Trade, Welfare, and International Labor Market Spillovers

Kyu Yub Lee

Job Market Paper

Abstract

This paper develops a two-country new trade theory framework with two types of labor (skilled and unskilled) and an imperfect labor market arising from country-specific real minimum wages. It examines welfare implications of trade liberalization and spillover effects of labor market shocks in a global economy. The model identifies two key forces that shape the results: i) external scale effects generating employment expansion among unskilled workers, and ii) wage effects arising from intensive use of skilled workers. The model provides three clear predictions. First, the introduction of trade results in a rise of wage inequality in both countries. Second, trade enhances welfare by lowering unemployment across countries only when external scale effects dominate. Last, when wage effects outweigh external scale effects, a country will be better off if the other country raises its minimum wage, but both countries will be worse off under unduly strong external scale effects in the global economy.

Keywords: External Scale Effects; Wage Effects; Welfare; Labor Market Spillovers

JEL-Codes: F11, F12, F16, L11

1 Introduction

Much attention has been paid to the impact of trade liberalization on wages and unemployment, yet relatively little is known about cross-country adjustments caused by shocks to labor

---

*I would like to express my sincere gratitude to Steven Matusz for his guidance throughout the process of this work. I am also indebted to Carl Davidson and Jay Wilson for many useful comments and suggestions. I would like to thank participants, in particular, Christian Ahlin, Jon Eguia, Tony Doblas Madrid, Arijit Mukherjee at the Theory Brown Bag seminar, held at Michigan State University, Lansing, 2014, and participants at the 2014 Midwest International Trade Meeting in Kansas, the 51st Annual Meeting of the Missouri Valley Economic Association in Missouri, and the 2015 Annual Meeting of the Midwest Economics Association in Minnesota. All remaining errors are mine.

†Department of Economics, Michigan State University, 110 Marshall-Adams Hall, East Lansing, MI 48824-1038 USA. Phone USA 517-515-0600. Email: leekyu8@msu.edu.
markets linked by trade for goods. This paper is interested in how trade interacts with labor market institutions to produce effects on distribution, employment and aggregate welfare. The main question is: How does variation of labor market institution, e.g. minimum wage, in one country affect its trading partners?

Consider a world comprised of Europe and the U.S. Well known is that Europe has a more rigid labor market than has the U.S. Davis (1998) and Meckl (2006) introduce minimum wages into the Heckscher-Ohlin model and shed light on insights that European minimum wages prop up U.S. wages. In other words, tightening labor market constraints in one country results in transmission of a positive labor market effect to the other country. This result has been the conventional view among scholars and politicians.

Recently, however, the same question has received attention in the monopolistic competition model with heterogeneous firms and country-specific minimum wages. Egger, Egger, and Markusen (2012) overturn the results by the aforementioned models and conclude that European minimum wages prop up U.S. unemployment. That is, stronger labor market institution in a country results in a negative labor market spillover effect on its trading partners. The model by Egger et al. (2012) hinges on the assumption of a single factor in which the real wage is subject to an exogenously specified minimum-wage. Hence, the assumption fails to address an adjustment mechanism via factor prices on external shocks. One may conjecture that when both worker types are indispensable in production, but skilled workers are used intensively, variations in the minimum wage would impact firms’ variable costs relatively lightly, while variations in the stock of skilled workers would impact heavily.

To explore this issue, this paper develops a two-country new trade theory framework with heterogeneous workers and labor market frictions arising from country-specific real minimum wages. The model’s novelty is the addition of two types of labor, skilled and unskilled, into the work by Egger et al. (2012) who emphasize external scale effects generating employment expansion. In this extension, this paper finds that their main result is not necessarily robust. By identifying two counteracting forces, namely wage effects and external scale

---

1 See also Krugman (1995) for detailed discussion of labor market institutions and trade.
2 In the earlier work, many models are built on the classic Heckscher-Ohlin model. For instance, Davidson et al. (1988, 1999) introduce search frictions into that model, which implies that if labor market frictions tighten in one country, then the unemployment rate rises. However, the trading partner benefits by a lower unemployment rate due to the terms-of-trade appreciation. In the spirit of Brecher (1974), Davis (1998) and Meckl (2006) develop a trade model with a flexible wage U.S. and a rigid wage Europe; they conclude that if Europe raises the minimum wage, it benefits the U.S. by increasing the U.S. wage. The key insight is that Europe determines U.S. wages via factor price equalization.
3 Melitz (2003) relies on a single factor with a flexible real wage among symmetric countries. Accordingly, the model developed herein may be viewed as a variant of Melitz (2003), incorporating flexible love-of-variety and multiple factors with an imperfect labor market.
4 External scale effects (following Ethier 1982) are known also as love-of-variety, variety gains, increasing
effects, this paper shows that shocks to a country’s labor markets exert either positive or negative spillover effects on its trading partners. When wage effects dominate external scale effects, the U.S. will be better off if Europe raises its minimum wage. Intuitions follow. Consider Europe raises its real minimum wage. An increase in the minimum wage directly increases all intermediate input firms’ variable costs. Due to worsened profitability, marginal firms begin to exit the market. Then, a reduced number of active firms lead to a fall in skilled workers’ wage, thus, European unemployment increases and welfare falls. Its trading partner the U.S. is impacted indirectly by a rise in European minimum wage. Two effects compete in this scenario. On one hand, potential entrants would benefit from positive wage effects or a lowered skilled workers’ wage. Lower variable cost enables those firms to enter the market. On the other hand, at the same time, those firms also encounter negative external scale effects or decreased demand for inputs by final output producers. If wage effects dominate, a rise in the minimum wage leads potential firms in the U.S. to enter the market. Entry of these firms expands employment (thus lowers unemployment for unskilled workers) and results in enhancing the U.S. welfare. When external scale effects dominate wage effects, the model has qualitatively the same prediction as in Egger et al. (2012).

The particular model considered here is where, in each country, the final output producers assemble various intermediate inputs according to the generalized Constant Elasticity of Substitution (CES) production technology which captures external scale effects. Each intermediate input firm with some market power has the ability to produce a unique variety valued by final output producers. Production by intermediate input firms involves both fixed and variable costs. Fixed costs are incurred as investment in units of the final good; variable costs use skilled and unskilled workers. Unskilled workers face the threat of unemployment due to a country-specific minimum wage, whereas skilled workers with flexible real wages are fully employed. Wage effects derive from intensive use of skilled workers. It then is possible to solve explicitly for autarky equilibrium assuming a Pareto distribution from which a firm draws random productivity.

In addition to focusing on how domestic labor market outcomes are affected by foreign labor market institutions, this paper also studies welfare implication of trade liberalization. It shows that the introduction of trade increases wage inequality between different skill-type workers in both countries. Thus, intuition of the result is: Without trade impediments, open-

---

5 As Ethier (1982) stresses, trade in intermediate inputs is important (see Yi 2003; Amiti and Konings 2007; and more recently, Amiti and Davis 2011). It occurs mostly among developed countries and represents respectively 56% and 73% of overall trade flows in goods and services (see Miroudot et al. 2009; Sturgeon and Memedovic 2010).
ing to trade expands market size for all intermediate input firms, compared to autarky. Those firms’ increased aggregate demand for skilled results in a rise of skilled workers’ wages since the stock of skilled workers is exogenous. In the model, increased wage for skilled workers implies a rise of wage inequality since the real minimum wage for the unskilled is binding. The result of widened wage inequality is consistent with the literature on trade liberalization and wage inequality (see, e.g., Yeaple 2005; Helpman et al. 2010; Harrigan and Reshef 2015; Burstein and Vogel 2010).

The other main result is that trade yields two opposite forces: positive external scale effects generating employment expansion, but negative wage effects arising from intensive use of skilled workers. Gains from trade materialize only when the former effects dominate. From the perspective of intermediate input firms, profitability essentially reliant on market demand and variable production costs would determine if those firms exit or enter the market. With strong external scale effects, liberalized trade leads to entry of potential firms. An increased aggregate demand for unskilled workers reduces the unemployment rate. Accordingly, total output and total labor income increase, thereby improving welfare across countries. Although average productivity falls in the product market with heterogeneous firms, gains from trade rise via positive employment expansion. Adjustment at the extensive margin of firms is crucial in the present paper, since selection effects are neutralized by the assumption of no trade cost. Interestingly, this paper shows a possibility that trade worsens both economies if negative wage effects outweigh positive employment effects. With the wage inequality result, we can say that by the opening to trade, skilled workers always become winners whereas unskilled workers may be losers due to unemployment or the threat thereof.

A long line of literature exists on trade and unemployment. Traditional international trade theory uses full-employment conditions in its simple and elegant Heckscher-Ohlin (HO) model. Although contributing to our understanding of trade patterns together with two important theorems (Stolper-Samuelson and Rybczynski), the model cannot be relied upon once attention is directed to unemployment issues. Many scholars extended the HO model by employing minimum wages (Brecher 1974; Davis 1998; Oslington 2002; Meckl 2006), implicit contracts (Matusz 1985, 1986), search frictions (Davidson et al. 1988, 1999), and fair wages (Kreickemeier and Nelson 2006). However, many economists have made the point of claiming that the HO model provides no explanation towards intra-industry trade as under the assumptions countries with identical factor endowments would not trade and produce goods domestically. Krugman’s new trade theory (1980) -and its generalized version by Melitz (2003)- successfully explain intra-industry trade patterns by emphasizing love-of-variety. In

This paper contributes to the literature on labor market interdependence between asymmetric countries. In the new trade theory, Matusz (1996) has shed light on the possibility of positive correlation of labor market outcomes. He constructs an intra-industry trade framework in the Ethier (1982) type model with homogeneous firms and an efficiency wage. His conclusion implies that relaxing constraints on the efficiency wage permits employment expansion in one country, boosting the world economy through trade, thereby expanding employment in the other. Recently, Egger et al. (2012) extends Matusz’s (1996) arguments in a variant of the Melitz (2003) model with country-specific minimum wages; they conclude that a fall in a country’s minimum wage has positive spillover effects for the trading partner via external scale effects. The model developed herein identifies wage effects and scale effects. In theory, one country’s labor market shocks can have both positive and negative spillover effects on the other country, and their possible coexistence is demonstrated in this paper’s framework.

The present paper relates to earlier models examining the link between welfare and trade liberalization. A theoretical possibility of negative welfare effects due to trade liberalization has been raised by Montagna (2001) who develops a monopolistic competition model with technical heterogeneity among firms and countries. She shows that trade reduces the minimum efficiency to survive in the more efficient country and argues that adverse welfare effects may prevail in an advanced technology economy if love-of-variety is sufficiently low. The present paper raises such a possibility of losses of trade if external scale effects are sufficiently low relative to wage effects.
This paper is complementary to Egger and Kreickemeier (2009) who examine the effect of trade liberalization on a labor market in the presence of positive variable trade costs and external scale effects. They conclude that a negative employment effect is triggered if variable trade costs are not too low and the external scale effects are moderate, whereas a positive employment effect can be expected if variable trade costs are negligible and the external scale effect is strong. This also is well in line with Matusz’s (1996) conclusions assuming full external scale effects and no trade cost.

