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Pollution, Financial Crises, and Minor Nuisances:
A Unified Theory of Regulation and Punishment

Carl Davidson, Lawrence W. Martin, and John D. Wilson
Department of Economics, Michigan State University; East Lansing, MI 48824

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Abstract: We study the use of fines and inspections to control production activities that create external damages. The model contains a continuum of firms, differing in their compliance costs, so that only high-cost firms evade the regulations. Modifying the usual Pigou rule for taxing externalities to account for costly inspections, the external damage from the marginal evader’s activities should exceed the expected fine by an amount equal to the resources expended to reduce the number of evaders a unit. According to Becker’s classic work on crime and punishment, however, these resources can be minimized by raising the fines to very high levels, while reducing costly inspections. We argue that the modified Pigou rule does not hold under such a policy, because it distorts capital markets. Firms caught evading the regulation will be bankrupted by the fines, and the possibility that they will not fully repay investors lowers their expected cost of capital. Investors will lend to all firms at an interest rate above the social opportunity cost of capital, to compensate for the risks of bankruptcy. The paper investigates the optimal choice between the Becker approach of high fines and few inspections, versus keeping fines low enough to eliminate capital-market distortions, in which case the modified Pigou rule holds. We derive conditions that determine how this choice should be resolved. In some case, welfare can be improved over the Pigou optimum with an equilibrium under which some regulation-evading firms risk bankruptcy, whereas others choose capital stocks low enough to eliminate such risks.
1. **Introduction**

Economic agents engage in a wide variety of activities that generate external effects. For example, drivers impose congestion costs on others when they use public roads and may endanger others by driving recklessly; homeowners may anger neighbors by listening to loud music or by allowing their property to deteriorate; firms may generate hazardous waste as part a byproduct of production or expose their workforce to unnecessary health risks by not taking sufficient care in designing their factories; and banks and other depositary institutions may accumulate the types and quantities of assets that increase the risks of financial crises. Society responds to such situations by attempting to regulating behavior and by punishing those who violate the established rules. Sometimes the behavior is criminalized (it is illegal to dump hazardous waste), while in other instances attempts are made to internalize the external damages (toll roads). In the economics literature there are two classic treatments of the issues that surround such activity, due to Pigou (1920) and Becker (1968), but the analyses differ in focus, and they offer solutions that have starkly different tones. Our goal in this paper is to offer a new approach that unifies the messages of Pigou and Becker by showing that the optimal policy prescription for activities that generate external costs can take on either form, and identifying the conditions that determine which form it takes.

Pigou addressed the issue of externalities in *The Economics of Welfare*. An externality arises whenever the social cost of an activity differs from the private cost. Pigou’s solution was to add a set of taxes to the price mechanism that would force individuals to internalize the full social costs. Thus, the Pigouvian solution is to set a tax which equals the marginal damage associated with the activity. If the external cost of the activity is low, the Pigouvian tax will be low; whereas activities that generate large external costs will be subject to large Pigouvian taxes. In this sense, the policy prescription proposed by Pigou is one in which the punishment fits the crime. Although Pigou (1954) acknowledged that there will be informational problems both in designing the optimal tax
scheme and implementing it, the issue of compliance played no role in his analysis. In addition, Pigou’s analysis did not emphasize the illegal nature of non-compliance.

In contrast, the illegal nature of non-compliance is at the center of Becker’s (1968) analysis of such issues in “Crime and Punishment: An Economic Approach.” Becker was interested in the question of how society should go about enforcing laws that criminalize activities that generate external costs. He focused on laws that are enforced by random inspection. The key policy parameters are the probability of detection, adjusted by increasing the rate of inspection, and the level of the fine imposed on those convicted of non-compliance. Becker’s goal was to find the optimal policy; the one that minimizes the cost of the illegal activity.¹ He argued that because detection is costly while fines are nearly costless, the fine should be raised all the way up to the full wealth of the perpetrator. This policy enables the regulation to be enforced with a low probability and low cost of detection. It is important to note that in Becker’s world, it is optimal to set the fine at a very high level, regardless of the costliness of detection and regardless of the extent of the external cost of the activity. Thus, with Becker’s policy prescription, the size of the punishment does not necessarily fit the crime – those found guilty of non-compliance are always driven to the edge of bankruptcy regardless of the extent of the external damage.

It is clear that economists were uncomfortable with the counter-intuitive policy prescription of drastically high fines and low audit rates put forth by Becker. In fact, this finding is sometimes referred to as the “Becker conundrum” because we rarely observe such harsh punishment, even though the argument in its favor is clear and compelling.² Since 1968, over 200 articles have been

¹ Becker recognized the need to correct marginal incentives. In fact, in the early part of his paper, he derived the optimal fine for a fixed inspection rate, showing that in the first-best outcome, the expected fine should be set equal to the harm (as noted by Polinsky and Shavell 2000, this result actually dates back to Bentham 1789). However, Becker’s focus was on enforcement. In particular, he argued that the existence of enforcement costs ensures that the marginal conditions that define the first-best outcome will not be satisfied. His solution of a high fine coupled with a low audit rate was designed to minimize the distortions created by such costs.

² In a survey of the literature on enforcement, Polinsky and Shavell (2000) provide a proof that the optimal fine is set at its upper limit when offenders are risk-neutral. Comparing this result with actual practice, they argue for
published on the economics of enforcement, with many targeted at conquering the Becker conundrum. In contrast, the robustness of Pigou’s main result is rarely questioned. Extensions have tended to focus on problems with implementation or complications that arise when Pigouvian taxes co-exist with other taxes.

In this paper we argue that for certain regulations, Becker’s analysis is too narrow, in the sense that it does not take into account the full implications of high fines. In particular, when firms must borrow or rent capital to produce, but face regulations that are imperfectly enforced, high fines may distort their choice of inputs and create inefficiencies in factor markets. The reason for this is that high fines alter the effective cost of capital that firms face. The costs of these distortions must then be balanced against the benefits from reduced detection costs associated with higher fines. Below we develop a model that explicitly takes these potential factor-market distortions into account and show that it is optimal to enforce some regulations with moderate fines and likely detection—the “Pigouvian approach”—while for others, a “Beckerian approach” is optimal, with fines that not only bankrupt some or all firms, but seize some or all of the assets that are involved in the illegal activity.

An interesting feature of our analysis is that there exist some fines and inspection rates under which the only equilibria contain ex ante identical non-compliant firms that make different higher fines. “Substantial enforcement costs could be saved without sacrificing deterrence by reducing enforcement effort and simultaneously raising fines.”

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3 For example, harsh fines are not optimal if agents are risk averse (Polinsky and Shavell 1979), because high fines impose an additional risk-bearing cost. In addition, if illegal activities can take on different gradations, it is optimal to impose moderate fines on less serious violations, thereby maintaining sufficient marginal incentives to deter more serious offenses (Sandmo 1981). Other approaches concern the optimal treatment of self-reported violations (Innes 1999), the structure of the criminal justice system (Rubenfeld and Sappington 1987; Malik 1990; Andreoni 1991; and Acemoglu and Verdier 2000), and heterogeneity among offenders (Babchuck and Kaplow 1993).

4 For important exceptions, see Buchanan (1969), Carlton and Loury (1980, 1986) and Kohn (1986). In addition, as is well known, Coase (1960) argued that when transactions cost are low, Pigouvian taxes will not be needed to reach an efficient outcome. He argued that as long as property rights are well defined, economic agents will be able to agree to the first-best outcome and split the surplus that will be created by eliminating distortionary behavior.

5 The double dividend literature stresses that in addition to correcting behavior, Pigouvian taxes generate revenue for the government. This creates a secondary benefit by allowing the government to reduce other taxes in the economy that may be creating distortions, but the modern literature has emphasized flaws in this argument (see, for example, Bovenberg and de Mooij 1994, Fullerton and Metcalf 1998, or Fullerton, Leicester and Smith 2010). The problems associated with collecting the information required to implement a Pigouvian tax (for example, measuring the true social cost) were stressed Baumol (1972) and a steady stream of related work has followed.
investment decisions: some choose to be overleveraged, meaning they are bankrupted if caught evading the regulations, whereas others have sufficient assets to pay the fine. Moreover, fines and inspection rates that generate these equilibria may be optimal.

In the next section, we sketch the basic framework of our model and provide the intuition for our key results. As we explain, there are three regimes of enforcement. In the first regime, fines are below the level that would drive a violator to bankruptcy, so that regulation is similar in tone to Pigou’s original design. In this regime, which is fully characterized in Section 3, firms use an efficient mix of inputs, and the price of capital for the relevant industry equals the economy-wide opportunity cost of capital. In the second regime, which has a tone consistent with Becker, the optimal fine exceeds each non-compliant firm’s ability to pay so that the government is forced to seize some of the assets owned by investors if the firm is convicted of non-compliance. When the fine is this high, we show in Section 4 that these firms over-employ capital. This factor market distortion is a direct result of the severity of the punishment scheme that the government uses for enforcement, and it generates additional costs to enforcing the regulation that have not been discussed in previous work on this topic. The third regime, which occurs for intermediate-valued fines, is the one where some but not all non-compliant firms are overleveraged. We call such equilibria “hybrid equilibria” and also describe them in Section 4.

