On the dynamics of asset prices and liquidity: the role of search frictions and idiosyncratic shocks*

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Abstract

We build a consumption asset pricing model with search frictions on the market for assets and idiosyncratic income shocks. Search is directed and a consumption-leisure decision drives the price-liquidity tradeoff: if consumption is marginally more desirable than leisure, a seller would be willing to wait longer for a higher price. Our model is consistent with two characteristics of the housing market, where search frictions are particularly important: i) procyclical liquidity (i.e. countercyclical time to sell) and ii) prices more sensitive to demand shocks.

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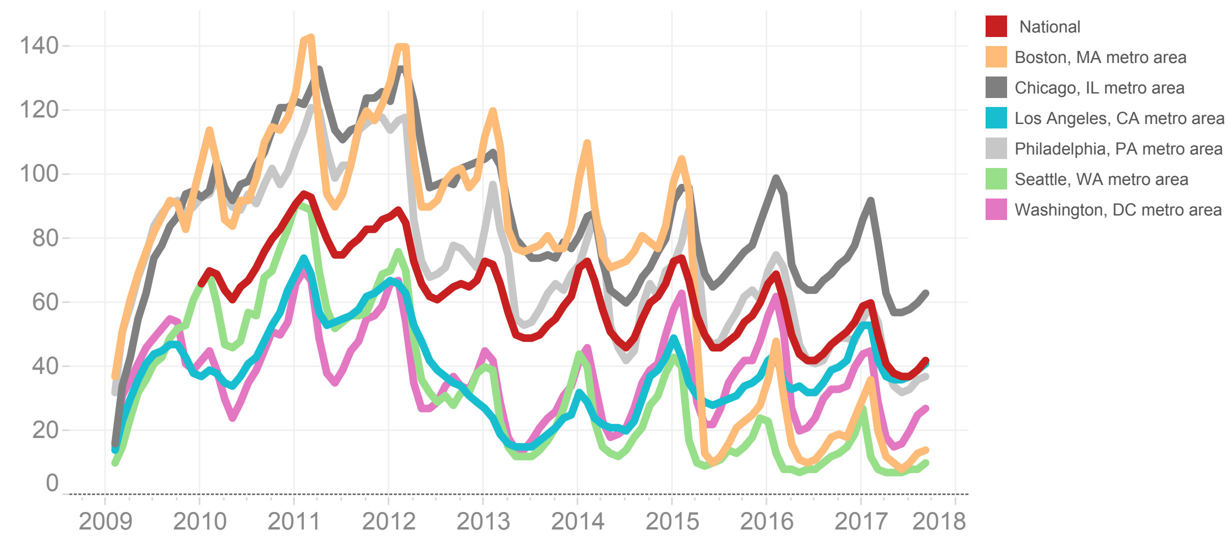

Figure 1: Average days on market (home sales); Source: Redfin

1 Introduction

The housing market exhibits two stylized patterns. First, this market is characterized by procyclical liquidity in the sense that homes sell more quickly in an expansion than in a recession. This is illustrated in Figure 1, which depicts the evolution of the average number of days it takes to sell a house advertised on Redfin—a residential real estate company that provides web-based real estate database and brokerage services.[1] The graph shows the monthly evolution from 2009 until now for six major cities in the US as well as the average in the whole country. It displays an increase in the time to sell following the recession, which is reversed after 2011, turning the evolution into a progressive decline. Appendix A of this paper shows that liquidity also appears procyclical when one looks at more disaggregated data such as at State or MSA level, controlling for time and fixed effects. Second, there is evidence that house prices are mostly driven by fluctuations in demand. The literature has documented a positive correlation between prices and sales volume for this market (**?**) and it has shown that the importance of demand for price fluctuation tend to be lower in areas where it is easy to build new homes (**?**, **?**, **?**, **?**).

The housing market also suffers from important search frictions. In this paper, we

[1]The data was downloaded from the Redfin webpage: `https://www.redfin.com/blog/data-center`. Similar data can be downloaded from the Zillow website (`https://www.zillow.com/research/data/`). We choose to report the Redfin data on Figure 1 because the time period is longer. Appendix A provides regression results at the US State and MSA level using data from Zillow.

build a simple consumption asset pricing model (CCAPM) with search and matching frictions on the financial market and idiosyncratic income shocks. Households are modeled using the large-family framework by **?**, while search is competitive as in **?**. The large-family framework allows the model to converge to the standard CCAPM when search frictions collapse.[2] In the model, the trading agents care about the price they pay (or receive) because they smooth consumption and they also care about the time it takes to sell an asset because of the associated leisure loss. Consequently a consumption-leisure decision drives the price-liquidity tradeoff: if consumption is marginally more desirable than leisure, a seller would be willing to wait longer for a higher price.

