Understanding the Distributional Impact of Long-Run Inflation[1]

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Abstract: The impact of fully-anticipated inflation is systematically studied in heterogeneous-agent economies with endogenous labor supply and portfolio choices. In stationary equilibrium, inflation nonlinearly alters the endogenous distributions of income, wealth and consumption. Small departures from zero inflation have the strongest impact. Three features determine how inflation impacts distributions and welfare: financial structure, shock persistence and labor supply elasticity. When agents can only self-insure with money, inflation reduces wealth inequality but may raise consumption inequality. Otherwise, inflation reduces consumption inequality but may raise wealth inequality. Given persistent shocks and an inelastic labor supply, inflation may raise average welfare.

Keywords: Money, Heterogeneity, Wealth, Consumption, Inequality.

JEL codes: E4, E5

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1 Introduction

Ongoing, massive liquidity injections by the U.S. central bank are raising fears of significant long-run inflation and, with it, questions about the likely economic impact. Studies based on representative-agent models of the U.S. economy typically point to a negative, monotonic association between long-run inflation and social welfare. Most studies find that a representative household would give up some consumption to live in a zero-inflation economy, though the deadweight loss from moderate inflation quantitatively amounts to a fraction of one percent of consumption. Hence, from the vantage point of a representative agent the optimal policy prescription is non-inflationary, but departures from this policy are not very costly.[2]

But field economies are not populated by representative agents, so distributional issues must be taken into account. For instance, survey evidence suggests that low-income households are more concerned about inflation than wealthier households (Easterly and Fischer, 2001), and there is some empirical support for the view that inflation is negatively associated to wealth inequality in the U.S. (e.g., Romer and Romer, 1998). Unfortunately, there is only a handful of studies about the impact of inflation in heterogeneous-agent monetary economies, and the results often differ on basic matters such as the impact on the distribution of wealth—which is the key aggregate state variable—whether it is optimal to target low inflation and, if so,

[2] Higher welfare costs are found in some OECD economies; see Boel and Camera (forthcoming).

what are the welfare gains, quantitatively.[3]

We report results from a study of the impact of long-run inflation in economies where inequality arises endogenously. The model is a production economy where labor supply and portfolio choices are endogenous. Ex-ante homogeneous households hold money to trade on spot markets *and* to self-insure against idiosyncratic productivity shocks. Precautionary savings needs can also be satisfied by holding riskless debt securities that are illiquid. There is no aggregate risk. In stationary equilibrium productivity shocks induce heterogeneity in earnings, income and wealth. Due to incomplete markets, consumption is heterogeneously distributed and the allocation is inefficient. Long-run inflation affects the (in)efficiency of the allocation by altering the equilibrium distributions of income, wealth and consumption. Inflation results from money supply expansions achieved via fully anticipated lump-sum injections. In equilibrium, a one percent increase of the rate of monetary expansion raises inflation exactly by one percent. Equilibrium is computationally studied for a model calibrated to the U.S. economy.

The study makes several contributions. First, it identifies three features of an economy that determine the distributional impact of monetary policy: financial structure, elasticity of labor supply, and the process underlying earnings shocks. Disparities in earlier results can be traced back to different assumptions about one

[3]E.g., in Akyol (2004) 10% inflation maximizes social welfare; in Molico (2006) average welfare increases in inflation, if inflation is small. In contrast, Chiu and Molico (2009) and Wen (2010) report that 10% inflation is worth, respectively, 0.6% and 8% of average consumption in the U.S.

or more of these features. Second, it provides evidence of a non-linear impact of inflation on the distributions of endogenous variables. Small departures from zero inflation have the greatest consequences because they strongly alter the incentives to self-insure, inducing a significant drop in the size and the liquidity of savings portfolios. Third, the study reveals that although a faster rate of monetary expansion may reduce wealth concentration, this can *magnify* consumption inequality (and vice-versa). It follows that inflation-induced reductions in the concentration of wealth do not necessarily result in improvements in average welfare.

There is clear intuition for these findings. When labor supply and portfolio choices are endogenous, there exists a trade-off between inflation-induced consumption *redistribution* and output *decline*. A faster rate of monetary expansion has the potential to reduce consumption inequality, by altering the distributions of wealth and income; however, it surely brings about a permanent output decline, due to endogenous labor choices. This mean-variance trade-off depends in meaningful ways on the assets available to self-insure against shocks (money, bonds), the persistence of earnings shocks, and the response of the labor supply to changes in the real wage. The first two elements control the extent of inflation-induced reallocation of consumption and the third impacts the overall consumption loss. We report that departures from a zero inflation policy may raise average welfare when earnings shocks are persistent and the labor supply is inelastic. This helps us reconcile different findings about the welfare-impact of monetary expansion in models where money is the *only* asset (e.g., as in Molico 2006, Chiu and Molico, 2010).

The financial structure matters for two reasons. First, inflation-induced wealth redistribution is tied to the composition of savings portfolios: When money is the only asset, a faster rate of monetary expansion acts as a progressive tax that lowers wealth inequality (e.g., as in Boel and Camera, 2009); When bonds can be traded, wealth inequality is less affected by inflation because the rich hold more illiquid portfolios than the poor. Second, the financial structure affects the ability to self-insure and to relax spending constraints. Higher inflation sharply lowers consumption of those who have tight spending constraints and cannot borrow. Shocks persistence directly affects the extent of inequality, while the labor supply elasticity impacts the inflation-induced output decline. An inelastic labor supply brings the model closer to an endowment economy where money injections simply induce mean-preserving consumption redistributions (e.g., as in Akyol, 2004).

The paper proceeds with a literature review in Section 2. Section 3 presents the model, Section 4 presents a partial characterization of equilibrium, and Section 5 presents the results from the computational analysis. Section 6 concludes.

2 Related literature

There is only a handful of studies about the impact of fully anticipated inflation in economies with heterogeneity. Of these, only very few perform a systematic investigation, i.e., very few studies consider a wide range of inflation rates instead of computing equilibria only for two or three inflation values (usually 0%, 5% and 10%; see Dressler, 2011, for a recent example). Though these more systematic studies differ in their motivation for money (cash-in-advance restrictions, market timing frictions, trading constraints, etc.) and in the availability of additional assets, in all studies money is valued, at least partly, because it allows agents to self-insure against some idiosyncratic shock. The methodology adopted is also similar, mostly computational. Differences in findings cannot be readily ascribed to the type of model adopted, or the role played by money in the model. For example, dissimilar results are reported in matching models of money, cash-in-advance models, and models where money has only a precautionary role.

In a pure-exchange economy, Akyol (1994) finds that as high as 10% inflation is *necessary* to maximize social welfare. Agents can trade bonds and only high-income agents hold money, for precautionary purposes. Inflation is optimal because it induces a bond demand, which improves risk-sharing, and redistributes wealth top to bottom through the inflation tax. The precautionary money demand model in Wen (2010) delivers an antithetic result: 10% inflation is worth at least 8% of per-capita consumption. The model is a production economy with capital, calibrated to match the distribution of money holdings in U.S. data by imposing random redistribution of net wealth.

In a random matching model, Molico (2006) shows that some inflation can improve social welfare because higher inflation can reduce wealth and price dispersion, but only if inflation is low. The opposite holds if shocks are not persistent. Chiu and Molico (2010) find that the welfare-improving effect of inflation vanishes when the search model in Molico (2006) is augmented with a market for money; inflation lowers average welfare, though the welfare cost is small. A small welfare cost of inflation emerges also from the matching model in Boel and Camera (2009), which shows how the financial structure matters a great deal for how this cost is distributed in the economy.

3 The model

Time is discrete, the horizon is infinite and there is a continuum of ex-ante homogeneous infinitely-lived households of measure one. Each household is a single economic decision unit composed of a shopper-worker pair. Thinking of each date as being divided into two subperiods (beginning and end), it is assumed that shopper and worker from each household are together only at the end of a period, and otherwise are apart and undertake separate economic activities. Households consume a single perishable good and geometrically discount future consumption at rate $\beta \in (0, 1)$. At the beginning of each period households operate on anonymous goods and labor markets, while they consume and operate on financial markets only at the end of each period.

On every date $t = 1, 2, ...$ a perishable good can be produced by a representative profit-maximizing firm. Labor is the only factor of production. The firm is owned by households in equal non-marketable shares and is defined by the production technology $Y : \mathbb{R}_+ \to \mathbb{R}_+$, strictly increasing and concave, and satisfying the Inada conditions. Though Y is not homogeneous of degree one, such a feature can be recovered by adding an "entrepreneurial" factor in fixed supply to redistribute profits as factor payments (see McKenzie, 1959). With this in mind, the firm can be considered representative. It is assumed that the firm can pay the wage bill after selling its output. However, since shopper and worker are apart, workers demand monetary compensation and shoppers carry money balances.

At the start of each date workers draw productivity shocks determining how much efficiency units of labor they can supply to the firm. For a worker from household n let $h_{n,t} \in \{h_L, h_H\}$ denote the amount of "effective" labor she can supply on date t per unit of time worked, i.e., the worker in the household can be either high- or low-productive with $0 \leq h_L < h_H < \infty$. It is assumed that for each household n and each date t the shock process follows a first-order Markov chain with transition probabilities $\Pr[h_{n,t+1} = h_j | h_{n,t} = h_j] = q$ for $j = L, H$ and $\Pr[h_{n,t+1} \neq h_j | h_{n,t} = h_j] = 1 - q$. The parameter $q \in (0, 1)$ affects the persistence of the shock, which is measured by the correlation coefficient $2q - 1$. Labor shocks introduce ex-post heterogeneity across households. The long-run distribution of labor productivity is invariant, with one-half households being low-productive and the other half being high-productive. Denoting by $\ell_{n,t}$ the labor supply of household n on date t, the per-capita supply of effective labor is $L_t = \int_n \ell_{n,t} h_{n,t} dn$ so that the per capita output supply is $Y(L_t)$.

