Why Forecasters Disagree? A Global Games Approach*

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Abstract

Two key features of economic forecasts are that they are based on incomplete information about the underlying fundamentals; and they reveal private information of the corresponding forecasters. In this paper, we use a global games approach to explain dispersion in economic forecasters' predictions. First, we analyze a stylized "beauty-contest" model to characterize conditions under which dispersion in forecasts persists through time. In particular, more precise private information raises dispersion when the relative precision of public information is sufficiently high. We then characterize an environment where forecasters are unreceptive to more transparent public information: substitutability of forecasts lessens the sensitivity of equilibrium forecasts to public information, contributing to persistent dispersion. Next we consider endogenous private information acquisition motives: more precise public information lowers the precision of private information acquired in equilibrium, raising dispersion when the relative precision of public information is sufficiently low. Finally, we examine an application to a two-stage game involving firms' production strategies in a Cournot market and analyze how dispersion of forecasts feeds into variation in output.

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1 Introduction

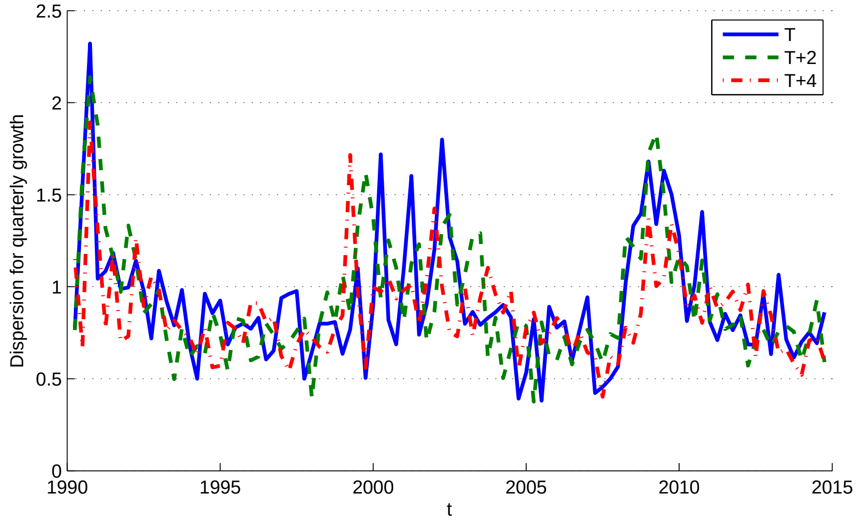
The development of information technology and mass media communication enabled economic agents to obtain more transparent and precise information. This information is easily accessible and thus disparity in information among agents seems to have diminished over time. However, dispersion in forecasters' predictions of economic variables tends to persist through time. For example, Figure 1 shows that the cross-sectional dispersion among professional forecasters in predicting GDP growth and inflation between 1990 and 2014 do not appear to fall through time. Hence, it is important to establish theoretically why disagreement among economic forecasters persists. To this end, we develop a novel approach for addressing the primary source of considerable persistence of dispersion in forecasts.

A global games approach is pertinent to our analysis if we take into account the following features of economic forecasts. First, the nature of forecasts stems from incomplete information about the underlying states of the economy, i.e., economic conditions.[1] Second, an agent's optimal forecast depends on his expectations of other agents' forecasts as well. Because different agents may have different (private) information about the fundamentals, each agent's forecast might reveal her private information. Furthermore, an individual agent may either want to mimic or deviate from others' forecasts depending on whether the forecasts are strategic complements or substitutes. These features of economic forecasts are exactly reminiscent of global games – games of incomplete information where players receive (possibly) correlated signals of the underlying state of the world. To the best of our knowledge, this paper represents a first attempt to explain the sources of persistent dispersion in forecasts theoretically by a global games approach.

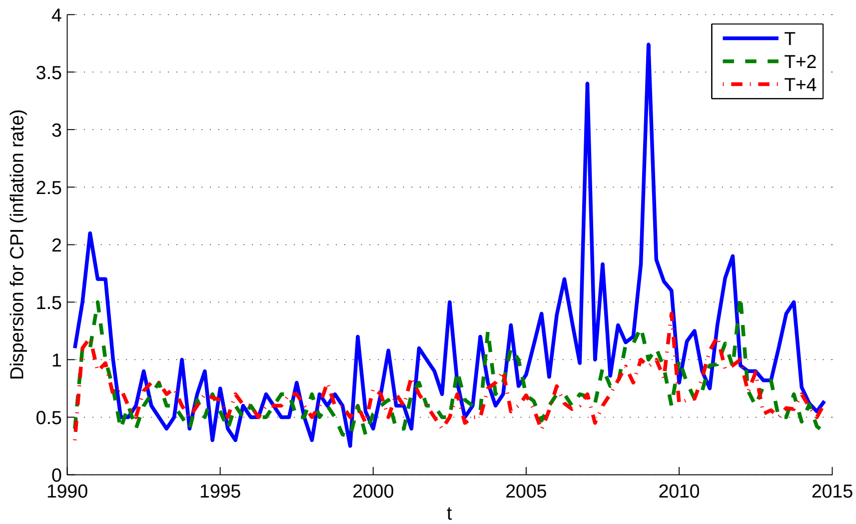
In Section 2, we present a benchmark model of forecasting game in which a large number of forecasters are making predictions about an uncertain exogenous state of the economy. The payoff depends on the fundamentals as well as on average forecast; forecasters receive two signals – private and public – about the true fundamentals. In Section 3, we characterize the best responses of forecasters: the individual optimal forecast takes the form of a linear combination of private and public signals. In Section 4, we examine the conditions under which dispersion of forecasts persists. In particular, more precise private information raises dispersion when the relative precision of private information is sufficiently low. In contrast, more precise public information raises aggregate volatility when the relative precision of public information is sufficiently low.

Section 4 also discusses related issues regarding complementarity or substitutability of forecasts, costs of obtaining information, and their effects on forecaster inattention to more

[1] For example, the exact output level of the next period is uncertain to economic agents; instead, agents observe various signals – media news from the stock market, public announcements of government agencies or the Federal Reserve, or private sources of information – about the fundamentals of the economy; and form expectations about future output growth based on information that one obtains from those signals.



(a) Dispersion in forecasts of real GDP growth rate



(b) Dispersion in forecasts of inflation rate (CPI)

Figure 1: Dispersion in Forecasts from Survey of Professional Forecasters

Note: (1) 75th Percentile Minus 25th Percentile of the Forecasts for Q/Q Growth
(2) CPI: 75th Percentile Minus 25th Percentile of the Forecasts for Levels

precise public information.[2] We show that a forecaster will be unreceptive to more transparent and precise public information when the forecasts are substitutes or when the costs

[2]Rudebusch and Williams (2009) find that "[f]or over two decades, researchers have provided evidence that the yield curve, specifically the spread between long- and short-term interest rates, contains useful information for signaling future recessions. Despite these findings, forecasters appear to have generally placed too little weight on the yield spread when projecting declines in the aggregate economy."

of obtaining better information are too high. This result implies that forecasters may not even bother to use the better information that is readily available, also contributing to the persistence of dispersion.

Section 4 closes with a consideration of endogenous information structure. Without endogenous information choice, more precise public information unambiguously lowers dispersion for given precision of private information and less precise private information raises dispersion for given precision of public information when the relative precision of private information is sufficiently high. However if we endogenize the forecaster's choice of private information, more precise public information crowds out the forecaster's acquisition of private information in equilibrium, in turn raising dispersion when the relative precision of private information is sufficiently high.

In Section 5, we consider an application of our model to a Cournot market by modeling a two-stage game in which each firm first chooses a forecast about the aggregate market demand and then subsequently chooses individual output. An interesting observable implication is that variation of actions in the first stage directly feeds into variation of actions in the second stage. We examine the effect of variation in forecasts on dispersion of firms' production decisions and volatility of aggregate output. Another way to appreciate our two-stage problem is to consider social welfare. Our results provide normative policy implications: when the agents have independent sources of information, it is always beneficial to encourage the collection of more precise private information; however, it is not always the case that more transparent disclosure of public information by policy makers is desirable. Specifically, when the private information of the agents is very precise, then increased precision of public information can reduce welfare.