We study the dynamics of liquidity (i.e. the speed at which agents sell assets) and asset prices in this framework. Our results are consistent with the two stylized facts on the housing market described above. We find that liquidity is procyclical when idiosyncratic income risk is countercyclical in the model. The intuition for this result comes from the consumption-leisure tradeoff. In a recession, a seller cares more about consumption as her marginal utility of consumption goes up. She is thus willing to wait longer on the financial market to sell her assets. Similarly, if income dispersion goes up in a recession, the marginal utility of consumption of a buyer is not affected much, implying that a buyer is willing to sacrifice consumption relatively more than a seller instead of incurring a larger leisure loss. As a consequence, time to sell goes up in a recession because of these two reasons.

The asset pricing literature based on heterogenous agent models with uninsurable idiosyncratic risk, such as e.g. **?** and **?**, uses countercyclical income risk as a solution to the equity premium puzzle. The idea is that this simply increases systematic risk. We show in this paper that the same assumption also generates procyclical liquidity as suggested by the data. A more recent contribution by **?**, with comprehensive data, argues that idiosyncratic risk is not countercyclical. Instead, they find that left-skewness increases during recessions. Our mechanism would also be robust under such a statistical framework because it only needs sellers to suffer relatively more the consequences of recessions than the buyers.

We also find that the presence of search frictions makes asset prices more sensitive to changes in the asset valuation of buyers, while frictions reduce their sensitivity to the the valuation of sellers. The intuition for this result is reminiscent of **?**. Indeed, a difference of our model with respect to the standard CCAPM is that a buyer has to pay a search cost on top of the price to acquire an asset. Hence, while a one percent

[2]Our large-family framework also simplifies the solution of the model in the sense that the distribution of prices and liquidity is degenerate in equilibrium. This is in contrast for instance with the framework by **?** who analyze the optimal portfolio choice in terms of price and liquidity.

increase in valuation yields a one percent increase in the price in the standard model, the same increase in valuation would imply a more than proportional increase in the price if the search cost is barely fixed. Hence, search frictions amplifies the impact of demand shocks in our framework.

Our paper is related to a growing literature that builds asset pricing models with procyclical liquidity. **?** introduce adverse selection in an asset pricing model, reducing the probability to sell an asset in a recession. **?** model an economy with search frictions in the market for used capital and obtain procyclical liquidity on the capital market through the entry of buyers. **?** introduce search on the market for assets in an RBC model.

Housing: **?**, **?**, **?**

Finally, the idea that agents use their time endowment to search for better price deals when leisure is abundant has been used in frameworks of the goods market. **?** studies the business cycle dimension. They build a model where unemployed workers in a recession have more time to search on the goods market. **?** studies the life cycle dimension. Their results suggest that old consumers have more time to search for cheaper products.

The paper is organized as follows. Section 2 introduces the model. The equilibrium is analyzed in Section 3, while the main results are exposed in Section 4. Section 5 tries to quantify the asymmetry between the price impact of the asset valuation of buyers and and the one of sellers.

2 Model

2.1 Assets and output

Time is discrete. We will use the index t to refer to each time period, but, in most of the presentation below, we will simply drop time indices and use primes to refer to variables in period $t+1$, while variables without prime are evaluated at time t.

There are two agents indexed by $i \in \{1, 2\}$ who discount time by a factor $\beta_i \in (0, 1)$. The discount factor is stochastic, as described below.[3] They derive instantaneous utility $u(c_i, l_i)$ from consumption c_i and leisure l_i in each period, with u being subject to standard properties.

[3] House prices are empirically more volatile than their fundamental value. Shocks to the discount factor allow to reproduce this fact. Moreover, considering shocks to the discount factor allows to analyze how asset prices may respond differently to exogenous changes in the asset valuation of buyers as compared to exogenous changes in the valuation of sellers.

Each agent is the owner of a farm that produces w_i units of real output per hours worked in the farm each period. The w_i's are subject to idiosyncratic shocks and we will sometimes refer to them as the *wages* of the economy. We denote by n_i the hours the agent i chooses to work in the farm. Agents can sell goods produced by the farm in the goods market, but they cannot sell claims on the farm.