Preferences are as follows. If household n consumes $c_{n,t} \geq 0$ goods and supplies $\ell_{n,t} \geq 0$ labor on any date t, then household's utility is $u(c_{n,t}) - g(\ell_{n,t})$ where the function u is twice continuously differentiable, strictly increasing and concave and g is convex with $g(0) = 0$. A government exists that is the sole supplier of fiat currency, of which there is an initial nominal stock $\bar{M} > 0$ evolving deterministically at gross rate π thanks to lump-sum transfers to households at the end of each period t. A bond market opens only at the end of each period. On t households sell or buy one-period nominal bonds (in zero net supply) that mature on $t + 1$ and pay gross interest $i_{t+1} \geq 0$. Firm's dividends are distributed at the end of each period. It is assumed that all parties can commit to fulfill their financial obligations and that all economic agents are price takers.

4 Stationary monetary allocations

To set the stage for the analysis, consider the allocation selected by a planner who treats agents identically. We call it the *efficient allocation*. The planner maximizes the expected lifetime utility of a representative agent subject to the physical and technological constraints. The optimal plan, which solves a dynamic problem analyzed in Appendix A, has the following characteristics. It is stationary, unique, implies constant individual consumption and individual state-contingent labor supply. Put simply, the efficient allocation perfectly insures each household, a result that motivates our focus on stationary allocations of the monetary economy.

Definition 1 *An allocation for a monetary economy is stationary if the distribution of consumption is time-invariant and the real money stock is positive and stationary.*

The efficient allocation can be decentralized by introducing a full set of contingent claims to be initially traded. However, the economy cannot achieve the efficient allocation because markets are assumed incomplete: households can self-insure only with money and bonds. Notice that in a stationary monetary allocation, money market clearing implies that inflation is pinned down by the money growth process. Let $\pi > 0$ denote the stationary growth rate of nominal prices.

4.1 The household's problem

Households maximize expected lifetime utility and since their problem is recursive we formalize it with a functional equation. Let V be the value function of a household at the start of a date *after* shocks are realized. Let $m \geq 0$ and $b \geq \underline{b} > -\infty$ denote the start-of-period household's portfolio of money and bonds, defined in real terms. Denote with a prime variables in the following period.

Consider a stationary distribution of wealth. Because households cannot trade state-contingent assets and can only trade goods on spot markets, then the relevant state of a household includes current productivity and portfolio of assets. In particular, the history of labor shocks s is relevant only if it impacts the current labor shock. Hence, let (m, b, h) denote the current state of the household.

Given (m, b, h), in an economy where the nominal price sequence grows at constant rate π, the household's problem is to choose $(c, \ell, m') \geq 0$ and $b' \geq \underline{b}$ to maximize expected lifetime utility. The problem has a recursive representation:

$$\begin{aligned} V(m,b,h) = \quad & \max\{u(c) - g(\ell) + \beta E V(m', b', h')\} \\ & \text{s.t.} \quad c + \pi(m' + b') \leq w\ell h + m + bi + \xi + \tau, \\ & \qquad\;\; c \leq m, \end{aligned} \tag{1}$$

where nominal variables have been normalized by the contemporaneous nominal price of goods. The household faces two constraints. The budget constraint accounts for uses of funds, i.e., consumption $c \geq 0$ and real savings in the form of $b' \geq \underline{b}$ bonds and $m' \geq 0$ money balances; the latter are adjusted by gross inflation π because both assets are nominal. Sources of funds include $w\ell h$ income from supplying ℓh efficiency units of labor to the market, bond interest payments bi, m real balances, a lump-sum real balance transfer τ, and a dividend payment ξ.[4] There is also a cash-in-advance constraint because the buyer must pay with money. Thus, disposable *income* includes $b(i-1) + \xi + \tau$ plus *earnings* $w\ell h$, while net *wealth* is $m + b$.

Conjecture that the function V exists and is differentiable. Let $V_x := \frac{\partial V}{\partial x}$ for

[4] ξ can be considered compensation to a second productive input called "entrepreneurial capital." This asset is in positive and identical net supply across households and is not tradeable.

$x = m, b$. Let $\mu \geq 0$ and $\lambda \geq 0$ be the multipliers on first and second constraint:

$$\begin{aligned} & u'(c) - \mu - \lambda = 0 \\ & g'(\ell) - \mu w h = 0 \\ & -\pi\mu + \beta E V_{m'} \leq 0 \\ & -\pi\mu + \beta E V_{b'} \leq 0 \end{aligned} \tag{2}$$

The Envelope theorem implies

$$V_m = \mu + \lambda \quad \text{and} \quad V_b = \mu i, \tag{3}$$

i.e., the marginal value of assets reflects the marginal utility of income μ, and the marginal utility of liquidity, λ.

The first order conditions reveal that heterogeneity in income and consumption depends on three elements. A household's labor supply depends not only on own productivity but also on wealth, through the marginal value of income μ. Consumption depends on wealth as well as on differences in the *liquidity premium* of money for that specific household, $u'(c) - \mu$. Liquidity premia will generally differ across households depending on level *and* composition of savings.

From (2) and (3) we have

$$\begin{aligned} & -\pi[u'(c) - \lambda] + \beta E[u'(c')] \leq 0 \qquad & (\text{with} = \text{if } m' > 0) \\ & -\pi[u'(c) - \lambda] + \beta i E[u'(c') - \lambda'] \leq 0 \qquad & (\text{with} = \text{if } b' > \underline{b}). \end{aligned} \tag{4}$$

Bonds, unlike money, cannot be immediately traded for consumption, so they are illiquid. The expressions in (4) indicate that, due to their illiquidity, bonds are held only if they dominate money in rate of return, i.e., if they pay a positive nominal interest rate. If money is held, then

$$u'(c) = \frac{\beta}{\pi} E[u'(c')] + \lambda \tag{5}$$

and if $b' > \underline{b}$, then

$$i = \frac{E[u'(c')]}{E[u'(c') - \lambda']}. \tag{6}$$

Instead, if $b' = \underline{b}$ then $i < \frac{E[u'(c')]}{E[u'(c') - \lambda']}$. Given uninsurable income shocks, the economy will generally exhibit heterogeneity in consumption, income and wealth. Households will differ also in the composition of their portfolios. While an analytical characterization of stationary monetary outcome is beyond the scope of this paper, the following Lemma provides some useful results.

Lemma 2 *In a stationary monetary outcome the following must hold: (i) $m' > 0$ so that (5) always holds; (ii) $\lambda > 0$ for at least some household; (iii) if $i \leq 1$, then $b' = \underline{b}$; and (iv) if $b' > \underline{b}$, then $E[\lambda'] > 0$.*

In a stationary monetary economy every household holds a positive fraction of their savings in cash. Though not everyone may hold bonds, which are illiquid, those who do hold bonds will never hold "enough" money to satisfy any desired consumption level. In fact, those who trade on the bond market will optimally choose money balances that leave them liquidity constrained in at least some possible future state. Put differently, when nominal interest rates are positive no household will fully insure against all their possible future liquidity needs; if this were not the case, then lenders should optimally consume more today and accumulate less wealth, while borrowers should borrow even more (see (6)).

To understand these results, consider that in this economy savings fulfill a precautionary need. Households are subject to uninsurable income shocks, which expose them to income risk. Though income cannot be spent on contemporaneous consumption, it can be saved for future purchases. A low income shock may constrain future consumption levels because it restricts current monetary savings. Therefore, households in this economy will generally want to hold extra savings as a precaution against low income shocks. Since households want to maximize the return from savings, wealthier households in general will hold only a fraction of their savings in cash. In particular, no household will optimally hoard enough cash to be unconstrained in their consumption in every possible state.

Availability of a bond market improves the efficiency of the allocation for two reasons. First, the possibility to *buy* bonds reduces the opportunity cost of holding precautionary savings. This especially matters to wealthy households who would otherwise suffer from a large inflation tax. Second, the possibility to *sell* bonds reduces the need to hold precautionary savings. This especially matters to poor households, which are more likely to be constrained. In the event of a low-income shock, these households can spend all their current cash savings on consumption, and borrow money for future consumption by selling bonds. As a result, opening a bond market will increase the velocity of money because a smaller portion of the money supply is kept idle, and it will also mitigate the impact of low-productivity shocks on spending patterns. In turn, this should improve consumption-smoothing and reduce consumption inequality.

Optimality of the firm's labor demand choice implies that in a stationary outcome the firm demands labor L that satisfies $w = F'(L)$. The firm distributes revenues in excess of labor compensation as dividends $\xi = Y(L) - wL$.

4.2 The distribution of savings and stationary equilibrium

Let the state of a household be denoted $\omega := (m, b, h) \in \Omega := M \times B \times H$, with $M = [0, \infty)$, $B = [\underline{b}, \infty)$ and $H = \{h_l, h_h\}$. Let $\mathcal{P}(H)$ denote the power set of H, $\mathcal{B}(M)$ and $\mathcal{B}(B)$ denote the Borel $\sigma-$algebra of M and B, respectively. Let $\mathcal{B}(\Omega) := \mathcal{B}(M) \times \mathcal{B}(B) \times \mathcal{P}(H)$ and define the subset of possible states $\mathbb{B}(\Omega) := (\mathbb{M}, \mathbb{B}, \mathbb{H}) \subseteq \mathcal{B}(\Omega)$. Finally, let $\{\Omega, \mathcal{B}(\Omega), \Phi\}$ define the probability space, where Φ is a probability measure.