2 The Model

We consider a forecasting game based on the "beauty-contest" type model following Morris and Shin (2002), Angeletos and Pavan (2007), and Róndina and Shim (2013). In the economy there is a continuum of agents (forecasters), indexed by i and uniformly distributed over the unit interval $[0, 1]$. Each agent i chooses a forecast $a_i \in \mathbb{R}$. His payoff is given by

$$u_i(a_i, A, \theta) = -\frac{1}{2}\left(a_i - (1-r)\theta - rA\right)^2, \tag{2.1}$$

whose components are interpreted as follows. $\theta \in \mathbb{R}$ is an exogenous random payoff-relevant variable (the underlying fundamentals). We assume that θ is drawn from an improper distribution over the real line. We let $A \equiv \int a d\Psi(a)$ denote the average forecast, where $\Psi(a)$ is the cumulative distribution function for individual forecasts across the population. Then an agent wants to choose a_i so as to minimize the distance between her forecast and a linear combination of the fundamentals and the average forecast. Finally, the parameter $r \in (-1, 1)$

captures whether the forecasting game exhibits complementarity ($r > 0$) or substitutability ($r < 0$). That is, an agent would want to align his forecast with other agents' forecasts when $r > 0$ while he prefers to differentiate his forecast from those of others when $r < 0$. For tractability, we assume a quadratic utility function to ensure linearity of best responses.[3]

We introduce incomplete information by assuming that agents do not observe the true state θ but instead observe noisy private and public signals about the underlying fundamentals. For each i, agent i observes a private signal x_i and a public signal p, respectively characterized as

$$x_i = \theta + (\alpha_{x,i})^{-1/2}\varepsilon_i \tag{2.2}$$

and

$$p = \theta + (\alpha_p)^{-1/2}\varepsilon, \tag{2.3}$$

where ε_i and ε are, respectively, idiosyncratic and common noises, independent of one another as well as of θ, and both follow $\mathbb{N}(0,1)$. Let $\alpha_{x,i}$ and α_p denote the precision of private and public signals, respectively.

3 Equilibrium

In line with the existing literature, we restrict our focus on symmetric linear equilibria – in which $\alpha_{x,i} = \alpha_{x,j}$ for $i \neq j$; and forecasts are linear functions of their respective signals.

Each agent chooses a_i so as to maximize $\mathbb{E}\left[u_i(a_i, A, \theta)|x_i, p\right]$. The solution to this utility maximization problem gives the best response for each agent. Because the private noises ε_i are assumed to be i.i.d. across agents, in any symmetric equilibrium the aggregate variable A is a function of (θ, p) alone. The following definition of a linear equilibrium is in line with Morris and Shin (2002) and Angeletos and Pavan (2007).

Definition 1. *A linear equilibrium is a strategy $a : \mathbb{R}^2 \to \mathbb{R}$ such that a is linear in x and p; and for all (x, p),*

$$a(x,p) = arg\max_{a'} \mathbb{E}\left[u(a', A(\theta,p), \theta)|x, p\right], \tag{3.1}$$

where $A(\theta,p) = \int_x a(x,p)d\bar{\Psi}(x|\theta,p)$ for all (θ,p), and $\bar{\Psi}(x|\theta,p)$ denotes the conditional cumulative distribution function of x given (θ,p).

Note that with complete information, the unique equilibrium entails $a_i = \theta$ for all i. Under incomplete information, agent i's best response is determined by the first-order condition:

$$a_i(x_i,p) = \mathbb{E}\left[(1-r)\theta + rA(\theta,p)|x_i,p\right], \ \forall(x_i,p) \tag{3.2}$$

[3] Another possibility is to think of the utility function as an approximation to the general concave function.

where $A(\theta, p) = \mathbb{E}[a(x,p)|\theta, p]$ for all (θ, p). This condition states that an agent's optimal forecast is an affine combination of his expectation of the complete information equilibrium and his expectation of average forecast. The following proposition then follows.

Proposition 1. *For any given value of (θ, p), a linear equilibrium exists and is unique. The equilibrium forecast of agent i is given by*

$$a_i(x_i, p) = \lambda x_i + (1-\lambda)p, \tag{3.3}$$

with $\lambda = \frac{\alpha_x}{\alpha_x + \frac{\alpha_p}{1-r}}$.

Proof. Given x_i and p, the first order condition for the optimization problem of the forecasting game, i.e.,

$$\max \mathbb{E}[u_i|x_i, p] = \mathbb{E}\left[-\frac{1}{2}(a_i - (1-r)\theta - rA)^2 | x_i, p\right],$$

gives the optimal forecast of

$$a_i(x_i, p) = (1-r)\mathbb{E}[\theta|x_i, p] + r\mathbb{E}[A(\theta, p)|x_i, p]. \tag{3.4}$$

Individual forecast is thus increasing in the expected level of the fundamentals and in the expected level of average forecast. Given the linearity, it is natural to look for equilibrium forecast decisions that are linear in x and p so that $a_i = \kappa_0 x_i + \kappa_1 p$, where κ_0 and κ_1 are constants determined in equilibrium. Then, $A(\theta, p) = \kappa_0\theta + \kappa_1 p$, and (5.5) reduces to

$$a_i(x_i, p) = (1-r)\mathbb{E}[\theta|x_i, p] + r\kappa_0\mathbb{E}[\theta|x_i, p] + r\kappa_1 p.$$

Substituting $\mathbb{E}(\theta|x_i, p) = \delta x_i + (1-\delta)p$, where $\delta = \frac{\alpha_x}{\alpha_x + \alpha_p}$,

$$a_i(x_i, p) = (1 - r + r\kappa_0)\delta x_i + ((1-r)(1-\delta) + r\kappa_0(1-\delta) + r\kappa_1)p.$$

It follows that $a_i(x_i, p) = \kappa_0 x_i + \kappa_1 p$ constitutes a linear equilibrium if and only if κ_0 and κ_1 solve $\kappa_0 = (1-r)\delta + r\delta\kappa_0$ and $\kappa_1 = (1-r)(1-\delta) + r\kappa_0(1-\delta) + r\kappa_1$. Equivalently, $\kappa_0 = \frac{\alpha_x}{\alpha_x + \frac{\alpha_p}{1-r}}$ and $\kappa_1 = 1 - \kappa_0$, which gives (3.3). Clearly, this is the unique symmetric linear equilibrium. Note that in the linear equilibrium, the aggregate output across agents is $A(\theta, p) = \lambda\theta + (1-\lambda)p$. □

Note that $\lambda = \frac{\alpha_x}{\alpha_x + \frac{\alpha_p}{1-r}}$ measures the relative sensitivity of the equilibrium to private information and $1-\lambda$ measures that to public information. We can easily see that this sensitivity also depends on r, which captures the private value agents assign to aligning their choices. When $r \neq 0$, equilibrium forecast is tilted toward public or privation information depending on whether forecasts are strategic complements or substitutes. In particular, complementarity lowers λ whereas substitutability raises λ. The reason is that, because public information is a relatively better predictor of other agents' forecasts than private information, an agent will adjust upward his reliance on public information when forecasts are strategic complements, and downward when forecasts are substitutes. From here on, we simply interpret λ as the relative precision of private information and $1-\lambda$ as the relative precision of public information for given degree of coordination.

4 Equilibrium Analysis

4.1 The Effect of More Transparent Information on Dispersion

One implication from the previous result regards the equilibrium level of dispersion in forecasts among agents. When information is complete, all agents would choose $a_i = \theta$. So there is no variation in individual forecasts. Incomplete information affects equilibrium behavior in a way that noisy signal generates dispersion, that is, variation in forecasts across agents.