It also closely relates to Egger et al. (2012) predicting that without trade impediments, trade liberalization always leads both economies to higher levels of welfare, reducing the unemployment rate due to positive employment effects via external scale economies. In addition, a country’s lowered minimum wage always yields positive spillover effects to its trading partner via the channel of external scale effects. While insightful, their model is an incomplete analysis. First, they rely on a single factor whose real wage is subject to an institutionally-set minimum, which shuts down the channel of wage effects. Second, it is impossible to consider effects of variations in the stock of skilled workers. Thus, introducing a second factor with flexible real wages complements many parts of the results of Egger et al. (2012). Key finding of the present paper is that the second factor revives the force of wage effects which acts to offset the force of external scale effects on external shocks. Eventually, only when external scale effects dominate, do gains from trade materialize and stronger foreign labor market institutions harm domestic workers. In addition, variations in foreign labor stock with flexible wages also would impact domestic workers.

In the following section, this author develops a baseline model in autarky. Section 3 characterizes autarky equilibrium in both the product market and the labor market. Section 4 discusses trade equilibrium with asymmetric labor market institutions. Section 5 examines the impact of a country’s labor market variations on the other: minimum wage shock and factor supply shock. Section 6 provides concluding remarks.

2 The Model: Autarky

Consider a world economy comprising two asymmetric countries indexed by $i$ and $j$. In each, two types of goods are produced: homogeneous final output and differentiated intermediate inputs. Assume that both countries share final and input production technology, but differ from each other in factor endowments (skilled and unskilled workers), size of real minimum wages, and mass of potential entrants. All notation is written in terms of country $i$. For country $j$, the subscript changes from $i$ to $j$. 
2.1 Production Technology and Firm Behavior

In the spirit of Dixit-Stiglitz (1975) and Ethier (1982), the final output used for consumption as well as investment is produced by assembling (without the use of labor) various intermediate inputs according to the generalized CES production technology.

\[ Y_i = M_i^{-\frac{\eta}{\sigma-1}} \left( \int_{v \in V_i} \left[ z_i(v) \right]^{\frac{\sigma-1}{\sigma-1}} dv \right)^{\frac{\sigma-1}{\sigma-1}} \]  

(1)

where \( V_i \) denotes the set of all available input varieties with measure \( M_i \), \( z_i(v) \) is the quantity of input variety \( v \) employed in the production of \( Y_i \).

Worth noting are a number of features of final goods technology in (1). First, \( \sigma > 1 \) is the constant elasticity of substitution between varieties. Second, \( \rho = \frac{\sigma-1}{\sigma} < 1 \) captures the degree of complementarity between input varieties. Third, two independent parameters, \( \eta \in [0, 1] \) together with \( \sigma \), measure the degree of external scale effects. To understand the third feature, consider the case where \( z_i(v) = z_i, \forall v \in V_i \). Final output becomes \( Y_i = M_i^{\frac{1-\eta}{\sigma-1}} z_i \). Define \( y_i = \frac{Y_i}{M_i z_i} \), which measures the final output gain derived from spreading a certain amount of production among \( M_i \) differentiated inputs instead of concentrating it on a single input (see Benassy 1996). Thus, the final output gain is \( y_i = M_i^{\frac{1-\eta}{\sigma-1}} \) with an elasticity \( \frac{1-\eta}{\sigma-1} \equiv \chi(\eta, \sigma) \). A marginal final output gain for additional input variety is called the external scale effect.

In the borderline case of \( \eta = 0 \), external scale effects are full, i.e., \( \chi(0, \sigma) = \frac{1}{\sigma-1} \) and the expression in (1) is equivalent to the standard Dixit-Stiglitz (1977) specification. As \( \eta \) rises, external scale effects become weaker, which is interpreted as an intermediate input per se becoming less important. If we set \( \eta = 1 \), final output is produced under constant returns to scale in the measure of inputs, thereby showing no external scale effects, i.e., \( \chi(1, \sigma) = 0 \).

Let \( Y_i \) be the numeraire. Taking input price \( p_i(v) \) as given, final goods producers choose input quantity \( z_i(v) \) in order to maximize their profits:

\[ \max_{z_i(v)} P_i Y_i - \int_{v \in V_i} p_i(v) z_i(v) dv \]

We observe that \( P_i Y_i = \int_{v \in V_i} p_i(v) z_i(v) dv \) in equilibrium since, in a perfectly competitive final

---

6In the unpublished version by Dixit-Stiglitz (1975), they include the product diversity gain (loss) with measure \( M_i \), interpreting as public good (bad). Later, Benassy (1996) and Montagna (2001) rediscover it in a consumption context. See also Neary (2004), Eckel (2008), Acemoglu, Antras, and Helpman (2007), Egger and Kreickemeier (2009), and recently, Egger et al. (2012).

7If we treat the final output technology as a utility function, \( \chi \) is similarly interpreted as a marginal utility gain from additional consumption variety, so called love-of-variety.

8Using data, Haveman and Hummels (2004) state that complete specialization model considerably overstates either the extent of specialization (the degree to which goods are differentiated) or the degree to which consumers value that differentiation. In the same vein, Ardelean (2009) reports that love-of-variety is forty two percent lower than is assumed in Krugman’s model, which implies existing studies may overstate the variety gains from trade liberalization.
output market, free entry drives profits of final goods producers to zero. Using $P_i = 1$ due to the choice of numeraire, the optimal input demand for variety $v$ is

$$z_i(v) = \frac{Y_i}{M_i} P_i(v)^{-\sigma}.$$  \hfill (2)

Each intermediate input firm with some market power has the ability to produce a unique variety valued by final output producers. Production by intermediate input firms involves fixed and variable cost. To operate, all intermediate input firms spend the same fixed investment cost $f$ normalized to one. Variable costs use skilled and unskilled workers with skill intensity $\beta \in [0, 1]$, but vary with a firm’s random productivity $\phi$. The variable cost function for a firm with productivity $\phi$ to produce $z_i$ amount of variety $v$ assumes the Cobb-Douglas form:

$$c_i(\phi) = \frac{z_i}{\phi} s_i^{\beta} w_i^{1-\beta}$$  \hfill (3)

where $s_i$ is the wage rate for skilled workers and $w_i$ is the minimum wage for unskilled workers in country $i$. The cost function in (3) has the convenience of generating unit labor demand for skilled and unskilled workers, respectively, as in Harrigan and Reshef (2015).\footnote{See Romalis (2004) who uses Krugman’s model in explaining how factor proportions determine the structure of commodity trade. See also Bernard et al. (2007).}

Original work by Egger et al. (2012) considers only a minimum-wage worker, \textit{i.e.}, $\beta = 0$. The homogeneous worker assumption, in particular one such as binding to a minimum wage, fails to address an adjustment mechanism \textit{via} factor prices. It naturally is conjectured that depending on skill intensity in production, heterogeneous firms may be influenced differently by exogenous shocks, such as variations in minimum wage or in factor supply. With a high skill intensity (high $\beta$), variations in the minimum wage would have relatively lighter impact on firms’ variable costs, while variations in the stock of skilled workers would impact heavily.

Taking the isoelastic demand by final goods producers in (2) and aggregate variables, an intermediate input firm maximizes its profit by setting its optimal price

$$p_i(\phi) = \frac{s_i^{\beta} w_i^{1-\beta}}{\rho \phi}.$$  \hfill (4)

The revenue and profit generated by an input firm with productivity $\phi$ are automatically calculated. Firm profit is then $\pi_i(\phi) = \frac{r_i(\phi)}{\sigma} - 1$ where $r_i(\phi)$ is firm revenue and $\frac{r_i(\phi)}{\sigma}$ is

\footnote{Thus, the index of intermediate goods prices is given as $P_i = M_i^{\frac{2}{1-\sigma}} \left( \int_{v \in V_i} p_i(v)^{1-\sigma} dv \right)^{\frac{1}{1-\sigma}}$.}
variable profit.

2.2 Input Firm Entry and Aggregate Variables

A firm draws a random productivity $\phi$ from Pareto distribution function with shape parameter $\kappa$.\(^{11}\) Productivity is distributed over $[1, \infty)$ according to

$$G(\phi) = 1 - \phi^{-\kappa}$$

where its corresponding density function is $g(\phi) = \kappa \phi^{-\kappa - 1}$. The shape parameter $\kappa$ measures the concentration of the firm’s productivity distribution. With a high $\kappa$, the input sector becomes more homogeneous in the sense that more input firms are located among the least productive firms. The Pareto distribution not only generates a good approximation of the distribution of firm size (see, e.g., Axtell 2001), but also provides analytical tractability, enabling a comparison of results in the closed economy with those from the open economy.

Denote $M_i$ as the mass of active firms and $\phi_i^*$ as a cutoff productivity. Assume that the total mass of potential entrants in the differentiated input sector is given by exogenous $\bar{N}_i$.\(^{12}\) Thus, only firms drawing productivity above $\phi_i^*$ engage in production. The cutoff productivity condition (CPC) yields

$$M_i = \bar{N}_i [1 - G(\phi_i^*)] = \bar{N}_i [\phi_i^*]^{-\kappa}. \quad (5)$$

By setting $\pi_i(\phi_i^*) = 0$ ($\iff r_i(\phi_i^*) = \sigma$), the zero-profit cutoff productivity (ZCP) indicates

$$\frac{Y_i}{M_i^{\beta/\rho} \left(\frac{1}{\rho} \phi_i^*\right)^{1-\sigma}} = \sigma. \quad (6)$$

Next, define the (weighted) average productivity as $\tilde{\phi}_i = \left(\int_{\phi_i^*}^{\infty} \phi^{\sigma-1} \mu_i(\phi) d\phi\right)^{1/(\sigma-1)}$ where $\mu_i(\phi) = \frac{g(\phi)}{1 - G(\phi_i^*)}$ is the productivity distribution of firms in equilibrium. Using the Pareto distribution, the average productivity condition (APC) is\(^{13}\)

---


\(^{12}\)The present paper can be seen as a steady-state model or a static variant model of the dynamic version in Melitz (2003). In Melitz (2003), there is an unbounded mass of potential entrants and free entry, resulting in expected zero profit in equilibrium. In the present paper, however, the assumption of exogenous $\bar{N}_i$ is adopted to simplify the analysis throughout without losing the main insights from Melitz (2003). See also Do and Levchenko (2009) and Chaney (2008).

\(^{13}\)Revenues of an average firm are proportional to revenues of a marginal firm, $\frac{r_i(\phi_i)}{r_i(\phi_i^*)} = \frac{\kappa}{\kappa - \sigma + 1}$. This fact will
\[ \frac{\phi_i}{\phi_i^*} = \left[ \frac{\kappa}{\kappa - \sigma + 1} \right]^{\frac{1}{\sigma-1}} \]  

(7)

where \( \kappa > \sigma - 1 \) is assumed.

Using the index of intermediate goods prices, the profit maximization condition (PMC) is

\[ M^X_i = p_i(\bar{\phi}_i). \]  

(8)

Other aggregate variables are obtained easily. For example, \( R_i = M_i r_i(\bar{\phi}_i) \) and \( \Pi_i = M_i \pi_i(\bar{\phi}_i) \) where \( R_i = \int_{\phi_i}^{\infty} r_i(\phi) M_i \mu_i(\phi) d\phi \) and \( \Pi_i = \int_{\phi_i}^{\infty} \pi_i(\phi) M_i \mu_i(\phi) d\phi \) represent aggregate revenue and profit, respectively.

### 2.3 Labor Market

Country \( i \) is endowed with \( \bar{H}_i \) of skilled workers and \( \bar{L}_i \) of unskilled workers. Labor markets for both types of worker are perfectly competitive. In the spirit of Brecher (1974), we consider here a very stark labor-market institution. Assume that the government sets a minimum wage \( w_i \) above the market clearing level for unskilled workers. Let \( U_i \) denote unemployment rate for unskilled workers.