There is a **?** tree delivering dividends π every periods. The vector $z \equiv (\beta_1, \beta_2, w_1, w_2, \pi)'$ follows a Markov chain with transition matrix Γ and ergodic mean $(\bar{\beta}, \bar{\beta}, \bar{w}, \bar{w}, \bar{\pi})'$. An agent i holds claims on the tree in quantities A_i. As we discuss below, the markets on which these assets are traded are characterized by search frictions, making them illiquid in the sense that a seller cannot immediately resell an asset at zero cost.

Goods are perfect substitutes whether they come from the tree or farms and exist to cover consumption needs. Aggregate output is the sum of the output realizations of all farms and the tree:

$$Y = \sum_{i=1}^{2} w_i n_i + \pi.$$

Through market clearing, Y is also the value taken by aggregate consumption,

$$C = \sum_{i=1}^{2} c_i,$$

since agents do not hold physical capital and no government habits this economy.

2.2 Search frictions

Agents trade assets in markets characterized by search frictions. In particular, we assume competitive search in the sense of **?**. An advantage of using this framework as opposed to a context with undirected search and Nash bargaining is that it eases the comparison of some results with the benchmark of the consumption CAPM model since agents are price takers in both frameworks.

A continuum of submarkets opens every period, where agents can exchange the assets. Since the amount of markets in equilibrium cannot be determined, we'll simply normalize this amount one. Each submarket may in principle be characterized by a different price, which we denote by p. Agents take these prices as given.

Each agent is endowed with L units of time every period. Agents can use time to search on financial markets, work on the farm or use it for leisure. Trade can occur only when search is successful. When a unit of time dedicated to search is successful, they can trade x units of assets. Search is directed: one freely chooses to spend time searching as a buyer or a seller on each one of the submarkets. Agents can potentially choose to search on several submarkets at the same time. We denote by b_i (and s_i) the time spent by agent i as a buyer (and a seller) on a submarket. Hence, each agent takes the following restriction into account:

$$l_i + b_i + s_i + n_i \leq L. \tag{1}$$

We denote by $\lambda_s(\theta)$ (and $\lambda_b(\theta)$) the probability that a seller (buyer) finds a partner to trade assets with on a given submarket (per unit of time effort invested in the search process).[4] It is an increasing (decreasing) function of the tightness $\theta = \frac{b}{s}$ and the property $\lambda_s(\theta) = \theta\lambda_b(\theta)$ holds. θ is a measure of liquidity: when θ is large, assets are sold quickly. Hence, agent i sells $s_i\lambda_s(\theta)$ assets and buys $b_i\lambda_b(\theta)$ assets on the same market.

We can write the value function characterizing the behavior of each agent i as follows:

$$V(A_i, z) = \max_{c_i, l_i, b, s, n_i} u(c_i, l_i) + \beta_i E\left[V(A_i', z')\right] \tag{2}$$

subject to (1), the budget constraint, i.e

$$n_i w_i + s_i x \lambda_s(\theta) p + A_i \pi \geq c_i + b_i x \lambda_b(\theta) p, \tag{3}$$

and the law of motions for asset holding:

$$A_i' = A_i - x s_i \lambda_s(\theta) + x b_i \lambda_b(\theta) \tag{4}$$

3 Equilibrium

3.1 First-order conditions

Agents buy or sell assets in order to smooth consumption. In equilibrium, each agent does not spend time searching as a buyer and as a seller at the same time. In times of liquidity needs, agents select themselves into the group of sellers, while those who prefer to save become buyer.

Call $\xi_{i,t,t+j} \equiv \beta_i^j \frac{\partial u(c_{i,t+j}, l_{i,t+j})/\partial c_{i,t+j}}{\partial u(c_{i,t}, l_{i,t})/\partial c_{i,t}}$ the stochastic discount factor that allows to discount units of output in period $t+j$ from the perspective of agent i in period t. In the Appendix B.1, we show that the following two asset pricing equations describe the

[4] These probabilities are obtained through a standard constant-return-to-scale matching function $\sigma(b, s)$: $\lambda_s(\theta) = \frac{\sigma(b,s)}{s} = \sigma\left(\frac{b}{s}, 1\right)$ and $\lambda_b(\theta_m) = \frac{\sigma(b,s)}{b} = \sigma\left(1, \frac{s}{b}\right)$.

behavior of sellers and buyers respectively:

$$p_t - \frac{\partial u(c_{i,t}, l_{i,t})/\partial l_{i,t}}{\partial u(c_{i,t}, l_{i,t})/\partial c_{i,t}} \frac{1}{x\lambda_s(\theta_t)} = \sum_{j=1}^{\infty} E_t \left[\xi_{i,t,t+j}\pi_{t+j}\right], \tag{5}$$

$$p_t + \frac{\partial u(c_{i,t}, l_{i,t})/\partial l_{i,t}}{\partial u(c_{i,t}, l_{i,t})/\partial c_{i,t}} \frac{1}{x\lambda_b(\theta_t)} = \sum_{j=1}^{\infty} E_t \left[\xi_{i,t,t+j}\pi_{t+j}\right]. \tag{6}$$

