Low-frequency Technology Shocks, Creative Destruction, and the Equity Premium

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1 Introduction. Beginning with Rietz [1988], a number of papers study the equity premium puzzle (Mehra and Prescott [1985]) in the context of models with large, but rare shocks (Mehra and Prescott [1988], Barro [2006, 2008, 2009], Gabaix [2008]). This line of research tends to use Lucas-tree and A-K models and to emphasize disruptions from wars, recessions, and natural disasters. The purpose of the present paper is to attempt to contribute a new modeling formulation to the literature. The new formulation features a neoclassical production sector and punctuated shocks to general purpose technologies (Laitner and Stolyarov [2003]).

We focus on low-frequency technology shocks. Our framework can be consistent with casual empirical evidence that occasional, severe stock-market declines heavily influence investor anxieties about owning equities. In our analysis, punctuated improvements in technology cause abrupt stock-market sell-offs, which are part of the process giving rise to the model's equity premium. We also show how nonlinearities can magnify the impact of high-amplitude shocks. Finally, some descriptions of long-term growth assign a prominent role to "seminal inventions," and we attempt to connect our analysis to the historical record. Our framework, in fact, may ultimately be able to aid in establishing more precise dates for the impacts of major technological innovations of the past.

Our modeling formulation offers several advantages, we believe, from a theoretical standpoint. First, it includes reproducible physical capital, a key element for dynamic analysis. Second, it allows us to determine equilibria from a sequence of nonstochastic Hamiltonian problems. Familiar phase diagrams characterize outcomes and facilitate computations and interpretations. Third, we show that we can distinguish the individual roles of different types of shocks.

In our model, equilibrium growth proceeds through a sequence of episodes of macroeconomic "creative destruction." We have a vintage specification of physical capital (Solow [1960], Laitner and Stolyarov [2003, 2004], Papanikolaou [2011]). Owners of old investment goods suffer capital losses at the arrival of a new general purpose technology (Helpman and Trajtenberg [1998], Howitt [1998]). Investment embodying the new technology offers, at first, a high marginal physical product, encouraging saving and reducing consumption. Over time, repetitions of capital losses and coincident low current consumption provide the basis for an equity premium.

Although the determinants of financial variables in our framework may, at first, seem more subtle than in the case of the tree model, our examples show that our baseline specification can explain 20-50% of the standard equity premium. Section 7's calibrated examples have major technological advances at 10-30 year intervals, on average, and a coefficient of relative risk aversion for investors in the range of 1-2. In Section 8, a combination of punctuated technological advances and business-cycle shocks can potentially account for the full empirical premium.

We show the extent to which a combination of convex marginal utility of consumption and large-amplitude shocks can amplify the equity premium. In examples, nonlinearities increase the equity premium by 25-50% for moderate shocks and can multiple it by 3-4 in the case of very large shocks.

We show that negative real interest rates can easily emerge on a temporary basis. The arrival of a new general purpose technology increases the marginal productivity of new capital, raising both the riskless and (expected) risky rates of return. A long period without the appearance of a new technology causes both rates to converge to stationary levels. In examples, the stationary-state riskless rate is often negative.

Our calibrations do not rely upon consumption data — in our formulation, technological change is exogenous, but consumption is endogenous. Nevertheless, the existing literature shows that the magnitude of a framework's aggregative-consumption fluctuations can be of interest regardless of the formulation's modeling approach (Mehra and Prescott [1985, 1988]). We interpret our model's representative agent as reflecting behavior at the top of the wealth distribution. The model's asset-price fluctuations can be large — dwarfing those in GDP — at times causing the representative agent to adjust his/her consumption steeply. We relate the size of the latter responses to corresponding consumption changes in the overall economy, and we attempt to gauge the effect of imposing constraints that limit the model's maximal aggregate-consumption fluctuations.

Although our benchmark specification emphasizes technological progress, there are many other low-frequency-shock possibilities: wars and depressions (Barro op. cit.), oil-price shocks (Baily [1981], Hamilton [2009, 2013], Kilian [2008]), international-trade changes accompanying the rapid growth of emerging market economies, etc. The simple structure of our model enables us to extend our analysis to different shock types. Section 8 provides preliminary examples.

The organization of this paper is as follows. Section 2 briefly reviews evidence from economic history on low-frequency technology shocks. Section 3 presents our analytic framework. Section 4 suggests a computational procedure, Section 5 generalizes the analytic framework to shocks of different magnitudes, and Section 6 derives formulas for the equity premium. Section 7 provides numerical examples for technology shocks, and Section 8 examines the combined effects of different shock types. Section 9 concludes.

2 Historical Record. This paper assumes that to implement a new general purpose technology, the economy needs new plant and equipment.[1] As new technologies make existing physical capital obsolete, owners suffer capital losses. For random disruptions occurring in a staggered fashion across industries, portfolio diversification could limit investor risk. Diversification provides much less protection against sweeping changes from new general purpose technologies, however.

[1] E.g., Abramovitz and David [1995, p.28]: "Even in a progressive economy ... the pace of actual incorporation may differ from the underlying rate of advance in practical knowledge. The main reason ... stems from the fact that a portion, probably a major portion, of advances in knowledge must be embodied in tangible equipment and structures and often placed in new locations."

Table 1: New Technologies	
Technology	Approximate Initial Date
1. Steam Power	1725
2. Iron Working	1750
3. Textile Mechanization	1775
4. Railroads	1825
5. Steel	1875
6. Chemicals	1875
7. Electricity	1900
8. Internal Combustion	1900
9. Aviation	1900
10. Petro-Chemicals	1950
11. Computers	1950
12. Networks	1975

Source: See text.

Table 1, derived from Cohen et al. [2000], lists 12 major technology changes over the last 250 years and approximate arrival dates. We might think of each row as designating a separate general purpose technology.[2] This would be consistent with a Poisson hazard rate for new technologies of about 5%/year.

We could argue that there are important omissions from the table — including, for example, Henry Ford's mass production, achievements in petroleum extraction and refining, and advances in agriculture. Citing a panel of 12 experts, Fallows [2013] lists 37 technological "breakthroughs" 1700-2000. See also Mokyr [1990], Jovanovic [], David [1990], and others. If Table 1 had more rows, the hazard for punctuated change would be higher.

Some historians suggest, on the other hand, that Table 1 has too many rows. In general, even among seminal inventions, some may have far wider impact than others (e.g., Edquist and Henrekson [2007]). Extensions of our model to allow TFP steps of different sizes — see Section 5 — may be especially useful in such circumstances.

Creative destruction is a foundation of our analysis. We assume that as new technologies replace old, the value of existing capital suffers a write-down. Landes [1969, p.42] observes: "Technological change is never automatic. It means the displacement of established methods, damage to vested interests, often serious human dislocations." In describing the ascendancy of Bessemer and open-hearth steel, he writes [p.259]:

> "Against this must be set the decline of wrought iron, long the frame of the industrial structure. At first, the older malleable form resisted: it was cheaper, and ... there was a fortune invested in puddling plant. ... Before long, however, steelmakers learned to correct the flaws in their product; and, improvements in efficiency wiped out enough of the price difference to make competition in most uses impossible.

[2] Cohen et al. would label them "successive Schumpeterian leading sectors."

In the same vein, Fishlow [1996, p.580, 582] describes the manner in which railroads supplanted canals and river boats:

> "Acceptance of the new technology cut short the profits on canals and led to the abandonment of many ... Greater speed, all-season utilization, less transshipment, and concentrated responsibility succeeded in capturing the trade not only in highly valued merchandise, but also in the less bulky agricultural commodities ....

In the twentieth century, trucks, automobiles, and airplanes began, in turn, to limit demand for rail services.[3] Horse-and-wagon transportation, which at first had complemented railroads, suffered obsolescence. Clark [2007, p.286] writes,

> "The population of working horses actually peaked in England ... in 1901 ... Though they had been replaced by rail for long-distance haulage ..., they still plowed fields, hauled wagons and carriages short distances, pulled boats on the canals, toiled in the pits, and carried armies into battle. But the arrival of the internal combustion engine ... rapidly displaced these workers ....

In the US, Sarle [1926, p.437] notes that even as the number of horses was falling, horse values declined 35% from 1918 to 1924, reaching a 60-year low. The agricultural sector as a whole was affected. Feed for horses represented an important market for farmers. Although harvested acres in the US remained constant after 1910, the 93 million acres (27% of the total) used to grow feed for horses and mules in 1915 diminished to 4 million by 1960. Loss of the market for feed presumably provided part of the explanation for the 33% drop in US farm relative to nonfarm household incomes 1920-40 (Gardner [2002, fig. 3.12]), the halving of the value of farm real estate per acre over the same period (Gardner [2002, fig. 3.14]), and the population outflow from the agricultural sector (that resumed, in force, after the Great Depression).

See also David's [1990] discussion of the electrification of factories, and Laitner and Stolyarov's [2003] discussion of the microprocessor chip.

It is important to note that we assume that the indirect impact of a new general purpose technology often greatly exceeds its contribution to value added in industries directly affected. In the case of 19th-century railroads, for instance, Rostow [1960, p.55] writes,

> "... the development of railways has lead on to the development of modern coal, iron, and engineering industries. In many countries the growth of modern basic industrial sectors can be traced in the most direct way to the requirements for building and, especially for maintaining substantial railway systems.

The automobile, to take another example, facilitated suburban living, a reduced agricultural sector (as we have just seen), and factories built on less expensive, non-urban land yet able to draw workers from a wide area. In the case of the microprocessor, Brynjofsson and McAfee [2011, p.41] note,

> "Studies ... found that a key aspect of SBTC [skill biased technical change] was not just the skills of those working with computers, but more importantly the broader

[3] Fishlow's Table 13.18 shows constant-dollar railroad investment falling 1902-1961, while highway investment rose 18-fold.

changes in work organization that were made possible by information technology. The most productive firms reinvested and reorganized decision rights, incentives systems, information flows, hiring systems, and other aspects of organizational capital to get the most from the technology. ... whole production processes, and even industries, were reengineered to exploit powerful new information technologies.