Starting from the optimal Pigou equilibrium, the remaining sections analyze the welfare effects of moving to a hybrid or Becker equilibrium. Section 5 presents an expression for the welfare change from marginal changes in the fine and inspection rate that create a hybrid equilibrium, and uses it to sign this welfare change in a variety of cases. Section 6 considers a large move, to a Becker equilibrium. Although it may be optimal for the fine to bankrupt firms, the optimal fine need not involve seizing all of the firm’s assets. Section 7 contains examples.
2. Framework and Intuition

Our model consists of a perfectly competitive industry, in which firms finance capital on a competitive market for loans. The firms face a government-imposed regulation of some sort. Our model is set-up to allow for a wide variety of regulations, including but not limited to, those that restrict the type and quantity of capital (we provide some examples of regulatory settings in Section 7). Compliance is costly, and we assume that the cost of compliance varies across firms. In equilibrium, some firms choose to comply with the regulation, whereas other firms operate illegally, risking detection and punishment, by evading the regulation. Neither the government nor potential investors can observe the firm’s behavior (or its cost of compliance) without monitoring, so investors cannot condition their investment decisions on the legal status of the firm. The government enforces the regulation by randomly inspecting firms and fining evaders. A firm’s capital is observed by the government’s auditor, so the fine will be allowed to vary with capital usage in our formal model. But we assume a fixed fine in this section, to identify and illustrate a central capital-market distortion created by fines. The government’s goal is to set the regulation parameters (the inspection rate, fines and possibly taxes) in a manner that maximizes social welfare.

The novel feature of our analysis is that we take into account the impact of regulation on factor market decisions. Thus, we begin by examining the firm’s choice of inputs. We assume that each risk-neutral firm produces a single unit of output \( x \) using two inputs, entrepreneurial activity \( e \) and capital \( k \), according to a production function, \( q(e, k) \), with neo-classical properties. Capital is provided by investors, who are promised that after all markets clear, they will be repaid the
principal of the loan along with interest at rate $r$. The principal consists of the unit of capital, which does not depreciate, and the cost of a unit of entrepreneurial activity is normalized at one.\(^6\)

The firm’s ability to repay investors will be determined by its choice of inputs, its behavior with respect the law, and the size of the potential punishment. In particular, since entrepreneurial assets are the residual claimants, the firm will have assets of $p + k$ to pay principal, interest and fines, where $p$ denotes the price of the product. If an evading firm chooses an input mix that ties up its liquidity, then the fine is paid first and any remaining assets go to investors. If investors receive less than the principal and interest owed to them, the firm is said to be “bankrupt.” Evading firms that leave themselves with more liquidity may be able to pay large fines without bankruptcy.

The firm’s input decision is depicted in Figure 1 with the convex curve representing the unit isoquant. For law-abiding firms, the isocost curve is a straight-line with a slope of $-r$ and, as is usual, the firm minimizes costs at the tangency of the two curves. These firms always use an efficient mix of inputs if $r$ equals the social opportunity cost of capital (denoted by $r^*$). Things are somewhat different for evaders; for them, the slope of the isocost curve will also depend on the regulation parameters. To see this, note that for any given level of the fine, $F$, there exists a critical level of capital, $k_F \equiv \frac{p-F}{r}$, such that an evader that selects $k \geq k_F$ will be bankrupt by the fine if caught violating the law. This firm will realize that it’s effective cost of capital changes at $k_F$. If the firm selects $k \leq k_F$, then it will carry sufficient liquidity to pay the fine and fully repay investors regardless of circumstances. In this range, the firm’s effective cost of capital is the same as it is for a law-abiding firm, $r$. However, if the firm selects $k_B \geq k_F$, it will fully compensate investors when it

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\(^6\) We ignore the effects of corporate and personal taxes on the cost of capital. In the absence of these considerations, the analysis does not depend on whether firms finance capital with equity or debt. In the case of equity financing, $r$ becomes a required return on equity that firms must pay if they are not bankrupt. If firms use both debt and equity at the margin, then the required returns may differ, if bankruptcy reduces payments to debt holders, but not equity holders. We avoid this complication, since it would require that we develop a theory of the firm’s financial structure.
successfully evades the law, but it will be able to pay investors only the amount $p + k_B - F$ if fined. If we use $\pi$ to denote the inspection rate, then the marginal cost of capital for evaders is $(1 - \pi)r$ for $k \geq k_F$. The basic idea is that increasing $k$ a unit, financed with borrowing, provides the firm with another unit of assets to pay back principal, but there are no additional assets to pay interest in the event the firm is fined; investors receive no interest income with probability $\pi$.\(^7\) A higher inspection rate lowers this marginal cost because it increases the probability that the interest on additional investment is effectively paid by the government through reduced fine payments, at no additional cost to the firm. As a result, the isocost curve facing an evader is kinked, with a slope of $-r$ for $k \leq k_F$ and $-(1 - \pi)r$ for $k \geq k_F$. Since the kink occurs at $k_F$, it will never be optimal for the firm to use the level of capital that leaves it exactly bankrupt when fined.

Figure 1 illustrates the case where an evading firm is indifferent between choosing low and high levels of $k$. In other words, the kinked isocost curve has two tangencies with the indifference curve, one on each side of the kink. More generally, when the when the fine is low, the kink occurs at a low value for $k$, and it is optimal for the firm to operate on the steep portion of the isocost curve, at a point such as A in Figure 1. However, when the fine is high, the kink occurs at a low value of $k$, and the firm will operate along the flatter portion of the isocost curve, at a point such as B. In other words, a high enough fine raises the marginal cost of capital from $r$ to $(1 - \pi)r$, causing the firm to increase its capital from $k_A$ to $k_B$, and insuring bankruptcy in the event of an inspection.

We assume that violation of the regulation produces some external costs that may vary with the type and quantity of capital and also with the level of output. In designing the optimal policy, the government then faces trade-off. If it uses low fines and frequent inspection, which we refer to as Pigouvian regulation, firms will use the proper mix of inputs. While there may be significant enforcement costs, factor markets will operate efficiently. Also, satisfying the constraints of

\(^7\)To be precise, for any given $F$, the expected cost of producing one unit of output is $e + rk + \pi F$ when $k \leq k_F$ and $e + \pi p + (1 - \pi)r k$ when $k \geq k_F$. Thus, the marginal cost of capital is $r$ for $k \leq k_F$ and $(1 - \pi)r$ for $k \geq k_F$. 

Pigouvian regulation may limit the achievable level of compliance. The other option, which we refer to as Beckerian regulation, is to use severe fines with a low rate of inspection. We show that this approach can attain more compliance, but it leads firms to distort their mix of inputs. This option has low enforcement costs, but this benefit must be weighed against the cost associated with inefficiency in the factor markets. Below we show that Beckerian regulation can be desirable under a variety of circumstances, including both low and high external costs from production, and high unit inspection costs. However, Pigouvian regulation may be preferred in cases where these external costs are associated with the usage of capital, not output, since it avoids the excessive use of capital described above.

3. Pigouvian Regulation

We are now ready to begin our formal analysis, which we divide into three parts. First, in this section, we confine our attention to situations in which the government finds it optimal to use low or modest fines, so that evading firms are not driven to bankruptcy if caught. In the next two sections, we consider the case of severe fines, and, finally, in Sections 5 and 6 we compare the two outcomes to find the globally-optimal enforcement mechanism.

Each of our perfectly competitive firms employs entrepreneurial activity $e$ and capital $k$ to produce a unit of output. The regulation both restricts $k$ to some socially-optimal level, $k^*$, and requires that firms reduce any external costs associated with production. In general these external costs will depend on both the level of $k$ and on the regulations involving the production process (e.g., emission controls in the case of pollution or restrictions on the use of particular financial instruments in the case of financial firms). But to simplify the analysis, we assume that firms that comply with the regulation produce no external costs, whereas those who evade the regulation generate $\beta k + \eta$ units of “external activity”. If the total output of the private good produced by non-compliant firms is $y_n$, then total external output is $x_n \equiv (\beta k + \eta)y_n$, which generates an external cost equal to $h(x_n)$. 
where $h$ is strictly convex. In other words, external costs are allowed to depend on both capital usage and output.