The optimal forecast can be rewritten as:

$$\begin{aligned} a_i &= \lambda\left(\theta + (\alpha_x)^{-1/2}\varepsilon_i\right) + (1-\lambda)\left(\theta + (\alpha_p)^{-1/2}\varepsilon\right) \\ &= \theta + \lambda(\alpha_x)^{-1/2}\varepsilon_i + (1-\lambda)(\alpha_p)^{-1/2}\varepsilon. \end{aligned} \tag{4.1}$$

The equilibrium level of dispersion for given realizations of θ and p can be characterized by the following expression:

$$Var(a|\theta,p) = \lambda^2(\alpha_x)^{-1} = \frac{\alpha_x}{\left(\alpha_x + \frac{\alpha_p}{1-r}\right)^2} \tag{4.2}$$

where $\lambda = \alpha_x\left(\alpha_x + \frac{\alpha_p}{1-r}\right)^{-1}$.

Now we can analyze how changes in the precision of public and private information affect dispersion of forecasts.

Proposition 2 (Sufficient Condition for Non-decreasing Dispersion of Forecasts)**.** *The dispersion of forecasts weakly increases if*

$$\left(\alpha_p - (1-r)\alpha_x\right)d\alpha_x \geq 2\alpha_x d\alpha_p. \tag{4.3}$$

Proof. A sufficient condition for non-decreasing dispersion is found by setting $dVar(a|\theta,p) \geq$

0. By total differentiating (4.2), we have

$$dVar(a|\theta,p) = \left[\frac{\left(\alpha_x+\frac{\alpha_p}{1-r}\right)^2 - 2\alpha_x\left(\alpha_x+\frac{\alpha_p}{1-r}\right)}{\left(\alpha_x+\frac{\alpha_p}{1-r}\right)^4}\right] d\alpha_x + \left[\frac{-2\frac{1}{1-r}\alpha_x\left(\alpha_x+\frac{\alpha_p}{1-r}\right)}{\left(\alpha_x+\frac{\alpha_p}{1-r}\right)^4}\right] d\alpha_p$$

By setting this to be greater than equal to zero, we obtain (4.3). □

Condition (4.3) can be rewritten as:

$$(1-2\lambda)d\alpha_x \geq \frac{\lambda}{1-r}d\alpha_p \tag{4.4}$$

The following corollaries immediately follow.

Corollary 1. *The dispersion of forecasts falls with an increase in α_p for given α_x.*

Proof. Obvious from equation (4.2). □

This result suggests that an increase in the absolute precision of public information for given precision of private information unambiguously decreases dispersion. The intuition is as follows. As public information gets more transparent, each agent puts more weight on public signal when making a prediction. As a result, forecasts of each agent become similar.

Corollary 2. *The dispersion of forecasts increases with an increase in α_x for given α_p if and only if $\alpha_p > (1-r)\alpha_x$ (or equivalently, $\lambda < \frac{1}{2}$).*

Proof. We differentiate $Var(a|\theta,p)$ with respect to α_x to obtain

$$sign\left(\frac{dVar(a|\theta,p)}{d\alpha_x}\right) = sign\left(\frac{\alpha_p}{1-r} - \alpha_x\right) \tag{4.5}$$

Then the corollary follows. □

Corollary 2 may explain the persistent dispersion of forecasts despite the improved transparency of private information. Interpreting λ as the relative precision of private information, $1-\lambda$ measures the sensitivity of equilibrium forecasts to public information. Then an increase in the transparency of private information (that is, an increase in the absolute precision of private information for given precision of public information) has the perverse effect of increasing dispersion of forecasts when the relative precision of private information is sufficiently low; in particular, $\lambda < \frac{1}{2}$.

To understand this result better, consider the effect of an increase in private signal precision on the variance through two separate channels: (1) an increase in the absolute precision, and (2) an increase in the relative precision of private information:

$$Var(a|\theta, p) = \underbrace{\lambda^2}_{(2)} \underbrace{(\alpha_x)^{-1}}_{(1)},$$

where the effects of a higher α_x can be decomposed as:

(1) lowers uncertainty from idiosyncratic noise ε_i;

(2) puts more weight on private signal (and thus on idiosyncratic noise).

That is, as private information becomes more precise, more weight would be put on less idiosyncratic noise. However, which channel dominates crucially depends on the initial precision of private information. If the agents' sensitivity to public information were so high such that $1 - \lambda > \frac{1}{2}$, then despite the increased precision of private information, the idiosyncratic noise would still get a relatively low weight; and thus uncertainty from private signal would still considerably contribute to dispersion.

4.2 The Effect of More Transparent Information on Volatility

We can also analyze the effect of more precise information on volatility, that is, variation in average forecast around the complete information equilibrium. In equilibrium, the average forecast can be written as:

$$\begin{aligned} A &= \lambda\theta + (1-\lambda)p \\ &= \theta + (1-\lambda)(\alpha_p)^{-1/2}\varepsilon, \end{aligned} \tag{4.6}$$

where $\lambda = \alpha_x \left(\alpha_x + \frac{\alpha_p}{1-r}\right)^{-1}$. The equilibrium level of volatility of average forecast for given realization of θ can be characterized by the following expression:

$$Var(A|\theta) = (1-\lambda)^2(\alpha_p)^{-1} = \frac{\frac{\alpha_p}{(1-r)^2}}{\left(\alpha_x + \frac{\alpha_p}{1-r}\right)^2} \tag{4.7}$$

Then we obtain the following results.

Proposition 3 (Sufficient Condition for Non-decreasing Aggregate Volatility)**.** *The volatility of average forecast weakly increases if*

$$\left[\alpha_x - \frac{\alpha_p}{1-r}\right] d\alpha_p \geq 2\alpha_p d\alpha_x. \tag{4.8}$$

Proof. (4.8) is obtained by total differentiating (4.7) and setting $dVar(Q|\theta) \geq 0$. □

Corollary 3. *The volatility increases with an increase in α_p for given α_x if and only if $\lambda > \frac{1}{2}$.*

Proof. Set $d\alpha_x = 0$ in (4.8). Then volatility increases iff $\alpha_x - \frac{\alpha_p}{1-r} > 0$, which is equivalent to $\lambda > \frac{1}{2}$. □

Corollary 4. *The volatility falls with an increase in α_x for given α_p.*

Proof. Obvious from (4.7). □

When the precision of private signal increases, each agent would depend more on private signal when making a prediction and thus less weight will be put on public signal. This lowers the exposure of average forecast to public signal and more weight is put on the fundamentals θ. Thus conditional on the realization of θ, variation in average forecast around the complete information level θ decreases unambiguously. Corollaries 2 and 4 together imply that decline in volatility of average forecast is not always associated with decline of dispersion in forecasts among individual agents.

4.3 Forecaster Inattention to More Transparent Public Information

The development of technology undoubtedly enabled economic agents to obtain more transparent public information. For example, government agencies and central bankers disseminate information about their policies through the mass media and this information is easily accessible. However, as been argued by Rudebusch and Williams (2009) and Binder (2014), economic agents may not fully seek available information when making predictions about the economy. In particular, Rudebusch and Williams (2009) argue that it is a puzzling observation that forecasters do not use yield curve information when making their predictions about the aggregate economy. Our framework may provide a way to understanding these observations, and thus provide an alternative explanation to why dispersion among forecasters persists.

Consider a situation where the Federal Reserve discloses more (or new) information about economic conditions through the media with high public visibility. Suppose that this information with a certain level of precision is *sought*[4] with additional costs. That is, though public information is in theory easily accessible, there is a cost to actively using that information in making predictions.

We shall need the following notations. Let c be a cost function that each agent faces, such that $c(\alpha_p)$ denotes the cost of seeking (or using) public signal with precision α_p. Suppose

[4]We use the words "seek" or "use" rather than "obtain" to distinguish between proactively seeking and using available public information versus being exposed to the information.

that agents receive $N \geq 1$ signals and one signal, say public signal p, is improved, i.e., α_p increases to α'_p where $\alpha_p < \alpha'_p$. Let us assume the following:

Assumption A cost function $c(\cdot)$ is increasing (and possibly convex) in α_p, i.e.,

$$c(\alpha_p) < c(\alpha'_p) \quad \text{when} \quad \alpha_p < \alpha'_p. \tag{4.9}$$

Let α_x summarize the information from other signals, and denote $u(\alpha_x, \alpha_p)$ to be the utility payoff of the agent if he uses public signal with precision α_p and other signals with precision α_x. We assume that the utility is increasing in the precision of signals.[5] Then an agent has an incentive to seek new (more transparent) public information if the following holds:

$$u(\alpha_x, \alpha'_p) - c(\alpha'_p) > u(\alpha_x, \alpha_p) - c(\alpha_p), \tag{4.10}$$

i.e., $\Delta c < \Delta u$ where $\Delta c = c(\alpha'_p) - c(\alpha_p)$ and $\Delta u = u(\alpha_x, \alpha'_p) - u(\alpha_x, \alpha_p)$. That is, the utility should increase more than the cost does in order for the agent to actively use more transparent information.