See also Abramovitz and David [1995, p.64].

3 The Model. This section describes our model. Throughout the remainder of this paper, we assume that all technological progress is embodied in new plant and equipment and occurs in a punctuated fashion. From the standpoint of the theoretical model, this streamlines the exposition but is otherwise inessential. For the numerical examples of Sections 7-8, on the other hand, it affects the fraction of the empirical equity premium that our formulation can explain. If a less stringent assumption is justified, the fraction of the actual premium attributable to our model will be lower.

Background Results We begin with 3 background results.

Start with a standard representative agent model with no technological progress. Let the utility flow from current consumption be

$$u(C) = \frac{[C]^\gamma}{\gamma}\,, \quad \gamma < 0\,. \tag{1}$$

The representative agent's total utility is

$$U(K\,,\,Z) = \max_{C_t} \sum_{t\geq 0} [\frac{1}{1+\beta}]^t \cdot u(C_t)\,, \quad \beta > 0 \tag{2}$$

$$\text{subject to:} \quad K_{t+1} = (1-\delta)\cdot K_t \,+ Z\cdot [K_t]^\alpha - C_t\,, \tag{3}$$

$$C_t \leq Z \cdot [K_t]^\alpha\,, \tag{4}$$

$$K_0 = K\,.$$

The physical capital stock is K_t. The aggregate production function has constant returns to scale:

$$Y_t = Z\cdot [K_t]^\alpha \cdot [L_t]^{1-\alpha}\,, \quad \alpha \in (0\,,\,1)\,. \tag{5}$$

Throughout this paper, fix the aggregate labor supply to 1:

$$L_t = 1 \quad \text{all} \quad t \geq 0\,. \tag{6}$$

The wear and tear depreciation rate on physical capital is $\delta \in (0\,,\,1)$. In anticipation of our vintage-model analysis below, constraint (3) requires physical investment to be irreversible. The subjective discount rate is β, with $\beta > 0$ giving us bounded total utility.

Lemma 1 shows that without loss of generality, we can restrict attention to capital stocks $K_t \in \mathcal{K}(Z)$ with

$$\mathcal{K}(Z) \equiv (0\,,\,K^U]\,, \quad K^U = [Z/\delta]^{\frac{1}{1-\alpha}}\,. \tag{7}$$

Lemma 1. *Suppose $(K_t\,,\,C_t)$ obey (3)-(4) all $t \geq 0$. Then $K_t \in \mathcal{K}(Z)$ implies $K_{t+1} \in \mathcal{K}(Z)$.*

Proof: See Appendix.

Thus, $\mathcal{K}(Z)$ functions as an invariant set: once the aggregate capital stock enters $\mathcal{K}$, it never leaves.

Lemma 2 derives a formula for de-trending model (2)-(4) in the event of a one-time improvement in technology:

Lemma 2. *Let $Z > 0$, $K > 0$, and $\theta \geq 1$ be given. Let*

$$a \equiv -\frac{1}{1-\alpha}\,. \tag{8}$$

Then

$$U(K\,,\,\theta \cdot Z) = \frac{1}{[\theta]^{a \cdot \gamma}} \cdot U([\theta]^a \cdot K\,,\,Z)\,. \tag{9}$$

Proof: See Appendix.

Lemma 2 provides a template for our analysis of repeated technology changes below.

A third background result demonstrates that even with persistent embodied technological change, an aggregate production function remains viable (see Solow [1960], Laitner and Stolyarov [2003]).

Assume there is a single consumption good at all times. Normalize its price to 1. Assume the economy interchangeably produces consumption and investment goods. Measure investment in physical units, say, tons. Each unit of capital goods embodies a specific technology. Suppose, for example, that K_t^j is the undepreciated physical capital at time t built for TFP $[\theta]^j \cdot Z$. Used with labor L_t^j, it provides output

$$[\theta]^j \cdot Z \cdot [K_t^j]^\alpha \cdot [L_t^j]^{1-\alpha}\,.$$

If J_t is the best j known at time t, all time-t investment is vintage J_t.

Aggregate output is

$$Y_t = \max_{\ell_t^j} \sum_{j \leq J_t} [\theta]^j \cdot Z \cdot [K_t^j]^\alpha \cdot [\ell_t^j]^{1-\alpha} \tag{10}$$

$$\text{subject to:} \quad \sum_{j \le J_t} \ell_t^j \le L_t \quad \text{and} \quad \ell_t^j \ge 0\,.$$

We have

Lemma 3: *Let J_t be the best j at time t — i.e., the best TFP is $[\theta]^{J_t}$. Define*

$$K_t \equiv \sum_{j \le J_t} b^{J_t - j} \cdot K_t^j\,, \tag{11}$$

where

$$b \equiv [\theta]^{-1/\alpha}\,. \tag{12}$$

Then if Y_t is as in (10), we have aggregate production function

$$Y_t = [\theta]^{J_t} \cdot Z \cdot [K_t]^{\alpha} \cdot [L_t]^{1-\alpha}\,. \tag{13}$$

Proof: See Appendix.

Instead of using exogenous price indices to convert new investment to "efficiency units," we use Lemma 3. Suppose a new general purpose technology arrives the moment after t_j , each $j = 0\,,\,1\,,\,2\,,\,....$ Let the economy's investment be I_t at time t. Let the rate of wear and tear depreciation be δ. Then letting b be as in (12), we have

$$J_{t+0} \equiv \begin{cases} J_t + 1\,, & \text{if } t = t_j \text{ some } j\,, \\ J_t\,, & \text{if } t \ne t_j \text{ any } j\,, \end{cases} \tag{14}$$

$$K_{t+0} = \begin{cases} b \cdot K_t\,, & \text{if } t = t_j \text{ some } j\,, \\ \int_{t_j}^{t} e^{-\delta \cdot (t-s)} \cdot I_s \, ds + b \cdot e^{-\delta \cdot (t - t_j)} \cdot K_{t_j}\,, & \text{if } t \in (t_{j+1}\,,\, t_j)\,. \end{cases} \tag{15}$$

Since (11) measures all components of K_t in the same efficiency units as K^{J_t}, the price per unit of K_t is 1. When the next general purpose technology arrives, undepreciated existing capital drops in resale value to b per physical unit times its previous price. In other words, at each t_j, the capital loss of K_t in (11) between t_j and $t_j + 0$ is $(1 - b) \cdot K_t$.

<u>New Modeling Formulation</u> Presentation of the new model is now straightforward.

The new formulation is as follows. We have a representative agent whose value function is $V(.)$ satisfying

$$V(K\,,\,Z) = \max_{C \in [0\,,\,Z \cdot [K]^{\alpha}]} \{u(C) + \frac{1}{1+\beta} \cdot [\lambda \cdot V(b \cdot \bar{K}\,,\,\theta \cdot Z) + (1-\lambda) \cdot V(\bar{K}\,,\,Z)]\} \tag{16}$$

$$\text{subject to:} \quad \bar{K} = (1-\delta) \cdot K + Z \cdot [K]^{\alpha} - C\,.$$

The aggregate production function is (4); the aggregate labor supply is (5); new general purpose technologies arrive at Poisson rate λ, each new technology causing the existing TFP level to be multiplied by $\theta \geq 1$. Lemma 3 defines b. On the right-hand side of (16), $V(b \cdot \bar{K}\,,\, \theta \cdot Z)$ registers the representative agent's total utility if an improvement in technology occurs next period, while $V(\bar{K}\,,\, Z)$ is total utility if no such change takes place.

"Equilibrium" is as follows.

Definition. *Fix $Z > 0$ and $\theta \geq 1$. A value function $V(K\,,\, Z)$, $K \in \mathcal{K}(Z)$, defines an "equilibrium" for our model (i) if $V(.\,,\, Z)$ is increasing and concave, (ii) if $V(.,.)$ satisfies (16), and (iii) if*

$$V(K\,,\, \theta \cdot Z) = \frac{1}{[\theta]^{a \cdot \gamma}} \cdot V([\theta]^{a} \cdot K\,,\, Z)\,. \tag{17}$$

Condition (17) manifests, and extends, Lemma 2. It leads to a time-autonomous framework of analysis. Put in other words, it provides an explicit method for de-trending the model. Once we substitute from (17) into the right-hand side term $\lambda \cdot V(b \cdot K\,,\, \theta \cdot Z)$ in (16), we can fix Z and use the Bellman equation to determine $V(.\,,\, Z)$, a univariate function of K.

We can provide a constructive proof of the existence of equilibrium:

Proposition 1. *Fix $Z > 0$ and $\theta \geq 1$. There exists $V(K\,,\, Z)$, $K \in \mathcal{K}(Z)$, which is strictly increasing and strictly concave, and which satisfies (16)-(17).*

Proof. See Appendix.

Our interest centers on the policy function of (16). We have

Proposition 2. *Fix $Z > 0$ and $\theta \geq 1$. Let $V(K\,,\, Z)$, $K \in \mathcal{K}(Z)$, be as in Proposition 1. Then the right-hand side of (16) uniquely defines a policy function*

$$C = \phi(K\,,\, Z)\,. \tag{18}$$

The function is continuous and

$$\phi(K\,,\, \theta \cdot Z) = \frac{1}{[\theta]^{a}} \cdot \phi([\theta]^{a} \cdot K\,,\, Z)\,. \tag{19}$$

If $\phi^{m}(K\,,\, Z)$ is the policy function for $V^{m}(K\,,\, Z)$ — which is recursively defined in the proof of Proposition 1 — we have

$$\lim_{m \to \infty} \phi^{m}(K\,,\, Z) = \phi(K\,,\, Z)\,. \tag{20}$$

Proof. See Appendix.

