Optimal Taxation and Social Networks

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Abstract

We study optimal tax policy in a model economy where workers find their jobs from their peers in a social network. The unemployment rate is determined by the dynamic of the labor market, which is governed by the social network. Unemployment results as individuals are unsuccessful in hearing about job opportunities themselves or through their peers in a network. The design of optimal tax policy follows the Ramsey approach. The optimal limiting capital tax rate is zero, independent of the labor market frictions. The optimal labor income tax is decreasing in the unemployment rate and the job network process parameters play an important role in determining optimal fiscal policy. We allow agents to invest some of their time on building links and connect to peers (endogenous network). The optimal tax is negatively related to the transmission rate of job information from peers in a particular network and it is lower in more connected job network economies.

WORK IN PROGRESS

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1 Introduction

The importance of social networks in labor markets has long been understood. Networking plays a critical role in job searching and in improving the quality of the match between firms and workers. The job network literature indicates that access to information about job opportunities is heavily influenced by social structure and that individuals use connections with others (e.g., relatives, friends, acquaintances) to build and maintain information networks.[1] Social networks have important implications for the dynamics of employment, as well as, the duration and persistence of unemployment (Calvó-Armengol and Jackson, 2004). As networks might affect economic outcomes, the relevance of social networks for the design of government policies must be recognized and explored.

The literature on optimal labor income taxation, however, has neglected the role of social networks in the labor market and has mainly focused on competitive or job search labor markets. Empirical research indicates that about half of jobs are obtained through networking and the other half are obtained through more formal methods (see Holzer, 1988; Montgomery, 1991; Topa, 2001, Gregg and Wadsworth, 1996; Addison and Portugal, 2001). Well-known results in the theory of optimal labor taxation are that tax rates on labor should be roughly constant, i.e., the optimal labor income tax rates are constant across time and states (Barro, 1979; Kyndland and Prescott, 1980; Chari and Kehoe, 1999), and labor taxes vary positively with employment (Zhu, 1992; Scott, 2007). In this paper we examine if these results survive when the labor market is governed by job networks.

We study optimal tax policy in a model economy where the informational structure of the job networking follows the classic epidemic diffusion model, surveyed recently in Vega-Redondo (2007). We apply the mean field approach, which assumes there are no correlations or neighborhood effects in information transmission, and a network is described by a degree distribution. Our approach amounts to assuming the average state of the network is replicated locally, for every agent, so that the proportion of an agent's peers who are employed is given by the employment rate. The mean-field approach is analytically simple and allows us to calculate well the long-run, average behavior of arbitrary networks, including power-law distributions and networks with the "small-worlds" properties of low diameter and high clustering. As expected,

[1] See Granovetter (1995) and Ioannides and Loury (2004) for a recent survey.

our model predicts that changes in the social network structure will alter the unemployment rate. For instance, an increase in the density of social ties generates lower unemployment level.

Information about job opportunities arrives randomly. All jobs are identical and the job arrival process is independent across agents. If the agent is unemployed, she will take the job. On the other hand, if the agent is already employed then he may pass the information along to a friend, relative or acquaintance who is unemployed. Each agent is connected to others through a network. Workers without jobs are in competition for the job information that their peers may pass them. The strength of social ties among workers determines the probability their peers pass job information along.

Unemployment results when individuals are unsuccessful in hearing about job opportunities themselves or through their peers in a network. Agents do not know the employment status of their peers and job information cannot be passed on if it is not needed; in this sense job information may be lost. The employment rate is then determined by the dynamic of the labor market, which is governed by the social network. That is, the flow of agents between employment and unemployment status depends on the job arrival and break-up probabilities and a worker's social network contacts. We will consider several different classes of network, and investigate their properties.

There are at least three main reasons for studying optimal labor income taxation in this environment. First, one of the most robust and best-studied roles of social networks concerns obtaining employment. There have been a number of studies of how social contacts matter in obtaining information about job openings. Second, labor income tax rates vary substantially over time and across countries and high labor taxes are often seen as one of the causes of high unemployment rates. And, third, our extensions to the specific models used by Lucas and Stokey (1983) and Chari, Christiano and Kehoe (1991), with the addition of social networks, enable us to provide a new insight into the relationship between taxes and the labor market dynamics.

This paper embeds the job network model into the general equilibrium framework and the design of optimal tax policy follows the Ramsey approach. This approach to optimal taxation is a solution to the problem of choosing optimal taxes given that only distortionary tax instruments are available. A social planner maximizes its objective function given that agents are in a competitive equilibrium and the optimal path of the planner's fiscal instruments are obtained such that the agent's utility is maximized. We follow the majority of the literature in assuming that there are institutions that effectively solve the time inconsistency problem so that the government can commit to its announced policy.

Our analysis proceeds in three stages. First, we characterize the long-run unemployment rate in the economy, as function of the underlying social network, and job transmission processes. Second, we consider an economy with a representative infinitely lived household. Each household consists of a continuum of family members, which either work or are unemployed. Employed workers receive a wage that is determined competitively, while agents without a job receive an unemployment benefit. Unemployed workers do not search for a job but rather learn about job opportunities through peers in their social network. Family members without a job can spend time to develop their social connections, increasing the strength of their ties to their peers. A stronger connection to their peers results in more job information from their employed peers, and will improve their chances of finding a job while unemployed. We assume that the time devoted to social networking affects the job transmission rate. That is, the rate at which job information is passed from employed workers to his unemployed peers in any period depends on how much effort agents spent on social networking in the previous period. This network effort intensity represents an additional trade-off for the agents. It improves their chances to become employed but at a (leisure) cost. Once a worker finds a job, he is beyond the social network dynamics and no effort is devoted to improve social contacts. We derive the optimal labor income tax and show that, under some conditions, the Ramsey optimal policy consists in making the labor income tax decreasing in the unemployment rate. Finally, we explore how different aspects of social networks can affect the design of the optimal tax policy via the determination of the unemployment rate in this economy.

Regardless the structure of the social networks and the dynamics of the labor market, the optimal limiting capital tax rate is zero as in Chamley (1986) and Judd (1985). The main reason for this result is that firms in our economy take the number of workers as given and choose optimally only the hours worked by these workers. If we had assume that firms could affect the employment rate we would be moving closer to the labor search literature. Domeij (2005) finds that the capital tax would be non-zero as long as the workers bargaining power is different than the elasticity of search in the matching function. In our model, if we endogenized the probability that an agent hears about a job opening, instead of the job information transmission rate, we would be able to replicate this same result.

In our economy, labor taxation is directly related to the unemployment rate, and so indirectly determined by the structure and properties of the job network. Although the government does not observe how job information is passed from one worker to another, and here we are not arguing that it should, it should recognize the relevance of job networks for the design of the optimal labor income tax, through their impact on the determination of the economy's equilibrium unemployment rate. In other words, the introduction of labor market frictions through job networks implies that the optimal tax policy should feature some response to unemployment. We show that labor income taxes vary negatively with unemployment and there is a positive relationship between labor income taxes and hours worked. Labor is more inelastically supplied when employment is high and, since the Ramsey planner is required to tax inelastic variables more heavily to minimize tax distortions, labor income tax rates vary positively with hours worked (Zhu, 1992; Scott, 2007).

