,  $r = r_C$  and  $F > 0$ , or  $F > \frac{\alpha V(rV - \frac{C_D}{2})}{b}$ , then there is no patenting in equilibrium [ $\omega_P^*(NT) = 0$ ]. We then have  $\omega_I^*(T) = \frac{V}{b}$ . The rate of invention is unaffected by (and is therefore weakly decreasing in)  $\alpha$ . Let  $r > r_C$ . Then, for  $F \leq \underline{F}^T$ , we have  $\omega_I^*(T) = \frac{V - \alpha(rV + \frac{C_D}{2})}{b}$ , which is strictly decreasing in  $\alpha$ . For  $F \in (\underline{F}^T, \bar{F}]$ , we have  $\omega_I^*(T) = \underline{\omega}_I^* = \frac{F}{\alpha(rV - \frac{C_D}{2})}$ , which is also strictly decreasing in  $\alpha$ . Hence, if  $r > r_C$  and  $F < \bar{F}$ , then the rate of invention is strictly decreasing in  $\alpha$ . It is also true that the rate of patenting is positive,  $\rho(T) > 0$ , if and only if  $r > r_C$  and  $F < \bar{F}$ .

**Proof of Proposition 6.** If  $F \leq \underline{F}^T$ , then  $\omega_P^*(T) = 1$  and is independent of  $\alpha$ . If  $F \in (\underline{F}^T, \bar{F})$ , then  $\omega_P^*(T) = \frac{\alpha V(rV - \frac{C_D}{2}) - Fb}{\alpha^2(rV - \frac{C_D}{2})(rV + \frac{C_D}{2})} \in (0, 1)$ . Differentiating, we find

$$\frac{d\omega_P^*(T)}{d\alpha} = \frac{[\alpha(rV - \frac{C_D}{2})(rV + \frac{C_D}{2})][2Fb - \alpha V(rV - \frac{C_D}{2})]}{[\alpha^2(rV - \frac{C_D}{2})(rV + \frac{C_D}{2})]^2},$$

which is positive iff

$$F > \frac{\alpha V(rV - \frac{C_D}{2})}{2b} = \frac{\bar{F}}{2}.$$

If  $\frac{\bar{F}}{2} < \underline{F}^T$ , then  $\frac{d\omega_P^*(T)}{d\alpha} > 0$  for any  $F \in (\underline{F}^T, \bar{F})$ , that is, for any case where  $\omega_P^*(T) \in (0, 1)$ . Simple algebra shows that  $\frac{\bar{F}}{2} < \underline{F}^T$  iff

$$\alpha < \frac{V}{2rV + C_D}.$$

Since our assumptions guarantee that  $r \leq 1$  and  $C_D < 2V$ , the right-hand side above can be no smaller than  $\frac{1}{4}$ . **QED**

**Some Additional Comparative Statics.** In section 3.1, we have  $\rho(NT) = \frac{\alpha V \left( rV - \frac{C_D}{2} \right) - Fb}{F \left[ \alpha \left( rV + \frac{C_D}{2} \right) \right]}$ .

It is obvious that this expression decreases with  $b$  and  $C_D$ . For the other parameters, we find

$$\begin{aligned} \frac{d\rho(NT)}{dV} &= \frac{\alpha^2 F \left[ rV \left( rV + \frac{C_D}{2} \right) + \frac{C_D}{2} \left( rV - \frac{C_D}{2} \right) \right] + \alpha b r F^2}{\left[ \alpha F \left( rV - \frac{C_D}{2} \right) \right]^2} > 0 \\ \frac{d\rho(NT)}{dr} &= \frac{\alpha^2 C_D V^3 F + \alpha b V^2 F}{\left[ \alpha F \left( rV - \frac{C_D}{2} \right) \right]^2} > 0 \end{aligned}$$

## References

- Aghion, Philippe and Peter Howitt. "A Model of Growth Through Creative Destruction," *Econometrica* 60, 1992, 323-51.
- Anton, James and Dennis Yao. "Finding 'Lost' Profits: An Equilibrium Analysis of Patent Infringement Damages." *Journal of Law, Economics and Organization* 27, 2007, 186-207.
- Ayres, Ian and Paul Klemperer. "Limiting Patentee's Market Power Without Reducing invention Incentives: The Perverse Benefits of Uncertainty and Non-Injunctive Remedies," *Michigan Law Review*, 97, 1999, 985-1033.
- Ayres, Ian and Gideon Parchomovsky. "Tradable Patent Rights," *Stanford law Review* 60, 2007, 863-94.
- Baumol, William. "Entrepreneurship: Productive, Unproductive and Destructive," *Journal of Political Economy* 98, 1990, 893-921.
- Bessen, James and Michael Meurer. Patent Failure: How Judges, Bureaucrats, and Lawyers Put Innovators at Risk. Princeton University Press: Princeton, NJ 2008.
- Bessen, James, Jennifer Ford and Michael Meurer. "The Private and Social Costs of Patent Trolls," Boston University School of Law Working Paper, 2011.