Given current productivity $h \in H$, $p(h'|h)$ denotes the conditional probability of reaching $h' \in H$ next period. The evolution of the distribution of the state ω can be characterized using a transition function $Q : \Omega \times \mathcal{B}(\Omega) \to [0, 1]$ defined by

$$Q(\omega, \mathbb{B}(\Omega)) = \begin{cases} \sum\limits_{h' \in \mathbb{H}} p(h'|h) & \text{if } (m'(\omega), b'(\omega)) \in \mathbb{M} \times \mathbb{B}, \\ 0 & \text{otherwise} \end{cases}$$

for all $\omega \in \Omega$ and all $\mathbb{B}(\Omega) \subseteq \mathcal{B}(\Omega)$. Put simply, the function Q allows us to calculate the probability of realizing any level of productivity $h' \in \mathbb{H}$ tomorrow, given that (i) today's state is $\omega \in \Omega$ and given that (ii) tomorrow's portfolios are restricted to be an element of $(\mathbb{M}, \mathbb{B})$. In particular, if portfolios are not in $(\mathbb{M}, \mathbb{B})$, then the function Q assigns probability zero to reaching the set of productivity $\mathbb{H}$. Let ϕ denote the joint probability density associated to the probability space $\mathcal{B}(\Omega)$. It is a mixed density, with a discrete random variable h, and two continuous random variables, m and b. Given the transition function defined in Q, the law of motion for the probability measure Φ is given by

$$\Phi'(\mathbb{B}(\Omega)) = \sum_{h} \int_{m} \int_{b} Q(\omega, \mathbb{B}(\Omega))\phi(\omega) dm db.$$

A stationary monetary equilibrium is a time-invariant distribution of consumption, labor supplies, real money balances and real bond holdings across the population of households, such that in each date the optimal plan of a household in state $\omega = (m, b, h) \in \Omega$ involves $c(\omega)$ consumption, $\ell(\omega)$ labor, $m'(\omega)$ and $b'(\omega)$ monetary and bonds savings that solve the household problem (1), given that wages maximize the firm's profit, that all markets clear (goods, money, bonds and labor), and given that the distribution of states (wealth and productivity) and the real value of the money supply are stationary. A formal definition is in Appendix A.

Clearly, stationary equilibrium, if it exists, is characterized by wealth and consumption inequality because markets are incomplete (Lemma 2). Characterization of equilibrium is an analytically intractable task because the distribution of money and bonds, which is the key aggregate state variable, is analytically intractable. Therefore, analysis is conducted using a computational methodology. Details of the numerical procedure, which is burdensome, are described in Appendix B.

5 Main findings

This section reports findings from a quantitative analysis of a yearly model calibrated to the U.S. for the sample period 1950-2006. During that period the average annual CPI inflation rate over that period was 3.9% with a maximum of 14% and a minimum of -0.7%. Appendix B discusses the calibration strategy adopted, which is standard. Results are reported for the baseline calibration that adopts a unit target for labor elasticity, following the literature on social security and estimates in the recent study in [9].

We first present results for economies where agents exclusively self-insure with money. This facilitates comparisons with the welfare cost of inflation literature, which is largely based on models where money is the *only* asset. It also helps in clarifying the role played by bond/credit markets, which we introduce subsequently. For expositional clarity, findings are reported as separate "Results," and reference to stationary equilibrium and to baseline case are omitted when understood.

5.1 Money is the only available asset

This section reports findings regarding stationary equilibrium allocations when the bond market is shut down. This means households exclusively self-insure with money and money balances represent the totality of savings, i.e., $b = 0$.

Result 1 *Equilibrium exhibits endogenous inequality in income, wealth and consumption. Wealth and consumption inequality increase with the persistence of shocks.*

Table 1 and Figures 1a-1b provide supporting evidence.[5] Equilibrium inequality originates from idiosyncratic shocks to productivity. In particular, consumption inequality is tied to market incompleteness and the stochastic process responsible for productivity shocks, which are persistent in the baseline calibration (see Appendix B). To understand the impact of persistence, we computed a version of the model in which we retain the same unconditional (long run) mean productivity of the baseline calibration, but assume iid shocks. We find that greater persistence reduces mobility across classes of wealth because it makes sudden earnings variations unlikely; wealth accumulation slows down for poor households and speeds up for rich households. More persistent shocks also generate a stronger desire to hold precautionary savings, which in turn affects consumption patterns. Wealthy households deplete their savings more slowly when they suffer a negative shock that is more persistent; poor households more slowly accumulate wealth when they face a positive shock that is more persistent.[6] Result 1 thus suggests that the redistributive impact of

[5] Wealth and consumption inequality are reported across inflation rates by means of Gini coefficients and, for 2% inflation, using Lorenz curves. A Lorenz curve represents inequality in the distribution of wealth by reporting the cumulative share of wealth against the cumulative share of the population. A curve coinciding with the 45^0 degree line corresponds to no inequality.

[6] Income inequality decreases with persistence (Table 1). Labor supply decisions are more extreme when shocks are iid; highly productive households supply more labor in order to take advantage of their temporarilyy high productivity; the opposite holds for low productive agents. As a result, the variance of effective labor hours is higher under iid than persistent shocks. Overall, the responses of the two types of workers do not offset each other, so the per-capita supply of labor is higher and the wage rate lower under iid than persistent shocks.

monetary policy crucially depends on the process of earnings shocks.

Quantitatively, the money-only model—which is calibrated to match persistence of earning shocks in the U.S. data (Appendix B)—can account for roughly half of the income inequality in the U.S. data but only for one-third of the wealth inequality. In addition, the money-only model cannot fully account for the feature that wealth is substantially more concentrated than income. Díaz-Gímenez et al. (2010) report Gini coefficients for income and non-housing wealth, respectively, 0.548 and 0.861 in 1998, 0.575 and 0.881 in 2007. At 2% inflation, which roughly corresponds to the experience in those years, the model generates Gini coefficients of 0.249 and 0.313 for income and wealth.

Result 2 *A faster rate of monetary expansion lowers income inequality and output.*

Table 1 provides supporting evidence. Income inequality primarily falls because the monetary expansion is accomplished with lump-sum injections. Households whose wealth m is below per-capita (or average) wealth $\bar{m}$ receive a net transfer $(\pi - 1)(\bar{m} - m)$, while all others are taxed. A faster rate of monetary expansion induces a permanent output decline because the rate of return on money falls with inflation, which raises the opportunity cost of savings. As the incentive to save declines, so does the incentive to supply labor. Hence, given a labor demand that is independent of inflation, equilibrium output falls; the severity of such a decline is a function of the labor supply elasticity. By market clearing, per-capita output equals per-capita consumption, so an increment in inflation induces a *permanent* decline in per-capita consumption. This decline is the key feature of representative-agent models with production, which explains why the representative household is typically found to be willing to give up some consumption to avoid *any* inflation. The message of Result 2 is that with heterogeneity, instead, inflation-induced output declines do not necessarily render inflation socially undesirable because inflation also redistributes income and, as we report in the next two results, wealth and consumption.

FIGURES 1a-1b & TABLE1 HERE

Result 3 *Wealth and wealth inequality decline with inflation, non-linearly. Small departures from zero inflation generate the steepest declines.*

The message here is that expansionary monetary policy can be a tool for re-distributing wealth. Table 1 and Figure 2a,b provide evidence. We report that per-capita savings rapidly fall as inflation grows above zero, and then slowly decline as inflation grows above 5% in the baseline calibration. A wealth decline occurs because the opportunity cost of precautionary savings grows with inflation. On the one hand, inflation lowers the self-insurance value of money, which reduces the desire to hold savings that exceed transactions needs (precautionary savings). Incentives to self-insure with money against earnings shocks are maximized when the opportunity cost of money is minimized, i.e., at zero inflation. In addition, lump-sum injections provide some insurance. Consequently, per-capita wealth monotonically falls with inflation, until it equals per-capita expenditure. The pattern is non-linear because the decline in precautionary savings is mostly concentrated among wealthy households who, unlike poor households, have significant precautionary savings. This is also explains why wealth concentration declines, and why this decline is significant at low inflation rates; that is when inequality is greater (Table 1). Figure 2a illustrates this phenomenon.

FIGURE 2a,b HERE

Wealth inequality declines *also* because lump-sum money injections directly redistribute income (Result 2). To appreciate this phenomenon, consider the trajectory of wealth by quartiles, across inflation rates (Figure 2b, Table 1). At low inflation, we identify three classes of households: those with a long history of similar income shocks, who occupy the tails of the wealth distribution, and households "in transition" (= the middle class). As inflation increases, precautionary savings decline, hence wealth levels increasingly reflect the household's most recent productivity shocks, so the middle class shrinks. From this point on, lump-sum money creation becomes the driving force behind inflation-induced wealth redistribution. If inflation is sufficiently high, then precautionary savings are virtually zero (Table 1). At that point, the wealth distribution becomes bimodal because only current earnings shocks matter, hence income and wealth inequality coincide.