At the steady state, the number of newly employed agents is exactly equal to the number of newly unemployed agents, and the economy will remain at this level of employment indefinitely. This long run prevalence of employment we take to be the economy's employment rate. This in turn defines the steady state unemployment rate. This steady state unemployment rate is decreasing in the job arrival probability, the job information transmission probability, and is increasing in the job break up probability. Since the optimal labor income tax is decreasing in the unemployment rate, it is positively related to the transmission rate of job information from peers in a particular network. In less connected economies, the unemployment is inefficiently high and the planner faces a tradeoff that calls for responding to unemployment (which reduces households' welfare) by reducing labor income taxation. We show that the optimal income tax is higher in more connected job network economies.

The paper proceeds as follows. In section 2, we present the model economy and characterize a labor market dynamics governed by social networks and exogenous job separation. We discuss how the unemployment rate is affected by job networking where the informational structure of the job networking follows the classic epidemic diffusion model. Section 3 and 4 studies the economy in steady state. We first characterize the long run employment in different network structures and then derive the optimal labor tax as function of the unemployment rate. We show shows how job networking can affect the optimal labor tax via the unemployment rate, when this rate is exogenously given in the agent's problem or it is affected by agent's effort to strength social ties. Section 6 concludes.

2 The Model Economy

There is a continuum of infinitely lived agents whose total measure is normalized to one. The economy is populated by households who consume, save and work. Each agent can be either employed or unemployed. Employed workers receive a wage that is determined competitively, while agents without a job receive an unemployment benefit. The labor market is characterized by social networks, meaning that unemployed workers learn about job opportunities through peers in their social network. The informational structure of the job networking follows the classic epidemic diffusion model, surveyed in Vega-Redondo (2007). The flow of agents between employment and unemployment status depends on a worker's social network contacts and on an exogenous job separation rate. We consider the role of social networks as a manner of obtaining information about job opportunities and study its implications for the dynamics of employment and the structure of optimal capital and labor income taxation.

2.1 Network Structure and Employment Rate Determination

There are two classes of agents in this economy: employed and unemployed workers. Time evolves in discrete periods indexed by t and information about job opportunities arrives randomly. All jobs are identical and the job arrival process is independent across agents. Each agent hears about a job opening with probability $\gamma \in [0,1]$. If the agent is unemployed, she will take the job. On the other hand, if the agent is already employed then she may pass the information along to a friend, relative or acquaintance who is unemployed. The rate at which an employed worker passes information to each of her unemployed peers is given by $v \in [0,1]$. We will elaborate more on this transmission rate when we present the household problem. The rate v is distinct from γ, and need not be directly derived from it. Let ρ be the exogenous job break up probability, which is independent across agents.

Each agent may have peers to whom she passes information when employed, and from whom she may receive information when unemployed. These peers are connected to one another in a social network. A *network* is described by a symmetric matrix M, where $m_{ij} \in \{0, 1\}$ denotes whether a link exists between agents i and j. That is, $m_{ij} = 1$ indicates that i and j know each other and $m_{ij} = 0$ otherwise. We assume that $m_{ij} = m_{ji}$, meaning that the relationship between i and j is reciprocal. The structure of this network m will determine how information flows throughout the network, and will have a large impact on each agent's employment status.

We are concerned with *large* networks, that is, a network among the continuum of agents, so that there are infinitely many nodes in this network. A key property of a network is its *degree distribution* $\{D_z\}_{z=1}^{\infty}$, where D_z is the proportion of agents who have z peers. A network's degree distribution summarizes much of its structure: whether there a some workers with many links, or not, and the relative prevalence of highly connected workers.[2] In general, there may be many networks m consistent with a particular degree distribution $\{D_z\}_{z=1}^{\infty}$. As networks grow large, much local information ceases to matter, so focussing on degree distributions is appropriate (Vegas-Redondo 2007). In other words, we are not concerned about particular network structures but rather focus on large classes of networks sharing the same degree distribution. We may think of the actual network m as being a random draw from the set of networks having degree distribution $\{D_z\}_{z=1}^{\infty}$. This is called the *random network* approach. In particular, we study the empty, regular, power-law and geometric degree distributions.

The employment rate may be different for agents with different number of links (peers) z. The average employment rate can then be expressed as follows:

$$n_t = \int_{z=1}^{\infty} (n_{zt} D_z)\, dz,$$

where n_{zt} is the employment rate among agents with z links. Agents who have more links may expect to hear about jobs from their peers more often, and their employment status will evolved differently than that of an unemployed agent with fewer links.

To analyze the dynamics of employment, we apply the mean field approach, which assumes

[2] Some important network properties, however, are not captured by the degree distribution, such as detailed local structures and clustering. For example, if workers who have a common peer are also likely to be connected themselves, this fact will not be captured by the degree distribution.

there are no correlations or neighborhood effects in information transmission. Our approach amounts to assuming the *average* state of the network is replicated *locally*, for every agent, so that the proportion of an agent's peers who are unemployed is given by the unemployment rate (Vega-Redondo 2007).[3] The mean field approach relies on an assumption of *homogenous mixing*, i.e., there are no systemic differences between each worker's local neighborhoods. This could be justified by imagining that a worker with z links does not have the same peers period after period, but continually draws new peers, randomly from the network. In that case, because the network is large, he could not infer anything about their employment status beyond the average in the network, and the mean-field approach is correct. Even without that formal assumption, the mean field approach has been shown in simulations to give good answers for the long-run dynamics in the networks we will consider (Vega-Redondo 2007, Jackson 2008).

Following the mean-field approach, and suppressing the subscript t when there is no confusion, we can determine the law of motion for employed workers as follows:

$$\dot{n}_z = -\rho n_z + (1 - n_z)[\gamma + (1 - \gamma)(1 - (1 - v\theta)^z)] \tag{1}$$

where employment is given by total labor force, normalized to one, minus the number of unemployed workers, i.e., $n_t = 1 - u_t$. The change in the level of employment has three main components. First, ρ percent of agents who are employed will lose their jobs. Second, a fraction γ of the unemployed agents will hear of a job themselves. Third, of those unemployed workers who do not hear of a job opportunity themselves, each of their z peers is employed with probability θ, and passes job information at rate v; the probability that at least of of their z peers passes them information is $(1 - (1 - v\theta)^z)$. We will consider v exogenous for the moment, but will late require workers to invest time and effort into maintaining their social relationships, so that v is a function of effort, e_t.