Firms are identical in all aspects except one, the cost of compliance. We use $\alpha$ to denote a firm’s cost of complying with the regulation, and we assume that this firm-specific parameter is drawn after the firm enters the market from a continuous distribution function, denoted by $G(\alpha)$. Since a complier, or “legal firm,” generates no external costs, it is always socially optimal for this firm to choose its capital and entrepreneurial inputs to minimize costs at the social opportunity cost of capital, denoted $r^*$. Letting $c^*_f$ denote this minimized costs, the total cost of production and compliance is $c^*_f + \alpha$ for a legal firm with compliance cost $\alpha$.  

Evaders choose their own capital levels and do not incur compliance costs, but they risk detection and punishment. The probability of detection is the inspection rate, $\pi$, which is the same for all firms. The total fine depends linearly on the firm’s capital, $F = tk + T$. Thus, the expected total cost for an evader firm is $c_n(r + \pi t) + \pi T$, where $r$ is the interest rate that investors charge the firm, and the cost of capital now includes the expected marginal fine on capital. We assume that investors obtain capital at the economy-wide rate (opportunity cost) of $r^*$. In the Pigouvian equilibrium, firms that evade the regulation choose to carry enough liquidity to repay investors fully, in which case investors charge all firms the interest rate $r = r^*$.

A firm that is indifferent between complying and not complying with the regulation has a compliance cost, $\bar{\alpha}$, that satisfies

$$
\bar{\alpha} = \pi T + c_n(r^* + \pi t) - c^*_f
$$

All firms with $\alpha \leq \bar{\alpha}$ prefer to operate legally; and all firms with $\alpha \geq \bar{\alpha}$ prefer to evade the regulation.

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8 Note that the unit cost function includes only payments for capital and entrepreneurial effort.
To complete the model, we now describe the timing of decisions. In the initial stage, ex ante identical firms decide whether to enter the market. In stage two, $\alpha$ is revealed and firms make their input and compliance decisions. In particular, they decide on the mix of entrepreneurial and capital inputs and sign contracts with investors. In stage three, production occurs and the product market clears. In stage four, the regulatory authority randomly inspection firms, detects non-compliance, and assesses fines, which must be paid immediately. Finally, in the last stage, investors are paid. The crucial assumption here is that the government collects fines before investors are paid. If the fine is set at a high level, then there may not be sufficient assets available to repay the investors if the firm is detected cheating.

We solve the model by backwards induction. The solution the firm’s compliance decision is as determined by (1). For the entry decision, since the firms do not know their value of $\alpha$ before entry, their expected profits from production are given by

$$E\Pi(p) = \int_0^{\alpha^*} [p - c^*_e - \alpha]dG(\alpha) + \int_{\alpha^*}^{\alpha^m} [p - c_n(r + \pi t) - \pi T]dG(\alpha) - S,$$

where $p$ is the price of the product, $S$ is a sunk cost of entry, and $\alpha^m$ is the maximum value of $\alpha$ among firms that produce. For any given set of enforcement parameters $(T, t, \pi)$, there is a unique value of $p$ at which expected profits are zero. For all higher $p$, all firms enter and there will be excess supply in the product market; for all lower $p$, no firm produces. Solving $E\Pi(p) = 0$ for $p$ and using (1) yields the market-clearing price:

$$p = c^*_e + \alpha^* [1 - G(\alpha^*)] + \int_0^{\alpha^*} \alpha dG(\alpha) + S.$$

We assume that the government also collects revenue from consumers by imposing a sales tax of $\tau$ on this good, so that the price paid by consumers for each unit is $q \equiv p + \tau$. The assumption here is that while some firms evade the regulation, all firms pay the tax. For example, a regulation

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9 Firms with higher values of $\alpha$ might enter the market, but they will then exit without producing, because they are unable to cover their variable production and compliance costs.
concerning a production process may be evadable, while no good possibilities exist for selling the product without paying a sales tax.\textsuperscript{10}

On the demand side of the product market, the representative consumer has the following quasi-linear utility function:

\begin{equation}
U(x, q) = E - q + v(x) - h(x_n),
\end{equation}

where $E$ denotes the consumer’s lump-sum income and $x$ is total output.\textsuperscript{11} Income $E$ consists of an endowment of the numeraire good, plus a government transfer financed by tax revenue and fines. The consumer treats $x_n$ as fixed and chooses $x$ to maximize utility. Thus, $x$ satisfies the following first order condition,\textsuperscript{12}

\begin{equation}
v'(x) = q = p + \tau
\end{equation}

Summarizing the product market, the producer price of output, $p$, is determined by the free-entry condition and is given by (3). Total output, $x$, is determined by the sales tax $\tau$ and the solution to the consumer’s maximization problem, given by (5). Since each firm produces one unit of output, $x$ also denotes the number of firms with $y_n = 1 - G(\bar{\alpha})$ of these firms evading the regulation.

If evaders choose a relatively low level of capital (so that they carry sufficient liquidity to fully repay investors in all cases), their expected costs are $c_n(r^* + \pi \tau) + \pi T$, as previously discussed. But a higher level of capital (as depicted by B in Figure 1) results in expected costs of $c_n[(1 - \pi)r^*] + \pi p$, since the fine bankrupts the firm.\textsuperscript{13} As described in the previous section, the higher level of capital entails a lower effective cost of capital and leads to a lower payment by the firm when

\textsuperscript{10} For analyses of the welfare effects of black-market activities undertaken to evade taxes, see Davidson et al. (2005, 2007).
\textsuperscript{11} Production by law-abiding firms creates no external costs because these firms comply with the regulation.
\textsuperscript{12} We assume that $E$ is large enough that (5) is satisfied for all relevant $q$.
\textsuperscript{13} At point B in Figure 1, the firm pays $e_B$ to entrepreneurs, $(1 + \tau)k_B$ to capital owners, including principal, when not inspected (which occurs with probability $1 - \pi$) and $k_B + p - F$ to capital owners, when inspected (which occurs with probability $\pi$). Thus, expected production costs at B (excluding principal) are $e_B + (1 - \pi)r k_B + \pi(p - F)$. In addition, the firm faces an expected fine of $\pi F$. Summing to get total expected costs, we obtain $e_B + (1 - \pi)r k_B + \pi p = c[(1 - \pi)r] + \pi p$. 

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caught evading the regulation. It is important to note that the expected marginal fine on capital, \( \pi t \), no longer enters the cost of capital. Evaders who are not inspected pay no fine, and evaders who are inspected surrender all of their assets to the government and investors. A rise in \( k \) may increase the amount owed to the government, but the firm does not care about the split of its assets between investors and the government; costs would not change if the government were given all of the firm’s assets, leaving investors with none.

For the lower level of capital to be optimal for the firm, as required for a Pigou equilibrium, it must lead to lower or the same expected costs, which occurs when

\[
\pi(p - T) \geq c_n(r^* + \pi t) - c_n[(1 - \pi)r^*]
\]

Thus, bankruptcy will not occur in equilibrium if (6) is satisfied by the government’s chosen regulation parameters. We refer to (6) as the “Pigou constraint.”

We now turn to the government’s problem of optimal enforcement. In addition to the external cost of \( h(x_n) \), the government must also be concerned about the resources that it devotes to enforcement. This cost is given by \( p_\alpha \pi x \), where \( p_\alpha \) denotes the cost of inspecting one firm and \( \pi x \) is the total number of inspections that are carried out. Social welfare (\( W \)) is given by

\[
W = v(x) - h(x_n) - x\left[c'_n G(\bar{a}) + c_n^*(r + \pi t)(1 - G(\bar{a}))\right] + \int_0^{\bar{a}} \alpha dG(\alpha) + p_\alpha \pi + S
\]

where \( x_n = [\beta k(r + \pi t) + \eta]x[1 - G(\bar{a})] \) and \( c_n^*(r + \pi t) = e(r + \pi t) + r^*k(r + \pi t) \); that is, the asterisk indicates that we are evaluating the evader’s profit-maximizing inputs at the social opportunity cost of capital, \( r^* \). We assume that lump-sum transfers are available to balance the government budget. Using (1) and (3), we may rewrite the Pigou constraint, given by (6), as follows:

\[
\pi\{c'_n + \bar{a}[1 - G(\bar{a})] + \int_0^{\bar{a}} \alpha dG(\alpha) + S\} - \bar{a} \geq c_n^* - c_n[(1 - \pi)r^*].
\]

The government’s problem is to select the policy variables \( \pi, T, t \) and \( \tau \) to maximize social welfare, subject to the Pigou constraint and the market equilibrium conditions. But the equilibrium
conditions have already been used to state the problem as the maximization of (7), subject to (8). Note first that the fine $T$ does not appear in the problem. Rather, it is replaced with the marginal compliance cost, $\bar{\alpha}$, as a control variable. Second, the sales tax is replaced by output $x$. Thus, the control variables are $\pi$, $t$, $\bar{\alpha}$, and $x$. After solving for their optimal values, we can return to the equilibrium conditions and find the values of $T$ and $\tau$ that support the equilibrium.