Let us find the labor demand for each type of worker to produce one unit of output within a firm. Using (3), the marginal cost of a firm with productivity \( \phi \) is then

\[ mc_i(\phi) = \frac{s_i^{\beta} w_i^{1-\beta}}{\phi}. \]  

(9)

Denote \( h_i(\phi) \) and \( l_i(\phi) \) as the unit labor demand for skilled and unskilled workers, respectively. Similarly to Harrigan and Reshef (2015), applying Shephard’s lemma yields

\[ h_i(\phi) = \frac{\beta}{\phi} \left( \frac{s_i}{w_i} \right)^{\beta-1}, \]  

(10)

\[ l_i(\phi) = \frac{1-\beta}{\phi} \left( \frac{s_i}{w_i} \right)^{\beta}. \]  

(11)

Within a firm with productivity \( \phi \), the unit labor cost \( s_i h_i(\phi) = \frac{\beta}{\phi} s_i^{\beta} w_i^{1-\beta} \) is incurred when hiring skilled workers and \( w_i l_i(\phi) = (1-\beta) s_i^{\beta} w_i^{1-\beta} \) when employing unskilled work-

be used later in the welfare analysis.
ers. The total unit labor cost a firm with productivity \( \phi \) spends is just \( mc_i(\phi) \) in (9). For a firm to produce \( z_i(\phi) \) amount of goods, it spends \( mc_i(\phi)z_i(\phi) \) which equals to \( \rho r_i(\phi) \). Hence,

\[
c_i(\phi) = \rho r_i(\phi).
\]

Denote \( W_i \) as the total labor cost expenditures: \( W_i = \int_{\phi_i^*}^{\infty} c_i(\phi)M_i \mu_i(\phi) \, d\phi \) from the sum of wage payments by all intermediate input firms. Integrating both sides in (12) over \( \phi \in [\phi_i^*, \infty) \),

\[
\int_{\phi_i^*}^{\infty} c_i(\phi)M_i \mu_i(\phi) \, d\phi = \rho \int_{\phi_i^*}^{\infty} r_i(\phi)M_i \mu_i(\phi) \, d\phi.
\]

The relation in (13) implies that \( W_i = \rho M_i r_i(\bar{\phi}_i) = \rho R_i \). Due to the choice of final output as numeraire,

\[
W_i = \rho Y_i.
\]

In equilibrium, \((1 - U_i)\bar{L}_i\) unskilled workers will be employed, whereas \( \bar{H}_i \) skilled workers will be fully employed since the skilled wage is assumed to be flexible. The aggregate labor demand for skilled workers is obtained by integrating over the total labor demand of firms with productivity \( \phi \in [\phi_i^*, \infty) \):

\[
\int_{\phi_i^*}^{\infty} h_i(\phi)z_i(\phi)M_i \mu_i(\phi) \, d\phi.
\]

Similarly, the aggregate labor demand for unskilled workers is

\[
\int_{\phi_i^*}^{\infty} l_i(\phi)z_i(\phi)M_i \mu_i(\phi) \, d\phi.
\]

Using input demand in (2), demand for skilled workers in (10), and demand for unskilled workers in (11), the two labor market equilibrium conditions then are

\[
\bar{H}_i = \int_{\phi_i^*}^{\infty} \left( \frac{\beta}{\phi} \left( \frac{s_i}{w_i} \right)^{\beta - 1} \right) \frac{Y_i}{M_i} d_i(\phi)^{-\sigma} M_i \mu_i(\phi) \, d\phi,
\]

\[
(1 - U_i)\bar{L}_i = \int_{\phi_i^*}^{\infty} \left( \frac{1 - \beta}{\phi} \left( \frac{s_i}{w_i} \right)^{\beta} \right) \frac{Y_i}{M_i} d_i(\phi)^{-\sigma} M_i \mu_i(\phi) \, d\phi.
\]

The sum of wage incomes for unskilled and skilled workers is equal to total labor cost expenditures, \( i.e., \), \( W_i = (1 - U_i)w_i\bar{L}_i + s_i\bar{H}_i \).

Manipulating (14)-(16), the unemployment rate is

\[
U_i = 1 - \frac{(1 - \beta)W_i}{w_i\bar{L}_i}
\]

and one simple relation is obtained

\[\text{The demand ratio of skilled to unskilled workers to produce a unit of input variety is inversely related to the relative wage of skilled workers, i.e., } \frac{h_i(\phi)}{l_i(\phi)} = \frac{\beta}{1 - \beta} \frac{w_i}{s_i}.\]
\[
\beta = \frac{s_i \bar{H}_i}{\rho Y_i}.
\]  

Equation (18) states that a fraction \( \beta \) of the total labor cost payments \( \rho Y_i \) is given to skilled workers, which must be equal to the total labor income for skilled workers, \( s_i \bar{H}_i \). As \( \beta \) rises, wage payments to skilled workers out of total labor costs rise accordingly. In the present paper, **wage effects** arise from intensive use of skilled workers. The magnitude of the effects will be determined by the parameter \( \beta \). Together with external scale effects \( \chi(\eta, \sigma) \), \( \beta \) will play a significant role throughout the paper.

### 3 Analysis of Autarky Equilibrium

Equilibrium variables in autarky are characterized completely by deriving the solution for cutoff productivity and skilled workers’ wage \( (\phi^*_i, s_i) \). Given the solution, all the other aggregate variables \( (\hat{\phi}_i, M_i, Y_i, W_i) \) in the product market and \( U_i \) in the labor market are determined endogenously.

Substituting CPC (5) and APC (7) into PMC (8), cutoff productivity is increasing in skilled workers’ wage with an assumption of \( \theta \equiv 1 - \kappa \chi > 0 \)

\[
\phi^*_i = \lambda \left[ \frac{w_i^{1-\beta}}{N_i^{(\kappa+1)/\kappa}} \right]^{-\frac{1}{\theta}} s_i^{\frac{\beta}{\sigma}},
\]  

where \( \lambda = \left[ \frac{1}{\rho} \left( \frac{\kappa-\sigma+1}{\kappa} \right)^{\frac{1}{\sigma-1}} \right]^{-\frac{1}{\theta}} > 0. \)

Plugging CPC (5) and \( Y_i \) of (18) into ZCP (6), cutoff productivity is decreasing in skilled workers’ wage with an assumption of \( \beta \leq \frac{1}{\sigma-1} \)

\[
\phi^*_i = \mu \left[ \frac{w_i (1-\beta)(\sigma-1)}{H_i} \right]^{-\frac{1}{\theta}} s_i \left( \frac{1}{\sigma} \right) \left( \frac{\beta}{\sigma-1} \right)
\]  

---

15 The condition \( 1 > \kappa \chi \) is \( 0 \leq \frac{1-\eta}{\sigma-1} < \frac{1}{\chi} \). That is, the elasticity of final output gain with respect to the measure of variety \( \chi(\eta, \sigma) = \frac{1-\eta}{\sigma-1} \) has a upper bound \( \frac{1}{\chi} \). Technically, it is known that if \( \phi \) is Pareto distributed, then \( \ln \phi \) is exponentially distributed with standard deviation of \( \frac{1}{\chi} \). Recall that \( \kappa > \sigma - 1 \) due to the assumption of Pareto distribution. Thus, \( 0 \leq \chi(\eta, \sigma) < \frac{1}{\chi} < \frac{1}{\sigma-1} \) which completes the restrictions of the elasticity.

16 Factor intensity \( \beta \) should be in \([0, 1]\) and elasticity of substitution \( \sigma \) is larger than one. In a general form, \( \beta \leq \min \left\{ \frac{1}{\sigma-1}, 1 \right\} \) should be satisfied. When \( \sigma > 2 \), factor intensity \( \beta \) should decrease to keep \( 0 \leq \beta \leq \frac{1}{\sigma-1} \). Depending on \( \sigma \), factor intensity has a upper bound \( \frac{1}{\sigma-1} \). If \( \beta(\sigma-1) > 1 \), we are no longer able to consider the case of \( \beta = 0 \) because this would imply that \( 0 > 1 \) (contradiction). Lastly, at \( \beta(\sigma-1) = 1 \), the curve in (20) becomes horizontal.
Note: Following Bernard et al. (2007) and Harrigan and Reshef (2015), it is assumed that $\kappa = 3.4$ for Pareto shape parameter, $\sigma = 3.8$ for elasticity of substitution. From Ardelean (2009), I adopt $\eta = 0.42$ for external scale parameter. For the rest of parameters, I set $w_i = 7.25$ for minimum wage, $\bar{N}_i = 100$ for the mass of potential entrants, and $\beta = 0.15$ for factor intensity. The mass of skilled workers, $\bar{H}_i$, is assumed to be sufficiently small.

with $\mu = \left[ \frac{\beta(\sigma - 1)}{\rho^\sigma - 1} \right]^{\frac{1}{\sigma}} \geq 0$ and $\vartheta = \sigma - 1 + \kappa \eta > 0$.

**Proposition 1.** A unique autarky equilibrium exists in which cutoff productivity and skilled workers’ wage are determined.

$$
\phi_i^* = \psi \left[ \bar{N}_i^{\beta - \chi(\eta, \sigma)} \left( \frac{w_i^{1-\beta}}{\bar{H}_i^\beta} \right) \right]^{\frac{1}{\sigma + \beta \kappa}}
$$

with $\psi = \left( \lambda^{\theta(1-\beta(\sigma-1))} \mu^{\theta} \right)^{\frac{1}{\sigma + \beta \kappa}} > 0$ and

$$
\phi_i = \left( \frac{\mu}{\lambda} \right)^{\frac{\theta \vartheta}{\sigma + \beta \kappa}} \left[ \frac{\bar{N}_i}{w_i^{1-\beta}} \frac{1}{\bar{H}_i^\theta} \right]^{\frac{1}{\sigma + \beta \kappa}}
$$

**Proof.** See Appendix A and Figure 1.

An increase in the stock of skilled workers $\bar{H}_i$ reduces both skilled workers’ wage $s_i$ in (22) and cutoff productivity $\phi_i^*$ in (21). A rise in minimum wage $w_i$ decreases skilled workers’ wage $s_i$ in (22) and raises cutoff productivity $\phi_i^*$ in (21). Clearly, an increase in the

\[17\] A necessary and sufficient condition for $\phi_i^* \geq 1$ is $\bar{N}_i \leq \left\{ \psi^{\theta + \beta \kappa} \frac{w_i^{1-\beta}}{\bar{H}_i^\theta} \right\}^{\frac{1}{\sigma + \beta \kappa}}.$
exogenous mass of potential firms $\bar{N}_i$ raises skilled workers’ wage $s_i$ in (22).

How does an increase in the exogenous mass of potential firms $\bar{N}_i$ affect the cutoff productivity level? To answer, we carefully should look at the solution in (21). An increased mass of potential firms $\bar{N}_i$ exhibits two adjustment channels: \textit{wage effects} $\beta$ and \textit{external scale effects} $\chi(\eta, \sigma)$. Relative forces between them determine sign$\{\beta - \chi(\eta, \sigma)\}$ which points to the direction of the change in cutoff productivity. If wage effects dominate external scale effects, \textit{i.e.}, $\beta > \chi(\eta, \sigma)$, cutoff productivity rises as $\bar{N}_i$ rises. To understand this result, we focus on a marginal firm. An increased mass of potential firms implies a higher demand for skilled workers in the labor market, thereby raising the wage for skilled workers. That increased wage means higher variable costs, forcing marginal firms to exit the market. Thus, wage effects drive cutoff productivity upward. Contrarily, an increased mass of potential firms raises $M_i$, thereby expanding final output through scale economies. Expanded final output indirectly raises demand for intermediate inputs, resulting in an increase in the returns to all intermediate input firms. Thus, external scale effects drive cutoff productivity downward.