The probability an unemployed worker's peers are employed (θ) will also depend on the employment status of the network as a whole. According to the mean field approach, we can

[3] This is not necessarily true, in general. Calvo-Armengol and Jackson (2004) showed that each worker's employment status is correlated with that of his peers, so an agent who remembers his past status could infer the expected employment rates of his peers, and this need not be equal to the average state of the network.

define this probability in the following way:

$$\theta = \int_{z=1}^{\infty} (n_z \psi_z)\, dz, \tag{2}$$

where ψ_z is the probability an agent's peer has z links, which is given by

$$\psi_z = \int_{z=1}^{\infty} \left(\frac{zD_z}{\int_{z=1}^{\infty} (zD_z)\, dz} \right) dz = \int_{z=1}^{\infty} \left(\frac{zD_z}{\langle z \rangle} \right) dz,$$

where $\langle z \rangle = \int_{z=1}^{\infty} (zD_z)\, dz$ is the average degree in the network. Note that $\psi_z \neq D_z$, i.e., the probability your peers have z links is not equal to the proportion of the population that has z links. This is because agents with many peers, and a large z, are disproportionately likely to be your peers. Plugging ψ_z into the definition of θ, equation (2), we have

$$\theta = \frac{1}{\langle z \rangle} \int_{z=1}^{\infty} (z n_z D_z)\, dz.$$

This implies that the probability an agent's peers are employed (θ) depends on the average degree in the network $\langle z \rangle$, the number of links each of these peers have, z, the proportion of agents who have z peers (D_z) and the employment rate among agents with z links (n_z).

Hence, in this economy the employment rate n_t follows a stochastic process and it is a function of the state of the network S, represented by the break-up probability (ρ), the job arrival probability (γ), the job transmission rate (v) and the degree distribution (D_z). Moreover, as the job transmission rate will depend on the time allocated to social networking e_t, the employment rate is also affected by agents' allocation decisions, which we discuss in details in the next section.

2.2 Households and Firms

In a typical household there is a measure n_t of employed family members and a measure $1 - n_t$ of unemployed individuals. Employed members supply labor hours l_t and unemployed members spend time e_t in developing their social connections, increasing the strength of their ties to their peers. A stronger connection to their peers results in more job information their employed peers, and will improve their chances of finding a job while unemployed.

We assume that the time devoted to social networking affects the job transmission rate. That is, the rate at which job information is passed from employed workers to his unemployed peers in time t depends on how much effort (e_{t-1}) agents spent on social networking in period $t-1$, i.e. $v = v(e_{t-1})$. The job transmission rate is determined according to the following decreasing returns to scale relationship technology:

$$v(e_{t-1}) = e_{t-1}^{1-\lambda},$$

where λ measures the efficacy of this technology. When λ is close to 1, workers are able to build strong relationships with relatively little cost, in terms of time and foregone leisure. When λ is close to 0, maintaining social relationships is more difficult, and requires a greater invest of time. The effort intensity e represents an additional trade-off for the agents. It improves their chances to become employed but at a (leisure) cost. Once a worker finds a job, he is beyond the social network dynamics and no effort is devoted to improve social contacts. Viewing v as a function of the investment in social ties implies that the entire long run level of employment in the economy is also a function of it, i.e., $n_t = n(e_{t-1})$.

Preferences are represented by the following utility function

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \tag{3}$$

where the momentary utility function u is increasing, concave and differentiable and β is the discount rate which lies in $(0, 1)$. The variable c_t is family consumption and the time endowment is normalized to 1 so that leisure is $h_t = 1 - n(e_{t-1})l_t - (1 - n(e_{t-1}))e_t$.

The timing of the model is as follows. At the beginning of each period, unemployed family members choose how much time they invest on social network e. Next employed family members - those that started the period with a job and those that just heard about and got a job - choose l. Then goods consumption c is determined. Households have two options with output they do not consume: they can invest in capital (k) or purchase government bonds (B). Next, employed family members are paid a wage (w) for the labor services, the unemployed receives unemployment benefits (b) and the family receives (tax free) interest (R) earnings on bonds and rental rate (r) of capital. The household takes as given government determined tax rates on labor (τ^l) and capital (τ^k) income. As in Calvó-Armengol and Jackson (2004), we interpret the timing as one where job break-up occurs, essentially, at the beginning of the period.

The sequence of real budget constraints reads as follows

$$c_t + k_{t+1} + B_{t+1} = n(e_{t-1})(1-\tau_t^l)w_t l_t + (1-n(e_{t-1}))b_t + (1-\tau_t^k)r_t k_t + (1-\delta)k_t + B_t R_t \quad (4)$$

where δ is the rate at which capital depreciates each period, R_t and r_t are the real rate of return on bonds and capital, respectively, and

$$n_t = n(e_{t-1}) = \int_{z=1}^{\infty} (n_z(e_{t-1})D_z)\, dz,$$

where $n_z(e_{t-1})$ is the employment rate among agents with z links in period t. The total household income is divided evenly among all individuals, so that family member perfectly insure each other against variation in labor income (Domeij, 2005). Or, alternatively, we can assume that agents can insure themselves against earning uncertainty and unemployment and, for this reason, wage earnings are interpreted as net of insurance costs (Merz, 1995; Andolfatto, 1996; Faia, 2008). Employed and unemployed family members consume the same amount and capital allocation and bonds purchase is a family decision.

Firms produce a single good and maximize profit taking factor prices as given. Production technology is a constant returns Cobb-Douglas specification so that output (y) is[4]

$$y_t = F(k_t, l_t) = (k_t)^{\alpha} (n(e_{t-1})l_t)^{1-\alpha} \quad (5)$$

where $\alpha \in (0,1)$ is the capital income share. Firms operate under perfect competition and earn zero profits in equilibrium. Factors of production are paid their marginal products, i.e. $F_k(t) = r_t = \alpha (k_t)^{\alpha-1} (n(e_{t-1})l_t)^{1-\alpha}$ and $F_l(t) = w_t = (1-\alpha)(k_t)^{\alpha} (n(e_{t-1})l_t)^{-\alpha} n(e_{t-1})$, where $F_k(t)$ and $F_l(t)$ denote the marginal product of capital and labor, respectively, and r_t is the rental rate of capital and w_t the wage rate for labor. Our specification of the technology is in line with the approach presented in Chari, Kehoe and McGrattan (2001). Differently than models of search that allow firms to influence the number of workers to be hired, through for

[4] Here we specify the production function although we keep utility function general. We will analyze how different preferences affect optimal taxation.

instance the number of vacancies posted (Domeij, 2005), here the number of workers employed n_t is entirely defined by the dynamics of the network and workers' time allocation decision process.

2.3 The Government and Aggregate Resources

The government faces the budget constraint

$$g_t + (1 - n(e_{t-1}))b_t = n(e_{t-1})\tau_t^l w_t l_t + \tau_t^k r_t k_t + B_{t+1} - B_t R_t. \tag{6}$$

where g_t denotes government consumption, which is assumed to be exogenously specified. The government finances its expenditures by levying taxes on labor and capital and issuing government bonds.