Maximizing (7) over $x$ yields the following first-order-condition

$$
v'(x) - h'(x_n)(\beta k_{na} + \eta)(1 - \bar{G}) - [c_i^* \bar{G} + c_{na}^*(1 - \bar{G}) + \int_0^\bar{\alpha} a dG(\alpha) + p_a \pi + S] = 0 \tag{9}
$$

where, to shorten notation, we have defined $\bar{G} = G(\bar{\alpha})$, $c_{na}^* = c_n^*(r + \pi t)$, and $k_{na} = k(r^* + \pi t)$. The subscript “na” indicates a non-compliant firm’s input decision in the case where the firm has sufficient leverage to pay the fine and investors in full (think “a” for an A rated investment grade). If we use (5) to substitute for $v'(x)$, (3) to substitute for $p$, and then solve for $\tau$, we obtain

$$
\tau = [1 - \bar{G}][h'(x_n)(\beta k_{na} + \eta) - \bar{\alpha}] + p_a \pi. \tag{10}
$$

The tax is positive for two reasons. First, when another firm produces, expected inspections rise, with an expected cost equal to $p_a \pi$. Second, the additional firm generates an expected external cost equal to $[1 - \bar{G}]h'(x_n)(\beta k_{na} + \eta)$, where $1 - \bar{G}$ is the probability that the entrant will fail to comply with the regulation. But there is a decline in expected total compliance costs equal to $[1 - \bar{G}]\bar{\alpha}$. We show below that the excess of this external cost over the reduced compliance cost is positive when inspections are costly. Hence, a positive sales tax is needed to internalize this excess external cost, plus the additional inspection cost.

Note next that the marginal fine on capital, $t$, enters only the objective function. Thus, we may differentiate the objective function with respect to $t$, and obtain the following first-order condition:

$$
t \pi = h'(x_n)\beta \tag{11}
$$
This is the usual Pigouvian rule: the tax on another unit of an externality-producing activity should equal the marginal external cost from that activity. Here the activity is additional investment. Note that inspection costs do not alter the rule, because \( t \) can be adjusted without altering the total number of inspections or the number of firms that choose to evade the regulation, simply by offsetting any change in \( t \) with a change in \( T \), which is the component of the fine that is independent of the firm’s capital. In fact, if the optimal total fine, \( F_{na} = tk_{na} + T \), is low, then it may be necessary for \( T \) to be negative to support the optimality conditions for both \( \bar{a} \) and \( t \).

The remaining control variables are \( \bar{a} \) and \( \pi \), which relate to our central concern: what are the relative uses of fines and inspections in optimal punishments? Since inspections are costly, the government will clearly want to minimize their use, but faces the Pigou constraint (eq. 8). Thus, this constraint holds with equality, and we can use it to define the inspection rate as a function of the marginal compliance cost, \( \pi(\bar{a}) \), thereby eliminating the constraint from the optimization problem and leaving \( \bar{a} \) as the remaining control variable. Implicit differentiation of the Pigou constraint, noting that the term in curly brackets is \( p \), yields

\[
(12) \quad \frac{d\pi}{d\bar{a}} = \frac{1 - \pi(1 - \bar{a})}{p - r^*k_{nb}},
\]

where \( k_{nb} \equiv k[(1 - \pi)r^*] \). The subscript “\( nb \)” indicates that this is a non-compliant firm that chooses the higher level of capital that bankrupts it in the event of a fine.

Differentiating objective function (7) with respect to \( \bar{a} \) now gives the first-order condition

\[
(13) \quad [h'(x_n)(\beta k_{na} + \eta) - \bar{a}]\bar{g} = p a \frac{d\pi}{d\bar{a}},
\]

where \( \bar{g} \equiv G'(\bar{a}) \). As previously described, the term in the square brackets is the net external benefit of additional compliance, recognizing that when another firm complies with the regulation, total compliance cost rise by \( \bar{a} \). If we use a higher fine \( T \) to increase the marginal compliance rate a unit, then \( \bar{g} \) additional firms comply, producing the marginal net external benefit on the left side of
(13). But to do so while still satisfying the Pigou constraint, we must raise the inspection probability by \( d\pi/d\tilde{a} \), generating a marginal cost of \( p_\alpha (d\pi/d\tilde{a}) \). At the optimum, the marginal benefit equals the marginal cost. This equality is illustrated in Figure 2, where the horizontal axis measures the reduction in external activity, \( \Delta x_n \). Note further that if external damages are large, the government will want to increase the severity of its policy to deter non-compliance.

To understand the determinants of \( d\pi/d\tilde{a} \), note that with a binding Pigou constraint (8), \( p = \tilde{a} + \{c_\epsilon^* - c_n[(1 - \pi)r^*]\}/\pi \). Thus, (12) may be rewritten as follows:

\[
(14) \quad \frac{d\pi}{d\tilde{a}} = \frac{\pi[1-\pi(1-\tilde{c})]}{\tilde{a} - (c_\epsilon^*[1-\pi(r^*)-c_\epsilon^*])},
\]

where once again an asterisk on the evaders cost function indicates that its profit-maximizing inputs are being evaluated at the social opportunity cost of capital. The expression in the curly brackets is the excess of this social cost over then minimized cost of production, evaluated at \( r^* \), which is the same cost paid by legal firms, \( c_\epsilon^* \). This is the usual definition of deadweight loss from a tax or subsidy distortion. We assume that a firm’s capital demand goes to infinity as the cost of capital goes to zero, in which case this deadweight loss goes to infinity as \( \pi \) goes to one. In this case, there will be a maximum feasible \( \tilde{a} \), denoted \( \tilde{a}^{max} \), at which \( d\pi/d\tilde{a} \) goes to infinity. We assume that \( G(\tilde{a}^{max}) < 1 \); that is, it is not feasible to obtain complete compliance as a Pigou equilibrium.

This deadweight loss may be approximated by the usual quadratic loss formula:

\[
(15) \quad L_{sb} = c_n^*[1-\pi r^*] - c_\epsilon^* = \frac{1}{2}\pi r^*[k_{nb} - k_\epsilon] = \frac{1}{2}\pi^2 r^* k_\epsilon \varepsilon,
\]

where \( k_\epsilon = k(r^*) \) and \( \varepsilon \) is the elasticity of demand for capital, evaluated at \( r^* \). The asterisk subscript indicates that this loss is calculated treating the socially-optimal cost of capital equal to \( r^* \),
since this is the way the loss enters (14). This approximation becomes exact when the capital demand curve is linear. Substituting (15) into (14) yields

\[
\frac{d\pi}{d\bar{a}} = \frac{\pi(1-\pi(1-\bar{c}))}{\bar{a}^{1-\pi}r^*k\epsilon}
\]

It may seem strange that the marginal cost of compliance depends positively on the capital distortions associated with overleveraged firms -- that is, firms that would be bankrupted by fines -- because there are no overleveraged firms in the Pigou case. But the presence of these distortions can be explained by noting from the binding Pigou constraint (eq. 8 with an equality) that when \(\bar{a}\) rises by some marginal amount, the required rise in \(\pi\), \(d\pi\), lowers production costs \(c_n[(1-\pi)r^*]\) by \(k_{nb}r^*d\pi\) for overleveraged firms, which by itself makes the overleverage option more attractive and therefore makes a given marginal rise in \(\pi\) less effective in restoring indifference about becoming overleveraged. The greater the fine and inspections distort the overleveraged firm’s choice of capital, given by \(k_{nb} - k\epsilon\), the more \(\pi\) must be raised. The deadweight loss variable is positively related to the same \(k_{nb} - k\epsilon\).

The value of \(\pi(\bar{a})\) is determined by the differential equation given by (14), once initial conditions are specified. We know that \(\pi(0) = 0\), but this alone does not determine \(d\pi(\bar{a})/d\bar{a}\) because the numerator and denominator in (16) are both zero at \(\bar{a} = 0\). Rather, we can use (8) to find an expression for the limiting value of \(\bar{a}\) as \(\pi\) goes to zero: \(\bar{a} = \pi[e(r^*) + S]\). Substituting this expression into (16) and taking the limit as \(\pi\) goes to zero gives

\[
\frac{d\pi(0)}{d\bar{a}} = \frac{1}{e(r^*) + S}
\]

Thus, a rise in \(e(r^*) + S\) lowers the initial value of \(d\pi/d\bar{a}\), presumably leading to lower inspection costs at positive values of \(\bar{a}\); that is, Pigouvian regulation becomes more attractive. The basic idea is

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14 But this measure of deadweight loss does not recognize the external costs associated with additional \(k\). Another measure is introduced below when we examine changes in social welfare resulting from overleveraged firms.
that the higher are fixed costs and entrepreneurial returns, the higher is the equilibrium price of output, and this higher price enables a given rate of compliance to be maintained with a higher fine and lower inspection rate.\footnote{The analysis could be generalized without altering the results by assuming that the initial owners of the firm own some amount of the capital. In this case, the value of this capital at \(r^*\) would be added to the denominator of (17) and higher fines would be possible in a Pigou equilibrium.}

4. Becker and Hybrid Equilibria

We now consider enforcement policies that bankrupt at least some inspected evaders. Fines are high enough to bankrupt such firms when Pigou constraint (6) is reversed; that is,

\[(18) \quad \pi(p - T) \leq c_n(r + \pi t) - c_n[(1 - \pi)r],\]

where \(r\) may now exceed \(r^*\) to compensate investors for the possibility of bankruptcy. We refer to (18) as the “Becker constraint.” When it holds with a \textit{strict} inequality, evading firms and legal firms will \textit{always} use different amounts of capital to minimize their cost of production. In particular, as described in Section 2, evaders will choose a higher level of capital, because they realize that if they fined, the marginal capital will be costless. Firms that operate at this point cannot pay their debts when fined, a situation we have referred to as “overleveraged.” Equilibria where all evaders are overleveraged are referred to as “Becker equilibria.”