Summing up, two lessons have emerged so far. First, we have found that if money is the *only* source of self-insurance, then increments in fully-anticipated inflation reduce income and wealth concentration, but also lower per-capita income, output, and wealth. Second, the impact of inflation is nonlinear: small departures from zero inflation have the strongest redistributive impact because the incentives to hold precautionary savings quickly vanish. Beyond a low inflation threshold, lump-sum money creation becomes the engine of inflation-induced redistribution. This threshold is very low in the baseline calibration; per-capita savings exceed consumption by 270% at zero inflation, and by only 6% at 5% inflation.

Results 2 and 3 might suggest that inflation necessarily reduces consumption inequality. In fact, this is not so.

Result 4 *A faster rate of monetary expansion may elevate consumption inequality.*

Tables 1 and Figures 3a, 3b and 4 provide supporting evidence. We report that, when households can only self-insure with money, consumption inequality grows with inflation if inflation is sufficiently low. At low rates, inflation has the potential to redistribute consumption shares *away* from the middle class towards those on the wealth distribution's tails (Figure 3a). At high rates, consumption shares are redistributed top to bottom. This non-monotone association between inflation and consumption inequality is observed also when inequality is lower, i.e., when earnings shocks are not very persistent (Result 1); see Table 1. The message from Result 4 is thus very simple: inflating to decrease wealth concentration does not necessarily imply a decrease in the concentration of consumption.

FIGURES 3a,b

Result 4 emerges because increments in inflation generate heterogeneous wealth and substitution effects. Lump-sum money transfers do induce strong wealth effects for the poorest households. But inflation also unequally alters spending constraints, which are heterogeneously tight as a result of wealth inequality. As a result, an increase in inflation significantly raises the marginal value of money for households with tight liquidity constraints (poor and low middle-class), so their consumption falls. Figure 4 illustrates this substitution effect when shocks are iid. The marginal value of money (V_m) nonlinearly declines in wealth because it reflects the level of wealth as well as the severity of the household's liquidity constraints (the multipliers μ and λ in (3)). Moving from 0% to 10% inflation substantially increases the marginal value of money for poor households, while it minimally changes, or slightly lowers it for everyone else. A similar pattern emerges if shocks are persistent.

FIGURE 4

Summing up, if money is the only available asset, then increments in fully anticipated inflation are sure to reduce per-capita consumption and may also increase consumption inequality. The efficiency consequences are next reported.

Result 5 *Per-capita welfare is non-linearly associated with inflation. The association is non-monotone when shocks are persistent.*

Tables 3a and 3b report the (average) welfare cost of $x\%$ inflation, as opposed to no inflation, for different specifications of the model (additional data are reported in Appendix C). Figure 5 reports welfare costs for the baseline case and inflation up to 100%.[7] The main point here is that qualitative and quantitative differences emerge based on the persistence of shocks, and the elasticity of labor supply. In a nutshell, labor elasticity and shocks affect the consumption mean-variance trade-off associated to long-run inflation.

TABLES 3a-b and FIGURE 5 HERE

With iid shocks, the welfare cost (of inflation) is positive, and it monotonically increases with inflation (Table 3b). This is qualitatively in line with findings from representative-agent models, and some heterogeneous-agent models. Quantitatively, the welfare cost is several times that found in representative agent models. With persistent shocks the average welfare cost remains positive but it becomes non-monotone in inflation (Table 3a); it initially increases as inflation rises above zero, then falls, and finally the average welfare cost rises again.

This non-monotonicity suggests caution should be taken in making qualitative and quantitative policy assessments. First, assessing the welfare impact monetary policy based on measurements for a few inflation targets, as is often done in the lit-

[7] Clearly, the average welfare cost obtained by comparing the monetary allocation to the efficient allocation is always positive and large because markets are incomplete. For example, at 2% inflation with persistent shocks, the average households would give up almost 15% consumption to be at the efficient allocation.

erature, can lead to troubling conclusions.[8] Second, with heterogeneity, the welfare impact of inflation hinges on the money-injection mechanism in meaningful ways. In a sense, assuming lump-sum injections maximizes the possible welfare-enhancing impact of inflation, as opposed to assuming injections mechanisms that need not redistribute wealth in a socially desirable manner (e.g., open market operations, asymmetric transfers).

The non-monotonicity is even more prominent when output is less responsive to inflation because in this case some inflation may raise welfare; e.g., consider $\delta = 3$ in Table 3a, which corresponds the case where the elasticity of labor supply doubles relative to the baseline case. To understand the role of the elasticity of labor supply, consider that a planner would be willing to dissipate some output to reduce consumption inequality. Shock persistence and labor supply elasticity alter this trade-off between inflation-induced output decline and redistribution. Persistent shocks magnify inequality, hence the desirability of redistribution (Result 1); an inelastic labor supply brings the model closer to an endowment economy where output is unresponsive to inflation (as in [1, 10]). Hence, a sufficiently inelastic labor supply allows moderate inflation to improve average welfare by redistributing consumption "top to bottom,", without generating excessive output declines (Result

[8] For instance, Imrohoroglu (1992) finds that there is a welfare cost of inflation, it is larger than in representative agent models, and it is higher at 10% than at 5% inflation. However, infinite inflation is optimal in that setting. Large inflations push the economy closer to the planner's allocation without affecting mean consumption (the aggregate endowment is assumed fixed).

2). This inflation rate is generally bounded away from zero, due to the initial increase in consumption inequality (Result 4).

Tables 3a-3b also report welfare costs by wealth quartiles. In the baseline calibration (Table 3a, $\delta = 2$), every household dislikes inflation except for those in the bottom quartile. Interestingly, the welfare costs can be non-monotone across wealth levels. For instance, households in the second quartile would give up 9% consumption to avoid 5% inflation, which is twice what households in the next quartile would give up. This is tied to Result 4: middle-class households lose consumption shares as inflation rises above zero. Table 3b shows that every household dislikes inflation when shocks are iid, because there is less inequality than under persistent shocks.

5.2 Introducing a credit market

This section reports findings when we add to the model the possibility to borrow and lend by trading risk-free bonds. One could think of this as introducing a financial innovation. For comparison purposes, the economies are calibrated to the same parameter values used before; Tables 2, 3 and 4 report the results.

Start by observing that the introduction of a bond market alters per-capita output; with no inflation, per-capita output in the bond economy is 0.9% higher than in the money-only economy. Output is higher because demand is stronger due to the fact that agents can now borrow to smooth their consumption. Inflation has still a negative impact on per-capita wealth and output. However, the output decline associated with expansionary monetary policy is slower relative to the money only economy; at 40% inflation output is almost 3% higher than in the money-only economy. In other words, with the introduction of bonds the price of inflation—in terms of lost output—declines.

Equilibrium is still characterized by endogenous wealth and consumption inequality because the introduction of debt securities is not sufficient to complete the market. Importantly, introducing a bond market alters both *composition* and *distribution* of portfolios in a meaningful way.

Result 6 *When money is not the only asset, the liquidity of portfolios declines with inflation and household's wealth.*

Table 4 gives evidence on how the financial innovation we have considered affects portfolios' composition. When households can self-insure with money and bonds, they reduce their exposure to the inflation tax by minimizing their money balances and holding bonds, hence the illiquid share of savings increases with inflation.

TABLE 4 HERE

Illiquid bonds form the bulk of precautionary savings because bonds dominate money in rate of return (Lemma 2). Consequently, money primarily has a transactions role. Since wealthy agents hold the bulk of precautionary savings, the monetary share of portfolios declines in the household's wealth. In the benchmark calibration, for instance, households in the bottom wealth quartile choose to hold entirely only money to use for transactions purposes. This finding is qualitatively in line with U.S. data from the Survey of Consumer Finances, which reports that poor households hold essentially cash. We next report how these features alter the distribution of endogenous variables and, consequently, the distributional impact of inflation, relative to the model without money.

Result 7 *Consumption inequality is lower and wealth inequality is greater when households can access a credit market, as opposed to when they cannot.*

Table 2 provides supporting evidence.

TABLE 2 HERE

Introducing the possibility to borrow and lend improves risk-sharing, which in equilibrium lowers consumption inequality, raising average welfare. Wealth inequality primarily increases relative to the money-only economy because poor households now can borrow. Because precautionary savings are now mostly composed of bonds (Result 6), per-capita money balances more closely track per-capita consumption, unlike the money-only economy.[9] Put simply, the point here is that observing an increase in wealth concentration should not lead us to conclude that consumption has also become more skewed.

[9]Consumption is more dispersed than money holdings because money is partially used for precautionary purposes. Most individuals hold small money balances. However, the liquidity of their asset portfolios differ. Individuals who are not very wealthy hold mostly liquid assets and may not spend their entire money balances, holding some for precautionary purposes. As inflation increases money is never held for precautionary purposes, hence the Gini coefficients of consumption and monmey holdings become identical.

Introducing a bond market is a step in the right direction in terms of getting concentration measures that are closer to the U.S. data. At 2% inflation, we obtain Gini coefficients 0.280 and 0.732 for income and wealth. The first measure still falls short in matching the income concentration seen in the data, but wealth concentration now accounts for about 85% of that observed in the data for 1998. As a result, ratio of concentrations in wealth and income exceeds that in the data.[10]

The possibility to self-insure with interest-bearing assets has important implications for inflation-induced redistributions and the welfare cost of inflation.

Result 8 *When money is not the only asset, a faster rate of monetary expansion reduces consumption inequality but does not decrease wealth inequality.*

Financial innovation weakens the redistributive impact of inflation. Table 2 reports evidence. When households can trade bonds, money plays a minor role in consumption smoothing, which makes inflation less effective in redistributing wealth. An increment in the money growth rate may increase wealth inequality because households can borrow to relax increasingly tight liquidity constraints. In addition, because monetary shares decline in wealth, a monetary expansion amounts to imposing a regressive tax on wealth. Consumption inequality monotonically falls with a faster rate of money growth for two primary reasons. First, money is injected lump-sum, which is a very effective way to reduce income inequality (Result

[10] In order to match the data better, households in the model should be given the possibility to over-accumulate wealth, or to experience substantial wealth losses. An alternative, is to consider financial market access constraints as in [4].