However, if final output firms use production technology that generates weak external scale effects and intermediate input firms use skill-intensive technology, \textit{i.e.}, a sufficiently small $\chi$ relative to $\beta$, marginal firms in the differentiated input sector cannot offset increased variable costs by larger market demand, thus \textit{exit} the market. If $\beta < \chi(\eta, \sigma)$, then potential firms \textit{enter} the market.

\textbf{Egger et al.} (2012) show that as $\bar{N}_i$ increases, cutoff productivity would not change at all if there is no external scale effect. In contrast, cutoff productivity increases due to prevailing wage effects even without external scale effect, as long as $\beta > 0$. In \textbf{Egger et al.} (2012), the effect of an increase in $\bar{N}_i$ depends only on external scale effects, driving cutoff productivity downward. This results because they only consider minimum-wage workers. In general, though, an additional stabilizing force exists by means of increasing factor prices in response to a larger number of potential entrants.\footnote{\textbf{Egger et al.} (2012) point out this matter in footnote 7 on page 777 but they seem to believe that the assumption of a single factor is innocent but helps keep the analysis tractable as stated in footnote 3 on page 772.}

In the next section, to facilitate comparison between autarky and trade equilibrium, it is useful to explicitly solve for aggregate product market variables. In the goods market, average productivity is derived by plugging (21) into APC (7),

$$\tilde{\phi}_i = \frac{\psi}{\rho \lambda \theta} \left[ \bar{N}_i^{\beta-\chi(\eta, \sigma)} \left( \frac{w_i^{1-\beta}}{\bar{H}_i^\beta} \right) \right]^{\frac{1}{\theta+\rho \kappa}} \tag{23}$$

Plugging (21) into CPC (5), the mass of active firms is
\[ M_i = \frac{\bar{N}_i^{\frac{1}{\kappa - \sigma + 1}} \bar{H}_i^\beta}{\psi^\kappa \bar{w}_i^{1 - \beta}}. \tag{24} \]

Since \( Y_i = M_i r_i(\tilde{\phi}_i) = M_i \frac{\kappa \sigma}{\kappa - \sigma + 1} \), the total final output produced \( Y_i \) becomes

\[ Y_i = \left( \frac{\kappa \sigma}{\kappa - \sigma + 1} \right) \bar{N}_i^{\frac{1}{\kappa - \sigma + 1}} \left( \frac{\bar{H}_i^\beta}{\bar{w}_i^{1 - \beta}} \right) \frac{\psi^\kappa}{\bar{w}_i^{1 - \beta}}. \tag{25} \]

Next, the labor market equilibrium in autarky is briefly discussed. Using \( W_i = \rho M_i r_i(\tilde{\phi}_i) \) and (25), the total labor costs \( W_i \) are

\[ W_i = \left( \frac{\kappa(\sigma - 1)}{\kappa - \sigma + 1} \right) \bar{N}_i^{\frac{1}{\kappa - \sigma + 1}} \left( \frac{\bar{H}_i^\beta}{\bar{w}_i^{1 - \beta}} \right) \frac{\psi^\kappa}{\bar{w}_i^{1 - \beta}}. \tag{26} \]

Using (17), the rate of unemployment is

\[ U_i = 1 - \frac{(1 - \beta)\kappa(\sigma - 1)}{(\kappa - \sigma + 1)\psi^\kappa \bar{L}_i} \left( \frac{\bar{N}_i\bar{H}_i^\beta}{\bar{w}_i^{\theta + \kappa}} \right) \frac{1}{\bar{w}_i^{1 - \beta}}. \tag{27} \]

Obviously, the unemployment rate is one when \( \beta = 1 \). The Egger et al. (2012) model is a special case of the present paper when \( \beta = 0 \). A higher minimum wage \( w_i \) (or a lower stock of skilled workers \( \bar{H}_i \)) lowers the total mass of input firms \( M_i \), total output \( Y_i \), and total labor income \( W_i \) from (24)-(26) with a rise in the rate of unemployment \( U_i \) from (27). This completes the analysis in autarky.\(^\text{19}\)

## 4 Introduction of Trade

Consider free international trade between country \( i \) and \( j \). Assume both are in all respects identical except in the minimum wage level, i.e., \( w_i \neq w_j, \bar{L}_i = \bar{L}_j, \bar{H}_i = \bar{H}_j, \) and \( \bar{N}_i = \bar{N}_j. \)^\text{20}\n
This assumption will be relaxed in the next section. Assume throughout that no trade costs are

\(^{19}\)Examining the effect of a rise in \( \bar{N}_i \) is not the primary focus of our analysis. However, it is interesting to know that a larger \( \bar{N}_i \) stimulates entry everywhere in the productivity distribution and thus, a larger number of high productive firms operate in the market. If \( \beta > \chi(\eta, \sigma) \), then \( \phi^*_i \) in (21) rises. However, it is clear to predict that as \( \bar{N}_i \) rises, total mass of input firms, total output, and total labor income increases from (24)-(26). Unemployment rate clearly falls in (27). Consequently, a rise in \( \bar{N}_i \) will have a positive impact on welfare in the long-run.

\(^{20}\)Egger et al. (2012) point out that Krugman’s type model with homogeneous firm fails to have binding minimum wages in both countries after trade. The assumption of heterogeneous firms, however, enables both countries to have different minimum wages.
present, so that all firms are exporters and charge the same price in both domestic and foreign markets.\footnote{By the assumption of no trade cost, selection effects in Melitz (2003) are neutralized in this paper. It is crucial to focus on the adjustment at the extensive margin of firms throughout the paper.} In each country, final goods producers do not discriminate among intermediate inputs produced in different countries. Last, all notation is written in terms of country $i$. For country $j$, the subscript changes from $i$ to $j$ and from $j$ to $i$.

### 4.1 Firm Behavior between Asymmetric Countries

Denote $M_{it}$ as the mass of active firms in country $i$ where the subscript $t$ refers to free trade. Final output is produced by assembling both domestic and foreign input varieties according to the CES production technology,\footnote{The price indices are equalized across countries. Due to the choice of $Y$ as numeraire, we set $P = 1$.}

$$Y_{it} = Y_{jt} = M_{it}^{-\frac{n}{\sigma - 1}} \left( \int_{v \in V_i} [z_{it}(v)]^{\frac{\sigma - 1}{\sigma}} dv + \int_{v_s \in V_j} [z_{jt}(v_s)]^{\frac{\sigma - 1}{\sigma}} dv_s \right)^{\frac{\sigma}{\sigma - 1}}$$

(28)

where $M_t \equiv M_{it} + M_{jt}$, $V_j$ denotes the set of all available varieties with measure $M_{jt}$, and $z_{jt}(v_s)$ is the quantity of variety $v_s$ produced by country $j$.

Taking intermediate input demands by final goods producers and other aggregate variables, intermediate goods firms solve their profit maximization problem. Each firm with different productivity $\phi$ follows the constant mark-up pricing rule:

$$p_{it}(\phi) = \frac{s^{\beta} w_{1}^{1-\beta}}{\rho \phi}, \quad p_{jt}(\phi) = \frac{s^{\beta} w_{j}^{1-\beta}}{\rho \phi}.$$  

(29)

Similar to the closed economy, cutoff productivity conditions (CPCs) yield

$$M_{it} = \bar{N}_i[\phi_{it}^*]^{-\kappa}, \quad M_{jt} = \bar{N}_j[\phi_{jt}^*]^{-\kappa}.$$  

(30)

Two zero-profit cutoff productivity conditions (ZCPs) are

$$r_{it}(\phi_{it}^*) = \sigma, \quad r_{jt}(\phi_{jt}^*) = \sigma.$$  

(31)

Using the common index of intermediate goods prices, profit maximization conditions (PMCs) yield

$$M_{it}^\chi = p_{it}(\hat{\phi}_{it}), \quad M_{jt}^\chi = p_{jt}(\hat{\phi}_{jt}).$$  

(32)
where $\tilde{\phi}_i$ and $\tilde{\phi}_j$ (APCs) are defined similarly to in the closed economy case.\textsuperscript{23}

Equation (31) implies that the ratio of two cutoff productivities can be expressed as the ratio of variable costs from both countries: $\phi^*_j / \phi^*_i = s_{\beta_j} w_j^{1-\beta} / s_{\beta_i} w_i^{1-\beta}$.\textsuperscript{24} In each the government imposes a different minimum wage so that it affects a firm’s price setting by influencing variable costs. In Egger et al. (2012), the ratio of two cutoff productivities equals the ratio of two minimum wages, i.e., $\phi^*_j / \phi^*_i = w_j / w_i$. Unlike the Egger et al. (2012) model, the generalized version of the model here proposed entails skilled workers’ wages. In this case, we take account of endogenously-determined skilled workers’ wage of each country in computing the ratio of two cutoff productivities. An interesting finding is it is unnecessary to do so in the present model because skilled workers’ wages in open economies are equalized. Intuitively, this result makes sense. Suppose country $j$ sets a higher minimum wage than that of country $i$. As discussed in autarky, country $j$ has a smaller goods market than has country $i$. In the autarky equilibrium, $s_i > s_j$ (22). Upon opening to trade, firms in country $j$ encounter a larger foreign market than that in country $i$. Given the same use of skill intensity in production across countries, the increase in the magnitude of labor demand for skilled workers in country $j$ should be higher proportionally than in country $i$. Due to the perfectly competitive labor market for skilled workers, wages for skilled workers adjust to clear the labor markets and turn out to be equal between the two countries. Indeed, we can derive this result using two zero profit conditions (ZPCs) and two profit maximization conditions (PMCs).

**Lemma 1.** Skilled workers’ wages are equalized in open economies, $s_{it} = s_{jt}$.\textsuperscript{25}

**Lemma 2.** $M_t = \left[ 1 + \left( \frac{w_i}{w_j} \right)^{(1-\beta)\kappa} \right] M_{it}$

**Proof.** See Appendix C.

### 4.2 Trade Equilibrium

Equilibrium variables in free trade between asymmetric countries are characterized completely by deriving the solution for cutoff productivity and skilled workers’ wage in each

\textsuperscript{23}As Egger et al. (2012) point out, in the open economy it is necessary to distinguish between the average productivity of domestic firms, $\phi_u$, and the average productivity in the market, $\phi_h$, with domestic production as well as imports from foreign firms. $\phi_h$ and $\phi_u$ are identical only if the negative lost-in-transit effect and the positive export-selection effect are of the same size. Since we abstract from any trade impediments and all firms export, $\phi_u = \phi_h$. For more details, see Egger and Kreickemeier (2009).

\textsuperscript{24}Due to the constant relationship between cutoff and average productivity, $\tilde{\phi}_j / \tilde{\phi}_i = s_{\beta_j} w_j^{1-\beta} / s_{\beta_i} w_i^{1-\beta}$ should be satisfied.