4 Computation. We propose the following computational approach for quantitative analysis.

Letting the time intervals in our discrete-time model approach 0, replace (1)-(3) with

$$\mathcal{V}^0(K\,,\,Z) = \max_{C_t} \int_0^\infty e^{-\beta\cdot t}\cdot\frac{[C_t]^\gamma}{\gamma}\,dt\,,\quad \gamma<0\,,\quad \beta>0 \tag{21}$$

$$\text{subject to:}\quad \dot{K}_t + \delta\cdot K_t = Z\cdot[K_t]^\alpha\cdot[L_t]^{1-\alpha} - C_t\,,\quad \alpha\in(0\,,\,1)\,,$$

$$C_t \le Z\cdot[K_t]^\alpha\,, \tag{22}$$

$$K_0 = K\,.$$

Back substituting for $V(\bar{K}\,,\,Z)$ on the right-hand side of (16), recursively define

$$\mathcal{V}^{m+1}(K\,,\,Z) = \max_{C_t}\int_0^\infty e^{-(\beta+\lambda)\cdot t}\cdot[\frac{[C_t]^\gamma}{\gamma} + \lambda\cdot\mathcal{V}^m(b\cdot K_t\,,\,\theta\cdot Z)]\,dt \tag{23}$$

$$\text{subject to:}\quad \dot{K}_t + \delta\cdot K_t = Z\cdot[K_t]^\alpha - C_t\,,$$

$$C_t \le Z\cdot[K_t]^\alpha\,, \tag{24}$$

$$K_0 = K\,,$$

for $m = 0\,,\,1\,,\,....$ As in Propositions 1-2, conditions (17) and (19) hold for every m and

$$0 \ge \mathcal{V}^{m+1}(K\,,\,Z) \ge \mathcal{V}^m(K\,,\,Z)\,.$$

Hence,

$$\mathcal{V}(K\,,\,Z) \equiv \lim_{m\to\infty}\mathcal{V}^m(K\,,\,Z)$$

is a continuous-time analogue to $V(K\,,\,Z)$.

Our main interest is Section 3's policy function $\phi(.\,,\,.)$. In our examples below, problem (21) has the familiar phase diagram as in Figure 1. We have a saddle-path solution for C_t as a function of K_t, say,

$$C = \varphi^0(K\,,\,Z)\,.$$

We proceed inductively to $\varphi^{m+1}(.,.)$. After applying (17) on the right-hand side of (23), the Hamiltonian is

$$\mathcal{H}^{m+1} = e^{-(\beta+\lambda)\cdot t} \cdot \big[\frac{[C_t]^\gamma}{\gamma} + \lambda \cdot b_1 \cdot \mathcal{V}^m(b \cdot b_0 \cdot K_t \, , \, Z)\big] + \mu_t \cdot \big[Z \cdot [K_t]^\alpha - C_t\big] \, ,$$

$$b = [\theta]^{-1/\alpha} \, , \quad b_0 = [\theta]^a \, , \quad b_1 = \frac{1}{[\theta]^{a\cdot\gamma}} \, , \quad a = \frac{-1}{1-\alpha} \, .$$

The first-order condition for C_t yields

$$e^{-(\beta+\lambda)\cdot t} \cdot [C_t]^{\gamma-1} = \mu_t \, . \tag{25}$$

The costate equation is

$$\dot{\mu}_t = -e^{-(\beta+\lambda)\cdot t} \cdot \lambda \cdot b_1 \cdot \frac{\partial \mathcal{V}^m(b_0 \cdot b \cdot K_t \, , \, Z)}{\partial K_t} - \mu_t \cdot \big[\alpha \cdot Z \cdot [K_t]^{\alpha-1} - \delta\big] \, . \tag{26}$$

The envelope theorem yields

$$\frac{\partial \mathcal{V}^m(b_0 \cdot b \cdot K_t \, , \, Z)}{\partial K_t} = b_0 \cdot b \cdot [\varphi^m(b_0 \cdot b \cdot K_t \, , \, Z)]^{\gamma-1} \, . \tag{27}$$

Substituting from (25) and (27) into (26), we have

$$\dot{C}_t = \frac{C_t}{1-\gamma} \cdot \{\lambda \cdot b_1 \cdot b_0 \cdot b \cdot [\frac{\varphi^m(b_0 \cdot b \cdot K_t \, , \, Z)}{C_t}]^{\gamma-1} + \alpha \cdot Z \cdot [K_t]^{\alpha-1} - (\beta+\lambda+\delta)\} \, . \tag{28}$$

The constraint from (23) yields

$$\dot{K}_t = Z \cdot [K_t]^\alpha - \delta \cdot K_t - C_t \, . \tag{29}$$

Construct a new phase diagram using differential equations (28)-(29). Provided the phase diagram remains qualitatively the same, we can calculate the new stationary point $(K^{*,m+1} \, , \, C^{*,m+1})$, and characterize the stable arm with $C = \varphi^{m+1}(K \, , \, Z) \, , \; K \in \mathcal{K}(Z)$. In every example below, the qualitative similarity of the phase diagram holds and uniform convergence emerges

$$\varphi(K \, , \, Z) = \lim_{m\to\infty} \varphi^m(K \, , \, Z) \, . \tag{30}$$

We use $\varphi(.,.)$ as our stand-in for $\phi(.,.)$ from Proposition 2.

In practice, the irreversibility constraint (i.e., (22) and (24)) is superfluous since our analysis can focus exclusively on the saddle path to the left of the stationary point. (See Figure 1.) Since $\mathcal{V}^m(. \, , \, Z)$ and $\varphi^m(. \, , \, Z)$ are continuously differentiable and since convergence in (30), in practice, is uniform in the derivative $\partial\varphi^m/\partial K$ as well, $\varphi(K \, , \, Z)$ is continuously differentiable in K (Apostol [1974, Thm. 9.13]).

A great convenience of this section's approximation is that we can utilize a standard 2-point boundary-value algorithm to solve the system (28)-(29) during the required iterations.[4]

[4] The computations below use the IMSL Fortran subroutine DBVPFD. We obtain very

5 Multiple TFP Shock Sizes. We can extend the analysis of Sections 3-4 to include multiple TFP-step sizes.

Assume new general purpose technologies continue to arrive according to a Poisson process with hazard λ. At each Poisson realization, however, suppose the economy's TFP level is multiplied by $\theta(i) \geq 1$ with probability $p(i)$, $i = 1, \ldots, i_0$. In other words, there are i_0 different TFP-step sizes at every Poisson realization, and the specific steps occur with exogenously given frequencies.

The analog for Propositions 1-2 is

Proposition 3: *Fix any $Z > 0$. Let $\theta(i) \geq 1$, $i = 1, \ldots, i_0$; $p(i) \geq 0$; $\sum p(i) = 1$; and, $b(i) = [\theta(i)]^{-1/\alpha}$. Then there exists $V(K, Z)$, $K \in \mathcal{K}(Z)$, which is strictly increasing and strictly concave, which satisfies (17), and which satisfies*

$$V(K, Z) = \max_{C \in [0, Z \cdot [K]^\alpha]} \{\frac{[C]^\gamma}{\gamma} + \frac{1}{1+\beta} \cdot [\lambda \cdot \sum_i \frac{p(i)}{[\theta(i)]^{a \cdot \gamma}} \cdot V(b(i) \cdot [\theta(i)]^a \cdot \bar{K}, Z) + (1-\lambda) \cdot V(\bar{K}, Z)]\} \quad (31)$$

$$\textit{subject to:} \quad \bar{K} = (1-\delta) \cdot K + Z \cdot [K]^\alpha - C.$$

The right-hand side of (31) uniquely defines a policy function

$$C = \phi(K, Z), \quad (32)$$

which is continuous and satisfies (19).

The proof is strictly analogous to Propositions 1-2. For a given Z, the solution to (31) is, as before, a univariate function $V(., .Z)$. As before, constructive steps recursively define a sequence of functions converging to $V(K, Z)$ and a corresponding sequence of policy functions converging to $\phi(K, Z)$. The computational steps of Section 4 remain valid if we replace the criterion of (23) with

$$\mathcal{V}^{m+1}(K, Z) = \max_{C_t} \int_0^\infty e^{-(\beta+\lambda) \cdot t} \cdot [u(C) + \lambda \cdot \sum_i \frac{p(i)}{[\theta(i)]^{a \cdot \gamma}} \cdot \mathcal{V}^m(b(i) \cdot [\theta(i)]^a \cdot K_t, Z)] \, dt. \quad (33)$$

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low relative errors with 15 iterations on m. On a desktop computer, the CPU time for one set of iterations is roughly 0.1 seconds.

6 The Equity Premium. This section derives formulas for the riskless and risky rates of return in the model, and for the equity premium. Because we are interested in the quantitative difference that large-magnitude shocks make, we compute our equity premium both with and without linear approximations, comparing the outcomes.

Our quantitative examples use the computational procedure of Section 4. In all cases, policy function convergence in (30) is uniform in both levels $\varphi^m(.,.Z)$ and derivatives $\partial\varphi^m(.,.Z)/\partial K$; hence, $\varphi(.,Z)$ is continuously differentiable. The results below correspond to those of Sections 3 and 5 if the discrete time intervals are short and the rates of return reflect continuous compounding.

Riskless Rate of Return A riskless rate of return is implicit in our model. We can characterize its magnitude from an Euler equation.