The economy as a whole faces the following aggregate resource constraint

$$c_t + k_{t+1} + g_t = F(k_t, l_t) + (1 - \delta)k_t. \tag{7}$$

2.4 The Network Competitive Equilibrium

A representative household, taking prices, taxes and the social network structure as given, chooses $\{c_t, k_{t+1}, l_t, e_t, B_{t+1}\}$ to solve

$$\max_{\{c_t, k_{t+1}, l_t, e_t, B_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \tag{P.1}$$

subject to

(1) $c_t + k_{t+1} + B_{t+1} = n(e_{t-1})(1 - \tau_t^l)w_t l_t + (1 - n(e_{t-1}))b_t + T_t k_t + B_t R_t$ for all $t \geq 0$,

(2) $n_z(e_t) = (1 - \rho)n_z(e_{t-1}) + (1 - n_z(e_{t-1}))[\gamma + (1 - \gamma)(1 - (1 - v(e_{t-1})\theta)^z)]$ for all $t \geq 0$,

(3) $k_0, B_0,$ and n_0 given.

where $T_t = [1 + (1 - \tau_t^k)(r_t - \delta)]$ is the gross return on capital after taxes and depreciation and $n_z(e_t)$ is the employability rate of an agent who has z peers in period $t + 1$ ($n_{z,t+1}$). Let $u(t) = u(c_t, h_t)$ and likewise for $u_i(t)$, where $i = 1$ for consumption and $i = 2$ for leisure. The equilibrium conditions form the family's problem can be represented as

$$u_2(t) = u_1(t)(1-\tau_t^l)w_t \tag{8}$$

$$R_t = (1-\tau_t^k)r_t + (1-\delta) \tag{9}$$

$$(1-n(e_{t-1}))u_2(t) = \beta n'(e_{t-1})\left[u_1(t+1)\left((1-\tau_{t+1}^l)w_{t+1}l_{t+1} - b_{t+1}\right) - u_2(t+1)(l_{t+1}-e_{t+1})\right] \tag{10}$$

Equation (8) is the standard equation showing how income labor tax affects the labor-leisure choice and equation (9) is the no-arbitrage condition for capital and bonds. Equation (10) states that the utility cost of network effort (LHS) equals the discounted (expected) gain from successfully finding a job, where the gain of one additional worker equals the additional consumption gain in period $t+1$ less the leisure cost of working and not spending time in social networking.

A network competitive equilibrium is a policy $\Upsilon = \{\tau_t^l, \tau_t^k\}_{t=0}^{\infty}$, government spending $\bar{G} = \{g_t, b_t\}_{t=0}^{\infty}$, household's allocations $x = \{c_t, k_{t+1}, l_t, e_t, B_{t+1}\}_{t=0}^{\infty}$, a price system $\bar{P} = \{w_t, r_t, R_t\}_{t=0}^{\infty}$ and the state of the network variables $\{\rho, \gamma, v, D_z\}$ such that given the policy, government spending, the price system and the state of the network, the resulting household's allocation choice maximizes the consumer's utility and satisfies the government's budget constraint, the economy's resource constraint and market clearing conditions.

2.5 Ramsey Equilibrium

At the beginning of each period, the government announces its program of tax rates and individuals behave competitively. The objective of the social planner is to choose values of its fiscal instruments such that the agent's utility is maximized. The problem is constrained by the households' and firm's optimization behavior and by the budget of the government. The status of the network, reflected in the economy's employment rate, also constrains the planner's problem. The social planner does not directly control the agent's allocations, and the problem is of second-best because the social planner chooses the fiscal instrument that satisfies the optimization restrictions of the private agent, i.e. the first-order conditions of the private agent's problem. The planner only observes the result of the network process and cannot influence it.

The Ramsey problem is a programming problem of finding optimum within a set of allocations that can be implemented as a competitive equilibrium with distorting taxes. In other words, the Ramsey problem is to choose a process for tax rates $\{\tau_t^l, \tau_t^k\}$, which maximizes social welfare and satisfies (4) and an implementability constraint (see Chari and Kehoe, 1999). In this paper the unemployment benefit is exogenously given and the planner does not choose it optimally. We follow the majority of the literature in assuming that the government can commit to follow a long-term program for taxing labor income. We assume that there are institutions that effectively solve the time inconsistency problem so that the government can commit to the tax plan it announces in the initial period.

To derive the implementability constraint, we use family's first order conditions and the intertemporal budget constraint, which yields the following expression (see Appendix for derivation details):

$$\sum_{t=0}^{\infty} \beta^t \left(u_1(t)\,(c_t - b_t) - u_2(t) n(e_{t-1})(l_t - e_t) - u_2(t)(1 - n(e_{t-1})) \frac{n(e_t)}{n'(e_t)} \right) = A_0 \qquad (11)$$

where $A_0 = u_1(0)\left(n(e_{-1})(1-\tau_0^l)w_0 l_0 - (1-n(e_{-1}))b_0 + T_0 k_0 + R_0 B_0\right) + u_2(0) n(e_{-1})(l_0 - e_0)$.

A Ramsey equilibrium in this economy is a policy Υ, an allocation rule x and a price rule $\bar{P}$ that satisfy the following two conditions: (i) the policy Υ maximizes (3) subject to the government budget constraint (6) and the state of the network $\{\rho, \gamma, v, D_z\}$ with allocations and prices given by x and $\bar{P}$ and (ii) for every Υ', the allocation $x(\Upsilon')$, the price rule $\bar{P}(\Upsilon')$ and the policy Υ' constitute a network competitive equilibrium.

Proposition 1 *The household's allocations and the date* 0 *policy* Υ_0*, in a network competitive equilibrium satisfy the economy's resource constraint* (7)*, the law of motion for employed workers* (1)*, the implementability constraint* (11) *and a constraint on labor income taxes*

$$u_1(t+1) b_{t+1} - u_2(t+1) e_{t+1} + u_2(t) \frac{1 - n(e_{t-1})}{n'(e_t)} = 0 \qquad (12)$$

Furthermore, given household's choices and Υ_0*, prices and policies can be constructed for all dates, which together with the choices and date* 0 *policies constitute a network competitive equilibrium.*

Proof. See Appendix. ■

The planner's maximization problem can thus be written as follows:

$$\max_{\{c_t,k_{t+1},l_t,e_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t, 1-n(e_{t-1})l_t - (1-n(e_{t-1}))e_t) \tag{P.2}$$

subject to

(*i*) $\sum_{t=0}^{\infty} \beta^t \left(u_1(t)\,(c_t - b_t) - u_2(t)n(e_{t-1})(l_t - e_t) - u_2(t)(1-n(e_{t-1}))\frac{n(e_t)}{n'(e_t)} \right) = A_0$

(*ii*) $u_1(t+1)b_{t+1} - u_2(t+1)e_{t+1} + u_2(t)\frac{1-n(e_{t-1})}{n'(e_t)} = 0$

(*iii*) $c_t + k_{t+1} + g_t = F\,(k_t, l_t) + (1-\delta)k_t.$

equation (1) and $\bar{g}$, τ_0^l, τ_0^k, k_0, n_0 given. The first order conditions of the planner's problem are not the same in the first period and subsequent periods which implies that this Ramsey problem is non-stationary. Since our goal is to study this economy in the steady state, we will focus our attention on the first-order conditions for period 1 and onwards. To save space, the first-order conditions for period zero are not presented.