If the Becker constraint holds with equality, then evaders will be indifferent between points A and B in Figure 2, and it is possible to have an equilibrium in which a fraction of evaders, \(\gamma < 1\), are overleveraged, with the remainder operating at a point like A. We refer to such equilibria as “hybrid equilibria.” In this case, only those inspected evaders that are overleveraged will be driven to bankruptcy by the fine. To summarize, \(\gamma = 1\) in a Becker equilibrium, \(\gamma \in (0,1)\) in a hybrid equilibrium, and \(\gamma = 0\) in a Pigou equilibrium.

When the government inspects an overleveraged evader, it will now lay claim to some income owed investors in an attempt to collect the unpaid fines. These anticipated seizures will
distort the capital market and lead to a higher price of capital for the regulated market. In
equilibrium, the profits earned by investors from supplying capital to this industry must exactly offset
losses associated with the expected seizures. The government inspects a particular firm with
probability \( \pi \) and seizes \( F - (p - rk) \) units of assets from that firm if it has not complied with the
regulation. Since the fraction of firms that evade the regulation is \( 1 - \bar{G} \) and \( \gamma \) is the fraction of
evaders that are overleveraged, it follows that expected seizures are \( \pi \gamma [1 - \bar{G}] (F - p + rk_{nb}) \). All
law-abiding firms employ the socially-optimal capital level, \( k_{e} = k(r^{*}) \), a fraction \( (1 - \gamma) \) of all
evaders employ \( k_{na} = k(r + \pi t) \) units of capital, and a fraction \( \gamma \) employ \( k_{nb} = k[(1 - \pi)r] \) units. Thus, since the investors pay \( r^{*} \) for the capital, their expected profits from supplying capital to this
industry at rate \( r \) are given by \( (r - r^{*})\{k_{e}\bar{G} + (1 - \bar{G})[(1 - \gamma)k_{na} + \gamma k_{nb}]\} \) in the absence of
seizures. The equilibrium \( r \) is determined by the requirement that these expected profits equal
expected seizures:

\[
(19) \quad (r - r^{*})\{\bar{G}k_{e} + (1 - \bar{G})[(1 - \gamma)k_{na} + \gamma k_{nb}]\} = \pi \gamma (1 - \bar{G})\{T - p + (r + t)k_{nb}\},
\]

noting that the expression in the curly brackets is the excess of the fine on overleveraged firms, \( F_{nb} \),
over the difference between total assets, \( p + k_{nb} \), and money owed to investors, \( (1 + r)k_{nb} \). Since
the right-hand-side of (19) is positive in a Becker or hybrid equilibrium, it must be the case that
\( r > r^{*} \) in any such equilibrium. Thus, capital is paid a premium in the regulated industry.

This excess of \( r \) over \( r^{*} \) is a major difference between a Pigou equilibrium and a hybrid or
Becker equilibria. Note, however, that since \( r \) depends on the total fine, not its composition, we are
free to change this composition without causing changes in \( r \). In the case of a hybrid economy,
differentiation of the objective function with respect to the marginal expected fine on capital gives a
modified Pigou rule:

\[
(20) \quad t\pi = h'(x_{n})\beta - (r - r^{*}),
\]
which says that the expected fine should be reduced by the excess of $r$ over $r^*$ to offset any investment distortions from the higher interest rate. In other words, the government has the tools to maintain efficient investment incentives for evaders who are not overleveraged. But such tools do not exist for overleveraged firms, since the fine becomes lump-sum in the case of bankruptcy. This is a major shortcoming of large fines.

Consider now the determination of the marginal compliance cost. Although legal firms are required to use the socially-efficient capital, $k_\ell$, they now must pay $r$ for this capital, so their costs rise to the level, $c_\ell(r) = e(r^*) + rk(r^*)$, which exceeds $c_\ell^* (r - r^*)k(r^*)$. In a hybrid equilibrium, the marginal compliance cost equates $c_\ell(r) + \bar{\alpha}t$ to the common total cost for all evaders:

$\bar{\alpha} = \pi T + c_n(r + \pi t) - c_\ell(r).$  

But in a Becker equilibrium, evaders strictly prefer to be overleveraged, in which case we have seen that their expected cost is $\pi p + c_n[(1 - \pi)r]$. In this case, the marginal compliance cost is

$\bar{\alpha} = \pi p + c_n[(1 - \pi)r] - c_\ell(r).$  

Turning to the product market, we can use this equality between costs for marginal legal and evader firms and write the equilibrium price in a form that is similar to (3), modified to reflect the higher interest rate:

$p = c_\ell(r) + \bar{\alpha}[1 - \bar{\alpha}] + \int_0^{\bar{\alpha}} \alpha dG(\alpha) + S.$  

Finally, output and the number of firms are determined, as in the previous section, by the demand side of the product market – in particular, (5).

Social welfare in a Becker or hybrid equilibrium is given by a form similar to (7), modified to reflect the social costs for overleveraged firms, $c_n^*((1 - \pi)r)$, and the higher interest rate $r$ faced by all evaders:
\begin{equation}
W = v(x) - h(x_n) - x[C^*(r, \bar{\alpha}) + \int_0^{\bar{\alpha}} \alpha d\tilde{G}(\alpha)] - p_a \pi x;
\end{equation}

where \( C^*(r, \bar{\alpha}) \equiv c^i \tilde{G} + (1 - \tilde{G})\{(1 - \gamma)c^i_n(r + \pi t) + \gamma c^i_n[(1 - \pi)r]\} \) is the expected social cost of production (that is, the cost of capital is evaluated at its opportunity cost, \( r^* \)). The only difference between this welfare expression and the welfare expression for a Pigou equilibrium is that overleveraged firms now employ an inefficient input mix.

\section*{5. Is a Hybrid Equilibrium Better than the Pigou Optimum?}

We now investigate the conditions under which welfare can be improved by moving from the Pigou optimum to a fine structure and inspection rate that causes some firms to be overleveraged; that is, the economy moves to a hybrid equilibrium. In this case, the introduction of overleveraged firms raises both \( \gamma \) and \( r \), from \( \gamma = 0 \) and \( r = r^* \). But the change in \( r \) does not directly cause any marginal distortions, because all existing firms in the Pigou equilibrium are choosing their socially-optimal capital levels. Rather, only the increase in \( \gamma \) matters, and it lowers welfare because each new overleveraged firm is creating a deadweight loss, \( L_{ab} \), by using too much capital. The subscript \(^{ab}\) indicates that the socially optimal use of capital would be the one chosen by an evader that was not overleveraged, \( k_{na} = k(r^* + h'(x_n)\beta) \), whereas instead the overleveraged firm chooses \( k_{nb} = k[(1 - \pi)r^*] \). Thus, the socially-excessive resource cost is

\begin{equation}
L_{ab} = \{e_{nb} + [r^* + h'(x_n)\beta]k_{nb}\} - c_n(r^* + h'(x_n)\beta),
\end{equation}

and the marginal welfare change per unit of output \( x \) is

\begin{equation}
\frac{dW}{x} = (h'(x_n)(\beta k_{na} + \eta) - \bar{\alpha})\tilde{g} d\bar{\alpha} - p_a d\pi - (1 - \tilde{G})L_{ab} d\gamma.
\end{equation}

The first and second terms in (26) apply also to the Pigou equilibrium and would be equated to zero if the changes in \( \pi \) and \( \bar{\alpha} \) were required to satisfy the binding Pigou constraint. But now \( \bar{\alpha} \) can be increased with a lower increase in costly inspections. However, raising \( \bar{\alpha} \) with fewer additional
inspections means a greater increase in fine, which generates the movement of some firms to the overleveraged status. The third term in (26) gives the resulting welfare loss. If we now define
\[ k_\mu = \tilde{G} k_\ell + (1 - \tilde{G}) k_{na} \] as the average capital used by firms at the Pigou optimum, we can use this welfare expression to obtain:

**Proposition 1:** Starting from the Pigou optimum, a small increase in the fine and inspection rate that causes some firms to become overleveraged is desirable (undesirable) if
\begin{align*}
(27) \quad & \frac{(1 + \frac{\pi}{1 - \alpha}) (k_\mu - k_\ell)}{[h'(x_n)(\beta k_{na} + \eta) - \alpha \tilde{g}]} < (>) 1.
\end{align*}

**Proof:** Start with the optimal \((\pi, \alpha)\), determined by (13) and (14):
\begin{align*}
(28) \quad & h'(x_n)(\beta k_{na} + \eta) \tilde{g} = \alpha \tilde{g} + p_a \frac{d\pi}{d\alpha}, \quad \frac{d\pi}{d\alpha} = \frac{\pi[1 - \pi(1 - \tilde{g})]}{\alpha - L_{sb}}.
\end{align*}

Next, implement a perturbation in the inspection rate and fine that involves increasing \(\alpha\) a marginal unit, but with a rise in \(\pi\) that is an amount \(\delta\) less than the amount needed to remain in the Pigou regime:
\begin{align*}
(29) \quad & \frac{d\pi}{d\alpha} = \frac{\pi[1 - \pi(1 - \tilde{g})]}{\alpha - L_{sb}} - \delta.
\end{align*}

With evaders indifferent about becoming overleveraged in the hybrid equilibrium, the Becker constraint (18) holds with equality, and (21) can be used to rewrite it in the same form as Pigou constraint (8), modified to reflect a possible \(r > r^*\):
\begin{align*}
(30) \quad & \pi\{c_\ell(r) + \alpha - \alpha(1 - \tilde{G}) + \int_0^{\alpha} \alpha dG(\alpha) + S\} - \tilde{g} = c_\ell(r) - c_n[(1 - \pi)r].
\end{align*}

We may differentiate (30) to find the change in \(r\) from \(r^*\) needed to keep evaders indifferent about becoming overleveraged, following the changes in the expected fine and inspection rate satisfying (29):
Differentiating the condition for capital market equilibrium, given by (19), we obtain the marginal effect of a rise in \( r \) from \( r^* \) on the fraction of firms that choose to become overleveraged:

\[
\frac{d\gamma}{dr} = \frac{\delta(p-rk_{nb})}{(k_{nb}-k_{\ell})(1-\pi)}
\]

where \( k_{\mu} \) is defined in the proposition, and the second equality uses the binding Becker constraint (18), evaluated at \( r = r^* \), with the definition of deadweight loss \( L_{ab} \).

Multiplying (31) and (32) together then gives

\[
\frac{d\gamma}{d\alpha} = \frac{\delta k_{\mu}(p-rk_{nb})}{(1-\delta)(1-\pi)(k_{nb}-k_{\ell})L_{ab}}.
\]

At the Pigou optimum, we know that the welfare change given by (26) equals zero when the change in the inspection rate satisfies (29) with \( \delta = 0 \). In this case, there is no change in \( \gamma \), since we are moving along the binding Pigou constraint. Thus reducing \( d\pi/d\alpha \) by a positive \( \delta \), thereby moving into a hybrid equilibrium, as described by (32), allows us to rewrite the welfare change in (26) as

\[
p_{a}\delta - \frac{d\gamma}{d\alpha} L_{ab} (1 - \tilde{G}) = p_{a}\delta - \frac{\delta k_{\mu}(p-rk_{nb})}{(1-\pi)(k_{nb}-k_{\ell})}.
\]

Substituting for the numerator from the optimality conditions (12)-(13), we then find that welfare rises (falls) if

\[
\delta p_{a} - \frac{\delta p_{a}[1-\pi(1-\delta)]k_{\mu}}{[h'(x_{n})(\beta k_{na} + \eta) - \alpha\bar{\gamma}(k_{nb}-k_{\ell})(1-\pi)]} > (<) 0
\]

Rearranging (35) proves (27). Q.E.D.

We have previously described the term in the denominator of (27) as the net external benefit from additional compliance, where additional compliance is measured by increasing \( \tilde{\alpha} \) a marginal
unit. Let us denote this marginal benefit by $MB_{\bar{a}}$. In a first-best economy with no inspection costs, $MB_{\bar{a}}$ would equal zero. Here, inspection costs imply that it is positive at the Pigou optimum, as determined by first-order condition (13). Thus, $MB_{\bar{a}}$ measures the extent to which the Pigou constraint binds, which helps explain why Proposition 1 indicates that breaking the constraint by moving to a hybrid equilibrium is desirable if $MB_{\bar{a}}$ is high, all else equal.

On the other hand, the numerator of (27) is related to the costs involved in moving into the hybrid region. Consider the term, $k_{nb} - k_\ell$. As $\bar{a}$ increases into the hybrid region, the market interest rate $r$ rises to support the higher $\bar{a}$ by increasing the effective punishment of detected evaders who are overleveraged. It does so by raising their expected costs, $\pi p + c_\alpha [(1 - \pi)r]$, relative to the rise in the costs for the marginal legal firm, $\bar{a} + c_\ell(r)$. The difference between these two marginal cost increases is $(k_{nb} - k_\ell)(1 - \pi)$. If $k_{nb}$ is close to $k_\ell$, a large increase in $r$ is then required to support the higher $\bar{a}$, and the greater excess of $r$ over $r^*$ implies a greater influx of overleveraged firms into the market; that is, $d\gamma/d\bar{a}$ rises as $k_{nb} - k_\ell$ falls, indicating that the rise in $\bar{a}$ causes firms to become overleveraged at a greater rate, increasing the deadweight losses associated with overleveraged firms. Indeed, $d\gamma/d\bar{a}$ goes to infinity as $k_{nb} - k_\ell$ goes to zero, which happens as the inspection rate goes to zero. As a result, we can prove:

**Proposition 2.** For any unit inspection cost, $p_a' > 0$, there exists a marginal compliance cost, $\bar{a}'$, such that if $p_a > p_a'$ and the inspection and external cost parameters, $(p_\alpha, \beta, \eta)$, imply a Pigou optimum with positive marginal compliance cost $\bar{a} < \bar{a}'$, then any marginal policy change from this optimum that creates overleveraged firms must lower welfare.

**Proof:** The parameter restrictions allow us to set $\bar{a}$ arbitrarily close to zero at the Pigou optimum. But as $\bar{a}$ goes to zero, $\pi$ goes to zero, causing $k_{nb} - k_\ell$ to go to zero. Thus, the numerator of (27) goes to infinity. On the other hand, (17) and optimality condition (13) show that the denominator
stays bounded from below by some positive number, provided \( p_a \) does not go to zero, as assumed. Proposition 2 then follows from Proposition 1. Q.E.D.

Thus, if inspection costs are high enough, or external costs are low enough, to imply a low level of compliance at the Pigou optimum, then it is not possible to raise welfare by moving to a hybrid equilibrium. This conclusion may seem counter-factual, because a high \( p_a \) suggests that the need to raise the inspection rate to satisfy the Pigou constraint carries a large social cost. Note, however, that this proposition is driven by the costs of moving into the hybrid region, not the benefits, and these costs are large if we are starting from a situation where there is very little compliance.

Suppose next that external costs are high enough to move the Pigou-optimal marginal compliance cost close to its maximum feasible level, denoted \( \bar{\alpha}^{max} \). In this case, welfare improvements from the Pigou optimum are now possible.\(^{16}\)

**Proposition 3.** For any unit inspection cost, \( p_a > 0 \), there exists a marginal compliance cost, \( \bar{\alpha}' < \bar{\alpha}^{max} \), such that if external cost parameters, \( \beta \) and \( \eta \), imply a Pigou optimum with marginal compliance cost between \( \bar{\alpha}' \) and \( \bar{\alpha}^{max} \), then any marginal policy change from this optimum that creates overleveraged firms must raise welfare.

Proof. Let \( \bar{\alpha}^{max} \) denote the maximum feasible \( \bar{\alpha} \) under the Pigou constraint. Under our assumption that capital demand goes to infinity as the cost of capital goes to zero, (14) tells us that \( \pi(\bar{\alpha}^{max}) < 1 \). Otherwise, the denominator of (14) would be negative, implying that greater compliance can be obtained with a lower inspection rate. Since this denominator goes to infinity as \( \bar{\alpha} \) goes to \( \bar{\alpha}^{max} \) from below, we know that the marginal benefit expression on the left side of optimality condition

\(^{16}\) There are no similar results for a sufficiently low \( p_a \) because reducing \( p_a \) enough will produce a corner solution, where there is complete compliance at the Pigou optimum, and therefore no scope for increasing compliance. Moreover, it can be shown in this case there are no marginal changes in the fine and inspection rate that increase welfare by lowering compliance.
(13) goes to infinity. Thus, the denominator of (27) goes to infinity, whereas the numerator stays bounded from above. Thus, the expression in (27) goes to zero, proving the proposition. Q.E.D.