Remark 1. *Rudebusch and Williams (2009)'s finding is not a puzzle if we take into account the above consideration: If the additional gain from using more transparent information is not large, then it is optimal for the forecasters to use only the previously available information.*

Remark 2. *Consider an environment where ex-ante, everything is equal across agents but the available resources for using signals are heterogeneous (i.e. income differences). Let w_i be the (net) income that agent i uses for receiving information. Then, even when $\Delta c < \Delta u$, the agent might not be receptive to better information if $c(\alpha_p) \leq w_i < c(\alpha'_p)$. Hence, an incentive to use better information is increasing in income level. Similarly, if $c_i(\alpha_p) < c_j(\alpha_p)$ for agents i and j where agent j's education level is higher than agent i, then less-educated agent might be inattentive to better information, consistent with Binder (2014)'s finding.*

We apply the above arguments to our model. Recall that a forecaster i solves the expected utility maximization problem in which the payoff function is given by

$$u_i = -\frac{1}{2}\left(a_i - (1-r)\theta - rA\right)^2.$$

Let $\mathbb{E}(u_i)$ be the ex-ante expected equilibrium payoff – the unconditional expectation of the

[5] One can think of CARA utility function; mean-variance form of the CARA function implies that utility is necessarily increasing in precision of signals.

payoff function for given α_x and α_p. Then

$$
\begin{aligned}
\mathbb{E}(u_i) &= \mathbb{E}\left(-\frac{1}{2}\left(a_i-(1-r)\theta-rA\right)^2\right) \\
&= -\frac{1}{2}\mathbb{E}\left(\frac{(1-r)(1-\lambda)}{\sqrt{\alpha_p}}\varepsilon+\frac{\lambda}{\sqrt{\alpha_x}}\varepsilon_i\right)^2 \\
&= -\frac{1}{2}\left(\frac{(1-r)^2(1-\lambda)^2}{\alpha_p}+\frac{\lambda^2}{\alpha_x}\right) \\
&= -\frac{1}{2}\frac{(1-r)^2(\alpha_x+\alpha_p)}{((1-r)\alpha_x+\alpha_p)^2}, \qquad (4.11)
\end{aligned}
$$

where the second equality is obtained by substituting in the equilibrium forecast and average forecast, the third equality follows because $\mathbb{E}(\varepsilon^2)=\mathbb{E}(\varepsilon_i^2)=1$, and the last equality comes from $\lambda=\frac{\alpha_x}{\alpha_x+\frac{\alpha_p}{1-r}}$.

Let $\bar{u}\equiv\mathbb{E}(u_i)$. Then we can show how changes in α_p affect the ex-ante expected utility of a forecaster, which determines the marginal revenue from using more transparent public information.

Proposition 4. *If α_p increases, then $\bar{u}$ increases when $r>0$, but decreases when $r<0$.*

Proof. Differentiating $\bar{u}$ with respect to α_p, we obtain

$$
\frac{\partial\bar{u}}{\partial\alpha_p}\propto r\frac{\alpha_x}{((1-r)\alpha_x+\alpha_p)^4} \qquad (4.12)
$$

Thus the sign of r determines the sign of $\frac{\partial\bar{u}}{\partial\alpha_p}$. □

The intuition behind this result is straightforward. If the precision of public information increases, a forecaster would put more weight on public signal relative to private signal. Note that in equilibrium the public signal is a relatively better predictor of average forecast than the private signal. Then as long as other agents also put more weight on public signal, aligning his forecast with other agents' forecasts would be beneficial when the forecasts are complements. However, when the forecasts are substitutes, the increased dependence on common public information by all agents would perversely affect each agent's utility.[6] The following corollary immediately follows.

Corollary 5 (Conditions for Non-Utilizing More Transparent Public Information)**.** *Suppose that two public signals p and p' with precisions α_p and α'_p, respectively, are available to forecasters. Then a forecaster is unreceptive to more transparent public information p' if, for any given α_x, either*

[6] It is easy to show that the effect of changes in α_x on $\bar{u}$ is exactly the opposite, where the intuition for the result can be similarly explained.

1. $r < 0$*; or*

2. $\bar{u}(\alpha_x, \alpha_p') - \bar{u}(\alpha_x, \alpha_p) < c(\alpha_p') - c(\alpha_p)$ *where* $\alpha_p' > \alpha_p$*.*

Corollary 5 implies that forecasters do not actively seek to use better public information even when it is readily available if the forecasting game exhibits substitutability or the cost of using better information is too high; rather, forecasters would stick to worse public information. The result justifies the findings by Rudebusch and Williams (2009): If forecasters wish to differentiate their choices from other forecasters' predictions about the aggregate economy, then they would rather be unreceptive to more precise information if that information is "publicly" available. This result questions whether enhanced dissemination of public information or more transparent communication by government agencies and central bank is always socially desirable.

4.4 Endogenous Private Information Acquisition

In Subsection 4.1, we showed that $Var(a|\theta, p)$ is necessarily decreasing in α_p for given α_x. In this subsection, we study the effect of endogenizing information choice as in Hellwig and Veldkamp (2009), Colombo, Femminis, and Pavan (2014), and Róndina and Shim (2013). In doing so, we add an information-acquisition stage prior to the forecasting game as follows:

$$\text{Initial Stage:} \quad \max_{\alpha_x} \mathbb{E}(u_i) - \frac{(1-r)^2}{2} C(\alpha_x), \tag{4.13}$$

where $\mathbb{E}(u_i) = -\frac{(1-r)^2}{2} \frac{\alpha_x + \alpha_p}{((1-r)\alpha_x + \alpha_p)^2}$, $C(0) = 0$, $C' > 0$, and $C'' > 0$. The term $\frac{(1-r)^2}{2}$ on $C(\alpha_x)$ is for normalization. Hence, each agent chooses α_x so as to maximize her expected utility subject to convex cost. Then the first order condition of the initial stage problem is given by

$$\frac{(1-r)\alpha_x + (1-2r)\alpha_p}{((1-r)\alpha_x + \alpha_p)^3} = C'(\alpha_x). \tag{4.14}$$

Let $G(\alpha_x) \equiv \frac{(1-r)\alpha_x + (1-2r)\alpha_p}{((1-r)\alpha_x + \alpha_p)^3}$. We restrict our attention to $r < \frac{1}{2}$ so that $G(0) > 0$. This assumption suffices for the equilibrium to exist in the initial stage problem.

Lemma 1. *Suppose that* $r < \frac{1}{2}$*. Then there exists a unique equilibrium to the initial stage problem* (4.13)*.*

Proof. It is easy to show that $\lim_{\alpha_x \to \infty} G(\alpha_x) = 0$ and thus the continuity of $G(\alpha_x)$ ensures the existence of equilibrium. In addition, $G'(\alpha_x) > 0$ for $\alpha_x < \frac{(3r-1)\alpha_p}{1-r}$ and $G'(\alpha_x) < 0$ for $\alpha_x > \frac{(3r-1)\alpha_p}{1-r}$. By the fixed point theorem, there is a unique equilibrium that solves (4.14). □

Let $\alpha_x^* > 0$ be the precision of private information acquired in the unique equilibrium of the initial stage. Then,

Proposition 5 (Effect of Changes in Public Precision on Endogenous Information Choice)**.** *The equilibrium precision of private information falls with an increase in the precision of public information. Formally,*

$$\frac{d\alpha_x^*}{d\alpha_p} < 0. \tag{4.15}$$

Proof. Differentiate $G(\alpha_x)$ with respect to α_p to obtain $\frac{\partial G(\alpha_x)}{\partial \alpha_p} < 0$. Hence the optimal choice of α_x falls with an increase in α_p. □

Proposition 5 is in line with Colombo, Femminis, and Pavan (2014)'s result on the crowding-out effect of public information: More precise public information reduces the agents' incentives to obtain better private information, and thus crowds out the agents' acquisition of private information in equilibrium.[7] Then the next proposition on dispersion of forecasts follows.