2). Second, the availability of a bond market allows households to relax increasingly binding spending constraints through borrowing. Clearly, when shocks are not persistent, inequality is already small so inflation-induced consumption redistribution is minimal. The availability of a credit market has also implications for the impact of inflation on average welfare, which we summarize as follows.

Result 9 *When shocks are persistent and the labor supply is inelastic, expansionary monetary policy may raise average welfare.*

Tables 3c and 3d provide supporting evidence. Introducing a bond market generally lowers the average welfare cost of inflation because households can better self-insure and can also partially shield their precautionary savings from the inflation tax. As a consequence, the positive redistributive effect of lump-sum money injections is heightened. Whether inflation improves average welfare depends on the specifics of the economy in terms of shocks and labor supply.

With iid shocks, the welfare cost of inflation is positive and it monotonically increases with inflation (Table 3d). In this case, the association inflation-welfare cost is qualitatively comparable to that observed in the economy where money in the only asset (Table 3b), but the welfare cost is now quantitatively smaller. For instance, moving from 10% to no inflation was worth 3.67% consumption in the money-only economy, while it is worth only 0.07% in the economy with bonds. The message here is that the economy with money, bonds and iid shocks is qualitatively and quantitatively in line with representative-agent model findings.

TABLES 3c-d HERE

With persistent shocks, the average welfare cost is non-monotone; this is consistent with Result 5. However, now some inflation may improve average welfare, and this may occur when the concentration of wealth also increases (Table 3c). For example, the average household in the money-only economy would *give up* 5.3% consumption to avoid 10% inflation, but would *request* 1.11% additional consumption in the bond economy, even if wealth is more concentrated than at zero inflation. The welfare cost is U-shaped because inflation reduces both per-capita output and income inequality. The first (negative) effect is dominated by the second (positive) at low inflation rates if the labor supply is sufficiently inelastic and shocks are persistent. In this case income redistribution benefits a majority of households.

To see this, note that now the wealthiest households may also prefer some inflation (Table 3c, $\delta = 2$). The reason is simple. Unlike the money-only economy, wealthy households can now avoid the inflation tax by holding illiquid portfolios. In addition, lump-sum money injections provide some insurance against earnings risk, which can be valuable enough to overcome any decrease in current utility from lower consumption and less leisure). The value of this "additional" insurance increases with the persistence in earning shocks.[11]

[11] There are variations in current utility and continuation utility as inflation increases. Inflation

The results are qualitatively similar if households can fully relax their liquidity constraints through borrowing. In a further analysis, we allowed households to borrow *after* productivity shocks are realized and before the good market opens. In this case, liquidity constraints no longer bind for anyone, unless money and bonds are equivalent assets or households can borrow insufficient amounts (see Appendix A). When the model is calibrated to the same parameters used above, we find that consumption inequality is lower, and dispersion in money holdings simply reflects income disparities; no differences in qualitative results emerge.

6 Final comments

This study has shown that whether inflation can be used as an instrument to improve social welfare crucially depends on the persistence of shocks to earnings, financial structure and the elasticity of labor supply. Persistent shocks magnify inequality, hence the desirability of redistributive policies from an average welfare perspective. The financial structure determines the extent to which households with different wealth use money for self-insurance purposes, which has implications for how effectively inflation can redistribute wealth. The elasticity of labor supply affects the

distorts labor supply decisions. With higher inflation all households supply less labor. Lump-sum injections of money decrease income inequality, in addition. As a result, current utility increases for the poor, because they consume more and work less. The welfare gain for the poor is significant because of their high marginal utility of consumption. Their continuation utility also increases. On the other hand, the rich suffer a current utility loss because they consume less, but not very much. This utility loss is limited because the rich have low marginal utility of consumption and high marginal utility of leisure. Moreover, the rich's continuation utility may increase because the continuation utility from going to a low productivity state is now higher. Hence, the improvement in continuation utility can outweigh the loss in current utility, in which case the rich can also benefit from higher inflation.

inflation-induced output loss.

In the model, long-run inflation results from fully anticipated lump-sum money transfers. When households can only self-insure with money, increments in inflation have been shown to reduce wealth disparities. However, this does not necessarily make inflation socially desirable. In fact, we report that inflation can *increase* consumption inequality. When agents can access to a credit market and can choose the composition of their portfolios, inflation becomes ineffective at redistributing wealth. Overall, we report that a faster rate of fully-anticipated monetary expansion may increase average welfare if shocks are persistent and the labor supply is inelastic.

Additional intuition could be gathered by studying the model when money creation is the result of open market operations and credit markets are not equally accessible to every household.

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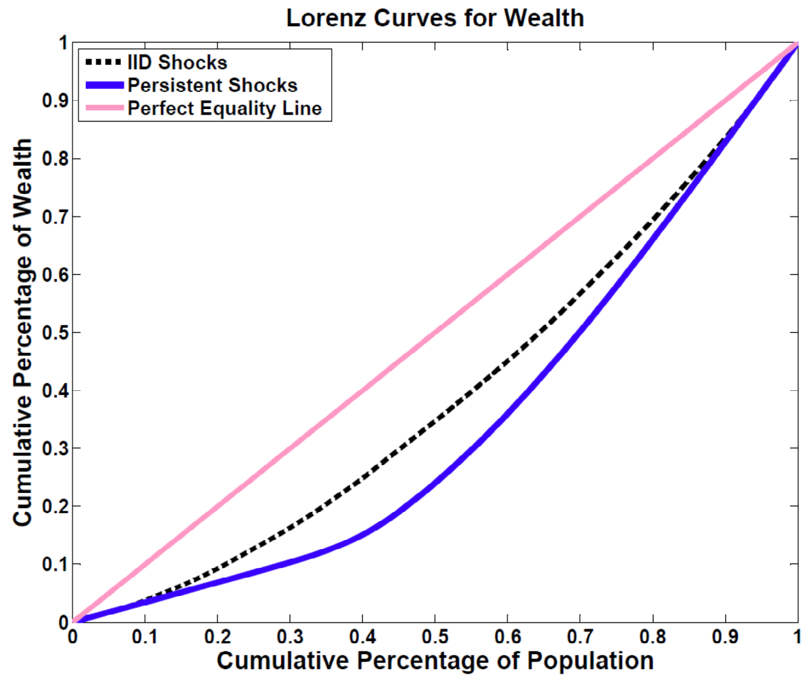
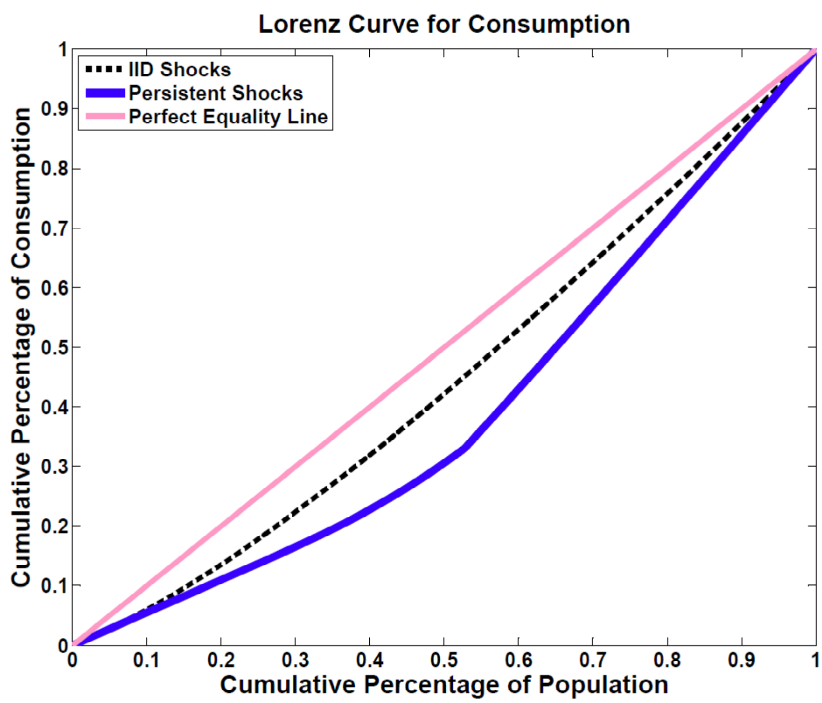
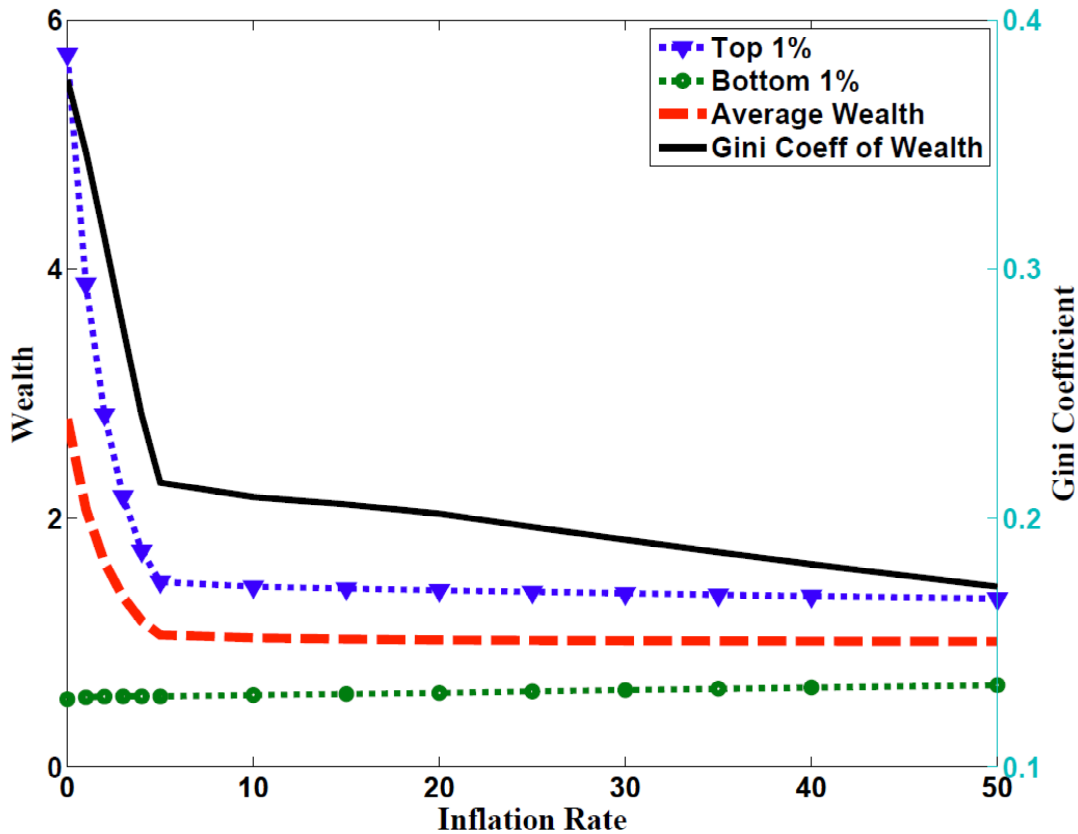
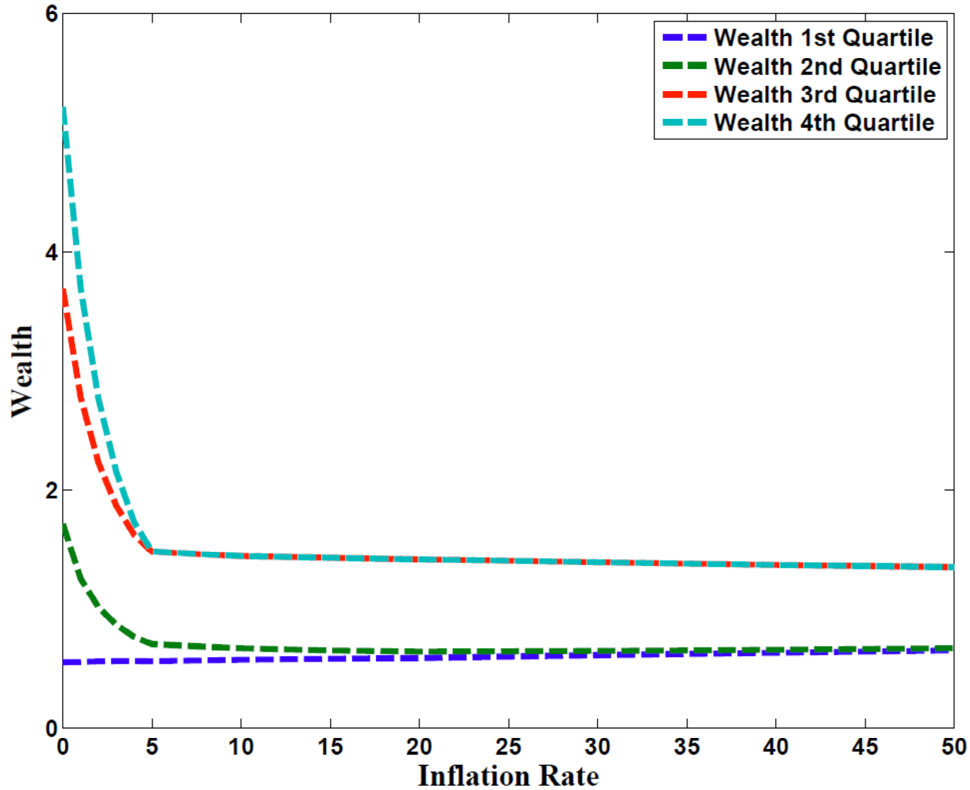