In the standard consumption CAPM model, the price of an asset is equal to the discounted sum of the expected dividends that the asset delivers: this benchmark corresponds to the case when x tends to infinity. When x is finite, the first-order conditions (5) and (6) show that search frictions act as a tax on financial transactions. This generates a gap between the effective price paid by buyers and the effective price received by sellers. For example, equation (5) subtracts the disutility of searching from the price a seller receives after a transaction has been made (the second term on the left-hand side). This cost considers the time spent searching for a buyer to sell the asset (the duration $1/\lambda_s(\theta^a_{m,t})$) weighted by the marginal rate of substitution between leisure and consumption (the seller sacrifices leisure to get liquidity which allows her to consume). A similar interpretation applies to equation (6), with the difference that, in this case, buyers have to pay an extra disutility cost on top of the price they pay to sellers.

The equilibrium is also characterized by a standard labor supply condition:

$$\frac{\partial u(c_i, l_i)/\partial l_i}{\partial u(c_i, l_i)/\partial c_i} = w_i \tag{7}$$

This relation will be useful a tool below to identify the marginal rate of substitution between consumption and leisure from the exogenous variable w_i.

3.2 The price-liquidity tradeoff

Equations (19) and (20) describe indifference curves between θ and p for the buyers and sellers respectively for given marginal utilities. These are shown on Figure 2. The (SS) convex locus refers to the implicit relation between p and θ given by equation (20), while the (BB) concave locus corresponds to the relation given by (19). Sellers prefer indifference curves characterized by high price and high liquidity: these curves would be located in the upper-right corner of the graph. Buyers prefer indifference curves characterized by low price and low liquidity (located in the lower-left corner of the graph).

Under competitive search, the set of prices in active submarkets is exhaustive:

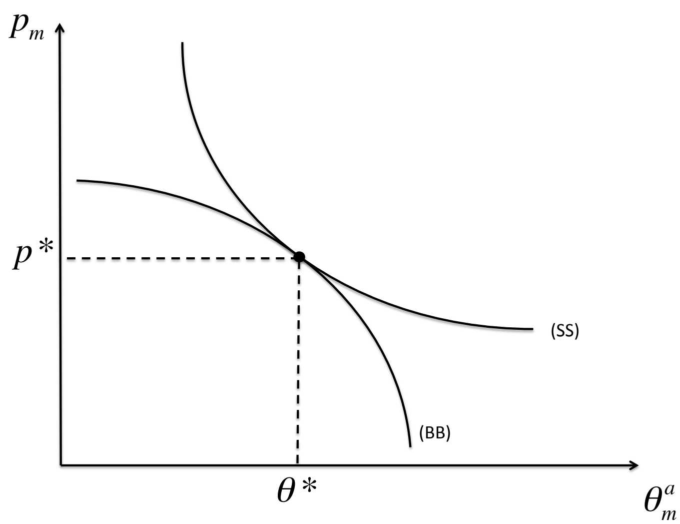

Figure 2: Joint determination of price and liquidity

there exist no other price such that a submarket would have participants at that price. This implies that the two indifference curves (SS) and (BB) have to be tangent in equilibrium, as shown on Figure 2. In the Appendix B.2, we show that the tangency implies the condition given in the following Proposition:

Proposition 1. *Denote by c_s and l_s the consumption and leisure levels of sellers in a submarket and by c_b and l_b the ones of buyers. Denote by $\eta(\theta)$ the elasticity of the matching function with respect to the mass of sellers. Liquidity reads as*

$$\theta = \frac{1-\eta(\theta)}{\eta(\theta)} \frac{\partial u(c_s, l_s)/\partial l_s}{\partial u(c_s, l_s)/\partial c_s} \frac{\partial u(c_b, l_b)/\partial c_b}{\partial u(c_b, l_b)/\partial l_b}. \tag{8}$$

The liquidity in a given submarket thus depends on two elements: i) the marginal rate of substitution between leisure and consumption of sellers relative to the one of buyers; and ii) the elasticity of the matching function with respect to the mass of buyers relative to the elasticity with respect to the mass of sellers. The intuition for the presence of the first factor is the following. When sellers choose towards which type of market to direct their search, they compare the utility they obtain through consumption by selling the asset with the disutility in terms of leisure they suffer through search. If the marginal utility of consumption is relatively large, they are willing to spend a large amount of time searching to sell the asset. A similarly reasoning applies to the behavior of buyers. This is why the ratio of marginal rates of substitution appears in equation (8).