Prop. 3 may seem counter-intuitive for two reasons. First, we should expect a high marginal external cost from capital, measured by \( \beta \), to imply a large deadweight loss from overleveraged firms, since the possibility of bankruptcy then essentially subsidizes capital. The explanation is that the impact of \( \beta \) on the deadweight loss created by one overleveraged firm is exactly offset by the impact of \( \beta \) on the number of firms that become overleveraged, \( d\gamma/d\bar{\alpha} \) (see eq. 33), leaving the change in total deadweight loss unchanged. A similar observation explains the second counter-intuitive feature of Proposition 3: why a high capital demand elasticity does not similarly prevent welfare gains. In fact, \( d\gamma/d\bar{\alpha} \) is inversely related to the square of this elasticity.\(^{17}\) But we next show that very different welfare results apply to comparisons between Becker equilibria and the Pigou optimum.

Finally, we note the role of the capital demand elasticity in Proposition 1. An increase in this elasticity increases \( k_{nb} - k_\ell \) in the numerator of (27). This difference has been shown to be negatively related to the rate at which firms become overleveraged, \( d\gamma/d\bar{\alpha} \). Since the numerator in Proposition 1 can be varied between zero and infinity by altering this elasticity, the elasticity alone determines the potential for welfare improvements, all else equal. High elasticities will imply that welfare can be raised by moving into the hybrid region, while low elasticities will imply that welfare declines. However, we will see in the next section that the capital demand elasticity has very different implications for the desirability of increasing the fine to the point where all firms are overleveraged.

\(^{17}\) The terms \( \left(k_{nb} - k_\mu \right) \) and \( L_{ab} \) in the denominator of (33) both depend on how elastic capital is with respect to its marginal cost.
6. Is a Becker Equilibrium Better than the Pigou Optimum?

Once we move to a Becker equilibrium, the impacts of the fine and inspection rate on the share of evaders that are overleveraged is no longer an issue, because all evaders are now overleveraged. We can generally conclude that moving from the Pigou optimum to the Becker optimum will be beneficial if deadweight losses created by the move, $L_{ab}$ for each evader, are sufficiently small, which will be the case for small capital elasticities. Thus, looking over all three types of equilibria, the global optimum will be a Becker equilibrium for sufficiently small capital elasticities.

Combined with the Proposition 2, we then find that starting from the Pigou optimum, inducing a small number of firms to become overleveraged may be welfare-reducing, while inducing all evaders to become overleveraged can then improve welfare. In other words, the welfare effects can be non-monotonic.

The following proposition states that a Becker equilibrium is superior if external damages are large enough to justify expenditures on enforcement, but not too large.

**Proposition 4:** There exists a marginal compliance cost, $\bar{\alpha}'$, and marginal external cost of capital, $\beta'$, such that for all $\beta < \beta'$, if the inspection and external cost parameters, $(p_a, \beta, \eta)$, imply a Pigou optimum with positive marginal compliance cost $\bar{\alpha} < \bar{\alpha}'$, then there exists Becker equilibria with the same, lower, and higher values of $\bar{\alpha}$, but higher welfare than the Pigou optimum.

**Proof:** Assume initially no marginal external cost from capital ($\beta = 0$), in which case $r^*$ is the social cost of capital. Starting from the Pigou optimum, increase the fine. If firms become overleveraged, their cost of capital changes to $(1 - \pi)r$, but the equilibrium interest rate $r$ rises above $r^*$ to compensate investors for the risks of bankruptcy. We next show that it is always the case that a high enough fine will raise $r$ so much that $(1 - \pi)r = r^*$, implying that overleveraged firms
make efficient investment decisions. In particular, the condition for capital market equilibrium, given by (19), implies:

\[(36) \quad r^* - (1 - \pi)r = \pi r \left( 1 - \frac{[1-G] F - p + r k_{nb}}{r k_{\mu}} \right).\]

Setting \( F = p + k_{nb} \), so that the fine takes all of the firm’s assets, (36) becomes

\[(37) \quad r^* - (1 - \pi)r = \pi r \left( 1 - \frac{[1-G] (1+r) k_{nb}}{r k_{\mu}} \right).\]

By choosing a sufficiently low value of \( \eta \) or high value of \( p_{in} \), but not one that produces a corner solution where all firms evade at the Pigou optimum, we have \( \tilde{G} \) almost equal to zero in (37), and \( k_{nb} \) almost equal to \( k_{\mu} \). But then right side is almost equal to minus \( \pi \), implying \( r^* - (1 - \pi)r < 0 \). But by (36), setting \( F = p + r k_{nb} \) implies \( r^* - (1 - \pi)r < 0 \). It follows that there is then some intermediate fine that produces equality between \( r^* \) and \( (1 - \pi)r \), in which case overleveraged firms use the efficient level of capital. Moreover, the rise in \( r \) does not distort the behavior of legal firms, since their investment decisions are efficiently regulated.

To conclude, we have shown that there is an increase in the fine that raises \( \bar{\alpha} \) without causing any social costs. Since we know that \( h'(x_n)(\beta k_{na} + \eta) - \bar{\alpha} > 0 \) at the Pigou optimum, welfare will rise if \( \bar{\alpha} \) does not increase too much. But we can achieve any smaller increase in \( \bar{\alpha} \) with the fine high enough to satisfy \( r^* = (1 - \pi)r \) by also reducing \( \pi \). Thus, it is possible to raise \( \bar{\alpha} \) in a way that causes welfare to rise while inducing all evaders to become overleveraged. Moreover, by reducing \( \pi \) enough, we can also achieve a rise in welfare with no change, or a small drop, in \( \bar{\alpha} \). Although we have assumed \( \beta = 0 \), the same argument clearly applies to sufficiently small positive values of \( \beta \). Q.E.D.

The basic idea here is that if fines are large enough to induce firms to become overleveraged, than even larger fines may be preferable. In fact, Proposition 3 shows that a small number of
overleveraged firms is harmful when compliance is low, whereas Proposition 4 shows that inducing all evaders to become overleveraged is beneficial under the similar conditions on compliance. The reason is that large fines drive up the market interest rate, \( r \), thereby undoing the implicit capital subsidy associated with bankruptcy. If fines are large enough for \( r \) to rise to the point where
\[
(1 - \pi)r = r^* + \beta h'
\]
then there will be no capital market distortion because overleveraged firms face the social cost of capital. This increase in \( r \) is possible if \( \beta \) and \( G(\tilde{a}) \) are not too high. A high \( G(\tilde{a}) \) would imply that few firms are evading the regulation, in which case it is not necessary for \( r \) to rise much above \( r^* \) to compensate investors for bankruptcy. The proposition limits the optimal value of \( G(\tilde{a}) \) by assuming sufficiently low external costs or high unit inspection costs.\(^{18}\)

Note that this argument also provides a reason for why it may not be desirable for fines to take all of an evader’s assets in cases where firms should be overleveraged. Large fines may raise the market interest rate enough to cause \( (1 - \pi)r \) to exceed \( r^* + \beta h' \), implying an effective tax on investment in overleveraged firms, relative to the first best. Then further increases in the fine will raise \( r \) further, worsening the resulting deadweight loss from this tax. For highly elastic capital demand, it may be preferable to incur the resource costs associated with a greater inspection rate, rather than a higher fine.

Consider the case of large external costs, which drives up the optimal compliance level, under both the Pigou constraint and unconstrained global optimum. In this case, there is a relatively large number of legal firms available to offset expected investor losses in legal firms, so \( r \) need not rise much over \( r^* \) to achieve equilibrium in the capital market. Thus, there may be no way for increased fines to bring about equality between the cost of capital for overleveraged firms, \( (1 - \pi)r \), and the social opportunity cost, \( r^* + \beta h' \), in the Becker region, without lowering compliance to an

\(^{18}\) Note that Proposition 4 does not tell us that the globally-optimal \( \tilde{a} \) will exceed the Pigou-optimal \( \tilde{a} \), since it does not show how the marginal cost of \( \tilde{a} \) changes as a result of the move to a Becker equilibrium.
unacceptably low level. Any resulting deadweight losses from moving to a Becker equilibrium are given by $(1 - G(\bar{a}))L_{ab}$, where the loss $L_{ab}$ is defined in (25) and may be approximated use the usual quadratic loss formula:

$$L_{ab} = \frac{1}{2} [\beta h' + r^* - (1 - \pi)r]^2 k',$$

where $k'$ is the capital demand derivative. Since the “effective subsidy” on capital is squared in the deadweight loss formula, large values of $\beta$ may imply a huge deadweight loss, relative to any gains from using fines that bankrupt firms to raise $\bar{a}$ beyond its maximum Pigou value. We then have a tradeoff: going from the Pigou-optimal compliance level to a level in the Becker region reduces the number of firms that are creating external costs, but by increasing the capital used by the non-compliant firms, it increases the external cost per firm. If either $\beta$ or the capital demand elasticity are low, then the latter consideration is unimportant, but a high capital demand elasticity may actually raise external costs so much that for high $\beta$, the Pigou optimum remains preferable to any Becker equilibrium.