3 Long Run Employment in Networks

In this section, we study the economy in a steady state. Assuming that the economy converges to a steady state implies that the change in the level of employment for each type of work is equal to zero, i.e., $\dot{n}_z = 0$ for all z. The number of newly employed agents of each type z is exactly equal to the number of newly unemployed agents, and the economy will remain at this level of employment indefinitely. We consider this *long run prevalence* of employment to be the economy's employment rate (Vega-Redondo 2007). Setting $\dot{n}_z = 0$ in equation (1), we find that the steady state level of employment (n_z^*) satisfies

$$n_z^*(e^*) = 1 - \left(\frac{\rho}{1+\rho-(1-\gamma)(1-\theta^* v(e^*))^z} \right) \tag{13}$$

where e^* is the steady state level of investment in social ties. According to equation (2), θ^* is given by

$$\theta^* = \frac{1}{\langle z \rangle} \int_{z=1}^{\infty} (z n_z^*(e^*) D_z)\, dz. \tag{14}$$

Together, the solution to these two equations for each k gives n_z^*, which can be used to define the long run steady state employment rate

$$n^*(e^*) = \int_{z=1}^{\infty} (n_z^*(e^*) D_z) \, dz, \tag{15}$$

and the associated unemployment rate $u^*(e^*) = 1 - n^*(e^*)$. Here it is interesting to observe that the government takes as given the network dynamics, except the information transmission rate which is affected by the agent's networking effort. In choosing allocations, the planner chooses the optimal network effort e^* that might be different depending on the kind of network structure we are studying. For instance, less effort might be optimal if agents are connected to more peers and information flows faster. This network effort intensity represents an additional trade-off for the agents. While it improves their chances to become employed, it is a risky and costly (in terms of leisure) investment.

The structure of the social network will influence the efficacy of investment in social relationships, the ease with which employed workers find jobs, and the long run level of employment in the economy. For different degree distributions D_z, the long run steady state employment rate, equation (15), may have different solutions, with different characteristics and implications for optimal labor tax.

We focus of several well known classes of large, complex networks. As a baseline, we consider *regular* networks, that is, networks where every agent has the same number of peers, k. For $k = 0$, this is the *empty* network, and may be taken as a worst case scenario, where each worker must hear of a job themselves, at the exogenous arrival rate γ. For these networks, $D_z = 1$ for $z = k$, and $D_z = 0$ for all other z, and every worker is exactly the same.

Regular networks are not very realistic, however, for the simple reason that they exhibit no heterogeneity among workers and no large scale structure. We consider two alternative models of large networks with heterogeneous workers, *power-law* and *geometric* degree distributions. Many models of social networks described as deriving from linear growth in the number of agents in a society, and preferential attachment in link formation as these agents arrive. In these models, we imagine the network growing over time. Workers arrive and choose to form some number of links to the workers already present in the network, with a preference for having links to workers with many links already. This preference is easy to justify, as well connected peers are more likely to be employed themselves, and thus prove to be a valuable source of job information. The limit of this process, as the number of workers goes to ∞, results in a power law degree distribution. A few workers end up with many, many links, while most have relatively few. These networks have a number of attractive features, that match well many empirical social networks (Vega-Redondo 2007, Jackson 2008). In the power-law networks, the degree distribution has the following form: $D_z = (a-1)z^{-a}$. These networks exhibit a thick tail, with a relatively high proportion of agents who have many links.

Geometric networks are derived from a similar growth process, but where agents do not have a preference for links to agents with many links already. In this model, links are simply formed randomly among those agents already present, and in the limit, the degree distribution has the following form: $D_z = \log \nu \nu^{1-z}$. These networks have a thinner tail than power-law networks.

Let n_z^q, θ^q and n^q denote the long run employment rate among agents with z links, the probability an unemployed worker's peers are employed and the economy's employment rate for a network q, respectively. We consider three classes of networks, namely, Regular ($q = R$), Power-Law ($q = PL$) and Geometric ($q = G$). The employment rate of agents with k links in a network $q = R$, PL and G is given by

$$n_z^{*q}(e) = 1 - \left(\frac{\rho}{1 + \rho - (1-\gamma)(1 - \theta^q v(e^*))^z} \right) \tag{16}$$

Table 1 presents the (steady state) expressions for θ^q and n^q for each network considered here.

Table 1 - Probability an unemployed worker's peers are employed

and the economy's employment rate for a network

	θ^q	n^q
Regular	$\frac{1}{k} k n_k^R(e) = n_k^R(e)$	$n_k^R(e)$
Power − Law	$\frac{1}{\langle z \rangle} \int_{z=1}^{\infty} \left(z n_z^{PL}(e)(a-1)z^{-a} \right) dz$	$\int_{z=1}^{\infty} \left(n_z^{PL}(e)(a-1)z^{-a} \right) dz$
Geometric	$\frac{1}{\langle z \rangle} \int_{z=1}^{\infty} \left(z n_z^{G}(e)\nu(e)^{1-z} \log \nu \right) dz$	$\int_{z=1}^{\infty} \left(n_z^{G}(e)\nu(e)^{1-z} \log \nu \right) dz$

The economy's employment rate for a regular network n^R is equal to the employment rate of the agents with k links, i.e., $n^R = n_k^R$, where n_k^R is solution of the following expression:

$$n_k^{*R}(e) = 1 - \left(\frac{\rho}{1+\rho-(1-\gamma)(1-n_k^R(e)v(e^*))^k}\right) \tag{17}$$

For the empty network, $k=0$ and this expression simplifies to $n_{k=0}^R = \gamma/(\rho+\gamma)$. Unfortunately, for the power-law and geometric networks, no closed form solution to this system of equations (n_k^q, θ^q) exists, and it must be characterized numerically.

For each of these possible networks, the behavior of unemployment with respect to the job information process is straightforward. The unemployment rate is decreasing in both the job opportunities arrival probability (γ) and the effort which an employed worker puts into passing information to each of her unemployed peers (e). And, there is a positive relationship between the exogenous job break up probability (ρ) and the equilibrium unemployment rate.

The equilibrium employment rate also depends on the average number of links k of agents in the network; in networks with a higher k, there are more agents with a higher number of peers, who have a higher (individual) employment rate. Employment is therefore higher for more connected networks.