Liquidity is also decreasing in the elasticity of the matching function with respect to the mass of sellers. When $\eta(\theta)$ is low, the probability of selling an asset quickly converges towards zero as more sellers enter the market. This means that it quickly becomes attractive for sellers to create an extra market and start searching there. This explain why only few sellers operate on a market and liquidity is high when $\eta(\theta)$. A similar reasoning applies when the elasticity of the matching function with respect to the mass of buyers $(1-\eta(\theta))$ is low: the incentive for buyers to operate on another market quickly increases as new buyers enter the market, explaining why only few buyers stay in a market when $(1-\eta(\theta))$ is low.

Another interpretation of condition (8) is the following. In a context with frictionless trading, a social planner would like to equalize the marginal utility of consumption between agents. Markets could achieve this, but frictions generate a wedge that does not allow such equalization. This wedge depends on congestion on the market and the relative leisure loss of transporting a unit of output from one agent to the other one.[5] This is suggested by simply rewriting (8) as follows:

$$\frac{\partial u(c_b,l_b)/\partial c_b}{\partial u(c_s,l_s)/\partial c_s}=\theta\frac{\eta(\theta)}{1-\eta(\theta)}\frac{\partial u(c_b,l_b)/\partial l_b}{\partial u(c_s,l_s)/\partial l_s}.$$

An appealing characteristic of (8) is that it is a static condition. In directed search models with free-entry of buyers such as e.g. **?**, the liquidity of a market also depends on the value of reselling the asset in the future. With two-sided free entry, as in our case, this component disappears, making the condition a static one.[6]

4 Income, liquidity and asset prices

The intra-temporal condition (7) is useful as it allows to identify all the equilibrium marginal rates of substitution between consumption and leisure with the exogenous variables w_i's. The equilibrium tightness and the Euler equations can be rewritten as

$$\theta=\frac{1-\eta(\theta)}{\eta(\theta)}\frac{w_s}{w_b}, \tag{9}$$

[5]We thank Guido Menzio for pointing this out to us.

[6]Interestingly, **?** obtain a similar condition in a context with undirected search, with some differences though. First, since the Hosios-Pissarides condition is not necessarily met, their condition includes the bargaining power of sellers instead of the elasticity of the matching function. Second, because agents are risk-neutral in their model, the marginal utility of consumption is absent and the opportunity cost of search (the marginal utility of leisure in our case) is constant.

$$p - \frac{w_s}{x\lambda_s(\theta)} = E(v'_s) \tag{10}$$

and

$$p + \frac{w_b}{x\lambda_b(\theta)} = E(v'_b) \tag{11}$$

respectively, where $E(v'_s)$ and $E(v'_b)$ are the present-discounted values of dividends when considering the stochastic discount factors of the seller and the buyer respectively (as they appear in the right-hand sides of (5) and (6)).

It is straightforward to see from (9) that the cyclicality of the wage ratio between sellers and buyers is what drives the cyclicality of liquidity in the model. For example, a procyclical ratio generates procyclical liquidity as long as the elasticity of the matching function is not too decreasing in the tightness. This would happen when wage dispersion is correlated negatively with the aggregate shock π. This result is summarized in the next paragraph:

Result 1. *Define* $\Gamma(\theta) \equiv \theta\frac{\eta(\theta)}{1-\eta(\theta)}$. *Assume* $\Gamma'(\theta) > 0$. *If* $COV\left(\frac{w_s}{w_b}, \pi\right) \lesseqgtr 0$, *then* $COV(\theta, \pi) \lesseqgtr 0$.

Equations (10) and (11) now have the interpretation that the economic price that a seller receives (or a buyer pays) is net of all the income she gives up while searching for a buyer (or a seller) for the asset. This is because a negative term appears on the left hand side of equation (10) next the price, which is the income of the seller would have received (per unit of time she would have worked) multiplied by the average search duration for a seller. A similar interpretation applies to equation (11) from a buyer's perspective.