\textsuperscript{25}However, if one country has a higher mass of skilled workers than the other, factor price equalization fails. See Section 5.
country. Given the solution, all the other aggregate variables \((\tilde{\phi}_{it}, M_{it}, Y_{it}, W_{it}, \tilde{\phi}_{jt}, M_{jt}, Y_{jt}, W_{jt})\) in product markets and \((U_{it}, U_{jt})\) in labor markets are determined across countries.

Using CPCs (30), ZCPs (31), PMCs (32), APCs, and Lemma 1-2, two key equations are derived for each country.

\[
\phi^{*}_{it} = \lambda \left[ \frac{w^{1-\beta}_{i}}{\bar{N}_{it}} \right] \frac{\beta}{\tau + \beta s_{it}}, \tag{33}
\]

\[
\phi^{*}_{it} = \mu \left[ \frac{w^{(1-\beta)(\sigma-1)}_{i} N_{it} \eta}{\bar{H}_{i} \beta} \right] \frac{1}{\tau + \beta s_{it}} \left[ \frac{\beta - \chi(\eta, \sigma)}{\sigma - 1} \right]. \tag{34}
\]

**Proposition 2.** A unique trade equilibrium exists in which cutoff productivity and skilled workers’ wage are determined in each country,

\[
\phi^{*}_{it} = \psi \left( \frac{1}{\bar{H}_{i} \beta} \right)^{\frac{1}{\sigma + \beta \kappa}} \left[ \frac{\beta - \chi(\eta, \sigma)}{\sigma - 1} \right] \tag{35}
\]

with \(\psi > 0\) as defined in Proposition 1 and

\[
s_{it} = \left( \frac{\mu}{\lambda} \right) \frac{\theta \rho}{\sigma + \beta \kappa} \left[ \frac{\beta - \chi(\eta, \sigma)}{\sigma - 1} \right]. \tag{36}
\]

**Corollary 1.** Using (21) in Proposition 1 and (35) in Proposition 2,

\[
\frac{\phi^{*}_{it}}{\phi^{*}_{i}} = \left[ 1 + \left( \frac{w_{i}}{w_{j}} \right)^{(1-\beta)\kappa} \right] \frac{1}{\sigma + \beta \kappa} \left[ \frac{\beta - \chi(\eta, \sigma)}{\sigma - 1} \right]. \tag{37}
\]

**Corollary 2.** Using (22) in Proposition 1 and (36) in Proposition 2,

\[
\frac{s_{it}}{s_{i}} = \left[ 1 + \left( \frac{w_{i}}{w_{j}} \right)^{(1-\beta)\kappa} \right] \frac{1}{\sigma + \beta \kappa} \equiv \Phi_{i}. \tag{38}
\]

Trade liberalization induces an increase of skilled workers’ wage due to an increase of aggregate labor demand for skilled workers (Corollary 2). As illustrated in Figure 2, with constant external scale effects \(\chi\), the dotted line of skilled workers’ wage in trade equilibrium is always above the solid line in autarky equilibrium regardless of \(\beta\). Moreover, wage inequality between different skill-typed workers grows after trade liberalization at any given \(\beta\). Due to the assumption of binding minimum wage, there is no change in (real) wage for unskilled workers so that a larger market size ought to tie with an increased wage inequality.
It is easy to show the ratio of wage inequality before and after free trade: \( \frac{w_i}{w_j} \geq \Phi_i > 1 \) where \( \Phi_i \) is defined in Corollary 2. This result is consistent with the literature on trade liberalization and wage inequality (see, e.g., Yeaple 2005; Helpman et al. 2010; Harrigan and Reshef 2015; Burstein and Vogel 2010). Last, one can compare wage inequality before and after free trade between country \( i \) and \( j \). Given an assumption that both countries are identical in all respects other than the minimum wage level, wage inequality in country \( i \) in free trade compared with one in autarky is less severe than one in country \( j \) if and only if \( w_i < w_j \). This result is straightforward in Corollary 2 because \( w_i < w_j \) implies \( \Phi_i < \Phi_j \).

Figure 3 illustrates Corollary 1 in which with constant external scale effects \( \chi \), the dotted line of cutoff productivity in trade equilibrium crosses from below the solid line in autarky equilibrium as \( \beta \) rises. Based on parameter values as noted in Figure 1, one additional parameter is imposed \( w_j = 8.5 \) for minimum wage in country \( j \). In Corollary 1, sign\( \{ \beta - \chi(\eta, \sigma) \} \) is important and governs the scale of \( \phi_i^*/\phi_i^* \). Clearly, \( \phi_i^*/\phi_i^* = 1 \) at \( \beta = \chi(\eta, \sigma) \). If \( \beta > \chi(\eta, \sigma) \), then \( \phi_i^*/\phi_i^* > 1 \) and vice versa. As noted in autarky, positive sign\( \{ \beta - \chi(\eta, \sigma) \} \) is interpreted similarly on the opening of trade. On one hand, trade liberalization expands the goods market via external scale economies, thus leads potential entrants into the market. On the other, it raises labor demand for both skilled and unskilled workers. Firms have to bear higher payments to workers, resulting in higher variable costs (negative wage effect). If the latter effects always dominate the former, i.e., \( \beta > \chi(\eta, \sigma) \), liberalized trade induces
Figure 3: Cutoff Productivity and Factor Intensity in Trade Equilibrium

Note: Parameter values are as noted in Figure 1. Vertical line is given at $\chi = \frac{1 - \eta}{\sigma - 1} = \frac{1 - 0.42}{3.8 - 1}$ (approximately, 0.207). Solid and dashed lines refer respectively to autarky and trade equilibrium.

Marginal firms to exit the market, resulting in $\phi_{it}^*/\phi_i^* > 1$. If $\beta < \chi(\eta, \sigma)$, then potential firms enter the market, $\phi_{it}^*/\phi_i^* < 1$.

Figure 4 illustrates how the current paper relates to several others in terms of firm exit/entry, depending on wage effects and external scale effects. Melitz (2003) assumes both a full love-of-variety ($1/(\sigma - 1)$) and a perfectly competitive labor market ($\beta = 1$), and predicts that, absent trade costs, cutoff productivity does not change at all after trade liberalization. At $\beta = 1$, there is no employment for unskilled workers in the world, regardless the existence of minimum wages, because input firms’ production technology obviates hiring any. The two countries become symmetric, thereby producing equal mass of varieties in each. Thus, his model corresponds to the upper-right corner in Figure 4. Matusz (1996) considers a trade model with efficiency wage and full external scale effects ($1/(\sigma - 1)$). He predicts that trade liberalization relaxes constraints on efficiency wage in the labor market and permits employment expansion in both countries. Although his model deals with homogeneous firms, entry of more firms after free trade can be interpreted as a reduction in the cutoff productivity in the framework of the present paper. Since his model has labor market frictions from efficiency wage, it corresponds to $\beta = 0$ and $1/(\sigma - 1)$ in the upper-left corner in Figure 4. Egger et al. (2012) explicitly consider flexible external scale effects but minimum-wage workers. They predict that potential firms always enter the market after trade liberalization.

26 For example, $M_t = 2M_{i0} = 2M_{jt}$. 

20
Note: To have the interval of skill intensity $\beta \in [0, 1]$, elasticity of substitution between variety should be $\sigma \leq 2$ which satisfies one of the stability conditions (see footnote 15). Region A shows $\phi_t < \phi_i$ if $\beta < \chi(\eta, \sigma)$ while Region C shows $\phi_t > \phi_i$ if $\beta > \chi(\eta, \sigma)$. Line B indicates $\phi_t = \phi_i$ if $\beta = \chi(\eta, \sigma)$.

Thus, their model can be located on the vertical line at $\beta = 0$. The model developed herein illuminates the rest of the area in the parameter space $(\chi(\eta, \sigma), \beta)$ of Figure 4.

All other aggregate variables in the trade equilibrium are provided in Appendix B. Using $\Phi_i$ in Corollary 2, it is convenient to deliver the ratio of equilibrium variables before and after free trade: $\frac{\phi^*_t}{\phi^*_i} = \frac{\Phi_i}{\Phi_i} = \Phi_i^{\beta - \chi}$ and $\frac{M_t}{M_i} = \frac{Y_t}{Y_i} = \frac{W_t}{W_i} = \Phi_i^{\chi - \beta}$. When wage effects outweigh external scale effects, i.e., $\beta > \chi(\eta, \sigma)$, an increased cutoff productivity $\phi^*_t$ triggers the mass of active firms, total output, and total labor income to fall. In contrast, $\beta < \chi(\eta, \sigma)$ implies that potential firms would not worry much about having to pay skilled-workers’ wages since production technology for intermediate input firms less intensively uses skilled workers. Thus, potential firms enter the market, thereby raising the mass of active firms, total outputs, and total labor costs.

### 4.3 Unemployment and Welfare

From the discussion in the previous subsection, it is clear that $\text{sign}\{\beta - \chi(\eta, \sigma)\}$ determines firm performance and firm exit/entry. In particular, $\beta > \chi(\eta, \sigma)$ implies that the most productive firms in the interval $(\phi^*_t, \infty)$ increase its production volume by serving both domestic and foreign markets, resulting in higher profits. Relatively less productive firms in $(\phi^*_i, \phi^*_t)$ serve both markets but have lower profits compared with ones in autarky, while all input firms in $(\phi^*_i, \phi^*_t)$ exit the market (Corollary 1). The two types of worker are unevenly affected by
trade liberalization. Regardless of sign $\{\beta - \chi(\eta, \sigma)\}$, also clear is that the transition from autarky to free trade results in increased returns to skilled workers in both countries (Corollary 2). In contrast to the model by Egger et al. (2012), some unskilled workers may face unemployment despite liberalized trade’s positive employment effects.

Denote $U_{it}$ and $E_{it}$, of country $i$ in free trade $t$, as unemployment and employment rate for unskilled workers. By construction, $E_{it} = 1 - U_{it}$ ($E_i$ is defined similarly in autarky). Using the definition of $\Phi_i$, the following ratio holds:

$$\frac{E_{it}}{E_i} = \Phi_i^{-\kappa(\beta - \chi(\eta, \sigma))}$$

(37)

Positive sign $\{\beta - \chi(\eta, \sigma)\}$ implies that negative wage effects, deriving from an increased wage for skilled workers, outweigh positive employment effects to unskilled workers. Free trade shrinks employment of unskilled workers (37), resulting in a higher rate of unemployment in each country. Thus, free trade harms some unskilled workers who have worked in less productive firms.\footnote{In the borderline case, $\eta = 1$ shows no external scale effects, $\chi = 0$. Then, $P_t = 1$ implies $p_{it}(\Phi_{it}) = 1 = s_{it}^{1-\beta}w_i^{1-\beta}/\rho \Phi_{it}$. Since there is no change in price index, no change is made in nominal wages in both countries. But there is a larger demand for skilled workers in both countries, which drives skilled workers’ wage upward. This implies that variable cost $s_{it}^{1-\beta}w_i^{1-\beta}/\phi$ increases. In order for average firm’s price to be 1, average productivity level should rise compared with the one in autarky. This can be done by the exit of marginal firms. From the perspective of marginal firms, the unit cost $s_{it}^{1-\beta}w_i^{1-\beta}/\phi$ increases so that it cannot bear the costs anymore. A slightly higher productive firm is in the same position. The marginal firm’s productivity increases up to a certain level at which average productivity satisfies $p_{it}(\Phi_{it}) = 1$. Hence, cutoff and average productivity rises. This leads $Y_{it}$ and $M_{it}$ to falls and consequently unemployment $U_{it}$ rises. The same logic applies to the case in country $j$.}

If $\beta < \chi(\eta, \sigma)$, then both skilled and unskilled workers become winners.