Furthermore, for any given set of parameters that characterize the state of the network, we observe a much higher unemployment rate in a empty network than in the power-law and geometric, and an even lower unemployment rate in regular networks, in particular for greater number of links. In the case of power law and geometric networks, where there is heterogeneity in the number of links workers have, the equilibrium unemployment rate is decreasing in the number of links z that a particular worker has. For these networks, because of the presence of workers with many many links, job information is disseminated more easily, which reduces unemployment. However, in these network the distribution of links is very heterogenous - we still can find people with few links. On the other hand, in a regular network everyone has the same number of links and this homogeneity leads to higher steady state employment rates. Power-law and geometric networks can be derived from a growing network process, and are more consistent with empirical social networks, than the regular and empty networks (Vega-Redondo 2007, Jackson 2008). The following proposition summarizes these results.

Proposition 2 *For each network structure, the equilibrium level of employment n has the following properties:* (i) $\partial n/\partial e \geq 0$, (ii) $\partial n/\partial \gamma \geq 0$ *and* (iii) $\partial n/\partial \rho \leq 0$.

Proof. See Appendix. ■

Figure 1 illustrates the relationship between effort and the employment rate in different network structures, where we use the following parameter values: $\gamma = 0.4$, $\rho = 0.4$, $\nu = 2.5$, $\alpha = 3$, $\lambda = 0.6$ and each network has the same average number links, $k = 3$. Figures 2 illustrates other properties of employment in different networks.

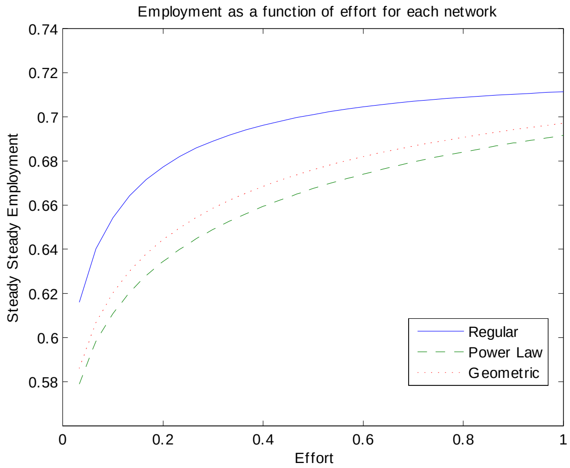

Figure 1 - Effort and Employment Rate

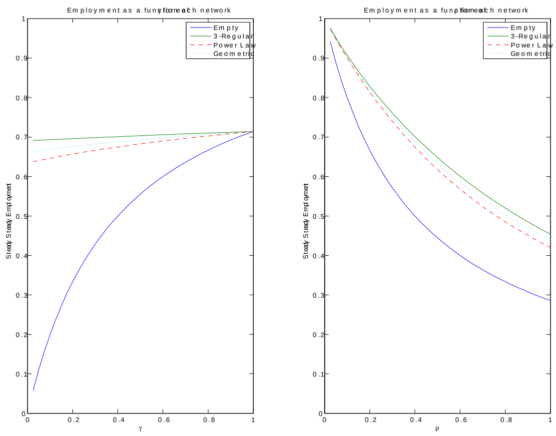

Figure 2 - Job Arrival and Break-up Probabilities

and Employment Rate

4 Optimal Tax Rates

Next, we investigate the impact of network dynamics for the optimal labor income and capital taxation. Suppose that the solution to the Ramsey problem converges to a steady state. Let $\{c, k, l, e\}$ and $\{\eta, \mu, \phi\}$ denote the associated allocations and Lagrange multipliers on the implementability constraint (11), condition (12) and resource constraint (7), respectively. We will assume constant government spending over time, i.e. $g_t = g$, $t = 0, 1, \ldots$ and write the planner's first-order conditions accordingly:

$$0 = (1+\eta)u_1 + \eta u_{11}(c-b) - u_{21}\left[\eta n(e)(l-e) - \frac{(1-n(e))}{n'(e)}(\eta n(e)-\mu)\right] - \phi \tag{18}$$

$$0 = -1 + \beta\left[F_k + (1-\delta)\right] \tag{19}$$

$$0 = -(1+\eta)u_2 - \eta u_{12}(c-b) - u_{22}n(e)\left[\eta n(e)(l-e) - \frac{(1-n(e))}{n'(e)}(\eta n(e)-\mu)\right] + \phi F_l \tag{20}$$

$$\begin{aligned}0 = &-u_2(1-n(e))\left[1 - \eta\frac{1}{(1-n(e))} + \eta\frac{n(e)n''(e)}{(n'(e))^2} - \mu\frac{n''(e)}{(n'(e))^2}\right] \\ &- \beta u_2 n'(e)\left[(1+\eta)(l-e) + \eta\frac{n(e)}{n'(e)} - \mu\frac{1}{n'(e)}\right] \\ &- u_{12}\left[n'(e)(l-e)\left[\beta\eta(c-b)+\mu b\right] - \eta(c-b)\right] \\ &- u_{22}(1-n(e))^2\left[\eta n(e)(l-e) - \frac{(1-n(e))}{n'(e)}(\eta n(e)-\mu)\right] \\ &- u_{22}n'(e)^2(l-e)\left[\beta\left(\eta n'(e)(l-e) - \frac{(1-n(e))}{n'(e)}\left[\eta n(e)-\mu\right]\right) + \mu\right] + \beta\phi F_l n'(e) l\end{aligned} \tag{21}$$

Because the sharpest analytical results hold for the case of labor taxes this is the main focus of our analysis. Notice however that the optimal limiting capital tax rate is zero as in Chamley (1986) and Judd (1985). The steady-state version of no-arbitrage condition, equation (9), becomes $1 = \beta\left[(1-\tau^k)F_k + (1-\delta)\right]$, which combined with equation (19), implies that $\tau^k = 0$. This result is obtained regardless the structure of the social networks and the dynamics of the labor market. The main reason for this result is twofold. First, in our economy the capital allocation decision is a family decision. That is, once it is determined the fraction of family members that have a job or don't the family decides what to do with the output they do not consume. After the social network mechanism had play the role of determining the economy's employment rate, it is not optimal to tax capital as the economy because it distorts capital and investment allocations. Second, firms in our economy take the number of workers as given and choose optimally only the number of hours worked by these workers. If we had assume that firms could affect the employment rate we would be moving towards the labor search literature. Domeij (2005) finds that the capital tax would non-zero as long as the workers bargaining power is different than the elasticity of search in the matching function. In our model, if we endogenized the probability that an agent hears about a job opening γ, instead of the job information transmission rate v, we would be able to replicate this same result.

In our economy, labor taxation is directly related to the unemployment rate, and so indirectly determined by the structure and properties of the job network. Although the government does not observe how job information is passed from one worker to another, and here we are not arguing that it should, it should recognize the relevance of job networks for the design of the optimal labor income tax, through their impact on the determination of the economy's equilibrium unemployment rate.

We start our discussion about optimal labor taxation and social network by considering the case where the network dynamics is completely exogenous. In this case, individuals and family units cannot affect the employment rate by their actions. This rate is determined by the dynamics of the labor market, which is governed by social network. In our model presented in Section 2, this implies shutting down the mechanism by which unemployed workers can invest time to improve social ties and hear about job opportunities. Nevertheless, the unemployment rate affects agents' optimal behavior and agents take into account the proportion of family members employed when they make decisions regarding consumption and leisure.