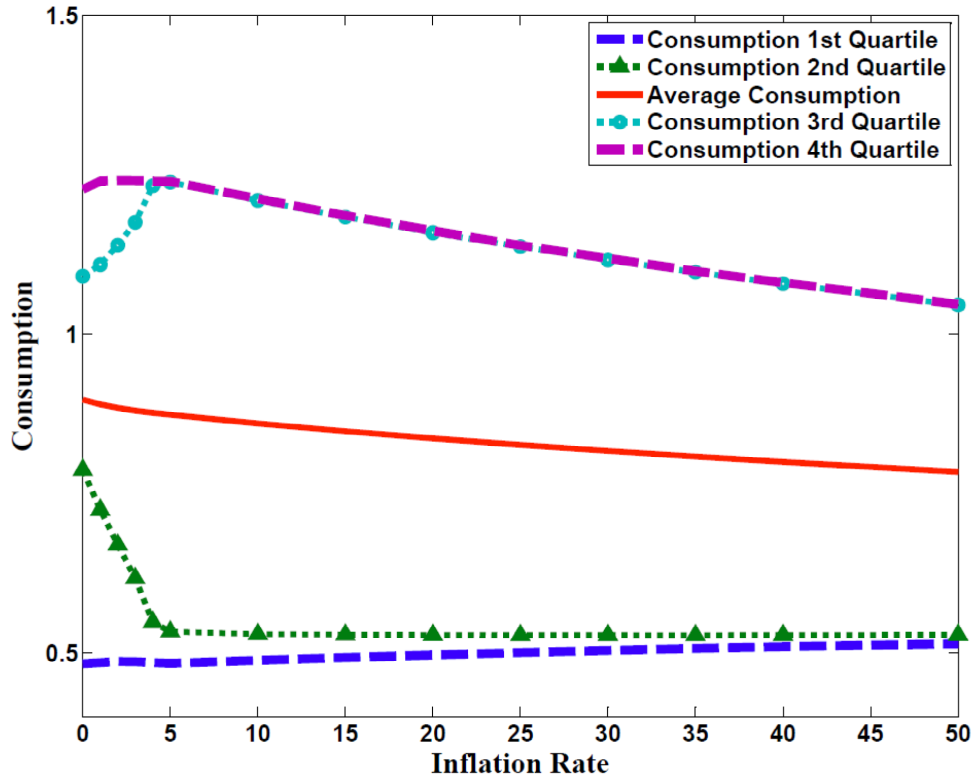
Notes to Figures 1a-1b: Lorenz curves are reported for stationary equilibrium at 2% inflation, baseline case (money only economy, persistent shocks, $\delta = 2$).



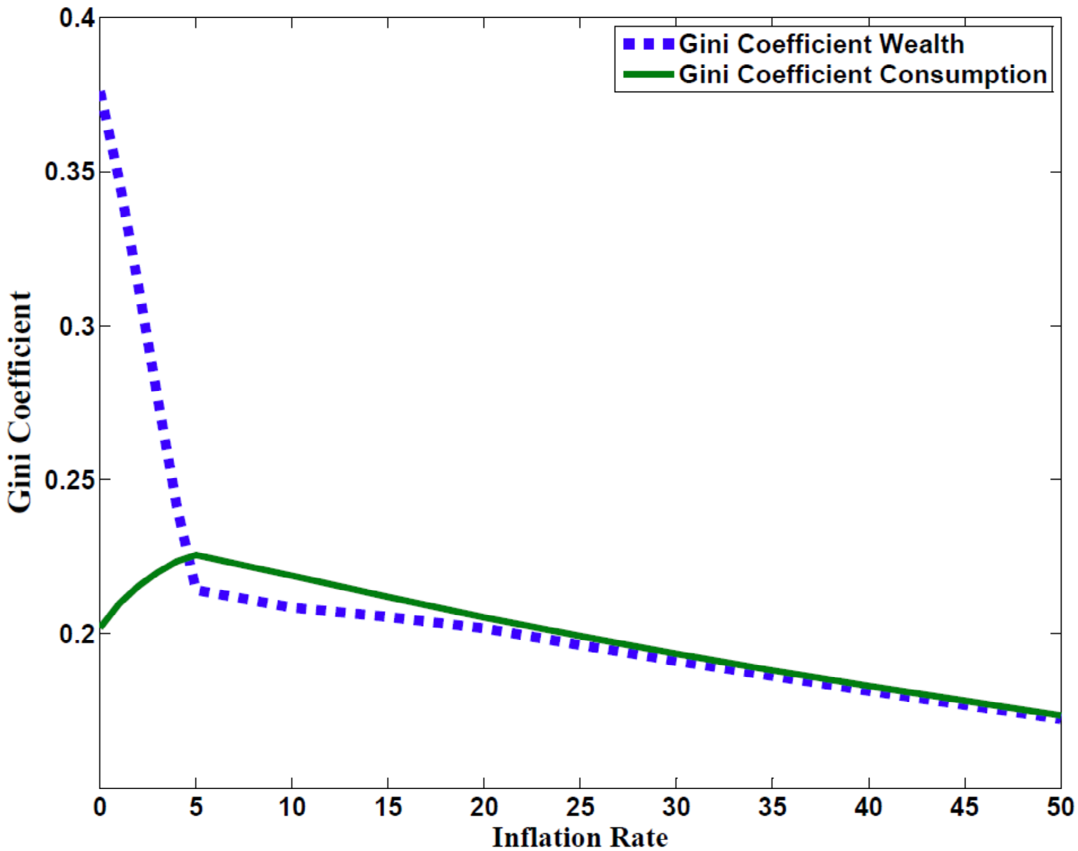
Notes to Figure 2a: The figure reports statistics regarding the distribution of money for inflation rates ranging from 0% to 50%, in single-digit increments, for the baseline case (money only economy, persistent shocks, $\delta = 2$). Statistics reported are: mean balances, average balances held by households in the top and in the bottom percentile of the wealth distribution, and Gini coefficient (scale on right vertical axis).