Proposition 6. *When endogenous private information choice is allowed, the dispersion of forecasts increases with an increase in α_p if and only if $\alpha_p < (1-r)\alpha_x$ (or equivalently $\lambda > \frac{1}{2}$).*

Proof. Because a higher α_p lowers equilibrium α_x by Proposition 5,

$$\frac{dVar(a|\theta,p)}{d\alpha_x}\underbrace{\frac{d\alpha_x}{d\alpha_p}}_{<0} > 0$$

if and only if $\frac{dVar(a|\theta,p)}{d\alpha_x} < 0$, which holds iff $\alpha_p < (1-r)\alpha_x$ (Corollary 1). □

Proposition 6 implies that, taking into account the endogenous information choice of private signal, more precise public information has an (indirect) perverse effect on dispersion of forecasts by lowering precision of private information acquired in equilibrium. If this effect dominates the (direct) effect of more precise public information on idiosyncratic variation of forecasts (i.e., when the relative precision of private information $\lambda > \frac{1}{2}$), then dispersion may increase with an increase in α_p in contrast to Corollary 1 (an unambiguous decrease of dispersion by more precise public information).

[7] In contrast, Róndina and Shim (2013) show that an increase in the precision of private information reduces the precision of public information.

5 Application: Two-Stage Game

5.1 Two-Stage Setup

In this section, we consider an application of our model to a setting in which firms – uncertain about the aggregate market demand – set optimal production strategies in two distinct decision-making stages: a forecasting stage and a production stage. In particular, we model a two-stage game of the following form: In the first stage, each firm chooses a forecast about the aggregate market demand (the underlying fundamentals of the economy) given private and public signals; then, in the second stage, each firm chooses a production level given the optimal forecast from the first stage. In modeling the second stage, we follow Vives (2008) and Róndina and Shim (2014).

Justification for our two-stage model comes from the fact that firms often have various divisions involved with different decision-making goals, such as research department, planning department, production and distribution department, marketing department, sales department, and so on. We focus on two consecutive decision-making steps, first in the research department and then in the production department; for which the key link that connects the two stages would be $\mathbb{E}(\theta|a) = a$.

The first stage is as described in Section 2. In the second stage, each firm produces a homogeneous output good of quantity q_i. The individual firm faces the inverse demand curve:

$$z = \theta - bQ,$$

where z is the unitary price of the firm's output, θ is a demand shock common but uncertain to firms, $Q \equiv \int_i q_i d_i$ is the aggregate output, and $b \geq 0$. Each firm faces a quadratic cost function $c(q_i) = \frac{1}{2}cq_i^2$, where $c > 0$. Then, for each i, the payoff function in the second stage is given by the individual firm's profit:

$$\pi_i(q_i, Q, \theta) \equiv zq_i - c(q_i) = (\theta - bQ)q_i - \frac{1}{2}cq_i^2. \tag{5.1}$$

At the time of choosing q_i, the individual firm does not know the uncertain state of the aggregate demand (θ) nor the unobserved aggregate production Q, but it receives a "signal" – its optimal forecast a_i from the first stage. That is, a_i effectively acts as the only signal that each firm observes at the start of the second stage, i.e., $\mathbb{E}(\theta|a_i) = a_i$.

5.2 Equilibrium

In the second stage, each firm chooses q_i given the optimal forecast a_i so as to maximize $\mathbb{E}\left[\pi_i(q_i, Q, \theta)|a_i\right]$.[8] Note that the information set of firm i in the second stage is given by the

[8]Note that we do not condition on private and public signals, x_i and p, because those information are already included in the forecast.

realization of a_i, whereas the state of the world is given by the realizations of θ, $(a_i)_{i\in[0,1]}$, and p. Then the aggregate output Q is only a function of (θ, p). A linear equilibrium is redefined as follows.

Definition 2. *A linear equilibrium of the two-stage game is a strategy profile $(a, q) : \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ such that a is linear in x and p, and satisfies (3.1) for all (x, p); q is linear in a, and for any given a,*

$$q(a) = arg\max_{q'} \mathbb{E}\left[\pi(q', Q(\theta, p), \theta)|a\right], \tag{5.2}$$

where $Q(\theta, p) = \int_a q(a)dP(a|\theta, p)$ for all θ, and $P(a|\theta, p)$ denotes the conditional cumulative distribution of a given (θ, p).

Note that when θ is known, the equilibrium output is given by $q_i = \frac{1}{b+c}\theta$ for all i. Under incomplete information, a strategy q_i constitutes an equilibrium if and only if, for all a_i, q_i satisfies the first-order condition:

$$q_i(a_i) = \mathbb{E}\left[c^{-1}\theta - bc^{-1}Q(\theta, p)|a_i\right], \tag{5.3}$$

where $Q(\theta, p) = \mathbb{E}\left[q(a)|\theta, p\right]$ for all (θ, p). The following proposition then follows.

Proposition 7. *For any given value of (θ, p), a linear equilibrium of the two stage game exists and is unique. The equilibrium forecast of firm i is given by (3.3), and the equilibrium output of firm i is given by*

$$q_i(a_i) = \frac{1}{b+c}a_i. \tag{5.4}$$

Proof. The first part follows from the proof of Proposition 1. Given a_i, the first order condition for the optimization problem of the second stage, i.e.,

$$\max \mathbb{E}[\pi_i|a_i] = \left[\mathbb{E}(\theta|a_i) - b\mathbb{E}(Q(\theta)|a_i)\right] q_i - \frac{1}{2}cq_i^2,$$

gives

$$q_i = \frac{1}{c}\left[\mathbb{E}(\theta|a_i) - b\mathbb{E}(Q(\theta, p)|a_i)\right]. \tag{5.5}$$

Individual firm output is thus increasing in the expected level of the fundamentals and decreasing in the expected level of aggregate output. Given the linearity, it is natural to look for equilibrium output decisions that are linear in a so that $q_i(a_i) = \kappa_2 + \kappa_3 a_i$, where κ_2 and κ_3 are constants determined in equilibrium. Then, $Q(\theta, p) = \kappa_2 + \kappa_3 A(\theta, p)$, and (5.5) reduces to

$$q_i(a_i) = \frac{1}{c}\left[\mathbb{E}(\theta|a_i) - b\mathbb{E}(\kappa_2 + \kappa_3 A(\theta, p)|a_i)\right]. \tag{5.6}$$

Note that $A(\theta, p) = \theta + (1-\lambda)(\alpha_p)^{-1/2}\varepsilon$. Suppose that $\theta \sim \mathbb{N}\left(\mu_\theta, \frac{1}{\alpha_\theta}\right)$. Then $\mathbb{E}(A) = \mu_\theta$ and

$\alpha_A = \frac{1}{\frac{1}{\alpha_\theta}+\frac{(1-\lambda)^2}{\alpha_p}}$. As $\alpha_\theta \to 0$ (improper prior), then $\alpha_A \to 0$. Because $a_i = A + \lambda(\alpha_x)^{-1/2}\varepsilon_i$, following the technical appendix of Vives (see page 15) under the assumption that $\alpha_\theta \to 0$, we arrive at

$$\mathbb{E}(A|a_i) = \mu_\theta + \frac{\alpha_a}{\alpha_A + \alpha_a}(a_i - \mu_\theta) = a_i.$$

Substituting $\mathbb{E}(\theta|a_i) = a_i$ and $\mathbb{E}(A(\theta,p)|a_i) = a_i$ in (5.6),

$$\begin{aligned} q_i(a_i) &= \frac{1}{c}\left[a_i - b(\kappa_2 + \kappa_3 a_i)\right]. \\ &= -\frac{b}{c}\kappa_2 + \frac{1-b\kappa_3}{c}a_i. \end{aligned}$$