Define by $\kappa_s \equiv \frac{w_s}{x\lambda_s(\theta)}$ and $\kappa_b \equiv \frac{w_b}{x\lambda_b(\theta)}$ the search costs incurred by the seller and the buyer respectively. As it is clear from (9), these two terms only depend on the exogenous variables w_i's. By log-linearizing the conditions (10) and (11) around the steady state, one can see that the presence of search frictions increases the elasticity of the price with respect to the buyer's valuation of the asset, while it reduces the elasticity with respect to the seller's valuation. Indeed, denote with hats log deviations from steady state and with asterisks steady state values, one gets

$$\hat{p} = \frac{p^* + \kappa_b^*}{p^*}\hat{v}'_b - \frac{\kappa_b^*}{p^*}\hat{\kappa}_b$$

and

$$\hat{p} = \frac{p^* - \kappa_s^*}{p^*}\hat{v}'_s + \frac{\kappa_s^*}{p^*}\hat{\kappa}_s.$$

In a standard consumption asset pricing model, these elasticities would both be equal to one because the price is simply equal to the agent's valuation of the asset in the standard framework. However, the presence of search frictions here pushes the buyer's elasticity above one and the seller's below unity. This result is summarized in the following paragraph:

Result 2. *Denote by* $\epsilon_b \equiv \frac{p^*+\kappa_b^*}{p^*}$ *and* $\epsilon_s \equiv \frac{p^*-\kappa_s^*}{p^*}$ *the elasticity of the price of the asset with respect to the buyer and the seller's valuation. The presence of search frictions imply* $\epsilon_b > 1$ *and* $\epsilon_b < 1$*.*

The aim of Section 5 is to evaluate quantitatively these elasticities.

5 Quantifying the multiplier

We now attempt to evaluate quantitatively the elasticities ϵ_b and ϵ_s mentioned in Result 2. The idea is to try to understand the importance of p^* with respect to κ_b^* and κ_s^*. We use two data sources for this exercise. The first one is data on time to sell homes, as described for example in Figure 1. This allows to identify the search duration for sellers in equation (10). We show below elasticities for average durations of 2 and 4 months. The second source is data on price to income ratio. According to Numbeo, home prices are about 3 years of median wages in the US. A disadvantage of this data is that we do not identify wages of buyers or sellers specifically. Wages of individuals trading houses are likely to be higher than the median. In this case, because the opportunity cost of search would be higher, one would obtain higher values for ϵ_b and lower values for ϵ_s exacerbating the difference.

The data allows to obtain the importance of κ_s with respect to p^*. In particular, if the average search duration for sellers is four months, then the price response to a change in the valuation of the seller is attenuated by 11.1% ($\epsilon_s = 0.889$) as compared to a standard CCAPM framework, while it would be attenuated by 5.6% in the case of a two-month duration.

Unfortunately the data does not allows to identify κ_b^* directly. We thus rely on a Cobb-Douglas specification of the matching function for this matter: $\sigma(b, s) = b^{1-\eta}s^{\eta}$. This specification allows to rewrite the Euler equations as

$$p - \frac{1}{x}\left(\frac{\eta}{1-\eta}\right)^{1-\eta} w_b^{1-\eta} w_s^{\eta} = E(v_s')$$

and

$$p + \frac{1}{x}\left(\frac{1-\eta}{\eta}\right)^{\eta} w_b^{1-\eta} w_s^{\eta} = E(v_b').$$

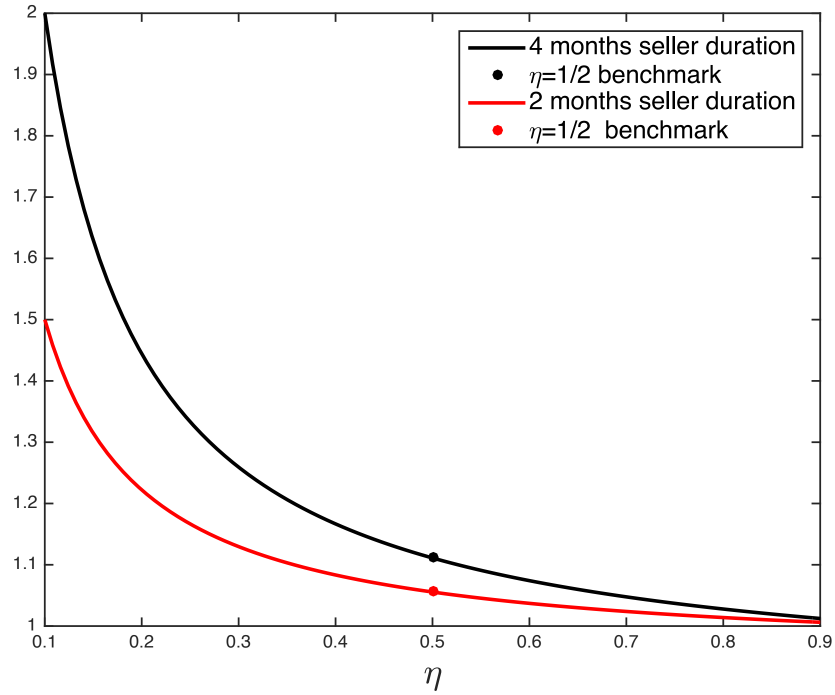

Figure 3: ϵ_b elasticity as a function of the elasticity of the matching function η