Consider, for example, the 2-technique case illustrate in Figure 3, where the technique with the low capital intensity is assumed to be socially efficient. Suppose that $\eta=0$ and $\beta > 0$. If the capital intensities for the two techniques are sufficiently far apart, while the lower cost of capital faced by non-compliant firms in the Becker region causes them to switch to the high capital-intensive technique, then external costs will actually be higher in the Becker region than in the Pigou region. On the other hand, similar intensities combined with a sufficiently high $\beta$ will ensure that a Becker equilibrium is superior to the Pigou optimum.

If external cost depend only on the component of external cost not dependent on the capital intensity ($\eta > 0; \beta = 0$), then increasing this component enables us to increase the marginal benefit of additional compliance, $MB_{\bar{a}} = [h'(x_n)(\beta k_{nb} + \eta) - \bar{a}]g$, while having no effect on deadweight loss, $L_{ab}$. Thus, sufficiently high $\eta$ and low $\beta$ will ensure that a Becker equilibrium is better than the...
Pigou optimum. We provide a more formal statement of this result for the case of an iso-elastic marginal damage function.

**Proposition 5.** Assume that the external damage function $h$ is isoelastic. Then there exists positive numbers $\eta'$ and $\delta$, such that if external cost parameter $\eta$ is greater than $\eta'$, while external cost parameter $\beta$ satisfies $\eta \beta < \delta$, then the globally-optimal fine and inspection rate supports a Becker equilibrium, where all evaders are overleveraged.

**Proof.** The isoelastic $h$ function may be written, $h = (x_\alpha)^\theta$, $\theta > 1$. Then $\beta h' = \theta [\beta^\theta + (\eta \beta)^{\theta-1}] y_\alpha^{\theta-1}$.

Thus, the constraints on $\eta$ and $\beta$ in the proposition place an upper bounds on deadweight loss, $L_{ab}$, under any given structure of fines and inspection costs. On the other hand $MB_\alpha$ can be increased without bound by raising $\eta$, insuring that increasing $\alpha$ into the Becker region will eventually increase social welfare. Q.E.D.

Finally, it is tempting to conjecture that a sufficiently low $p_\alpha$ insures that the Pigou optimum provides greater welfare than any Becker equilibrium, but this conjecture turns out to be wrong. The problem is that the $\alpha$ at which $d\pi/d\alpha$ goes to infinity does not depend on $p_\alpha$ so the maximum $\alpha$ that satisfies the Pigou constraint is independent of $p_\alpha$. If external costs are sufficiently high, it may be desirable to achieve higher compliance levels, at any unit inspection cost, including zero.

**7. Examples**

While we believe that our model applies to a broad spectrum of regulated activities, ranging from nuisances, such as product safety and illegal parking, to environmental, safety and financial. Not all of these regulation settings accord perfectly with the framework described above. Nevertheless, our main results with respect to the desirability of Pigouvian versus Becker regulation will carry through provided that the key features of the model remain in place. The key attributes of the model that we need are: a regulation that specifies some production technology, the existence of
alternative technologies, the rental of some portion of capital from investors, the inability of investors
to discern the legal status of their borrowers, random inspection and monetary fines that potentially
bankrupt evaders. In addition, violations of the regulation must generate external costs that may
depend upon the level of output, the type of capital or the amount of capital. To provide some
additional context for the analysis that follows, we close this section by providing four concrete
examples and with a brief description of how they satisfy the criteria needed to generate our results.

Ex. 1: Illegal parking by delivery firms.

Consider an industry of restaurants that deliver meals to households. When parking the
delivery vehicle, the firm faces a regulation about where it may park legally. Generally, the costs of
compliance will vary with the availability of legal spaces, congestion, the character of the
neighborhood, and the type of delivery car used. Any firm that evades the regulation risks a
monetary fine, which, if high enough, could lead to bankruptcy and the seizure of assets by the
parking authority. Investors considering loaning vehicles or lending money to firms in this industry
will demand a premium that compensates them for their expected losses. Firms that face bankruptcy
will perceive a lower marginal cost of capital and presumably substitute cheap capital for other
relatively expensive inputs. In this example, the external cost depends only upon the amount of
illegal parking. In order to reduce it, the government may use higher fines, more likely detection and
perhaps a tax on the activity itself. Since it is the production of output—meal delivery—not the
capital used to produce it—trucks, ovens, etc.—that produces the external cost, our analysis suggest
the optimality of high fines, rather than high detection rates, particularly if we view illegal parking as
belonging to the category of relatively “minor nuisances” (Proposition 4). This solution is based
solely on efficiency considerations and assumes risk-neutrality.
Ex. 2: **Pollution regulation**

Pollution regulations often require the installation of the “best available” pollution control equipment to mitigate the damage from emissions. A firm can evade the regulation by choosing a different technology. Depending upon the age of the plant, the complexity of the production process, the firm’s experience and the skills of its workforce, the cost savings will vary (failing to save these savings is equivalent, of course, to bearing the costs of compliance). High fines may expose the firm to bankruptcy if detected with the non-compliant equipment, but, as we have seen, they lower its marginal cost of capital, inducing the firm to use more capital-intensive production techniques than those employed by legal competitors. This factor market distortion is a cost of the severe fine. In addition, the external cost depends on the amount of non-compliant capital. If this particular cost is high, then low fines and frequent inspections may be desirable, so that that the fine structure can be adjusted to provide evaders of the regulation with the proper incentives to limit their capital usage. On the other hand, if there is little substitutability in the use of capital, then such incentives are unimportant (the deadweight loss $L_{ab}$ is low in our model), and high external costs then suggest the use of high fines, particularly if high inspection costs make it costly to control pollution with low fines.

Ex. 3: **Capital requirements in financial markets**

Bank and other depository institutions may be subject to a minimum capital requirement where the qualifying capital must be from a certain class of low-risk (“tier-one”) assets. Let the variable $e$ include the quantity of these low-risk assets along the vertical axis in Figure 1, and use $k$ to measure the level of loans along the horizontal axis. A risk-adjusted equal yield contour will have the shape indicated in Figure 1. That is, its slope will flatten as the bank substitutes lower-quality loans for tier-one assets. Compliant firms will meet the requirement and produce at point A, where

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19 FDIC, section 325.3 sets a 4% minimum leverage ratio for a class of banks. The qualifying assets must be tier 1 (or core) capital.
the firm pays 1 per unit of tier 1 assets and \( r \) per unit of deposits to be loaned. Depositors will not be able to monitor the portfolios of banks but will anticipate the risk of bankruptcy in the industry and demand a premium. Non-compliant firms, who are subject to high fines and bankruptcy if detected, will face a lower marginal cost of funds and choose the portfolio indicated at point B, paying (at the margin) \( r \) to depositors if not inspected but nothing if inspected. Thus, the expected cost for this firm is \((1-\pi)r\).

The external cost depends upon the quantity and type of assets held by the depository institution. Assume that too high a level of low-quality loans carries a high social cost, as demonstrated by the recent financial crisis. If the different types of assets are highly substitutable, then low fines may be desirable, so that the fine structure can be used to control the capital structure of non-compliant firms. On the other hand, if high fines and reasonable inspection costs can be used to achieve almost full compliance, then they may be desirable.

**Ex. 4: Licensing requirements**

Many industries have licensing requirements. For example, hair salons must hire licensed stylists. The licensing requirement typically specifies some minimal level of training (that is, a minimum level of industry-specific human capital).\(^{20}\) Measure the human capital on the vertical axis of Figure 1. Then a non-compliant firm will hire stylists with illegally low levels of training and substitute other forms of capital, such as furniture, design and equipment, for the regulated human capital. Facing potential bankruptcy, these firms borrowed funds at a rate of \((1-\pi)r < r\) and operate at a point such as B in Figure 1. Lenders will not perceive the compliance or non-compliance of the individual firm but will be aware that losses exist and demand the appropriate payment. The external cost will not depend upon the capital, but rather upon only the level of output. Assuming this external cost is not too high, Proposition 4 suggests the use of high fines.

\(^{20}\) In Michigan barbers must complete a 2,000 hour course of study. (Barbering Law Book, Michigan Department of Labor and Economic Growth, BCS-LDL-PUB-001 (02/06).
References


Figure 1: Choosing Inputs
Figure 2: Optimal Pigouvian Regulation
Figure 3: Two Production Techniques