It follows that $q_i(a_i) = \kappa_2 + \kappa_3 a_i$ constitutes a linear equilibrium if and only if κ_2 and κ_3 solve $\kappa_2 = -\frac{b}{c}\kappa_2$ and $\kappa_3 = \frac{1-b\kappa_3}{c}$. Equivalently, $\kappa_2 = 0$ and $\kappa_3 = \frac{1}{b+c}$, which gives (5.4). Clearly, this is the unique symmetric linear equilibrium. Note that in the linear equilibrium, the aggregate output across firms is $Q(\theta,p) = \frac{1}{b+c}A(\theta,p)$. □

5.3 Dispersion of Individual Output and Volatility of Aggregate Output

If information is complete, then all firms choose $q_i = \frac{1}{b+c}\theta$. So there exists no dispersion in output levels across firms. When θ is not known, each firm i chooses $q_i = \frac{1}{b+c}a_i$. It is easy to see that the optimal output in the second stage is directly linked to the optimal forecast from the first stage. Hence, idiosyncratic variation in forecasts due to noise would subsequently generate cross-sectional dispersion in output (that is, variation in output across firms) and volatility of aggregate output (that is, variation in aggregate output around the complete information level).

One way to appreciate our two-stage model is then to consider how the precision of public and private information relates to cross-sectional dispersion in output and volatility of aggregate output. The optimal output from the second stage can be rewritten as:

$$q_i = \frac{1}{b+c}\left(\theta + \lambda(\alpha_x)^{-1/2}\varepsilon_i + (1-\lambda)(\alpha_p)^{-1/2}\varepsilon\right), \tag{5.7}$$

and the aggregate output as:

$$Q = \frac{1}{b+c}\left(\theta + (1-\lambda)(\alpha_p)^{-1/2}\varepsilon\right). \tag{5.8}$$

The equilibrium levels of dispersion in output and volatility of aggregate output for a given realization of θ can be characterized by the following expressions, respectively:

$$Var(q|\theta) = \frac{1}{(b+c)^2}\left[\frac{\lambda^2}{\alpha_x} + \frac{(1-\lambda)^2}{\alpha_p}\right] = \frac{1}{(b+c)^2}\left(\alpha_x + \frac{\alpha_p}{(1-r)^2}\right)\left(\alpha_x + \frac{\alpha_p}{1-r}\right)^{-2} \quad (5.9)$$

$$Var(Q|\theta) = \frac{1}{(b+c)^2}\frac{(1-\lambda)^2}{\alpha_p} = \frac{1}{(b+c)^2}\left(\frac{\alpha_p}{(1-r)^2}\right)\left(\alpha_x + \frac{\alpha_p}{1-r}\right)^{-2} \quad (5.10)$$

The following proposition analyzes how changes in the precision of public and private information affect cross-sectional dispersion in output and volatility of aggregate output.

Proposition 8 (Sufficient Conditions for Non-decreasing Dispersion and Volatility)**.**

(i) The cross-sectional dispersion in output weakly increases if

$$\left[\alpha_x(2r-1) - \frac{\alpha_p}{1-r}\right]d\alpha_p \geq \left[(1-r)^2\alpha_x + (1+r)\alpha_p\right]d\alpha_x. \quad (5.11)$$

(ii) The volatility of aggregate output weakly increases if

$$\left[\alpha_x - \frac{\alpha_p}{1-r}\right]d\alpha_p \geq 2\alpha_p d\alpha_x. \quad (5.12)$$

Proof. *(i)* A sufficient condition for non-decreasing dispersion is found by setting $dVar(q|\theta) \geq 0$. By total differentiating (5.9), we have

$$\begin{aligned} dVar(q|\theta) = & \left[\frac{\left(\alpha_x + \frac{\alpha_p}{1-r}\right)^2 - 2\left(\alpha_x + \frac{\alpha_p}{(1-r)^2}\right)\left(\alpha_x + \frac{\alpha_p}{1-r}\right)}{\left(\alpha_x + \frac{\alpha_p}{1-r}\right)^4}\right]d\alpha_x \\ & + \left[\frac{\frac{1}{(1-r)^2}\left(\alpha_x + \frac{\alpha_p}{1-r}\right)^2 - 2\frac{1}{1-r}\left(\alpha_x + \frac{\alpha_p}{(1-r)^2}\right)\left(\alpha_x + \frac{\alpha_p}{1-r}\right)}{\left(\alpha_x + \frac{\alpha_p}{1-r}\right)^4}\right]d\alpha_p \end{aligned} \quad (5.13)$$

By setting this to be greater than equal to zero, we obtain (5.11). *(ii)* Similarly, (5.12) is obtained by setting $dVar(Q|\theta) \geq 0$. □

We can also look at the partial effect of an increase in the absolute precision of information on dispersion and volatility

Corollary 6 (More Transparent Public Information)**.**

(i) The cross-sectional dispersion in output falls with an increase in α_p for given α_x when $r < \frac{1}{2}$; and increases with an increase in α_p for given α_x when $r > \frac{1}{2}$ if and only if $\lambda > \frac{1}{2r}$ for $r > \frac{1}{2}$.

(ii) The volatility of aggregate output increases with an increase in α_p for given α_x if and only if $\lambda > \frac{1}{2}$.

Proof. *(i)* Set $d\alpha_x = 0$ in (5.13). Then

$$sign\left(\frac{dVar(q|a)}{d\alpha_p}\right) = sign\left((2r-1)\alpha_x - \frac{\alpha_p}{1-r}\right) \tag{5.14}$$

When $r < \frac{1}{2}$, it is always $(2r-1)\alpha_x - \frac{\alpha_p}{1-r} < 0$ and so $\frac{dVar(q|\theta)}{d\alpha_p} < 0$. When $r > \frac{1}{2}$, $\frac{dVar(q|\theta)}{d\alpha_p} < 0$ if $(2r-1)\alpha_x - \frac{\alpha_p}{1-r} < 0$ or $2r < \frac{\alpha_x + \frac{\alpha_p}{1-r}}{\alpha_x} = \lambda^{-1}$, but $\frac{dVar(q|\theta)}{d\alpha_p} > 0$ if otherwise. *(ii)* Set $d\alpha_x = 0$ in (5.12). Then volatility increases iff $\alpha_x - \frac{\alpha_p}{1-r} > 0$, which is equivalently to $\lambda > \frac{1}{2}$. □

Corollary 7 (More Transparent Private Information)**.**

(i) The cross-sectional dispersion in output falls with an increase in α_x for given α_p.

(ii) The volatility of aggregate output falls with an increase in α_x for given α_p.

Proof. *(i)* Set $d\alpha_p = 0$ in (5.13). Then, $\frac{dVar(q|\theta)}{d\alpha_x} < 0$. *(ii)* Obvious from (5.10). □

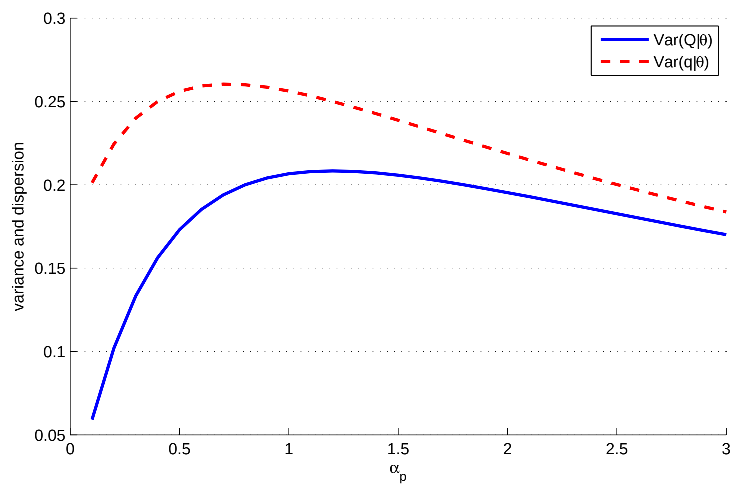


Figure 2: The effect of α_p on dispersion and volatility for $r = 0.8$ and $\alpha_x = 6$

Together with Corollaries 1 and 2, these results suggest that more transparent public information unambiguously lowers dispersion of forecasts, but may raise both dispersion in output across firms and volatility of aggregate output when complementarities are strong enough and the relative precision of private information is sufficiently high. For example,

Figure 2 illustrates the effect of α_p on dispersion in individual output and volatility of aggregate output for fixed values of $\alpha_x = 6$ and $r = 0.8$. On the other hand, more transparent private information may increase dispersion of forecasts when the relative precision of private information is sufficiently low, but unambiguously lowers both dispersion in output and volatility of aggregate output.