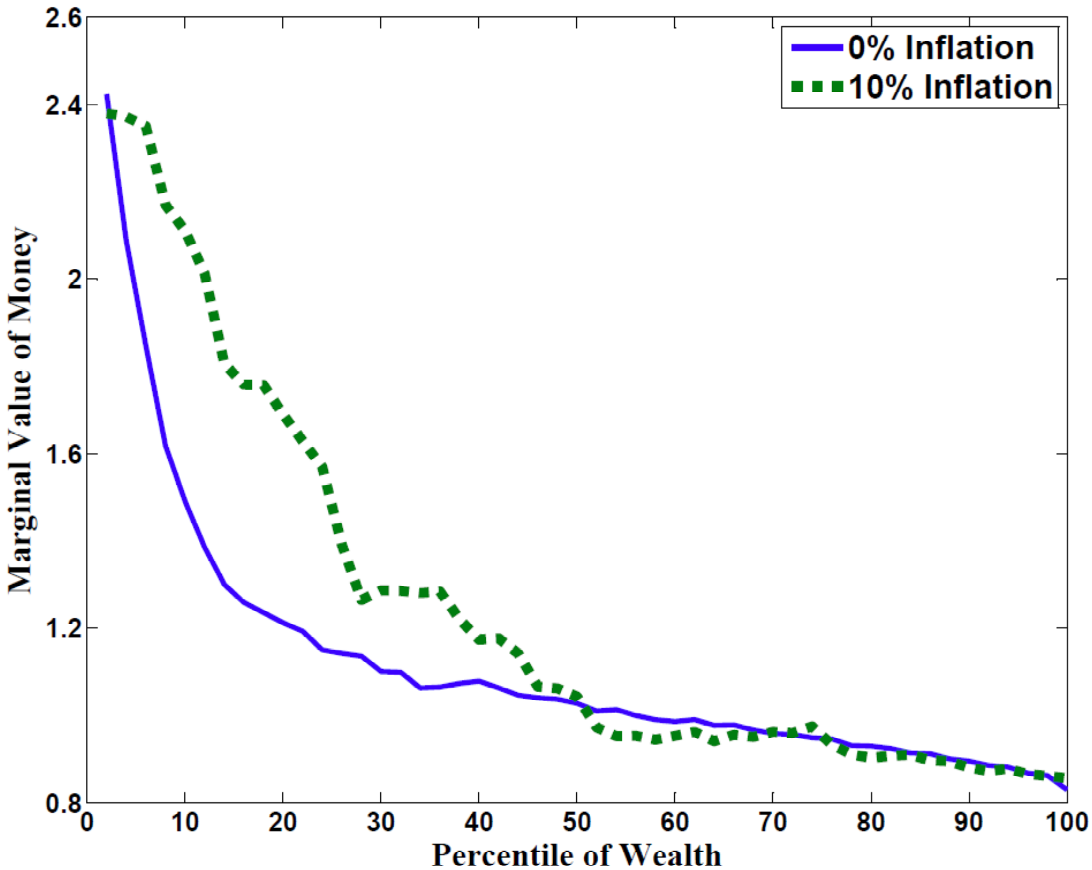
Notes to Figure 2b: The figure reports the equilibrium wealth level by quartiles of wealth , for inflation rates ranging from 0% to 50%, in single-digit increments, for the baseline case (money only economy, persistent shocks, $\delta = 2$). The figure provides an illustration of how inequality develops with inflation. As inflation reaches 5%, the wealth distribution becomes bimodal: agents with below-average wealth hold similar balances, and those with more-than-average wealth hold more than twice as many balances. The figure also illustrates the nonlinear association between inflation and wealth inequality.



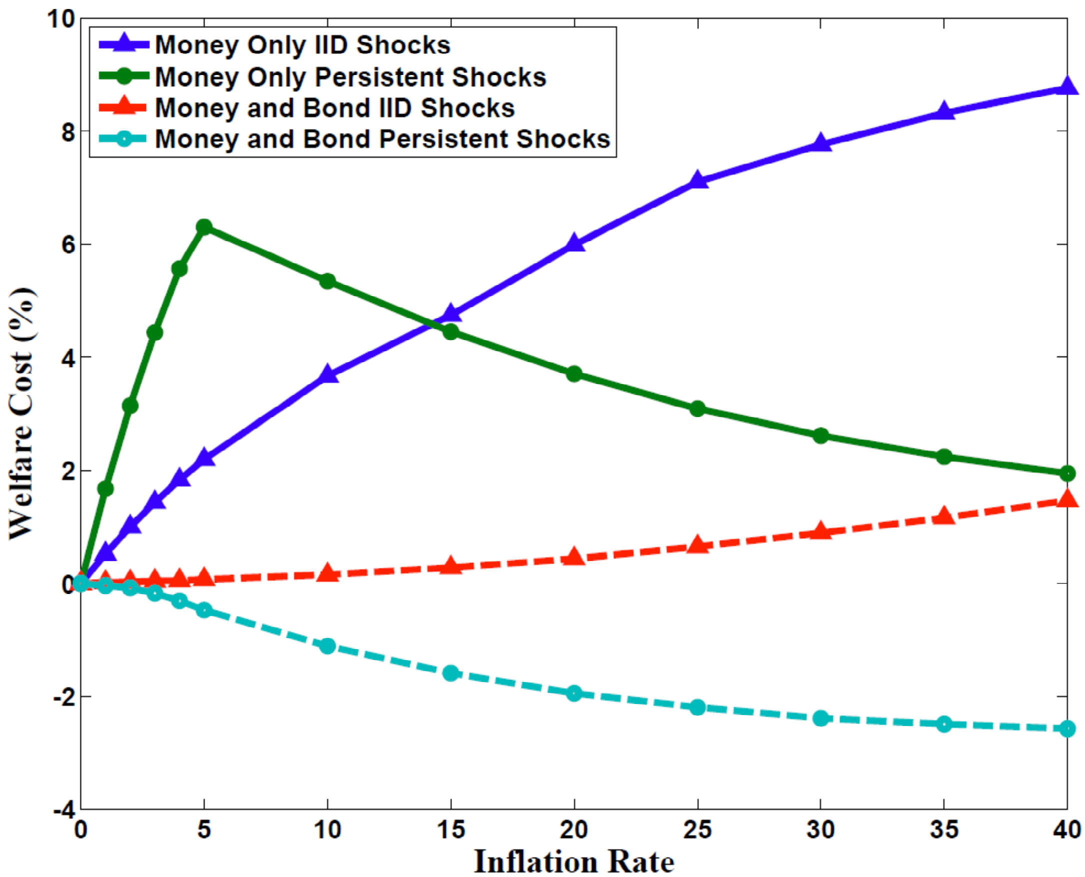
Notes to Figure 3a: The figure reports the equilibrium consumption level by quartiles of wealth , for inflation rates ranging from 0% to 50%, in single-digit increments, for the baseline case (money only economy, persistent shocks, $\delta = 2$).



Notes to Figure 3b: The figure reports the Gini coefficients of consumption and wealth for the baseline case (money only economy, persistent shocks, $\delta = 2$). Consumption and wealth inequality do not necessarily move in the same direction as inflation increases.



Notes to Figure 4: The figure reports the equilibrium marginal value of money by percentile of wealth, for inflation rates of 0% (solid line) and 10% (dashed line), for the baseline case (money only economy, persistent shocks, $\delta = 2$). The values reported are averages for two consecutive percentiles of wealth.