- Choi, Jay Pil. "Alternative Damage Rules and Probabilistic Intellectual Property Rights: Unjust Enrichment, Lost Profits, and Reasonable Royalty Remedies" *Information Economics and Policy* 21, 2009, 145-57.
- Cotropia, Christopher and Mark Lemley. "Copying in Patent Law," *University of North Carolina Law Review* 87, 2008, 1421.
- Cournot, Augustin. Recherches sur les Principes Mathematizues de la Theorie des Richesses, Paris, France: L. Hachette, 1838
- Dixit, Avinash and Joseph Stiglitz. "Monopolistic Competition and Optimum Product Diversity," *American Economic Review* 67, 1977, 297-308.
- Gallini, Nancy. "The Economics of Patent: Lessons from Recent US Patent Reform," *Journal of Economic Perspectives* 16, 2002, 131-54.
- Gans, Joshua and Scott Stern. "The Product Market and the Market for "Ideas": Commercialization Strategies for Technology Entrepreneurs," *Research Policy* 32, 333-50.
- Gilbert, Richard and Carl Shapiro. "Optimal Patent Length and Breadth," *RAND Journal of Economics* 21, 1990, 106-12.
- Hall, Bronwyn, Adam Jaffe and Manuel Trajtenberg. "The NBER Patent Citation Datafile: Lessons, Insights and Methodological Tools," NBER Working Paper 8498, 2001.
- Hardin, Garrett. "The Tragedy of the Commons," *Science* 162, 1968, 1243-48.
- Hayter, Earl. "The Patent System and Agrarian Discontent, 1875-88." *The Mississippi Valley Historical Review* 34, 1947, 59-82.
- Heller, Michael and Rebecca Eisenberg. "Can Patents Deter Infringement? The Anticommons in Biomedical Research." *Science* 280, 1 May 1998, 698-701.
- Henry, Matthew and John L. Turner. "Patent Damages and Spatial Competition," *Journal of Industrial Economics* 58, 2010, 279-305.
- Khan, Zorina and Kenneth Sokoloff. "Schemes of Practical Utility," *Journal of Economic History* 53, 1993, 298-307.
- Klemperer, Paul. "How Broad Should the Scope of Patent Protection Be?" *RAND Journal of Economics* 21, 1990, 113-30.
- Lamoreaux, Naomi and Kenneth Sokoloff. "Market Trade in Patents and the Rise of a Class of Specialized Inventors in 19th-Century United States," *AEA Papers and Proceedings* 91, 2001, 39-44.
- Lemley, Mark and Carl Shapiro. "Patent Hold-Up and Royalty Stacking," *Texas Law Review* 85, 2007, 1991-2049.

- Lerner, Josh and Jean Tirole. "Efficient Patent Pools," *American Economic Review* 94, 2004, 691-711.
- Melitz, Marc. "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica* 71, 2003, 1695-1725.
- Merges, Robert. "The Trouble With Trolls: Innovation, Rebt-Seeking and Patent Law Reform," *Berkeley Technology Law Journal* 24, 2009, 1583-1614.
- Merges, Robert and Richard R. Nelson. "On the Complex Economics of Patent Scope," *Columbia Law Review* 90, 1990, 839-916.
- Murphy, Kevin, Andrei Schleifer and Robert Vishny. "The Allocation of Talent: Implications for Growth," *Quarterly Journal of Economics* 106, 1991, 503-30.
- Nordhaus, William D. Invention, Growth and Welfare: A Theoretical Treatment of Technological Change, M.I.T. Press, 1969.
- Ramirez, Edith and Lisa Kimmel. "A Competition Policy Perspective on Patent Law: The Federal Trade Commission's Report on the Evolving IP Marketplace," *The Antitrust Source*, August 2011, 1-10.
- Reitzig, Markus, Joachim Henkel and Christopher Heath. "On Sharks, Trolls and Their Patent prey. Unrealistic Damage Awards and Firms' Strategies of 'Being Infringed'," *Research Policy* 36, 2007, 134-54.
- Reitzig, Markus, Joachim Henkel and Ferdinand Schneider. "Collateral Damage for R&D Manufacturers: How Patent Sharks Operate in Markets for Technology," *Industrial and Corporate Change* 19, 2010, 947-67.
- Rhodes, Kevin. "Comment: The Federal Circuit's Non-obviousness Standards: Theoretical Perspectives on Recent Doctrinal Changes," *Northwestern University Law Review* 85, 1991, 1051-1102.
- Romer, Paul. "Endogenous Technical Change," *Journal of Political Economy* 98, 1990, 71-102.
- Sabety, Ted. "Nanotechnology Innovation and the Patent Thicket: Which IP Policies Promote Growth?" *Albany Law Journal of Science and Technology* 15, 2005, 477.
- Schankerman, Mark and Scotchmer, Suzanne. "Damages and Injunctions in Protecting Intellectual Property." *RAND Journal of Economics*, 32, 2001, 199-220.
- Shapiro, Carl. "Navigating the Patent Thicket: Cross Licenses, Patent Pools and Standard Setting," in Jaffe, A., Lerner, J., Stern, S. eds. *Innovation Policy and the Economy 1*, 2001, MIT Press: Cambridge, MA.

- Shapiro, Carl. Injunctions, Hold-Up and Patent Royalties. *American Law and Economics Review* 12, 2010, 509-57.
- Spence, Michael. "Product Selection, Fixed Costs, and Monopolistic Competition," *Review of Economic Studies* 43, 1976, 217-235.
- Turner, John L. "Patent Thickets, Trolls and Idea Factories," University of Georgia Working Paper, 2011.
- United States Department of Justice and Federal Trade Commission. "The Evolving IP Marketplace: Aligning Patent Notice and Remedies with Competition," 2011, available at <http://www.ftc.gov/os/2011/03/110307patentreport.pdf>.
- Werden, Gregory, Luke Froeb and Lucian Beavers. "Economic Analysis: Lost Profits from Patent Infringement With and Without Non-infringing Substitutes," *AIPLA Quarterly Journal* 27, 1999, 305-34.