A common benchmark from the search and matching literature is $\eta = 1/2$. In this case, the equations above imply that $\kappa_b = \kappa_s$. Hence, under this benchmark, the price response to a change in the valuation of the buyer is amplified by 11.1% as compared to a standard CCAPM framework if the average search duration for sellers is four months, while it would be amplified by 5.6% in the case of a two-month duration. Lower values of η increase ϵ_b, while higher values tend ϵ_b to one. Figure 5 depicts how ϵ_b varies with η. For example, if one considers a value for η as low as 0.1, then ϵ_b reaches 2 if the search duration of sellers is four months.

Table 1: Evidence on procyclical liquidity

	Coef.	Time effects	State effects	MSA effects	Obs.	# of groups	Period	Source
(1)	2.12***	Yes	Yes	No	3,332	49	2012-2017	Zillow
	(0.297)							
(2)	2.18***	Yes	No	Yes	22,916	337	2012-2017	Zillow
	(0.183)							
(3)	1.11***	Yes	Yes	No	4,230	47	2010-2017	Zillow
	(0.226)							
(4)	0.98***	Yes	No	Yes	39,864	443	2010-2017	Zillow
	(0.105)							
(5)	2.80***	Yes	No	Yes	39,864	443	2010-2017	Zillow
	(0.129)							

A Appendix: evidence on procyclical liquidity

In this Appendix, we show that the evidence reported in Figure 1 is robust when looking at more disaggregated data, such as at state or MSA level, controlling for time and fixed effects, or considering data from other web-based real estate company.

Table 1 reports regression results for five specification using data from Zillow. In each regression, we use the unemployment rate as a regressor. A positive coefficient is thus associated with procyclical liquidity.

Regressions (1) and (2) consider the average number of days homes have been on the market (on the Zillow website). The first one considers the average in each US State, while the second one considers liquidity data at the MSA level (for 337 cities). In both regressions, the unemployment rate is the State-level one from the Bureau of Labor Statistics.

Because of incompatibilities in terms of MSA coding between Zillow and the BLS, we were unable to match the liquidity data from regressions (1) and (2) with the unemployment data from the BLS at the MSA level. However, regressions (3) to (5) considers a liquidity variable that can be matched with unemployment data at the MSA level. The liquidity variable is the average number of days it took to sell a home. A disadvantage of this variable is that it excludes information for homes that cannot be sold (unlike the liquidity variable in regressions (1) and (2)), but it can be combined with unemployment at the MSA level. Regression (3) is the equivalent of regression (1) with this alternative liquidity variable. It delivers a lower coefficient because the liquidity variable excludes information for homes that cannot be sold. Regression (4) considers data at the MSA level, with an estimated coefficient similar to regression (3). Finally, regression (5) is the weighted version of regression (4), where each MSA is weighted by its employment level.

B Appendix: proofs

B.1 First-order conditions

Denote by μ_1, μ_2 and μ_3 the multiplier associated with (1), (3) and (4) and write the following lagrangian:

$$\begin{aligned}\mathcal{L}(c_i, l_i, A_i', b_i, s_i, n_i) = u(c_i, l_i) + \beta_i E\left[V(A_i', z')\right] \\ +\mu_1\left[L - l_i - b_i - s_i - n_i\right] \\ +\mu_2\left[w_i n_i + (s_i\lambda_s(\theta) - b_i\lambda_b(\theta))\, xp + A_i\pi - c_i\right] \\ +\mu_3\left[A_i - A_i' - xs_i\lambda_s(\theta) + xb_i\lambda_b(\theta)\right].\end{aligned} \tag{12}$$

Derive with respect to each argument in the Lagrangian and set the derivative to zero:

$$\frac{\partial u(c_i, l_i)}{\partial c_i} = \mu_2 \tag{13}$$

$$\frac{\partial u(c_i, l_i)}{\partial l_i} = \mu_1 \tag{14}$$

$$\beta_i E\left[\frac{\partial V(A_i', z')}{\partial A_i'}\right] = \mu_3 \tag{15}$$

$$-\mu_1 - \mu_2 xp\lambda_b(\theta) + \mu_3 x\lambda_b(\theta) = 0 \tag{16}$$

$$-\mu_1 + \mu_2 xp\lambda_s(\theta) - \mu_3 x\lambda_s(\theta) = 0 \tag{17}$$

$$\mu_1 = \mu_2 w_i \tag{18}$$

B.1.1 Labor supply

The intratemporal condition (7) can easily be obtained by combining (13), (14) and (18).