At first, suppose $i_0 = 1$. Letting r_t be the riskless rate of return at time t, we have

$$u'(C_t) = [e^{\int_t^{t+\Delta t} (r_s - \beta)\, ds}] \cdot E[u'(C_{t+\Delta t})] \,,$$

with β the subjective discount rate. With first-order approximations,

$$\begin{aligned} u'(C_t) \approx (1 + (r_t - \beta) \cdot \Delta t) \cdot [(1 - \lambda \Delta t) \cdot \{u'(C_t) + u''(C_t) \cdot \dot{C}_t \Delta t\} + \\ (\lambda \Delta t) \cdot \{u'(C_t) + u''(C_t) \cdot \Delta C_t\}] \,, \end{aligned} \tag{34}$$

where ΔC_t is the drop in consumption following the arrival of a new general purpose technology. Canceling second-order Δt terms, and noting that $u''(C_t) = (\gamma - 1) \cdot u'(C_t)/C_t$, we have

$$r_t \approx r_t^1 \equiv (1 - \gamma) \cdot \{\frac{\dot{C}_t}{C_t} + \lambda \cdot \frac{\Delta C_t}{C_t}\} + \beta \,, \tag{35}$$

which is our "Solution 1" for the riskless rate.

A problem is that while the first Taylor series for $u'(C_{t+\Delta t})$ in (31) produces a good approximation as $\Delta t \to 0$, the second generally will not: even as $\Delta t \to 0$, ΔC_t remains finite. "Solution 2" avoids the difficulty, determining r_t from — see Propositions 1-3 —

$$r_t = r_t^2 \equiv (1 - \gamma) \cdot \frac{\dot{C}_t}{C_t} - \lambda \cdot \{[\frac{\bar{C}_t}{C_t}]^{\gamma - 1} - 1\} + \beta \quad \text{with} \quad \bar{C}_t = \frac{\varphi([\theta]^a \cdot b \cdot K_t \,,\, Z)}{[\theta]^a} \,. \tag{36}$$

If $i_0 > 1$, let $p(i)$ be the probability that we change from TFP level Z to $\theta(i) \cdot Z$ at the next Poisson event. Then

$$r_t^1 \equiv (1 - \gamma) \cdot \{\frac{\dot{C}_t}{C_t} + \lambda \cdot \sum_i p(i) \cdot \frac{\Delta C_t(i)}{C_t}\} + \beta \,, \tag{37}$$

$$r_t^2 \equiv (1 - \gamma) \cdot \frac{\dot{C}_t}{C_t} - \lambda \cdot \sum_i p(i) \cdot \{[\frac{\bar{C}_t(i)}{C_t}]^{\gamma - 1} - 1\} + \beta \,, \tag{38}$$

where

$$\bar{C}_t(i) \equiv \frac{\varphi([\theta(i)]^a \cdot b(i) \cdot K_t \,,\, Z)}{[\theta(i)]^a} \quad \text{and} \quad \Delta C_t(i) \equiv \bar{C}_t(i) - C_t \,. \tag{39}$$

<u>Risky Rate of Return</u> Let $\bar{R}_t$ be the expected value of the risky rate of return, at time t, for the interval $[t\,,\, t+\Delta t)$. First, assume $i_0 = 1$. Let m_t be the marginal revenue product on capital, say, M_t, less the wear and tear rate of depreciation, i.e., $m_t \equiv M_t - \delta$. Using the Jorgenson [1963] rate of return formula, over an interval of length Δt an investor expects to receive

$$\begin{cases} 1 + m_t \Delta t\,, & \text{with probability } 1 - \lambda\Delta t, \\ \\ 1 + m_t \Delta t + (b-1)\,, & \text{with probability } \lambda\Delta t, \end{cases} \tag{40}$$

where $b < 1$, as above, is the value of old capital after a new technology arrives. The average risky rate of return then satisfies

$$\bar{R}_t \Delta t \approx m_t \Delta t + (b-1)\cdot \lambda\Delta t \Longleftrightarrow \bar{R}_t = m_t + \lambda \cdot (b-1)\,, \tag{41}$$

with $\lambda \cdot (b-1)$ the expected capital loss.

Using an Euler equation, our "Solution 1" follows the pattern of (34):

$$\begin{aligned} u'(C_t) \approx\; & (1 + (m_t - \beta)\Delta t)\cdot(1-\lambda\Delta t)\cdot\{u'(C_t) + u''(C_t)\cdot \dot{C}_t \Delta t\} \\ & + (1 + (m_t-\beta)\Delta t + (b-1))\cdot(\lambda\Delta t)\cdot\{u'(C_t) + u''(C_t)\cdot \Delta C_t\}\,. \end{aligned}$$

Hence,

$$m_t + \lambda\cdot(b-1) = \bar{R}_t \approx \bar{R}^1_t \equiv (1-\gamma)\cdot\{\frac{\dot{C}_t}{C_t} + \lambda\cdot b\cdot\frac{\Delta C_t}{C_t}\} + \beta\,. \tag{42}$$

"Solution 2" dispenses with the linear approximation for ΔC_t, yielding

$$\bar{R}_t = \bar{R}^2_t \equiv (1-\gamma)\cdot\frac{\dot{C}_t}{C_t} - \lambda\cdot b\cdot\{[\frac{\bar{C}_t}{C_t}]^{\gamma-1}\} - 1\} + \beta\,. \tag{43}$$

If $i_0 > 1$, following the notation of (37)-(38), we have

$$\bar{R}^1_t \equiv (1-\gamma)\cdot\{\frac{\dot{C}_t}{C_t} + \lambda\cdot\sum_i p(i)\cdot b(i)\cdot\frac{\Delta C_t(i)}{C_t}\} + \beta\,. \tag{44}$$

$$\begin{aligned} \bar{R}^2_t &\equiv m_t + \lambda\cdot\sum_i p(i)\cdot[b(i)-1] \\ &= (1-\gamma)\cdot\frac{\dot{C}_t}{C_t} - \lambda\cdot\sum_i p(i)\cdot b(i)\cdot\{[\frac{\bar{C}_t(i)}{C_t}]^{\gamma-1} - 1\} + \beta\,. \end{aligned} \tag{45}$$

The Equity Premium The equity premium, e_t, is the difference between $\bar{R}_t$ and r_t.

Let $i_0 \geq 1$. Under Solutions 1 and 2, respectively,

$$e_t \approx e_t^1 \equiv (1-\gamma) \cdot \lambda \cdot \sum_i p(i) \cdot [b(i) - 1] \cdot \frac{\Delta C_t(i)}{C_t}\,, \tag{46}$$

$$e_t = e_t^2 \equiv \lambda \cdot \sum_i p(i) \cdot [1 - b(i)] \cdot \{[\frac{\bar{C}_t(i)}{C_t}]^{\gamma-1} - 1\}\,. \tag{47}$$

When $\Delta C_t(i) < 0$ — the case which always arises below — the equity premium is positive and the convexity of the marginal utility function makes (47) larger than (46).

Discussion Our approach contrasts to, for instance, Mankiw and Zeldes [1991].

Mankiw and Zeldes derive empirical estimates of $\Delta C_t/C_t$ (their "GC") and $\lambda \cdot (b-1)$ (their "$r^m - r^f$") — see Mankiw and Zeldes [1991, tab. 3-4]. If the empirical equity premium is, for instance, 6%/yr, they compute the required $1-\gamma$ using their analogue of (34). The present paper likewise employs estimates of λ and $1-b$. However, rather than attempting to determine $\Delta C_t/C_t$ from data as well, we deduce it from a full solution of our model, as in Propositions 1-3. In our case, data calibrate technology shocks, which we take to be exogenous. The coefficient of relative risk aversion is $1-\gamma$. In either Mankiw and Zeldes, or the present paper, the higher the representative agent's sensitivity to risk, the greater, *cet. par.*, the equilibrium risk premium.

But a higher $1-\gamma$ also implies a lower intertemporal elasticity of substitution, $1/(1-\gamma)$. The latter, in turn, implies, in our general equilibrium treatment, a more limited consumption response $\Delta C_t/C_t$ to a technology shock. That tends to reduce the equity premium. Our calculations reflect both factors.

Leverage In practice, businesses finance their plant and equipment with a mixture of short-term credit, long-term debt, and equities. In our stylized model, where the representative agent owns all of the physical capital, the structure of financing is irrelevant for Propositions 1-3. On the other hand, when we attempt to calibrate the model against security returns in the actual economy, we must be careful to choose securities from the model that correspond to those in our data.

Suppose the model's physical capital stock were financed with a share ω from short-term bonds, each with return r_t^2 as above, and the remainder from common stocks, each with average return $\bar{S}_t$. Assume no-arbitrage pricing. Our model's entire capital stock is financed with risky securities having average return $\bar{R}_t^2$. So,

$$\bar{R}_t^2 = \omega \cdot r_t^2 + (1-\omega) \cdot \bar{S}_t\,. \tag{48}$$

Hence,

$$\bar{S}_t = \frac{1}{1-\omega} \cdot [\bar{R}_t^2 - \omega \cdot r_t^2]\,. \tag{49}$$

Our empirical equity premium then corresponds to

$$\bar{E}_t = \bar{S}_t - r_t^2 = \frac{1}{1-\omega} \cdot \bar{R}_t^2 - \frac{\omega}{1-\omega} \cdot r_t^2 - \frac{1-\omega}{1-\omega} \cdot r_t^2 = \frac{e_t^2}{1-\omega} \,. \tag{50}$$

If we start at the stationary solution, call the leveraged premium E^*.