Rewriting the family's and Ramsey's problem to reflect this exogenous nature of the network dynamics (which will not be presented here for the sake of space and readability), we obtain the following expression for the optimal labor income tax

$$\tau_t^l = \frac{1}{n_t}\left[\frac{\eta\left[H_2(t) - H_1(t)\right] - (1 - n_t)\left[(1+\eta) + \eta H_2(t)\right]}{(1+\eta) + \eta H_2(t)}\right] \tag{22}$$

where $H_1(t) \equiv \left[u_{11}(t)C_t - u_{21}(t)(1-h_t)\right]/u_1(t)$, $H_2(t) \equiv \left[u_{12}(t)C_t - u_{22}(t)(1-h_t)\right]/u_2(t)$, $C_t = (c_t - (1-n_t)b_t)$ and η is the Lagrangian multiplier on the implementability constraint. Notice that (22) is not an explicit expression for the optimal tax rate, since the H_1, H_2 depend on endogenous variables.

The case of additively separable preferences is of particular interest because it allows us to solve for the optimal labor tax analytically. To simplify the presentation consider a quasi-linear utility function

$$U(c_t, h_t) = \ln c_t + \phi \ln h_t \tag{23}$$

When the utility function is additively separable ($u_{12}(t) = u_{21}(t) = 0$) in leisure, it implies that leisure is a normal good and labor is taxed.[5] Evaluating (22) for this functional form and assuming that, under the Ramsey plan, the allocations converge to a steady state, we have

$$\tau^* = \frac{1}{(1-u^*)}\left[\frac{\eta(1-h)}{\eta+h} - u^*\right] \tag{24}$$

where $u^* = (1-n^*)$ is a measure of unemployed family members in a steady state, n^* is defined as in (15) and we assume constant government spending over time, i.e. $g_t = g$, $t = 0, 1, \ldots$. Equation (24) suggests that there is a positive relationship between labor income taxes and hours worked $(1-h_t)$. Or conversely, labor taxes vary negatively with leisure (h_t). That is, $\partial\tau^*/\partial h = -\left[\eta(1+\eta)\right]/\left[(1-u^*)(\eta+h)^2\right] < 0$. Also from equation (24), labor taxes vary negatively with unemployment, or conversely are positively related to the economy employment rate, i.e., $\partial\tau^*/\partial u^* = -\left[h(1+\eta)\right]/\left[(1-u^*)^2(\eta+h)\right] < 0$.

Although in our setup we make a distinction between employment $n_t = (1-u_t)$ and hours worked $(1-h_t)$, the intuition for this result follows the same arguments presented by Zhu (1992) and Scott (2007), where the structure of the optimal labor income taxation depends on the elasticity of the labor supply. Labor is more inelastically supplied when employment is high and, since the Ramsey planner is required to tax inelastic variables more heavily to minimize tax distortions, labor income tax rates vary positively with employment and hours worked.

To see this relationship more clearly, note that family's problem first order conditions with respect to consumption and leisure form a system of equations such that c and h can be solved in terms of λ (the Lagrangian multiplier on the household's budget constraint) and ω, where $\omega = (1-\tau)w$. For our purpose we are interested in the compensated labor supply response with respect to a change in ω holding λ constant. We get

$$\frac{\partial(1-h_t)}{\partial\omega_t} = -\frac{(1-u_t)u_{11}(t)\lambda_t}{u_{22}(t)u_{11}(t) - u_{12}^2(t)} > 0 \tag{25}$$

This expression represents the compensated labor-supply response when the tax rate changes (the substitution effect). This substitution effect captures the distortionary effect of the labor-income tax. That is, a higher labor tax increases leisure and lowers labor supply $(1-h_t)$ and thus lowers the tax base.

[5] Basu and Renström (2007) study optimal taxation in an environment with indivisible labor supply, HARA class of preferences with nonseparable leisure.

For our additively separable utility function, equation (23), the compensate elasticity of labor supply is given by

$$\epsilon_t = \frac{\partial(1-h_t)}{\partial\omega_t}\frac{\omega_t}{(1-h_t)} = (1-u_t)\omega_t(1-h_t)\phi^{-1}\lambda_t \tag{26}$$

There are two effects of the unemployment on the elasticity of labor supply. The unemployment rate impacts this elasticity directly and implies that when unemployment is high, labor is more inelastically supplied. On the other hand, a high unemployment indirectly increases labor supplied, indicating that labor is more elastic. One can show that the net effect of a high unemployment rate on the elasticity of labor supply is negative. That is, labor is more inelastically supplied when unemployment is low (the indirect effect dominates the direct one). Hence, labor income tax rates vary negatively with unemployment.

Given the implications of the network process for the equilibrium unemployment rate, summarized in Proposition (2), we can study how different network characteristics might affect the design of the optimal labor income tax. In economies where the job information process is poor, i.e., low arrival probability, low rate of job information transmission among peers, high job break up probability or low numbers of links, the equilibrium unemployment rate is higher and the Ramsey planner is required to tax labor income at a lower rate. On the other hand, if information about job opportunities is well transmitted among peers, it increases the likelihood of an unemployed worker to hear and get a job. In such an economy, the unemployment rate tend to be lower and the government can implement a higher income tax. In sum:

Proposition 3 *When the labor market is governed by social network and the network dynamics is exogenous, the optimal labor income tax is higher in more connected job network economies.*

Proof. See Appendix. ■

5 Numerical Results

[TO BE COMPLETED]

When the network dynamics is exogenous and preferences are separable in consumption and leisure, the optimal labor income tax is higher in more connected job network economies.

Among the cases we study - empty, regular, power law and geometric - the optimal taxation in the presence of an empty network is a good illustration of one of the extremes faced by the government. In this case, the flow of information about job opportunities among agents is nonexistent. There are no peer effects (no information transmission) and information is lost (if an employed agent hear about another job opening) in this context. Unemployment is higher and, consequentially, it is optimal for the government to tax less those with jobs. To the extent that more and better information is transmitted from an employed worker to his/her unemployed peers, either because agents have more links or because the rate at which such information is transmitted is higher, the required labor income tax is higher. Figure 3 illustrates this result for the geometric network.

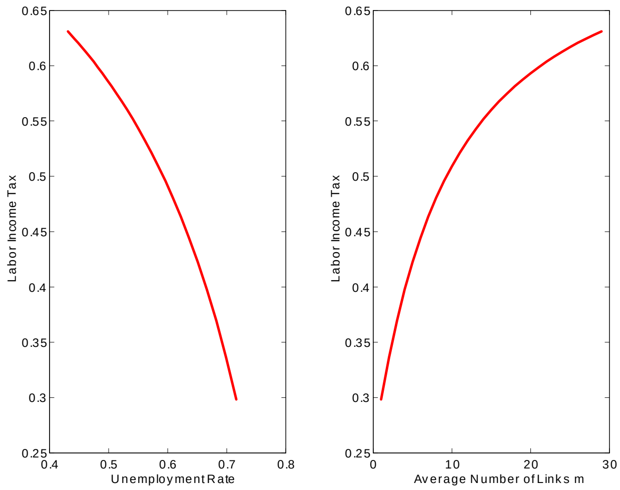

Figure 3 - Labor income tax, unemployment rate and number of links.