B.1.2 Euler equations

By combiging (13) to (16), one gets:

$$-\frac{\partial u(c_i,l_i)}{\partial l_i}-\frac{\partial u(c_i,l_i)}{\partial c_i}xp\lambda_b(\theta)+\beta_i E\left[\frac{\partial V(A_i',z')}{\partial A_i'}\right]x\lambda_b(\theta)=0,$$

which can be rewritten as

$$\frac{\partial u(c_i,l_i)}{\partial c_i}p+\frac{\partial u(c_i,l_i)}{\partial l_i}\frac{1}{x\lambda_b(\theta)}=\beta_i E\left[\frac{\partial V(A_i',z')}{\partial A_i'}\right], \tag{19}$$

By combiging (13) to (15) with (17), one gets:

$$-\frac{\partial u(c_i,l_i)}{\partial l_i}+\frac{\partial u(c_i,l_i)}{\partial c_i}xp\lambda_s(\theta)-\beta_i E\left[\frac{\partial V(A_i',z')}{\partial A_i'}\right]x\lambda_s(\theta)=0,$$

which can be rewritten as

$$\frac{\partial u(c_i,l_i)}{\partial c_i}p-\frac{\partial u(c_i,l_i)}{\partial l_i}\frac{1}{x\lambda_s(\theta)}=\beta_i E\left[\frac{\partial V(A_i',z')}{\partial A_i'}\right], \tag{20}$$

By applying the envelope theorem, one can calculate the right-hand side in (19) and (20):

$$\frac{\partial V(A_i,z)}{\partial A_i}=\mu_2\pi+\mu_3,$$

which can be rewritten as

$$\frac{\partial V(A_i,z)}{\partial A_i}=\frac{\partial u(c_i,l_i)}{\partial c_i}\pi+\beta_i E\left[\frac{\partial V(A_i',z')}{\partial A_i'}\right].$$

By iterative substitution, one can rewrite the equation above as:

$$\frac{\partial V(A_{i,t},z_t)}{\partial A_{i,t}}=E_t\left[\sum_{j=0}^{\infty}\beta_i^j\frac{\partial u(c_{i,t+j}l_{i,t+j})}{\partial c_{i,t+j}}\pi_{t+j}\right].$$

Replace the formula above into (19) and (20) and rearrange terms to get (6) and (5) respectively.

B.2 The price-liquidity tradeoff

A pair θ-p is determined such that the two curves (19) and (20) are tangent. Calculate the slope of the (SS) locus on Figure 2:

$$\frac{\partial p}{\partial \theta}|_{SS} = -\frac{\partial u(c_s, l_s)/\partial l_s}{\partial u(c_s, l_s)/\partial c_s} \frac{{\lambda_s}'(\theta)}{x\left[\lambda_s(\theta)\right]^2},$$

as well as the slope of the (BB) locus:

$$\frac{\partial p}{\partial \theta}|_{BB} = \frac{\partial u(c_b, l_b)/\partial l_b}{\partial u(c_b, l_b)/\partial c_b} \frac{{\lambda_b}'(\theta)}{x\left[\lambda_b^a(\theta)\right]^2}.$$

The subscripts s and b allow to respectively refer to the consumption and leisure levels of the sellers and buyers of the specific market.

By equating these two slopes we obtain

$$-\frac{\partial u(c_s, l_s)/\partial l_s}{\partial u(c_s, l_s)/\partial c_s} \frac{\partial u(c_b, l_b)/\partial c_b}{\partial u(c_b, l_b)/\partial l_b} {\lambda_s}'(\theta) = \theta^2 {\lambda_b}'(\theta).$$

Call $\eta(\theta)$ the elasticity of the matching function with respect to the mass of sellers. We can rewrite the equation above as:

$$\frac{\partial u(c_s, l_s)/\partial l_s}{\partial u(c_s, l_s)/\partial c_s} \frac{\partial u(c_b, l_b)/\partial c_b}{\partial u(c_b, l_b)/\partial l_b} = \frac{\eta(\theta)}{1-\eta(\theta)}\theta.$$

By rearranging the terms in this equation, one gets condition (8).