The model presented herein accords with the efficiency wage model developed by Matsuz (1996), analyzing labor market effects of trade liberalization without variable costs in an Ethier-type (1982) framework. He predicts that introduction of trade results in establishment of more firms in each country and commensurately-higher employment levels, thereby lowering unemployment. That is consistent with the result under full external scale effects, $\eta = 0$ (see also Figure 4). The present paper is complementary to the model by Egger and Kreickemeier (2009). Respective findings by Egger and Kreickemeier (2009) derive from consideration of the effect of trade liberalization on a labor market in the presence of positive variable trade costs and external scale effects. They conclude that a negative employment effect is triggered if variable trade costs are not too low and external scale effects are moderate, whereas a positive employment effect can be expected if variable trade costs are negligible and external scale effects are strong.
Next is economy-wide welfare. Define welfare measure $\Omega_{it}$ as the sum of aggregate labor income $W_{it}$ and aggregate profits of domestic firms $\Pi_{it}$:\textsuperscript{28} That is, $\Omega_{it} = W_{it} + \Pi_{it} = \rho M_{it} r_{it}(\bar{\phi}_{it}) + M_{it} [r_{it}(\tilde{\phi}_{it})/\sigma - 1]$. Hence,

$$\Omega_{it} = M_{it} [r_{it}(\tilde{\phi}_{it}) - 1].$$ (38)

The welfare measure in (38) is unlike Melitz (2003) which focuses mainly on welfare per worker, represented by the number of varieties consumed and the average productivity. The reason is that his model assumes free entry conditions so that the present value of aggregate profits equals total entry costs of firms in the productivity draw. Hence, only wage income is available for consumption. Meanwhile, the present paper assumes exogenous $N$ potential firms so that the welfare measure also includes aggregate profits. Since an average productivity firm’s revenue is constant, i.e., $r_{it}(\tilde{\phi}_{it}) = \kappa \sigma / \kappa - \sigma + 1$, the mass of active firms $M_{it}$ in (38) determines economy-wide welfare level in country $i$. The following proposition summarizes the results.

**Proposition 3. (Economy-wide Welfare)** Trade welfare is improving if and only if $\beta < \chi(\eta, \sigma)$.

This paper shows that when $\beta > \chi(\eta, \sigma)$, the possibility exists that welfare worsens due to trade liberalization (i.e., $M_{it}/M_i < 1$). Noteworthy here is that Neary’s statement (2004) that “Dixit-Stiglitz specification imposes too benign a view of product diversity. It clearly fails to capture one of the concerns of anti-globalization protesters: that liberalizing trade may reduce rather than increase variety.” In addition, a theoretical possibility of negative welfare effects due to trade liberalization has been raised by Montagna (2001) who develops a monopolistic competition model with technical heterogeneity among firms and countries. She shows that adverse welfare effects may prevail in an advanced technology economy if love-of-variety is sufficiently low. In contrast, Egger et al. (2012) conclude that trade liberalization always leads both economies to higher levels of welfare once external scale effects, i.e., $\eta < 1$, are in play. In their model, workers always benefit from free trade because of positive external scale effects (thus positive employment effects) and economy-wide welfare always improves. Their analysis is incomplete in the presence of heterogeneous workers.

\textsuperscript{28}The welfare definition follows Egger et al. (2012) who explain that total income can be used as a utilitarian welfare measure in each country, due to assuming one homogeneous final good.
5 Labor Market Shocks

This paper is motivated by the increasing interest in potential labor market spillovers between asymmetric countries. It is particularly relevant for the U.S. and Europe, or for nations within Europe, closely linked by trade. As long as policymakers in one country concern themselves with labor market reforms intended to lower unemployment, abundant debates easily occur between in-country proponents and opponents in their country and among their trading partners. Recently, labor market reforms -the so-called Hartz IV reforms in Germany- which aimed at lowering unemployment rates, have been criticized by other countries as being in effect beggar-thy-neighbor policies. However, recent empirical papers do not support this viewpoint (see, e.g., Felbermayr et al. 2012). Two possible questions of particular interest are (i) how variations in country $i$’s minimum wage affect country $j$ and (ii) how variations in country $i$’s factor endowment affect country $j$.

In what follows, both countries are asymmetric such as $w_i \neq w_j$, $L_i \neq L_j$, $H_i \neq H_j$, and $N_i \neq N_j$. Thus, Lemma 1 and 2 should be revised thus:

**Generalized Lemma 1.** Skilled workers’ wages are relatively equalized, $\frac{s_i}{s_j} = \frac{H_i}{H_j}$.

**Generalized Lemma 2.** $M_i = \left[ 1 + \left( \frac{N_i}{N_j} \right) \left( \frac{H_i}{H_j} \right) \left( \frac{w_j}{w_i} \right) \left( 1 - \beta \right) \right] M_H$.

The ratio of skilled workers’ wages between asymmetric countries is inversely related to the ratio of stocks of skilled workers. In addition, the total mass $M_i$ of firms in the world is characterized using relative minimum wages, relative endowments of skilled workers, relative mass of potential entrants between two asymmetric countries, and the mass of active firms in one country.

For later use, one additional Lemma is useful. We define $\Gamma_i$ as country $i$’s input share ($\frac{M_i}{M_t}$) among total input varieties used in final output production.

**Generalized Lemma 3.** $\Gamma_i + \Gamma_j = 1$.

\[
\Gamma_i = \left( \frac{N_i}{N_j} \right) \left( \frac{H_i}{H_j} \right) \left( \frac{w_j}{w_i} \right) \left( 1 - \beta \right) \left( 1 + \left( \frac{N_i}{N_j} \right) \left( \frac{H_i}{H_j} \right) \left( \frac{w_j}{w_i} \right) \right). 
\]

\[
\Gamma_j = \left( \frac{N_j}{N_i} \right) \left( \frac{H_j}{H_i} \right) \left( \frac{w_i}{w_j} \right) \left( 1 - \beta \right) \left( 1 + \left( \frac{N_j}{N_i} \right) \left( \frac{H_j}{H_i} \right) \left( \frac{w_i}{w_j} \right) \right). 
\]

**Proof.** See Appendix C.

By virtue of Generalized Lemma 1-2, equilibrium cutoff productivity and skilled workers’ wage in each country are produced as follows: For country $i$,
\[ \phi^*_i = \psi \left( \frac{w_i^{1-\beta}}{H_i^\beta} \right)^{\frac{1}{\theta + \beta \kappa}} \left[ \frac{\hat{N}_i}{w_i} \left( \frac{w_i}{w_j} \right)^{(1-\beta)\kappa} \right] \left[ \frac{\hat{H}_j}{\hat{H}_i} \right]^{\beta \kappa} \left( \frac{1}{\theta + \beta \kappa} \right) \left\{ \beta - \chi(\eta, \sigma) \right\}, \quad (39) \]

\[ s_{it} = \left( \frac{\mu}{\lambda} \right)^{\frac{\theta \beta}{\theta + \beta \kappa}} \left[ \frac{\hat{N}_i}{\hat{H}_i} \left( \frac{1}{w_i} \right)^{(1-\beta)\kappa} \right] + \left( \frac{\hat{N}_j}{\hat{H}_j} \right)^{\beta \kappa} \left( \frac{1}{\theta + \beta \kappa} \right)^{\theta} \left\{ \frac{\beta - \chi(\eta, \sigma)}{\beta (1-\beta) \kappa} \right\}, \quad (40) \]

For country \( j \), the subscript changes from \( i \) to \( j \) and from \( j \) to \( i \).

In the remainder of this paper, it should suffice to focus on cutoff productivity and skilled workers’ wage in each country. Crucial is focus on adjustment at the extensive margin of firms since selection effects are neutralized by assuming of no trade cost. Predictable is the direction of all other equilibrium variables responding to external shocks in labor markets. Equipped with (39), (40), and Generalized Lemma 3, a trade equilibrium as the starting point is ready to be examined.

### 5.1 Change in Minimum Wage Policy

We arrive at discussing how variations in one country’s minimum wage affects the other economy. In open economies, stronger labor market institutions in one country positively or negatively affect its trading partners? This is tackled in the seminal paper by Davis (1998) who explicitly considers international labor market linkages based on the Heckscher-Ohlin model. Davis (1998) concludes that high European minimum wages prop up U.S. wages. In contrast, Egger et al. (2012) raise a theoretical possibility of international negative spillover based on the intra-industry trade model. They conclude that high European minimum wages prop up the U.S. unemployment rate. Whenever the focus is on the international transmission of the consequences of labor market institutions between developed countries, should we expect a negative labor market spillover effect on the other country if one country raises its minimum wage? This paper shows that the main result by Egger et al. (2012) is not necessarily robust.

Define \( \varepsilon_{wi}^i \) as an elasticity of cutoff productivity in country \( i \) with respect to minimum wage in country \( i \).

\[ \varepsilon_{wi}^i = \left( \frac{1-\beta}{\theta + \beta \kappa} \right) \left[ 1 + \kappa(\beta - \chi) \Gamma_j \right] > 0. \quad (41) \]
Define $\varepsilon_{j}^{i}$ as an elasticity of cutoff productivity in country $j$ with respect to minimum wage in country $i$.

$$\varepsilon_{j}^{i} = -\left(\frac{1 - \beta}{\theta + \beta \kappa}\right) \kappa (\beta - \chi) \Gamma_i \geq 0 \iff \beta \lesssim \chi(\eta, \sigma).$$

(42)

Suppose country $i$ raises its real minimum wage. An increase in $w_i$ directly increases all intermediate input firms’ variable costs. Due to worsened profitability, marginal firms exit the market in (41). A reduced number of input varieties $M_{it}$ leads to a fall in skilled workers’ wage in (40). Consequently, unemployment increases and welfare falls in country $i$. Elsewhere, country $j$ is indirectly impacted by a rise in $w_i$ since there is no change of minimum wage policy in country $j$. Potential entrants in country $j$ would benefit from positive wages effects from a rise in $w_i$ (a fall in $s_{jt}$ in (40)). Lower variable cost enables those firms to enter the market. At the same time, however, those firms also encounter decreased demand for inputs by final output producers (negative external scale effects). If $\beta > \chi(\eta, \sigma)$, then a rise in $w_i$ leads potential firms in country $j$ to enter the market due to prevailing positive wage effects in (42). Entry of these firms, a rise in $M_{jt}$, expands employment (thus lowers unemployment for unskilled workers) and results in enhancing welfare in country $j$. Positive spillover effects of stronger labor market institutions to its trading partners are a possible channel under positive sign $\{\beta - \chi(\eta, \sigma)\}$. If negative external scale effects dominate positive wage effects, the fall in $M_{it}$ (due to a rise in $w_i$) leads marginal firms to exit the market (42), shrinks employment, thus worsens country $j$’s economy. Under negative sign $\{\beta - \chi(\eta, \sigma)\}$, the model has qualitatively the same theoretical prediction as the Egger et al. (2012). The following proposition summarizes possible spillover effects of a change in minimum wage.

**Proposition 4.** (Factor Price Shock) A rise in one country’s minimum wage harms its own economy. If $\beta > \chi(\eta, \sigma)$, it benefits its trading partners by expanding employment (lowering unemployment rate) and improving welfare and vice versa.