7 Baseline-case Examples. This section presents quantitative examples. Section 8 considers additional types of shocks.

<u>Calibration</u> We examine values (γ , β) with

$$(\gamma , \beta) \in \{-0.01 , -0.1 , -0.2 , ... , -4.0\} \times \{0.0001 , 0.001 , 0.002 , ... , 0.075\} \,. \tag{51}$$

To match Section 3's propositions, we assume $\gamma < 0$. This includes the most common parameterizations in the literature. To avoid implausibly high degrees of risk aversion, we focus on $\gamma \geq -4.0$ (see, for example, Mankiw and Zeldes [1991] and Kocherlakota [1996]). We need $\beta > 0$ to ensure bounded value functions. We consider values of β as high as 0.075 (e.g., Cooley and Prescott [1995] and many others). In all cases, normalize $Z = 1$.

We present outcomes for $\alpha = 0.25$ and 0.30. Gollin [2002] favors $\alpha = 0.30$; Krueger [1999] and Baily [1981] suggest $\alpha = 0.25$.[5]

Our model provides two ways of measuring technology shocks: (i) the size of the drop in the resale value on existing equipment as shocks arrive, and (ii) the long-run rise in per capita output in the economy.[6] We take advantage of both.

To use measurement (i), normalize the price of consumption goods to 1, and do the same for new investment goods. Lemma 3 shows that after the introduction of a new general purpose technology, the resale value of a unit of existing physical capital declines to

$$b = [\theta]^{-1/\alpha} \,.$$

Laitner and Stolyarov [2003, tab. 1] estimate $b = 0.3866$ (see also Section 8 below). Inverting the preceding formula, our calibration is

$$\theta = [0.3866]^{-\alpha} \,. \tag{52}$$

Turning to measurement (ii), Maddison [1991, tab. 3.3] suggests an average annual per capita GDP growth rate of $g = 0.021$ for 1870-1987. Suppose the average span between our TFP steps is T. In a growth accounting framework, g would be the rate of labor-augmenting technical progress. Since our model's growth is "balanced" in the long run, we approximate

[5] Figure 14.4 in Clark [2007] implies $\alpha \leq 0.225$, reflecting a correction for rents to land.

[6] A third approach would attempt to price investment goods in efficiency units on the basis of independent information (Greenwood et al. [1997]). Our approach deduces "efficiency units" indirectly, using Lemma 3 and measurements (i)-(ii). See also Gordon [1990] and Edquist and Henrekson [2007].

$$e^{(1-\alpha)\cdot g\cdot T} \approx \theta\,.$$

Hence,

$$T = \frac{1}{1-\alpha}\cdot\frac{\ln(\theta)}{g}\,. \tag{53}$$

The inverse of the Poisson hazard gives the average time between TFP shocks; so, we set

$$\lambda = 1/T\,. \tag{54}$$

The parameter δ measures wear and tear depreciation. Excluding housing, Laitner and Stolyarov [2003] provide an estimate 0.0752 for total depreciation 1953-2001 on remaining physical capital. They argue that the latter should correspond to wear and tear depreciation plus creative destruction; hence, for long-run averages,

$$\delta + \lambda\cdot(1-b) = \delta + \lambda\cdot(1-[\theta]^{-1/\alpha}) = 0.0752\,. \tag{55}$$

We set δ from the second equality.

For conformity with the literature (e.g., Boldrin et al. [2001], Papaikolaou [2011]), our baseline calibration sets

$$\omega = 0.40\,. \tag{56}$$

Flow of Funds data suggests that the ratio for nonfinancial corporate business of short-term liabilities plus mortgage debt to total debt plus equity (liability) averages 0.2372 for 1952-2012; hence, Section 8 also considers

$$\omega = 0.20\,. \tag{56'}$$

Limited Participation We want to interpret our representative agent using a "limited participation" specification (Mankiw and Zeldes [1991], Kocherlakota [1996], and others).

In the US economy, the wealthiest families (say, the top 3-5%) hold a disproportionate share of national wealth (Bricker et al. [2012], Laitner [2002]). These families seem likely to leave intentional bequests (Laitner [1992]) and, hence, seem appropriately modeled with a long-time-horizon utility function. Divide the economy into dynastic and life-cycle households. Assume this paper's representative agent models the former. Life-cycle households, we imagine, simply consume their labor earnings.[7]

Continue to set $L_t = 1$. Let the aggregate production function be

$$z\cdot[K_t]^{\alpha}\cdot[L_t]^{1-\alpha}\,.$$

[7] Examples of models with two agent types include Laitner [2001] and Muller and Woodford [1988]. Notice that evidence on the intentional nature of middle-class bequests is very mixed — i.e., Altonji et al. [1990, 1999].

Think of non-dynastic households as having a fraction ν of total labor income and no income from capital. Then to interpret the aggregate production function in our representative agent specification, let

$$Z = z \cdot \big(1 - \nu \cdot (1 - \alpha)\big) .$$

The representative agent's consumption is C_t. Non-dynastic households' consumption is

$$C_t^N = \nu \cdot (1 - \alpha) \cdot z \cdot [K_t]^\alpha .$$

At the introduction of a new general purpose technology, existing plant and equipment suffers obsolescence, lowering its market value. The representative agent's portfolio realizes a corresponding capital loss. On the other hand, new-vintage equipment temporarily has a very high marginal product — tempting the representative agent to restrict his consumption in order to invest heavily. Suppose the representative agent's current consumption declines by ΔC_t. Despite the arrival of a new technology, GDP manifests no immediate change — a change in GDP requires production from new physical capital, which must first be built. So, non-dynastic consumption temporarily remains unchanged. The percentage decline in aggregate consumption is

$$\Delta C_t / [C_t + C_t^N] . \tag{57}$$

Suppose our representative agent's share of total consumption does not exceed

$$\alpha \cdot (C + C^N) .$$

The latter seems a conservative estimate — in practice, the richest households are unusually heavy savers. Then

$$|\frac{\Delta C_t}{\alpha \cdot (C_t + C_t^N)}| \leq |\frac{\Delta C_t}{C_t}| \iff |\frac{\Delta C_t}{C_t + C_t^N}| \leq \alpha \cdot |\frac{\Delta C_t}{C_t}| .$$

Mehra and Prescott [1985, 1988] argue that measured fluctuations in aggregate consumption rarely exceed of 5% per year. In that light, we consider below a sufficient condition[8]

$$\alpha \cdot |\frac{\Delta C_t}{C_t}| \leq 0.05 . \tag{58}$$

[8] When Mankiw and Zeldes [1991] use PSID expenditure data to compare stockholders to households at large, they find the former display a higher correlation between consumption and the risky rate of return, as well as a higher variance of expenditure fluctuations (Mankiw and Zeldes [1991, tab. 3-4]). See also Vissing-Jorgensen [2002] and Attanasio et al. [2002].

Rates of Return We model neither currency holdings nor cash management behavior.

Using the time period 1949-2013 and the NIPA GDP price deflator, the average annual return on the S&P500 Index, with dividend reinvestment, was 8.4598%/yr.[9] Using the same deflator and time interval, the (real) prime interest rate was 3.6170%/yr, the (real) rate on short-duration commercial paper was 1.8509%/yr, and the (real) rate on 3-month Treasury Bills averaged 1.3192%/yr.[10] Our calculations use the prime and commercial paper rates, yielding an equity premium of 4.8428%/yr or 6.6089%/yr, respectively.[11] We think of the benchmark equity premium from the literature as 6%/yr.

Simulations Table 2 presents simulation results for the basic model. We consider 2 specifications. In columns 1-4, we have a single type of TFP shock, and we set $\lambda(1) = \lambda$ and $b(1) = b$, calibrating λ and b as in the previous sections. In columns 5-8, we have 2 types of TFP shock. We assume both are equally likely. The first, labeled $i = 1$, uses our calibration from the microprocessor revolution, $b(1) = b$. The second, labeled $i = 2$, causes a reduction in the resale value of existing physical capital only one-half as great, $b(2) = 1 - 0.5 \cdot (1 - b)$. Let the total number of shock types be i_0.

In a given column, we solve the model, as in Section 4, for every pair $(\gamma\,,\,\beta)$ satisfying (51). In each case, we compute the long-run average riskless real rate of return $\bar{r}$, i.e., the unconditional average r_t^2, using an approximation, as follows. In (36), set $\dot{C}_t/C_t = g = 0.02$ to reflect long-run average growth and set

$$\frac{\bar{C}_t}{C_t} \approx \frac{\varphi^{\infty}([\theta]^a \cdot b \cdot K^{*\infty}\,,\, Z)/[\theta]^a}{C^{*\infty}}\,. \tag{59}$$

As the latter shows, this paper's analysis starts from the stationary solution in evaluating the representative agent's consumption response to a shock.

For each β, we limit our attention to grid values γ yielding $\bar{r}$ levels bracketing our real return on commercial paper, 1.85%/yr., or prime rate, 3.62%/yr. From the resulting set of viable parameter combinations, we choose the one implying the highest equity premium E^*. We also report the stationary-state riskless real interest rate, r^*, and equity premiums with and without linear approximations, e^{1*} and e^{2*}.

The bottom of each column of Table 2 examines the effect of constraint (58). Using approximation (59) again, we re-phrase the constraint as

$$\alpha \cdot |\frac{\Delta C_t}{C_t}| \approx \alpha \cdot |\frac{\Delta C^*}{C^*}| \equiv \alpha \cdot |\frac{\varphi^{\infty}([\theta]^a \cdot b \cdot K^{*\infty}\,,\, Z)/[\theta]^a - C^{*\infty}}{C^{*\infty}}| \leq 0.05\,. \tag{60}$$

Table 2 reports the maximum equity premium on the subset of viable parameters satisfying (60). Under the additional constraint, we call the maximum premium E^{c*}.

[9] See http://www.bogleheads.org/wiki/S&P_500_index and http://en.wikipedia.org/wiki/S&P_500.

[10] See Homer and Sylla [1996] and Board of Governors [2014].

[11] See, for example, the discussion in Mehra and Prescott [2008].