Notes to Figure 5: The figure reports the average welfare cost of $x\%$ inflation as opposed to no inflation for inflation rates ranging from 0% to 100% for persistent shocks and iid shocks (money only economy, $\delta = 2$).

	a. Persistent shocks					**b. Iid shocks**				
$\pi - 1$	$\overline{m}$	$\overline{c}$	$Gini_c$	$Gini_m$	$Gini_I$	$\overline{m}$	$\overline{c}$	$Gini_c$	$Gini_m$	$Gini_I$
0%	2.507	0.897	0.202	0.376	0.274	1.960	0.954	0.091	0.222	0.317
1%	1.843	0.890	0.210	0.347	0.259	1.767	0.947	0.100	0.218	0.309
2%	1.454	0.884	0.215	0.313	0.249	1.633	0.941	0.108	0.215	0.302
3%	1.201	0.880	0.220	0.277	0.242	1.533	0.936	0.115	0.212	0.296
4%	1.028	0.877	0.224	0.241	0.236	1.455	0.931	0.121	0.210	0.290
5%	0.928	0.874	0.226	0.214	0.232	1.394	0.926	0.126	0.208	0.285
10%	0.891	0.860	0.219	0.208	0.223	1.205	0.904	0.143	0.197	0.264
15%	0.870	0.848	0.212	0.206	0.215	1.105	0.886	0.151	0.190	0.247
20%	0.853	0.836	0.205	0.202	0.208	1.026	0.870	0.160	0.181	0.233
25%	0.840	0.826	0.199	0.196	0.202	0.967	0.856	0.167	0.174	0.221
30%	0.828	0.816	0.194	0.191	0.195	0.932	0.843	0.167	0.171	0.211
35%	0.817	0.807	0.188	0.186	0.190	0.904	0.830	0.165	0.170	0.203
40%	0.807	0.799	0.183	0.181	0.184	0.883	0.819	0.162	0.168	0.195

Table 1: Money-only economy

Notes to Tables 1-2: $\gamma = 1.3$ and $\delta = 2$; $\pi - 1$ is the net inflation rate; $\overline{x}$ is the mean value and $Gini_x$ is the Gini coefficient associated to the equilibrium random variable x. We define m =money balances (in real terms), c =consumption, I =income net of taxes/transfers, $b =$ bonds balances (in real terms), wealth is $w = m + b$.

	a. Persistent shocks						**b. Iid shocks**					
$\pi - 1$	$\overline{m}$	$\overline{c}$	$Gini_c$	$Gini_w$	$Gini_m$	$Gini_I$	$\overline{m}$	$\overline{c}$	$Gini_c$	$Gini_w$	$Gini_m$	$Gini_I$
0%	0.980	0.906	0.191	0.724	0.158	0.287	0.988	0.968	0.055	0.446	0.042	0.332
1%	0.939	0.902	0.190	0.732	0.173	0.283	0.979	0.965	0.055	0.448	0.044	0.329
2%	0.917	0.899	0.189	0.736	0.183	0.281	0.972	0.962	0.054	0.448	0.045	0.326
3%	0.908	0.896	0.188	0.739	0.185	0.278	0.966	0.959	0.054	0.448	0.047	0.323
4%	0.903	0.894	0.186	0.741	0.185	0.276	0.962	0.956	0.053	0.447	0.048	0.320
5%	0.900	0.891	0.185	0.740	0.184	0.274	0.957	0.953	0.053	0.446	0.048	0.317
10%	0.885	0.879	0.179	0.738	0.179	0.264	0.941	0.939	0.051	0.440	0.049	0.302
15%	0.873	0.868	0.173	0.737	0.173	0.255	0.927	0.926	0.049	0.437	0.049	0.289
20%	0.861	0.858	0.168	0.736	0.168	0.247	0.915	0.915	0.047	0.435	0.047	0.277
25%	0.850	0.849	0.163	0.734	0.163	0.239	0.903	0.903	0.046	0.436	0.046	0.266
30%	0.841	0.840	0.158	0.731	0.158	0.231	0.893	0.893	0.044	0.436	0.044	0.256
35%	0.832	0.831	0.154	0.729	0.154	0.224	0.883	0.883	0.043	0.437	0.043	0.247
40%	0.824	0.823	0.150	0.724	0.150	0.217	0.873	0.873	0.042	0.439	0.042	0.238

Table 2: Money and bonds economy

	$\delta = 1.5$	$\delta = 2$					$\delta = 3$
$\pi - 1$	Δ_π	Δ_π	Q_1	Q_2	Q_3	Q_4	Δ_π
0%	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1%	2.213	1.674	−0.786	2.219	2.061	3.328	1.302
2%	4.155	3.143	−0.866	4.103	3.266	6.126	2.455
3%	5.866	4.438	−0.582	6.029	3.802	8.355	3.461
4%	7.389	5.569	−0.168	8.373	3.533	10.113	4.346
5%	8.739	6.300	0.096	9.020	4.281	11.251	4.336
10%	8.392	5.348	−0.918	7.856	3.827	10.888	3.373
15%	7.496	4.451	−1.821	6.739	3.479	10.388	2.499
20%	6.670	3.705	−2.636	5.833	3.184	10.053	1.774
25%	6.028	3.090	−3.284	5.000	3.014	9.776	1.185
30%	5.542	2.615	−3.783	4.289	2.912	9.631	0.711
35%	5.183	2.241	−4.219	3.725	2.884	9.547	0.307
40%	4.889	1.945	−4.571	3.230	2.888	9.533	−0.035

Table 3a: Distribution of welfare costs (money-only economy with persistent shocks)

Notes to Tables 3a-d: $\gamma = 1.3$ and $\delta = 2$ unless otherwise noted; $\pi - 1$ is the net inflation rate. The welfare cost Δ_π is reported in percent of current consumption the average household would give up to be at zero inflation (two-digit approximation; a negative number indicates a welfare gain). Q_i denotes the i^{th} quartile of the wealth distribution.

	$\delta = 1.5$	$\delta = 2$					$\delta = 3$
$\pi - 1$	Δ_π	Δ_π	Q_1	Q_2	Q_3	Q_4	Δ_π
0%	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1%	0.641	0.522	0.183	0.417	0.601	0.882	0.419
2%	1.220	0.998	0.423	0.793	1.164	1.603	0.794
3%	1.759	1.432	0.668	1.119	1.722	2.200	1.129
4%	2.266	1.833	0.928	1.510	2.078	2.790	1.427
5%	2.750	2.196	1.203	1.855	2.441	3.250	1.688
10%	4.710	3.668	2.358	3.374	3.768	5.097	2.753
15%	6.169	4.748	3.252	4.472	4.755	6.411	3.560
20%	7.753	5.989	4.302	5.733	5.924	7.851	4.538
25%	9.455	7.103	5.255	6.875	6.991	9.104	5.181
30%	10.536	7.763	5.813	7.573	7.651	9.813	5.430
35%	11.394	8.322	6.292	8.167	8.227	10.383	5.664
40%	12.034	8.764	6.655	8.637	8.664	10.874	5.902

Table 3b: Distribution of welfare costs (money-only economy with iid shocks)

	$\delta = 1.5$			$\delta = 2$			$\delta = 3$
$\pi - 1$	Δ_π	Δ_π	Q_1	Q_2	Q_3	Q_4	Δ_π
0%	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1%	0.029	−0.039	−0.206	0.083	−0.045	0.097	−0.049
2%	0.027	−0.078	−0.331	0.109	−0.046	0.117	−0.122
3%	−0.058	−0.171	−0.495	0.099	−0.080	0.049	−0.242
4%	−0.147	−0.306	−0.671	−0.017	−0.150	−0.018	−0.376
5%	−0.331	−0.471	−0.891	−0.157	−0.278	−0.058	−0.527
10%	−1.029	−1.109	−1.741	−0.847	−0.462	−0.334	−1.097
15%	−1.553	−1.588	−2.491	−1.352	−0.515	−0.494	−1.583
20%	−1.951	−1.949	−3.031	−1.832	−0.491	−0.578	−1.942
25%	−2.221	−2.196	−3.446	−2.128	−0.427	−0.634	−2.222
30%	−2.392	−2.390	−3.763	−2.392	−0.493	−0.522	−2.433
35%	−2.481	−2.494	−4.036	−2.567	−0.421	−0.371	−2.599
40%	−2.515	−2.571	−4.269	−2.668	−0.324	−0.278	−2.691

Table 3c: Distribution of welfare costs (bond economy with persistent shocks)

	$\delta = 1.5$			$\delta = 2$			$\delta = 3$
$\pi - 1$	Δ_π	Δ_π	Q_1	Q_2	Q_3	Q_4	Δ_π
0%	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1%	0.012	0.021	0.029	0.028	0.007	0.022	0.004
2%	0.041	0.026	0.052	0.026	0.002	0.033	0.008
3%	0.060	0.042	0.089	0.028	0.029	0.034	0.010
4%	0.076	0.049	0.071	0.027	0.037	0.079	0.017
5%	0.101	0.069	0.086	0.042	0.057	0.113	0.019
10%	0.238	0.152	0.103	0.103	0.172	0.266	0.075
15%	0.422	0.278	0.193	0.237	0.363	0.368	0.123
20%	0.657	0.437	0.352	0.415	0.543	0.496	0.245
25%	0.931	0.648	0.574	0.637	0.804	0.638	0.377
30%	1.254	0.890	0.838	0.904	1.093	0.785	0.531
35%	1.607	1.158	1.111	1.196	1.409	0.976	0.691
40%	1.992	1.464	1.418	1.559	1.733	1.203	0.918

Table 3d: Distribution of welfare costs (bond economy with iid shocks)

$\pi - 1$	Q_1	Q_2	Q_3	Q_4
0%	1.000	0.919	0.403	0.250
1%	1.000	0.907	0.393	0.250
2%	1.000	0.900	0.392	0.251
3%	1.000	0.902	0.393	0.251
4%	1.000	0.902	0.393	0.251
5%	1.000	0.901	0.393	0.250
10%	1.000	0.902	0.395	0.248
15%	1.000	0.905	0.397	0.246
20%	1.000	0.907	0.397	0.244
25%	1.000	0.907	0.397	0.242
30%	1.000	0.906	0.398	0.241
35%	1.000	0.905	0.399	0.240
40%	1.000	0.901	0.400	0.240

Table 4: Money/Total Asset ratios by wealth quartiles

Notes to Table 4: The table is drawn for the benchmark calibration of persistent shocks with $\gamma = 1.3$ and $\delta = 2$; $\pi - 1$ is the net inflation rate. Columns 2-5 report the average real money balances divided by the average total assets held by households in each of the four wealth quartiles; this ratio varies from zero to one.