### 5.2 Change in Factor Endowments

Suppose country $i$ receives unskilled immigrants. Due to the existence of binding real minimum wages, no impact on trade equilibrium outcomes is observed except for a proportional increase in the unemployment rate within country $i$. When there are unskilled immigrants in country $i$, country $j$ is wholly insulated from the shock and *vice versa*. With respect to unskilled immigration in the present paper, Davis’ (1998) insulation hypothesis holds in *both*
countries.

How would countries respond to skilled emigrants in either country? Define $\varepsilon^i_{\bar{H}_i}$ as an elasticity of cutoff productivity in country $i$ with respect to the mass of skilled workers in country $i$.

$$\varepsilon^i_{\bar{H}_i} = -\left(\frac{\beta}{\theta + \beta \kappa}\right) \left[1 + \kappa (\beta - \chi) \Gamma_j\right] < 0. \quad (43)$$

Define $\varepsilon^j_{\bar{H}_i}$ as an elasticity of cutoff productivity in country $j$ with respect to the mass of skilled workers in country $i$.

$$\varepsilon^j_{\bar{H}_i} = \left(\frac{\beta}{\theta + \beta \kappa}\right) \kappa (\beta - \chi) \Gamma_i \leq 0 \iff \beta \leq \chi(\eta, \sigma). \quad (44)$$

A fall in $\bar{H}_i$ directly and negatively impacts its economy. It raises skilled workers’ wage (40) thus induces an increase in variable costs throughout all intermediate input firms, resulting in marginal firms exiting the market (43). Thus, total mass of varieties declines, the unemployment rate rises, and welfare worsens in country $i$. Elsewhere, a fall in $\bar{H}_i$ leads skilled workers’ wage in country $j$ to fall (Generalized Lemma 1 and Equation (40)). Intermediate input firms in country $j$ observe a decreased variable cost (positive wage effects). At the same time, those same firms encounter decreased demand for inputs by final output producers. If $\beta > \chi(\eta, \sigma)$, then a fall in $\bar{H}_i$ leads potential entrants in country $j$ to enter its intermediate input market in (44). The entry of firms stimulates country $j$’s economy by increasing the total mass of variety and lowering unemployment. In this way, country $j$ benefits from a decrease in the mass of skilled workers in country $i$.

**Proposition 5.** *(Factor Supply Shock)* A fall in the mass of skilled workers in country $i$ raises its skilled workers’ wage, while it lowers country $j$’s. The economy in country $i$ is always harmed. If $\beta > \chi(\eta, \sigma)$, then a fall in $\bar{H}_i$ benefits country $j$ by lowering unemployment rate and thus enhancing welfare and vice versa.

### 5.3 Magnitude of Impact from Shocks

The direction of labor market spillover from one country to another is now clear. If $\beta > \chi(\eta, \sigma)$, then country $i$ will benefit if country $j$ either increases its minimum wage or has skilled emigrants (and *vice versa*). It is as if both countries are on a teeter-totter in which one country benefits from the other country’s bad shocks in its labor market. If country $i$ (or $j$)
either raises its minimum wage or has skilled emigrants, then both countries are worse off when \( \beta < \chi(\eta, \sigma) \). It is as if both countries are in the same boat in which one country is also harmed by the other country’s bad shocks in its labor market. However, it is less clear to understand the magnitude of impact from labor market shocks. For the sake of argument, assume that the minimum wage in country \( j \) is larger than the one in country \( i \) and both countries have similar endowments: \( w_i < w_j, \bar{H}_i \simeq \bar{H}_j, \) and \( \bar{N}_i \simeq \bar{N}_j \). From Generalized Lemma 3, it is easy to show that \( \Gamma_i > \Gamma_j \).

### 5.3.1 Direct impact within country

Which country will be most heavily impacted by labor market shocks within its own economy? Using (41)-(44), the following ratio is obtained

\[
\frac{\varepsilon_{w_i}}{\varepsilon_{w_j}} = \frac{\varepsilon_{\bar{H}_i}}{\varepsilon_{\bar{H}_j}} = \frac{1 + \kappa (\beta - \chi) \Gamma_i}{1 + \kappa (\beta - \chi) \Gamma_j},
\]

(45)

Depending on sign\( \{ \beta - \chi \} \), two cases can be detected. If \( \beta > \chi \), then \( \varepsilon_{w_j} > \varepsilon_{w_i} \) and \( \varepsilon_{\bar{H}_j} > \varepsilon_{\bar{H}_i} \) since \( \Gamma_i > \Gamma_j \). This result implies that country \( j \) with a high minimum wage has a strong incentive to reform its labor market, as any variations in the labor market to reduce unemployment will stimulate heavily its own economy. However, country \( i \) will face negative spillover effects as examined earlier. Consequently, there is a higher probability for country \( j \) with high minimum wages to implement a *beggar-thy-neighbor* policy when \( \beta > \chi \). Contrarily, \( \varepsilon_{w_j} < \varepsilon_{w_i} \) and \( \varepsilon_{\bar{H}_j} < \varepsilon_{\bar{H}_i} \) when \( \beta < \chi \). In this case, country \( i \) with low minimum wage has a strong incentive to reform the labor market. Unlike the earlier case, there is no conflict with country \( j \) in which positive spillover effects are present. Consequently, there is a higher probability for country \( i \) with low minimum wages to implement a *love-thy-neighbor* policy when \( \beta < \chi \).

### 5.3.2 Indirect impact into country

Compared with direct impact within country \( i \), how heavily will country \( i \) be impacted by labor market shocks from country \( j \)? Using (41)-(44), the following ratio is relevant

\[
\left| \frac{\varepsilon_{w_j}^i}{\varepsilon_{w_i}^i} \right| = \left| \frac{\varepsilon_{\bar{H}_j}^i}{\varepsilon_{\bar{H}_i}^i} \right| = \left| \frac{-\kappa (\beta - \chi) \Gamma_j}{1 + \kappa (\beta - \chi) \Gamma_j} \right| \in [0, 1).
\]

(46)

As indicated by (46), the magnitude of impacts of labor market shocks from country \( j \) to \( i \) cannot be larger than one within country \( i \). Using data from 20 OECD countries, Fel-
bermayr et al. (2012) suggest that, on average, the effect of foreign institutions on domestic unemployment amounts to about 10% of the effect of domestic institutions. Similarly, Heid and Larch (2013) show that using data from 28 OECD countries labor market reforms in one country have small spillover effects on trading partners.

5.3.3 Relative magnitude of spillover effects

The next question, then, is which country will have a heavier impact on its trading partner? Using (41)-(44), the following ratio is obtained:

\[
\frac{\varepsilon_i^{j}}{\varepsilon_{w,j}} = \frac{\varepsilon_i^{j}}{\varepsilon_{H,j}} = \frac{\Gamma_i}{\Gamma_j}.
\]  

(47)

From (47), it is clear that any variations in labor market in country \(i\) with a low minimum wage will heavily impact on country \(j\) since \(\Gamma_i > \Gamma_j\). This result is reminiscent of a striking result from Davis (1998), who maintains the U.S. is wholly insulated from factor supply shocks in Europe, whereas factor supply shocks in the U.S. powerfully affect Europe.

6 Concluding Remarks

This paper essentially extends the paper by Egger et al. (2012) to incorporate two types of labor in a two-country new trade theory framework with heterogeneous firms and country-specific minimum wages. It highlights the role of a second factor with flexible real wages when one factor faces the risk of unemployment. This paper finds that upon external shocks, wages effects arising from intensive use of skilled workers counteract external scale effects generating employment expansion. With respect to welfare implications of trade liberalization, gains from trade arise only when external scale effects dominate wage effects. This paper raises the possibility of losses of trade under strong wage effects relative to external scale effects in the absence of trade cost. Regarding spillover effects of labor market shocks, this paper further predicts that labor market shocks in a country can exert either positive or negative spillover effects on the trading partners. Whereas a higher foreign minimum wage harms domestic workers under strong external scale effects, it otherwise can benefit domestic workers under strong wage effects.

Although empirical evidence is hitherto scarce, in future empirical studies it should not be surprising to find both positive and negative estimates of labor market spillover effects. Last, the model developed in this paper can be extended to studies of other issues such as
Appendix

A. Proof of Proposition 1

We can solve for skilled workers’ wage, using (19) and (20). Let me write down the two equations:

\[ \phi_s = \lambda \left[ \frac{w^{1-\beta}}{N} \right] \frac{1}{\frac{1 - \beta}{s^{1-k}}} \]

and

\[ \phi_s = \frac{w^{1-\beta}N}{\bar{H}} \frac{1}{\frac{1 - \beta}{s^{1-k}}} \frac{\beta - \frac{1}{s^{1-k}}} {s^{\frac{1}{s^{1-k}}}}. \]

By equating two cutoff productivities,

\[ s = \left( \frac{\zeta}{\lambda} \right) \frac{1 - \beta}{\frac{1}{s^{1-k}}} \frac{1 - \frac{1}{s^{1-k}}}{s^{1-k}} \]

\[ \frac{1 - \beta}{s^{1-k}} \left[ \frac{N}{w^{1-\beta}(1-\beta)\bar{H}1-k} \right] \frac{1}{\frac{1}{s^{1-k}}} \frac{1 - \frac{1}{s^{1-k}}}{s^{1-k}}. \]

Similarly, we can solve for \( \phi_s \). Next, write down (19) and (20) as follows.

\[ s = \left( \frac{1}{\lambda} \right) \frac{1 - \beta}{\frac{1}{s^{1-k}}} \frac{1 - \frac{1}{s^{1-k}}}{s^{1-k}} \]

\[ \frac{1 - \beta}{s^{1-k}} \left[ \frac{N}{w^{1-\beta}(1-\beta)\bar{H}1-k} \right] \frac{1}{\frac{1}{s^{1-k}}} \frac{1 - \frac{1}{s^{1-k}}}{s^{1-k}}. \]

By equating two wages for skilled workers,

\[ \phi_s = \left( \frac{1}{\lambda} \right) \frac{1 - \beta}{\frac{1}{s^{1-k}}} \frac{1 - \frac{1}{s^{1-k}}}{s^{1-k}} \left( \frac{N}{w^{1-\beta}(1-\beta)\bar{H}1-k} \right) \frac{1}{\frac{1}{s^{1-k}}} \frac{1 - \frac{1}{s^{1-k}}}{s^{1-k}}. \]
After some algebra,

$$\phi_* = \psi \left[ \tilde{N}^{\beta - \chi(\eta, \sigma)} w_1^{1-\beta} \right]^{\frac{1}{1+, \kappa(\beta - \chi)}}$$

with

$$\psi = \left( \lambda^{(1-\kappa \chi)(\frac{1}{\sigma -1} - \beta)} \tilde{\zeta} (1+\frac{\kappa}{\beta - \chi}) \right)^{\frac{\sigma -1}{1+, \kappa(\beta - \chi)}} > 0.$$ We arrive at the results in Proposition 1.