Table 2. Equity Premium Simulations for Technology Shocks								
Variable	Specification with $i_0 = 1$				Specification with $i_0 = 2$			
	$\bar{r} = 0.0185$		$\bar{r} = 0.0362$		$\bar{r} = 0.0185$		$\bar{r} = 0.0362$	
	$\alpha = 0.25$ (1)	$\alpha = 0.30$ (2)	$\alpha = 0.25$ (3)	$\alpha = 0.30$ (4)	$\alpha = 0.25$ (5)	$\alpha = 0.30$ (6)	$\alpha = 0.25$ (7)	$\alpha = 0.30$ (8)
Choose $(\gamma\,,\,\beta)$ to maximize E^*, given $\bar{r}$ and constraint (51)								
γ	-0.4000	-0.1000	-0.6000	-0.2000	-0.5000	-0.1000	-0.6000	-0.3000
$\lambda(1)$	0.0663	0.0516	0.0663	0.0516	0.0471	0.0365	0.0471	0.0365
$\lambda(2)$	na	na	na	na	0.0471	0.0365	0.0471	0.0365
$b(1)$	0.3866	0.3866	0.3866	0.3866	0.3866	0.3866	0.3866	0.3866
$b(2)$	na	na	na	na	0.6933	0.6933	0.6933	0.6933
β	0.0160	0.0160	0.0310	0.0340	0.0140	0.0170	0.0310	0.0310
$\Delta C^*/C^*$	-0.2149	-0.2713	-0.1974	-0.2598	-0.2028	-0.2722	-0.1982	-0.2424
r^*	-0.0107	-0.0055	0.0030	0.0116	-0.0123	-0.0039	0.0035	0.0092
e^{1*}	0.0122	0.0094	0.0128	0.0099	0.0107	0.0082	0.0112	0.0086
e^{2*}	0.0164	0.0132	0.0172	0.0137	0.0139	0.0111	0.0146	0.0116
$E^*\,,\ \omega = 0.40$	0.0273	0.0219	0.0286	0.0229	0.0232	0.0185	0.0243	0.0193
Add constraint (58)								
γ^c	-0.6000	-0.8000	-0.6000	-0.9000	-0.6000	-0.8000	-0.6000	-0.9000
β^c	0.0120	0.0001	0.0310	0.0180	0.0120	0.0001	0.0310	0.0170
$\Delta C^{c*}/C^{c*}$	-0.1901	-0.1578	-0.1974	-0.1623	-0.1910	-0.1587	-0.1982	-0.1624
r^{c*}	-0.0146	-0.0186	0.0030	-0.0026	-0.0142	-0.0183	0.0035	-0.0032
$E^{c*}\,,\ \omega = 0.40$	0.0272	0.0191	0.0286	0.0211	0.0231	0.0162	0.0243	0.0178

Source: See text.

Remarks on Table 2   Observations on Table 2 include the following. Remarks (i)-(v) refer to columns 1-4.

(i) Mehra and Prescott's [1985] version of the average e_t^1 was about 0.004. In Table 2, e^{1*} is 2-3 times as large. In addition to our formulation's neoclassical production sector, two important differences from Mehra and Prescott are: our technology shocks, due to their low frequency, have large amplitude; and, on the basis of a limited-participation interpretation, we allow relatively large values of $|\Delta C^*/C^*|$.

The ratio e^{2*}/e^{1*} provides our measure of the nonlinear effect of large-amplitude shocks. In Table 2, the ratio is 1.3-1.5. In other words, this section's high-magnitude shocks increase the equity premium by 30-50%.

The bottom of Table 2 illustrates the impact of our restriction on $\Delta C^*/C^*$. We find a relatively modest effect, largest in column 2. (That is not to say, however, that our limited-participation interpretation does not have a vital role — none of Table 2's columns yield a parameter combination simultaneously satisfying (51), $|\Delta C^*/C^*| \leq 0.05$, and $\bar{r} = 0.0185$ or 0.0362.)

(ii) The ratio $E^*/e^{2*} = E^{c*}/e^{2c*} = 1/(1-\omega) = 1.67$ shows the effect of leverage assumption (56). As stated, this assumption is standard in the recent literature.

(iii) We can assess whether the values for b and ω are mutually consistent. This issue becomes especially important in Section 8 below.

Suppose we assume that after a punctuated change, the stock market falls so precipitously that a new infusion of equity financing can only follow at the bottom. If firm owners have limited liability in practice, bonds cannot be riskless unless declines in market values of existing plant and equipment always remain less than equity's financial share. In other words, we need

$$1 - b \leq 1 - \omega \, . \tag{61}$$

When $b = 0.3866$ and $\omega = 0.40$, inequality (61) is violated — though a marginal adjustment to b or ω will suffice to re-establish it. If the violation were more severe, we would need to rethink our calibrations.

(iv) Throughout Table 2, the coefficients of relative risk aversion, $1-\gamma$, are modest, varying from 1.1 to 1.9.

(v) Table 2 shows that our model provides a possible explanation for the sporadic, temporary appearance of negative real interest rates. Although we calibrate the average riskless rate to 0.0185 or 0.0362, the model's stationary riskless rate, r^*, is negative in columns 1-2. At the bottom of Table 2, r^{c*} is negative in columns 1, 2, and 4. Between the arrival of general purpose advances, the actual riskless rate converges toward r^*. If the next punctuated advance happens to be unusually slow in arriving, the economy can experience $r_t \approx r^*$.

Formula (36) shows why r^* can be much smaller than $\bar{r}$. In (36), $\beta > 0$ makes r_t larger, but precautionary saving against the chance λ of a severe capital loss makes r_t smaller. In normal times, a last term, $(1-\gamma) \cdot \dot{C}_t/C_t$, makes r_t larger as well. On average, $\dot{C}_t/C_t$ equals g. But at the stationary solution, $\dot{C}_t/C_t = 0$. We then have

$$r^* = \bar{r} - (1-\gamma) \cdot g \, .$$

(vi) Columns 5-8 use Section 5. We make half of the technology shocks one-half as severe in terms of creative destruction as Section-7 calibrations based upon the microprocessor. A comparison with columns 1-4 shows only a mild effect.

Two factors limit the reduction of the equity premium in columns 5-8. On the one hand, Jensen's inequality and the convexity of the marginal utility of consumption reduce the quantitative impact on consumers. On the other hand, to maintain consistency with Maddison's long-run growth rate, the calibrated frequency of the technology shocks must rise if their magnitude falls (see (53)).

Discussion In the end, Table 2 offers an explanation of 20-50% of our benchmark equity premium 0.06. Although the existing literature suggests that switching to Epstein-Zin preferences (Weil [1989]) could raise the model's average premium substantially (e.g., Barro [2008], Papanikolaou [2011]), the remainder of this paper takes the alternative tack of adding more types of shocks.

8 Additional Shocks. Barro [2006, 2008, 2009] (see also Blanchard and Constantinides [2008]) focuses on low-frequency shocks from wars or economic crises, using "tree" and "$A \cdot K$" specifications. The present section attempts to incorporate recessions into our formulation. The treatment is very stylized.

At the onset of a recession/depression, the economy typically experiences both an abrupt stock-market decline and reduction in GDP. We assume that a recession occurs when firms as a group discover that their capital is mismatched with demand or in the wrong location, that utilization rates on existing plant and equipment are likely to be disappointing, and that remedial investment is needed. For a recession starting at time T, we adjust the number of "effective" units of existing capital downward

$$K_{T+0} = \check{b} \cdot K_T \,, \quad \check{b} \in (0\,,\,1)\,. \tag{62}$$

The aggregate production function remains unchanged. Subsequent investment restores the "effective" size of the capital stock, ending the recession.

The fall in GDP at the beginning of a recession contrasts to the arrival of a punctuated technology shock, which brings no immediate change in output. The new investment that finally ends a recession does not embody a better technology. Moreover, recessions have no permanent effect. In Section 7, acceptance of Maddison's long-run rate of growth of output constrains the product of the magnitude and frequency of technology shocks to a constant — if we raise our calibration of one, our calibration of the other must decline. In the case of recessions, there is no corresponding restriction: we can set shock magnitudes and frequencies independently.

The logic of Lemma 3 shows that at time T, the resale value of existing capital declines from K_T to $\check{b} \cdot K_T$ and GDP falls from $Z \cdot [K_T]^{\alpha}$ to $\rho \cdot Z \cdot [K_T]^{\alpha}$, with

$$\rho = [\check{b}]^{\alpha}\,. \tag{63}$$

Concentrate, for the moment, exclusively on recessions. We can preserve most of the analytical framework of Sections 3-4 if we replace b with $\check{b}$, and θ with 1. The sequence of value functions in the proof of Proposition 1 now monotonically decreases. The proof will remain valid, however, if the sequence is bounded below at each $K \in \mathcal{K}(Z)$ (Lucas and Stokey [1989]). A sufficient condition is, for instance,

$$\lambda \cdot [\check{b}]^{\alpha \cdot \gamma} < 1 \,. \tag{64}$$

Suppose that we do, in fact, have a limiting value function

$$\check{V}(K\,,\,Z) > -\infty \quad \text{all} \quad K \in \mathcal{K}(Z)\,. \tag{65}$$

Given (65), re-introduce the TFP shocks of the Section 3-7. The total number of shock types is i_0. For simplicity, let there be one type of recession and one type of technology shock. So, $i_0 = 2$. Let $\theta > 1$ be as in Section 3, and set

$$b(i) \equiv \begin{cases} [\theta]^{-1/\alpha}\,, & \text{if } i = 1, \\ \\ \check{b}\,, & \text{if } i = 2, \end{cases} \tag{66}$$

$$\theta(i) \equiv \begin{cases} \theta\,, & \text{if } i = 1\,. \\ 1\,, & \text{if } i = 2, \end{cases} \tag{67}$$

Let $\bar{\lambda}$ be the hazard for a recession, and let λ as in Table 2. Define

$$\Lambda \equiv \lambda + \bar{\lambda}\,, \quad p(1) \equiv \lambda/\Lambda\,, \quad p(2) \equiv \bar{\lambda}/\Lambda\,. \tag{68}$$

Then

Proposition 4. *Fix any $Z > 0$. Let $\theta(i)$, $p(i)$, $b(i)$, and Λ be as in (66)-(68). Suppose $\check{V}(K\,,\,Z)$ is bounded as in (65). Then there exists $V(K\,,\,Z)\,,\ K \in \mathcal{K}(Z)$, which is strictly increasing and strictly concave, which satisfies (17), and which satisfies (31). The right-hand side of (31) uniquely defines a policy function as in (32). The policy function is continuous and satisfies (19).*

Proof. See Appendix.