The intuition behind these results is straightforward if we decompose the effect of an increase in signal precision on dispersion into three separable channels: (1) an increase in the absolute precision, (2) an increase (decrease) in the relative precision of private information, and (3) a decrease (increase) in the relative precision of public information.

Let's first consider an increase in α_x for given α_p. Note that

$$Var(q|\theta) \propto \underbrace{\lambda^2}_{(2)} \underbrace{(\alpha_x)^{-1}}_{(1)} + \underbrace{(1-\lambda)^2}_{(3)} (\alpha_p)^{-1}, \tag{5.15}$$

where the effects of a higher α_x can be decomposed as follows:

(1) lowers uncertainty from the individual error ε_i $\Rightarrow$ less idiosyncratic noise;

(2) raises the relative precision of private information ($\Rightarrow$ more weight on private signal) $\Rightarrow$ more weight on idiosyncratic noise;

(3) lowers the relative precision of public information ($\Rightarrow$ less weight on public signal) $\Rightarrow$ less weight on (given) common noise.

That is, as private information becomes more precise, *more* weight is put on *less* idiosyncratic noise, together with less weight on (given) common noise, in turn lowering cross-sectional variation in output.

Similarly consider an increase in α_p for given α_x:

$$Var(q|\theta) \propto \underbrace{\lambda^2}_{(2)} (\alpha_x)^{-1} + \underbrace{(1-\lambda)^2}_{(3)} \underbrace{(\alpha_p)^{-1}}_{(1)}, \tag{5.16}$$

where the effects of a higher α_p can be decomposed as follows:

(1) lowers uncertainty from the common error ε $\Rightarrow$ less common noise;

(2) lowers the relative precision of private information ($\Rightarrow$ less weight on private signal) $\Rightarrow$ less weight on (given) idiosyncratic noise;

(3) raises the relative precision of public information ($\Rightarrow$ more weight on public signal) $\Rightarrow$ more weight on common noise.

However, unlike the previous case, the effects of the above three channels are ambiguous: which channel dominates crucially depends on the strength of complementarity and on the initial precision of private information. For example, consider an environment where the complementarities between individual forecasts and average forecast are strong (e.g., $r = \frac{3}{4}$). Suppose that the precision of private signal is initially so high (relative to that of public signal) such that the common noise gets relatively low weight (e.g., $\alpha_x = 10$, $\alpha_p = 1$, and so, $\lambda = \frac{10}{14} = 0.7143 > \frac{1}{2r} = \frac{2}{3}$). Now suppose a small increase in the precision of public information (e.g., $\alpha_p^{'} = 1.1$ and so $\lambda = 0.6944 > \frac{2}{3}$). Then, we have the following changes through the three channels:

$$Var(q|\theta) \propto \underbrace{\underbrace{\underbrace{\lambda^2}_{0.5101\to0.4822} (\alpha_x)^{-1}}_{0.0510\to0.0482} + \underbrace{\underbrace{(1-\lambda)^2}_{0.0816\to0.0933} \underbrace{(\alpha_p)^{-1}}_{1\to0.9091}}_{0.0816\to0.0849}}_{0.1326\to0.1331}.$$

That is, despite the increased precision of public information and thus a higher weight is put on the common noise, the common noise still gets relatively low weight. Correspondingly, the idiosyncratic noise still gets relatively high weight although a bit lower than before. Then an increased but still relatively low weight on common noise perversely extracts proportionally more uncertainty from the common noise than before. This increased common variation due to noise in public information offsets the decreased idiosyncratic variation due to noise in private information. Thus cross-sectional dispersion in output increases, and the increased net dispersion in output comes from the common variation.

Note that noise in public information generates nonfundamental aggregate volatility only through channels (1) and (3). Taking the above arguments into account, the effect of more transparent public information on aggregate volatility depends on the relative precision of signals.

5.4 Discussion on Aggregate Utility

The welfare effects of public and private information in an economy with complementarities have been studied by Angeletos and Pavan (2004). They argue that, in an environment where the complementarities are weak, more transparent public information unambiguously increases welfare, whereas more transparent private information does not necessarily increase welfare. On the other hand, when complementarities are strong enough so that multiple equilibria are possible, more transparent public information is welfare enhancing if the market coordinates on the desirable equilibrium, whereas there is a critical threshold for the precision of public information above which more transparent public information can be welfare reducing if the market coordinates on the undesirable equilibrium.

In this subsection, we also examine the comparative statics of equilibrium welfare with respect to the information structure applied to our setting. In our setup, firms solve a two-stage problem, in which they first choose their optimal forecasts and subsequently choose their output given their forecasts about the underlying fundamentals. Recall that the individual firm's profit function in the second stage is given by $\pi = (\theta - bQ) - \frac{1}{2}cq^2$.

Suppose now that a social planner considers the aggregate utility maximization problem where social welfare is given by a utilitarian aggregator $U = \int_0^1 \pi_i di$. In equilibrium, social welfare is then given by

$$\begin{aligned}
\mathbb{E}[U] &= \mathbb{E}\left[\int_0^1 \pi_i di\right] = \mathbb{E}\left[\int_0^1 \left((\theta - bQ)q - \frac{1}{2}cq^2\right) di\right] \\
&= \mathbb{E}\left[\theta Q - bQ^2 - \frac{c}{2}\int_0^1 \underbrace{q^2}_{=\int_0^1 (q-Q)^2 di + Q^2} di\right] \\
&= \mathbb{E}\left[\theta Q - (b + \frac{c}{2})Q^2 - \frac{c}{2}\int_0^1 (q - Q)^2 di\right] \\
&= \mathbb{E}\left[\frac{\theta}{b+c}A - \frac{b + \frac{c}{2}}{(b+c)^2}A^2 - \frac{c}{2}\frac{1}{(b+c)^2}\frac{\lambda^2}{\alpha_x}\int_0^1 \varepsilon_i^2 di\right] \text{ using solution for } q \text{ and } Q \\
&= \kappa\mathbb{E}(\theta^2) - b\underbrace{\frac{1}{(b+c)^2}\frac{(1-\lambda)^2}{\alpha_p}}_{=\mathbb{V}(Q|\theta)} - \frac{c}{2}\underbrace{\frac{1}{(b+c)^2}\left[\frac{(1-\lambda)^2}{\alpha_p} + \frac{\lambda^2}{\alpha_x}\right]}_{=\mathbb{V}(q|\theta)} \text{ using } a \text{ and } A,
\end{aligned}$$

where κ is the collection of constants for $\mathbb{E}(\theta^2)$. Then, it follows that:

$$\mathbb{E}[U] \propto -b\mathbb{V}(Q|\theta) - \frac{c}{2}\mathbb{V}(q|\theta). \tag{5.17}$$

That is, social welfare decreases with both volatility in aggregate output, $\mathbb{V}(Q|\theta)$, and dispersion in individual quantity, $\mathbb{V}(q|\theta)$. Then, in order to improve social welfare, the utilitarian planner would want to lower both (if possible) variation in aggregate output and variation in the cross-section of the population.

Suppose that the planner can choose either a higher α_p or a higher α_x. For the sake of simplicity, let $b = 1$ and $c = 2$. Then (5.17) reduces to $\mathbb{E}[U] \propto -\mathbb{V}(Q|\theta) - \mathbb{V}(q|\theta)$.