B. Trade Equilibrium: Aggregate Variables

I show the case in country $i$. The average productivity is derived by plugging (35) into APC,

$$\tilde{\phi}_i = \frac{\psi}{\rho \lambda} \left[ \tilde{N}^{\beta - \chi(\eta, \sigma)} w_i^{1-\beta} \right]^{\frac{1}{\sigma + \beta \kappa}} \left[ 1 + \left( \frac{w_i}{w_j} \right)^{\kappa(1-\beta)} \right]^{\frac{\beta - \chi}{\sigma + \beta \kappa}}.$$

Plugging (35) into CPC, the mass of active firms is

$$M_i = \frac{\tilde{N}^{\frac{1}{\sigma + \beta \kappa}}}{\psi \lambda} \left( \tilde{H}_i^{\beta} \right)^{\frac{\beta}{\sigma + \beta \kappa}} \left[ 1 + \left( \frac{w_i}{w_j} \right)^{\kappa(1-\beta)} \right]^{\frac{\kappa(1-\beta)}{\sigma + \beta \kappa}}.$$

Since $Y_i = M_i r_i (\tilde{\phi}_i) = M_i \frac{\kappa \sigma}{\kappa - \sigma + 1}$, total final output produced $Y_i$ becomes

$$Y_i = \left( \frac{\kappa \sigma}{\kappa - \sigma + 1} \right) \frac{\tilde{N}^{\frac{1}{\sigma + \beta \kappa}}}{\psi \lambda} \left( \tilde{H}_i^{\beta} \right)^{\frac{\beta}{\sigma + \beta \kappa}} \left[ 1 + \left( \frac{w_i}{w_j} \right)^{\kappa(1-\beta)} \right]^{\frac{\kappa(1-\beta)}{\sigma + \beta \kappa}}.$$

Using $W_i = \rho M_i r_i (\tilde{\phi}_i)$, total labor costs $W_i$ are

$$W_i = \left( \frac{\kappa(\sigma -1)}{\kappa - \sigma +1} \right) \frac{\tilde{N}^{\frac{1}{\sigma + \beta \kappa}}}{\psi \lambda} \left( \tilde{H}_i^{\beta} \right)^{\frac{\beta}{\sigma + \beta \kappa}} \left[ 1 + \left( \frac{w_i}{w_j} \right)^{\kappa(1-\beta)} \right]^{\frac{\kappa(1-\beta)}{\sigma + \beta \kappa}}.$$

Unemployment rate, $U_i = 1 - (1-\beta) \frac{W_i}{w_i L_i}$, is

$$U_i = 1 - \frac{\kappa(1-\beta)(\sigma -1)}{(\kappa - \sigma +1) \psi L_i} \left[ \tilde{N}^{\frac{1}{\sigma + \beta \kappa}} \tilde{H}_i^{\beta} \left( \frac{w_i^{1-\beta}}{w_j^{1-\beta}} \right)^{\frac{1}{\sigma + \beta \kappa}} \left[ 1 + \left( \frac{w_i}{w_j} \right)^{\kappa(1-\beta)} \right]^{\frac{\kappa(1-\beta)}{\sigma + \beta \kappa}}.\right.$$  

For country $j$, the other two equations are derived by changing notations from $i$ to $j$ and from $j$ to $i$.  

31
C. Proof of Lemmas

C.1 Generalized Lemma 1

The model is built on the two country case. Two lemmas can be further generalized in many country case. In the case of two country case, write down four equations: two PMCs and two ZCPs.

\[ M_{i}^{\frac{1}{1-\eta}} = \frac{w_{i}^{-\beta}}{\rho} \left( \frac{\kappa}{\kappa - \sigma + 1} \right) \frac{1}{1-\sigma} M_{i}^{\frac{1}{\beta}} \phi_{i}^{*} \]  \hspace{1cm} (a)

\[ M_{j}^{\frac{1}{1-\eta}} = \frac{w_{j}^{-\beta}}{\rho} \left( \frac{\kappa}{\kappa - \sigma + 1} \right) \frac{1}{1-\sigma} M_{j}^{\frac{1}{\beta}} \phi_{j}^{*} \]  \hspace{1cm} (b)

\[ s_{H_{i}}^{\beta} = \left( \frac{\rho}{w_{i}^{-1}} \right) ^{\frac{\beta(\sigma-1)}{\beta(\sigma-1)-1}} \left( \frac{\rho \sigma p}{H_{i}} \right) ^{\frac{\beta}{1-\beta(\sigma-1)}} \left( M_{i}^{\frac{1}{\beta(\sigma-1)-1}} \right) ^{\frac{\beta(\sigma-1)}{\beta(\sigma-1)-1}} \left( \phi_{i}^{*} \right) ^{\frac{\beta(\sigma-1)}{\beta(\sigma-1)-1}} \]  \hspace{1cm} (c)

\[ s_{H_{j}}^{\beta} = \left( \frac{\rho}{w_{j}^{-1}} \right) ^{\frac{\beta(\sigma-1)}{\beta(\sigma-1)-1}} \left( \frac{\rho \sigma p}{H_{j}} \right) ^{\frac{\beta}{1-\beta(\sigma-1)}} \left( M_{j}^{\frac{1}{\beta(\sigma-1)-1}} \right) ^{\frac{\beta(\sigma-1)}{\beta(\sigma-1)-1}} \left( \phi_{j}^{*} \right) ^{\frac{\beta(\sigma-1)}{\beta(\sigma-1)-1}} \]  \hspace{1cm} (d)

Plugging (c) into (a),

\[ M_{i}^{\frac{1}{1-\eta} + \frac{\eta\beta}{\beta(\sigma-1)-1}} = \left( \frac{w_{i}^{-1}}{\rho} \right) ^{\frac{1}{1-\beta(\sigma-1)}} \left( \frac{\beta \sigma p}{H_{i}} \right) ^{\frac{1}{1-\beta(\sigma-1)}} \left( \frac{\kappa}{\kappa - \sigma + 1} \right) ^{\frac{1}{1-\sigma}} \left[ \phi_{i}^{*} \right] ^{\frac{1}{1-\beta(\sigma-1)-1}} \]

Similarly, plugging (d) into (b),

\[ M_{j}^{\frac{1}{1-\eta} + \frac{\eta\beta}{\beta(\sigma-1)-1}} = \left( \frac{w_{j}^{-1}}{\rho} \right) ^{\frac{1}{1-\beta(\sigma-1)}} \left( \frac{\beta \sigma p}{H_{j}} \right) ^{\frac{1}{1-\beta(\sigma-1)}} \left( \frac{\kappa}{\kappa - \sigma + 1} \right) ^{\frac{1}{1-\sigma}} \left[ \phi_{j}^{*} \right] ^{\frac{1}{1-\beta(\sigma-1)-1}} \]

By equating both equations right above,

\[ 1 = \left( \frac{H_{j}}{H_{i}} \right) ^{\frac{\beta}{1-\beta(\sigma-1)}} \left( \frac{w_{i}^{-1}}{w_{j}^{-1}} \right) ^{\frac{1}{1-\beta(\sigma-1)}} \left( \phi_{i}^{*} \right) ^{\frac{1}{1-\beta(\sigma-1)-1}} \left( \phi_{j}^{*} \right) ^{\frac{1}{1-\beta(\sigma-1)-1}} \]

That is,
The ratio of two cutoff productivities can be expressed as the ratio of variable costs from both countries: \( \frac{\phi^*_i}{\phi^*_j} = s_i^\beta w_{ij}^{1-\beta} / s_j^\beta w_{ij}^{1-\beta} \). This implies relative skilled workers’ wage equalization which is Generalized Lemma 1.

Using Equation (40),

\[
\frac{\bar{s}_i}{s_j} = \left( \frac{\bar{H}_i}{\bar{H}_j} \right)^{\kappa(1-\beta)} \frac{\bar{N}_i(1 - \kappa \chi)}{\bar{N}_j(1 - \kappa \chi)} + \frac{\bar{N}_j(1 - \kappa \chi)}{\bar{N}_i(1 - \kappa \chi)} \left( \frac{\beta}{\bar{H}_j} \right)^{\kappa(1-\beta)} \frac{1}{(\bar{w}_j)^{1-\kappa \chi}} + \frac{\bar{N}_i(1 - \kappa \chi)}{\bar{N}_j(1 - \kappa \chi)} \left( \frac{\beta}{\bar{H}_i} \right)^{\kappa(1-\beta)} \frac{1}{(\bar{w}_i)^{1-\kappa \chi}} \right]^{\frac{1}{\kappa(\beta - \chi)}}.
\]

Factoring \( \left( \frac{1}{\bar{H}_i} \right)^{1-\kappa \chi} \left( \frac{\beta}{\bar{H}_i} \right)^{\kappa(1-\beta)} / \left( \frac{1}{\bar{H}_j} \right)^{1-\kappa \chi} \) out inside the bracket,

\[
\frac{\bar{s}_i}{s_j} = \left\{ \left( \frac{1}{\bar{H}_i} \right)^{1-\kappa \chi} \left( \frac{\beta}{\bar{H}_i} \right)^{\kappa(1-\beta)} \right\}^{\frac{1}{\kappa(\beta - \chi)}} \left\{ \frac{\bar{N}_j(1 - \kappa \chi)}{\bar{N}_i(1 - \kappa \chi)} \left( \frac{\beta}{\bar{H}_j} \right)^{\kappa(1-\beta)} + \frac{\bar{N}_j(1 - \kappa \chi)}{\bar{N}_i(1 - \kappa \chi)} \left( \frac{\beta}{\bar{H}_i} \right)^{\kappa(1-\beta)} \right\}^{\frac{1}{\kappa(\beta - \chi)}}.
\]

Canceling the second bracket,

\[
\frac{\bar{s}_i}{s_j} = \left\{ \left( \frac{1}{\bar{H}_i} \right)^{1-\kappa \chi} \left( \frac{\beta}{\bar{H}_i} \right)^{\kappa(1-\beta)} \right\}^{\frac{1}{1+\kappa(\beta - \chi)}}.
\]

which is \( \frac{\bar{s}_i}{s_j} = \frac{\bar{H}_i}{\bar{H}_j} \). When \( \frac{\bar{H}_i}{\bar{H}_j} = 1 \), it indicates Lemma 1.

**C2. Generalized Lemma 2**

Using \( M_t = \left[ 1 + \left( \frac{\bar{N}_i}{\bar{N}_j} \right) \left( \frac{\phi^*_i}{\phi^*_j} \right)^\kappa \right] M_{it} \) and Generalized Lemma 1,
\[ M_t = \left[ 1 + \left( \frac{\bar{N}_j}{\bar{N}_i} \right) \left( \frac{\bar{H}_j}{\bar{H}_i} \right)^{\kappa \beta} \left( \frac{w_j}{w_i} \right)^{\kappa(1-\beta)} \right] M_t. \]

Similarly, the case in country \( j \) is derived. If we assume \( \frac{\bar{H}_j}{\bar{H}_i} = 1 \) and \( \frac{\bar{N}_j}{\bar{N}_i} = 1 \), it indicates Lemma 2.

**C3. Generalized Lemma 3**

\( \Gamma_i + \Gamma_j = 1 \). So, \( \Gamma_i = 1 - \frac{M_{jt}}{M_t} = \frac{M_t - M_{jt}}{M_t}. \) That is, \( \Gamma_i = \frac{M_t - M_{jt}}{M_t} \). Using the result in Generalized Lemma 2,

\[ \frac{M_t}{M_{jt}} = 1 + \left( \frac{\bar{N}_j}{\bar{N}_i} \right) \left( \frac{\bar{H}_j}{\bar{H}_i} \right)^{\kappa \beta} \left( \frac{w_j}{w_i} \right)^{\kappa(1-\beta)} \]. Therefore, \( \Gamma_i = \frac{1}{1 + \left( \frac{\bar{N}_j}{\bar{N}_i} \right) \left( \frac{\bar{H}_j}{\bar{H}_i} \right)^{\kappa \beta} \left( \frac{w_j}{w_i} \right)^{\kappa(1-\beta)}} \).

\( \Gamma_j \) is similarly derived.

**References**


36