We provide several illustrative examples.

Example 1 This section's first example has 2 types of shock. One is the punctuated technology shock from columns 1-4 of Table 2. The second is a recessionary shock with output and stock-market declines calibrated to the Great Depression 1929-1939.

We assume an average of one depression per century. If λ is as in Table 2, we then have $\Lambda = \lambda + 0.01$ and (recalling the notation of Section 5)

$$p(i) = \begin{cases} \lambda/\Lambda\,, & \text{if } i = 1, \\ 0.01/\Lambda\,, & \text{if } i = 2, \end{cases}$$

$$\lambda(i) = \Lambda \cdot p(i)\,,\ i = 1\,,\,2\,.$$

Table 3. Simulations with $i_0 = 2$: Shocks Calibrated to Punctuated Technological Change ($i = 1$) and the Great Depression ($i = 2$), with the Latter's Average Frequency of Occurrence Set to Once Per Century

Variable	$\bar{r} = 0.0185$				$\bar{r} = 0.0362$			
	$b(i)$ from (70)		$b(i)$ from (71)		$b(i)$ from (70)		$b(i)$ from (71)	
	$\alpha = 0.25$ (1)	$\alpha = 0.30$ (2)	$\alpha = 0.25$ (3)	$\alpha = 0.30$ (4)	$\alpha = 0.25$ (5)	$\alpha = 0.30$ (6)	$\alpha = 0.25$ (7)	$\alpha = 0.30$ (8)
Choose (γ, β) to maximize E^*, given $\bar{r}$ and constraint (51)								
γ	-4.0000	-2.6000	-4.0000	-3.4000	-3.6000	-4.0000	-3.7000	-2.8000
$\lambda(1)$	0.0663	0.0516	0.0663	0.0516	0.0663	0.0516	0.0663	0.0516
$\lambda(2)$	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100
b(1)	0.3866	0.3866	0.3866	0.3866	0.3866	0.3866	0.3866	0.3866
b(2)	0.1716	0.2302	0.1749	0.1749	0.1716	0.2302	0.1749	0.1749
β	0.0630	0.0010	0.0570	0.0750	0.0750	0.0390	0.0750	0.0750
$\Delta C^*/C^*$	-0.0607	-0.0454	-0.0590	-0.0641	-0.0721	-0.0333	-0.0700	-0.0824
r^*	-0.0868	-0.0568	-0.0868	-0.0746	-0.0604	-0.0690	-0.0617	-0.0434
e^{1*}	0.0292	0.0159	0.0286	0.0254	0.0293	0.0199	0.0298	0.0245
e^{2*}	0.1189	0.0430	0.1137	0.1197	0.1063	0.0816	0.1071	0.0935
$E^*,\ \omega = 0.4$	0.1981	0.0717	0.1895	0.1995	0.1771	0.1360	0.1785	0.1558
Add constraint (58) and $r^* \geq -0.03$								
γ^c	-1.3000	-1.2000	-1.2000	-1.0000	-1.9000	-2.0000	-2.1000	-2.1000
β^c	0.0320	0.0200	0.0220	0.0320	0.0520	0.0340	0.0510	0.0590
$\Delta C^{c*}/C^{c*}$	-0.1427	-0.1383	-0.1490	-0.1613	-0.1179	-0.0978	-0.1095	-0.1040
r^{c*}	-0.0299	-0.0274	-0.0276	-0.0236	-0.0246	-0.0268	-0.0289	-0.0285
$E^{c*},\ \omega = 0.4$	0.0754	0.0556	0.0720	0.0688	0.0954	0.0731	0.0998	0.1129
$E^{c*},\ \omega = 0.2$	0.0566	0.0417	0.0540	0.0516	0.0716	0.0548	0.0749	0.0847

Source: See text. ($\Delta C^*/C^*$ here refers only to shock 1 — see text.)

We calibrate $\check{b}$ for the new shock in 2 ways. The first is based upon output. Suppose recession j has peak real GDP q_{jt} and trough $q_{j,t+s}$. Then actual output at the trough as a fraction of the peak plus trend growth, say, $1 + \tau$, is

$$\rho(j) \equiv \frac{q_{j,t+s}}{q_{jt} \cdot (1+\tau)^s} \,. \tag{69}$$

In practice, trend growth includes population change as well as technological progress. We calibrate τ from BLS data for 1948-2013, which shows an average growth rate for real output of 0.8471%/quarter.[12] NIPA annual constant-dollar GDP then displays a decline 1929-1933, relative to trend, of 35.64%. Using (63) and letting b be as in Table 2, we set

$$b(i) = \begin{cases} b\,, & \text{if } i = 1, \\ [0.6436]^{1/\alpha}\,, & \text{if } i = 2. \end{cases} \tag{70}$$

A alternative calibration uses stock-market data. From September 1929 to June 1933, the SP500 index fell 82.51% relative to trend.[13] This provides a second, direct measure of $\check{b}$. Letting b remain as above, we set

$$b(i) = \begin{cases} b\,, & \text{if } i = 1, \\ 0.1749\,, & \text{if } i = 2. \end{cases} \tag{71}$$

Remarks on Table 3   Table 3 presents separate results for (70) and (71). During the Great Depression, aggregate consumption fell nearly in step with GDP (Cole and Ohanian [1999]). Thus, we assume that constraint (58) is unwarranted for shock 2. Table 3's $\Delta C^*/C^*$ measures the representative agent's response to shock 1. We find very low values of r^* can emerge. As above, on grounds of plausibility we impose $r^* \geq -0.03$ in the table's bottom panel.

Observations are as follows.

(i) The relatively low intertemporal rates of substitution in Table 3 cause consumption constraint (58) not to bind for shocks of type 1.

(ii) The constraint that does bind at the bottom of Table 3 is $r^* \geq -0.03$. It leads to substantial differences between E^* and E^{c*}. The floor -0.03 is arbitrary. The Remarks on Table 2 suggest why the new constraint can bind. They also suggest how the floor, in effect, can be re-interpreted as a lower bound for γ.

(iii) The very large magnitude of the type $i = 2$ shocks evidently strongly effects the model's equity premium. This is true despite the low probability for a depression that we assume.

The model's nonlinearity provides part of the explanation. Sections 6-7 propose using the ratio e^{2*}/e^{1*} to assess the impact of the marginal utility function's convexity

[12] See www.bls.gov/data/#productivity. Output comes from real NIPA GDP less general government, private households, and nonprofit institution output.

[13] We use Shiller's SP500 price index and Consumer Price Index monthly data. See http://www.econ.yale.edu/∼shiller/data.htm. Trend growth for the SP500 index in constant-dollar terms is 3.81%/yr.

on the equity premium. In Table 2, $e^{2*}/e^{1*} = 1.3$ or 1.5. In Table 3, in contrast, the ratio is 3-4.

However, the values of e^{1*} are also much bigger in Table 3. In Table 2, e^{1*} is 2-3 times as large as the literature's original premium estimate, 0.004; in Table 3, e^{1*} is 4-7 times the original. Of course, our new recessionary shock is larger than our technology shock, which should make e^{1*} higher in Table 3, perhaps linearly. Beyond that, in Table 3 a depressionary shock lowers GDP as well as raising the marginal product of capital on new investment, whereas our punctuated technology shock only does the latter. Thus, the Table-3 drop in the representative agent's consumption, ΔC^*, tends to be larger, raising e^{1*} for a second reason.

(iv) The Remarks on Table 2 suggest that under limited liability for equity owners, we may want to impose $\omega \leq b$. Table 3's low values of $b(2)$ then suggest that $\omega = 0.2$ is marginally possible but $\omega = 0.4$ is not. With $\omega = 0.4$, we had $E^*/e^{2*} = 1.67$. With $\omega = 0.2$, the ratio is only 1.25. We end up counterbalancing some of the increase in e^{1*} and e^{2*} with a decrease in ω.

(v) Nevertheless, the net effect of the $i = 2$ shock is large. Suppose $\omega = 0.2$. When $\bar{r} = 0.0362$, E^{c*} exceeds our benchmark empirical estimate of the equity premium, 0.06, in 3 out of 4 cases, and it exceeds 0.0484 (recall Section 7) in all 4.

(vi) Calibrations (70) and (71) yield similar values of $b(2)$.

<u>Discussion</u> In sum, Table 3 implies that even the small chance of an event as disruptive as the Great Depression can greatly increase the equity premium.

Outcomes would be qualitatively the same with technology shocks of varying magnitude resembling those of Table 2, columns 5-8. Using the latter technology shocks, Table 3, columns 5-8, for instance, generate values of E^{c*} , $\omega = 0.2$, of 0.0686, 0.0530, 0.0696, and 0.0827, respectively.

Finally, notice that while Table 3 treats technology and recession shocks as independent, there might, in practice, be connections. For instance, Section 2 described technology-induced creative destruction in agriculture that might have increased the economy's fragility just prior to the onset of the Great Depression.[14]

<u>Example 2</u> Our second example combines punctuated technological progress with recessions calibrated to US data 1948-2013.