Policy Choice: More Precise Private Information Corollary 7 suggests that it is always beneficial from a social viewpoint to increase α_x; because when α_x increases for given level of α_p, both $Var(q|\theta)$ and $Var(Q|\theta)$ unambiguously fall. Figure 3 illustrates the effect of α_x on welfare for fixed values of $\alpha_p = 1$ and $r = 0.3$. This simple comparative statics of equilibrium welfare with respect to private information suggests that government intervention that encourages the collection of private information will always increase welfare. Our result that more precise private information increases welfare contrasts with the result of Angeletos and Pavan (2004) that more precise private information has an ambiguous effect on welfare.

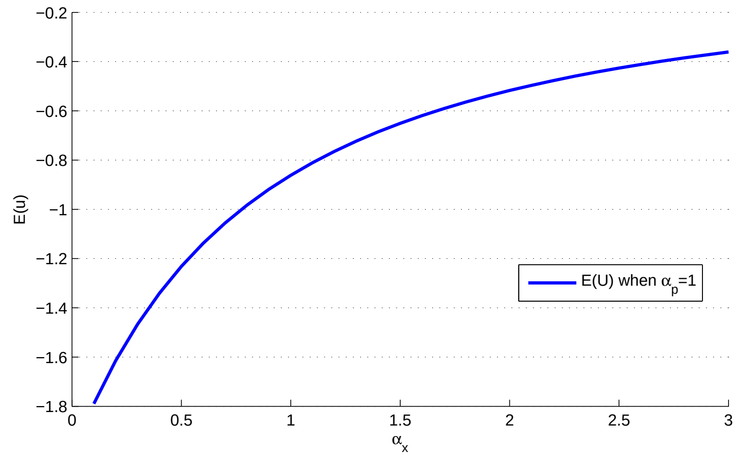


Figure 3: Expected aggregate utility relative to α_x for $b = 1$, $c = 2$, $r = 0.3$, and $\alpha_p = 1$

Policy Choice: More Precise Public Information Corollary 6 implies that if $\alpha_p < (1-r)(2r-1)\alpha_x$ (or $\lambda > \frac{1}{2r}$ for $r > \frac{1}{2}$), then a higher α_p unambiguously increases both $Var(q|\theta)$ and $Var(Q|\theta)$ (Refer the lower range of $\alpha_p < 0.72$ in Figure 2), and thus reduces the expected aggregate utility. That is, if the complementarity is sufficiently strong and the relative precision of public information is very low, an increase in the precision of public information reduces welfare. For example, Figure 4 shows that, for a fixed value of $\alpha_x = 6$, when the complementarity is pretty strong ($r = 0.8$), more precise public information is welfare-reducing if the precision of public information is below some threshold level (in this case, $\bar{\alpha}_p = 1$).[9]

Also, the expected aggregate utility decreases with an increase in α_p only if $\alpha_p < (1-r)\alpha_x$. Hence, for a very low level of precision of public information, welfare may decrease with more precise public information even when the complementarity is weak.[10] Figure 5 illustrates this effect of α_p on welfare for fixed values of $\alpha_x = 4$ and $r = 0.3$. We can see that, for relatively lower values of α_p compared to α_x, more precise public information is in fact welfare-reducing.

Our results that more precise public information may decrease welfare regardless of whether complementarities are strong or weak are in direct comparison with the results of Angeletos and Pavan (2004). They show that, when the complementarity is weak, an increase in the precision of public information always increases welfare (Proposition 4); when

[9]Note that in this example both dispersion and volatility increase when $\alpha_p < 0.72$ (or $\lambda > \frac{5}{8}$); dispersion decreases while volatility increases when $0.72 < \alpha_p < 1.2$ (or $\frac{1}{2} < \lambda < \frac{5}{8}$); and both dispersion and volatility decrease when $\alpha_p > 1.2$ (or $\lambda < \frac{1}{2}$). See Figure 2. The threshold beyond which the decrease in dispersion outweighs the increase in volatility so that the expected aggregate utility increases with an increase in α_p is $\bar{\alpha}_p = 1$ (or $\bar{\lambda} = 0.54$).

[10]This holds when the increase in $V(Q|\theta)$ due to an increase in α_p when $\alpha_p < (1-r)\alpha_x$ outweighs the decrease in $V(q|\theta)$ due to an increase in α_p when $r < \frac{1}{2}$.

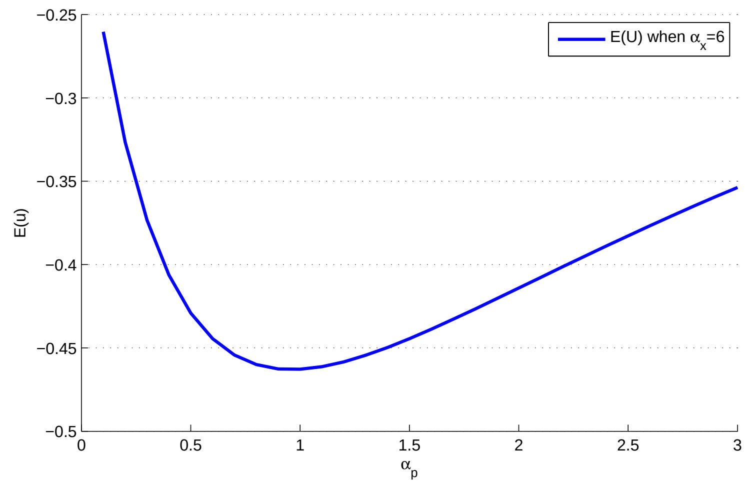


Figure 4: Expected aggregate utility relative to α_p for $b = 1$, $c = 2$, $r = 0.8$ and $\alpha_x = 6$

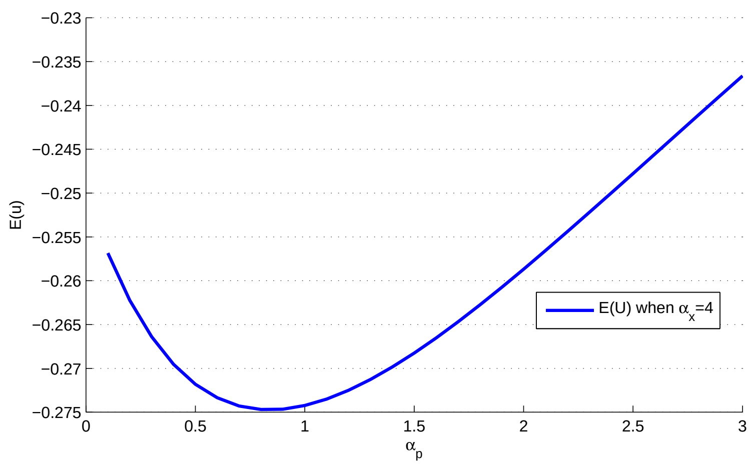


Figure 5: Expected aggregate utility relative to α_p for $b = 1$, $c = 2$, $r = 0.3$ and $\alpha_x = 4$

the complementarity is strong, an increase in the precision of public information below some threshold level of transparency increases welfare but an increase in the precision above the threshold may decrease welfare by increasing the probability that the "bad" equilibrium is played, in which case welfare is maximized at an intermediate level of transparency (Proposition 7).[11] Our results suggest the opposite: an increase in the precision of public information below some threshold level always reduces welfare.[12] Hence, our exercise provides a norma-

[11]If instead the "good" equilibrium is played, then welfare is necessarily increasing in the level of transparency, and thus full transparency of public information is desirable.

[12]The key source of difference comes from the fact that we condition only on θ when computing variance,

tive implication on policy maker's choice in improving social welfare: In the economy where the precision of public information is initially very low, enhanced dissemination of public information through the media or more transparent public disclosure by policy makers can reduce welfare. This is in line with the literature that argue that the welfare effect of increased public disclosures is ambiguous.[13]

6 Future Work (in progress)

- Estimation on r, using kalman filter (state space model)
- Relate to data on volatility and dispersion.

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while Angeletos and Pavan (2004) condition on both θ and p. (We will add more discussion on the justification of our choice of conditional expectations.)

[13]See Morris and Shin (2002). They show that the greater the precision of the agents' private information, the more likely it is that the greater precision of public information lowers social welfare.

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