The NBER identifies 11 US recessions 1948-2013, providing peak and trough dates for each.[15] For each, Table 4 calculates $\rho(j)$, $j = 1, \ldots, 11$, as in (69), using constant-dollar NIPA quarterly data. We derive $\check{b}(j)$ as in (63). In addition, as in Example 1, we use Shiller's SP500 data to calibrate each $\check{b}(j)$ in a second way (referring to the second estimate as $\check{b}^S(j)$).

In Table 4, alternative values of $\check{b}$ in the same row tend to be consistent with one another (as was the case in Table 3). However, the 1973 recession provides an exception, having an unusually large stock-market drop relative to the output decline. In fact, 1973 marked the punctuated technology shock that we atttributed to the microprocessor chip

[14] See Romer and Galbraith....

[15] See www.nber.org/cycles/cyclesmain.html.

Table 4. US Recessions 1948-2013				
peak year	ρ (NIPA Data)	$\check{b} = [\rho]^{1/\alpha}$ $\alpha = 0.25$	$\alpha = 0.30$	$\check{b}^S$ (SP500 Data)
1948	0.9574	0.8400	0.8648	0.8068
1953	0.9317	0.7536	0.7900	0.8573
1957	0.9279	0.7414	0.7793	0.8156
1960	0.9583	0.8434	0.8677	0.8700
1969	0.9615	0.8547	0.8774	0.6140
1973	0.9098	0.6852	0.7298	0.4328
1980	0.9544	0.8298	0.8560	0.8648
1981	0.9225	0.7242	0.7643	0.8336
1990	0.9631	0.8605	0.8823	0.8189
2001	0.9763	0.9086	0.9232	0.7492
2007	0.8864	0.6174	0.6691	0.4581

Source: See text.

revolution in Section 7's calibrations.[16] If punctuated technological progress caused the stock-market decline, corresponding financial disruptions (as with firms attempting to rebuild their equity-to-debt ratios) might temporarily have interfered with new investment, contributing to the GDP fall.[17] In any case, to avoid double counting, we exclude the 1973 row of Table 4 from our list of recessionary shocks.

The 2007 recession is another anomaly in Table 4. Its output decline was unusually large. Its stock-market decline was disproportionately great as well. We assign the 2007 recession to a separate category.

Example 2 has $i_0 = 3$. The punctuated technology improvements of Table 2, columns 1-4, are type $i = 1$ shocks. The 9 recessions 1948-2001, excluding 1973, are type $i = 2$ shocks. And, the 2007 recession is a type $i = 3$ shock. Let λ and b be as in Table 2. Noting that our new data covers 66 years, we set $\Lambda = \lambda + 9/66 + 1/66$ and

[16] Section 7's 1973 stock-market decline, determined from different data and a somewhat different framework of analysis, was 4-5 points greater.

[17] For another possible impediment to investment spending, see David's [1990] discussion of "network externality effects" and "issues of compatibility standardization" that can lead to "protracted ... diffusion" for new inventions.

Table 5. Simulations with $i_0 = 3$: Shocks Calibrated to Punctuated Technological Progress ($i = 1$), 1948-2001 recessions ($i = 2$), and 2007 recession ($i = 3$)

Variable	$\bar{r} = 0.0185$				$\bar{r} = 0.0362$			
	$b(i)$ from (73)		$b(i)$ from (74)		$b(i)$ from (73)		$b(i)$ from (74)	
	$\alpha = 0.25$ (1)	$\alpha = 0.30$ (2)	$\alpha = 0.25$ (3)	$\alpha = 0.30$ (4)	$\alpha = 0.25$ (5)	$\alpha = 0.30$ (6)	$\alpha = 0.25$ (7)	$\alpha = 0.30$ (8)
Choose (γ, β) to maximize E^*, given $\bar{r}$ and constraint (51)								
γ	-1.7000	-0.7000	-2.6000	-2.5000	-2.1000	-1.1000	-3.4000	-3.2000
$\lambda(1)$	0.0663	0.0516	0.0663	0.0516	0.0663	0.0516	0.0663	0.0516
$\lambda(2)$	0.1364	0.1364	0.1364	0.1364	0.1364	0.1364	0.1364	0.1364
$\lambda(3)$	0.0152	0.0152	0.0152	0.0152	0.0152	0.0152	0.0152	0.0152
b(1)	0.3866	0.3866	0.3866	0.3866	0.3866	0.3866	0.3866	0.3866
b(2)	0.8159	0.8440	0.8033	0.8033	0.8159	0.8440	0.8033	0.8033
b(3)	0.6173	0.6690	0.4581	0.4581	0.6173	0.6690	0.4581	0.4581
β	0.0280	0.0310	0.0290	0.0270	0.0430	0.0450	0.0410	0.0420
$\Delta C^*/C^*$	-0.1175	-0.1893	-0.0805	-0.0655	-0.1049	-0.0961	-0.0622	-0.0490
r^*	-0.0377	-0.0169	-0.0566	-0.0545	-0.0283	-0.0075	-0.0558	-0.0518
e^{1*}	0.0202	0.0144	0.0250	0.0213	0.0214	0.0158	0.0265	0.0227
e^{2*}	0.0253	0.0186	0.0342	0.0303	0.0268	0.0195	0.0377	0.0342
E^*, $\omega = 0.4$	0.0422	0.0310	0.0571	0.0505	0.0447	0.0326	0.0629	0.0570
Add constraint (58) and $r^* \geq -0.03$								
γ^c	-1.2000	-1.0000	-1.2000	-1.3000	-2.1000	-1.1000	-2.1000	-2.0000
β^c	0.0350	0.0260	0.0440	0.0410	0.0430	0.0450	0.0560	0.0560
$\Delta C^{c*}/C^{c*}$	-0.1488	-0.1572	-0.1512	-0.1396	-0.1049	-0.0961	-0.1082	-0.1039
r^{c*}	-0.0273	-0.0236	-0.0279	-0.0300	-0.0283	-0.0075	-0.0292	-0.0267
E^{c*}, $\omega = 0.4$	0.0419	0.0308	0.0518	0.0478	0.0447	0.0326	0.0590	0.0539

Source: See text. ($\Delta C^*/C^*$ here refers only to shocks 1-2.)

$$p(i) = \begin{cases} \lambda/\Lambda\,, & \text{if } i = 1, \\ 9/(\Lambda \cdot 66)\,, & \text{if } i = 2, \\ 1/(\Lambda \cdot 66)\,, & \text{if } i = 3, \end{cases}$$

$$\lambda(i) = \Lambda \cdot p(i)\,, \quad i = 1\,,\, 2\,,\, 3\,,$$

$$b(i) = \begin{cases} b\,, & \text{if } i = 1, \\ [\sum_{j \le 10, j \ne 6} \rho(j)/9]^{1/\alpha} = [0.9504]^{1/\alpha}\,, & \text{if } i = 2, \\ [\rho(11)]^{1/\alpha} = [0.8864]^{1/\alpha}\,, & \text{if } i = 3, \end{cases} \tag{73}$$

$$b^S(i) = \begin{cases} b\,, & \text{if } i = 1, \\ \sum_{j \le 10, j \ne 6} \check{b}^S(j)/9 = 0.8033\,, & \text{if } i = 2, \\ \check{b}^S(11) = 0.4581\,, & \text{if } i = 3. \end{cases} \tag{74}$$

Remarks on Table 5   Table 5 presents simulation outcomes, treating (73) and (74) separately. As in Table 3, we assume that constraint (58) is unwarranted for shock 3. Table 3's $\Delta C^*/C^*$ measures the representative agent's response to shocks 1-2. As in Table 3, we impose (58) and $r^* \ge -0.03$ in the bottom panel.

Observations include the following.

(i) Constraint (58) turns out to be relatively unimportant. In contrast, as in Table 3, the constraint $r^* \ge -0.03$ binds in most columns.

(ii) In Remarks on Tables 2-3, we discussed the potential need for $\omega \le b(i)$ all i. Violations seem minor in Table 5; hence, we maintain leverage assumption (56).

(iii) In general, outcomes for E^{c*} are intermediate to Tables 2-3. Even with $\omega = 0.40$, E^{c*} never reaches 0.06. However, columns 7-8 surpass 0.0484 (recall the empirical evidence in Section 7), and column 5 nearly reaches it.

Discussion   Several different interpretations seem possible.

On the one hand, a comparison of Examples 1 and 2 demonstrates the potential importance of large-magnitude shocks. Table 5 has mild recessions occurring on average every 7-8 years, and severe recessions every 66 years; Table 3 has only depressions at one-hundred year intervals. Yet, equity premiums are substantially larger in Table 3.

On the other hand, Example 5 shows that a combination of punctuated technological progress and recessions resembling those of the post-WWII era may be sufficient to generate an empirically plausible equity premium. Fear of another Great Depression may not be necessary.

9 Conclusion. This paper introduces a new equity premium model that combines a neoclassical production sector and punctuated, aggregative technology shocks. We show that the model can be solved recursively, and we illustrate a numerical procedure for deriving quantitative results. We show why creative destruction accompanying new general purpose technologies can lead to an equity premium, why a negative riskless interest rate may emerge sporadically, how the model's inherent nonlinearities can amplify the impact of large-magnitude shocks, and why creative destruction might sometimes lead to financial crises.

An advantage of our formulation is its ability to incorporate a variety of types of shocks. In fact, this paper's most successful explanations of the empirical equity premium utilize mixtures of technological change and recessions. For example, we find that a remote possibility of recurrence of the Great Depression could affect the equity premium substantially. More detailed treatments of different shock types should be possible in future work.