(Geometric Network)

6 Conclusion

This paper studies the optimal labor income taxation in the presence of social networks. The unemployment rate is then determined by the dynamic of the labor market, which is governed by the social network. Unemployment results as individuals are unsuccessful in hearing about job opportunities themselves or through their peers in a network. The optimal limiting capital tax rate is zero, independent of the labor market frictions. The optimal labor income tax is decreasing in the unemployment rate and the job network process parameters play an important role in determining optimal fiscal policy. We allow agents to invest some of their time on building links and connect to peers (endogenous network). The optimal tax is negatively related to the transmission rate of job information from peers in a particular network and it is lower in more connected job network economies.

Appendix

Derivation of the Implementability Constraint

To derive the implementability constraint, equation (11), first premultiply the family's budget constraint in period t with the associated Lagrangian multiplier $\beta^t \lambda_t$ and sum over all periods $t \geq 0$

$$\sum_{t=0}^{\infty} \beta^t u_1(t) \left[c_t + k_{t+1} + B_{t+1}\right] = \sum_{t=0}^{\infty} \beta^t u_1(t) \left[n(e_{t-1})(1-\tau_t^l) w_t l_t + (1 - n(e_{t-1}))b_t + T_t k_t + B_t R_t\right] \tag{27}$$

Use first-order conditions with respect to capital and bonds to eliminate the after-tax return on capital and bonds we obtain

$$\sum_{t=0}^{\infty} \beta^t u_1(t) \left[c_t\right] = \sum_{t=0}^{\infty} \beta^t u_1(t) \left[n(e_{t-1})(1-\tau_t^l) w_t l_t + (1 - n(e_{t-1}))b_t\right] + A_{00} \tag{28}$$

where $A_{00} = u_1(0) \left[T_0 k_0 + B_0 R_0\right]$. Multiplying equilibrium equation (10) by $\beta^{t+1} u_1(t+1)$ we get

$$\beta^{t+1} u_1(t+1)(1-\tau_{t+1}^l) w_{t+1} l_{t+1} = \beta^{t+1} u_1(t+1) b_{t+1} + \beta^{t+1} u_2(t+1)(l_{t+1} - e_{t+1}) + \beta^t u_2(t) \frac{(1-n(e_{t-1}))}{n'(e_t)}$$

and then multiply it by $n(e_t)$ yields

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^{t+1} n(e_t) u_1(t+1) \left[(1-\tau_{t+1}^l) w_{t+1} l_{t+1} - b_{t+1}\right] &= \sum_{t=0}^{\infty} \beta^{t+1} n(e_t) u_2(t+1)(l_{t+1} - e_{t+1}) \\ &\quad + \sum_{t=0}^{\infty} \beta^{t+1} n(e_t) u_2(t) \frac{(1-n(e_{t-1}))}{n'(e_t)} \end{aligned} \tag{29}$$

Notice that the right-hand-side of equation (28) can be written as

$$\begin{aligned} & u_0(0) \left[n(-1)(1-\tau_0^l) w_0 l_0 + (1-n(-1)) b_0\right] \\ & + \sum_{t=0}^{\infty} \beta^{t+1} u_1(t+1) n(e_t) \left[(1-\tau_{t+1}^l) w_{t+1} l_{t+1} - b_{t+1}\right] + \sum_{t=0}^{\infty} \beta^t u_1(t) b_t \end{aligned}$$

Substituting (29) into (28) and after some manipulation, we obtain the implementability constraint for this problem equation (11):

$$\sum_{t=0}^{\infty} \beta^t \left(u_1(t)\left(c_t - b_t\right) - u_2(t) n(e_{t-1})(l_t - e_t) - u_2(t)(1-n(e_{t-1})) \frac{n(e_t)}{n'(e_t)} \right) = A_0$$

where $A_0 = u_1(0) \left(n(e_{-1})(1-\tau_0^l) w_0 l_0 - (1-n(e_{-1})) b_0 + T_0 k_0 + R_0 B_0\right) + u_2(0) n(e_{-1})(l_0 - e_0)$.

Proof of Proposition 1

Proof. To show that any allocation that satisfy equations (7), (11) and (12) can be decentralized as a network competitive equilibrium we use these allocations together with the family's and firm's first-order conditions to construct the corresponding prices and taxes. The rental rate r_t is given by the firm's first-order condition with respect to capital. The capital tax τ_t^k is determined using the family's and firm's first-order condition with respect to capital, and implicitly defined by

$$\frac{u_1(t)}{\beta u_1(t+1)} - [1 + F_1(t+1) - \delta] = -\tau_t^k F_1(t+1) \tag{30}$$

The wage rate w_t and the labor tax rate τ_t^l are determined by substituting equation (10) into the firm's first-order condition with respect to labor, obtaining

$$\frac{1}{(1-\tau_{t+1}^l)} = u_1(t{+}1)F_2(t{+}1)l_{t+1}\left[u_1(t+1)b_{t+1} + u_2(t+1)(l_{t+1} - e_{t+1}) + u_2(t)\frac{(1-n(e_{t-1}))}{n'(e_t)}\right]^{-1} \tag{31}$$

The family's first-order condition with respect to labor for period $t+1$ is

$$\frac{1}{(1-\tau_{t+1}^l)} = \frac{u_1(t+1)}{u_2(t+1)}F_2(t+1) \tag{32}$$

Rewrite (31) and (32) as follows

$$(1-\tau_{t+1}^l) = \frac{1}{u_1(t+1)F_2(t+1)l_{t+1}}\left[u_1(t+1)b_{t+1} + u_2(t+1)(l_{t+1} - e_{t+1}) + u_2(t)\frac{(1-n(e_{t-1}))}{n'(e_t)}\right] \tag{33}$$

$$(1-\tau_{t+1}^l) = \frac{u_2(t+1)}{u_1(t+1)F_2(t+1)} \tag{34}$$

and rearranging we obtain

$$\Psi(c_t, l_t, e_t, c_{t+1}, l_{t+1}, e_{t+1}) = u_1(t+1)b_{t+1} - u_2(t+1)e_{t+1} + u_2(t)\frac{1-n(e_{t-1})}{n'(e_t)} \tag{35}$$

which is equivalent to equation (12). The labor tax τ_t^l is implicitly defined by both (33) and (34) and to ensure that the labor taxes implied by these two conditions coincide the constraint (35) is imposed in the Ramsey problem.

To show that any network competitive equilibrium allocations satisfy equations (7), (1), (11) and (12), we proceed as follows. (1) The resource constraint, equation (7), is implied by the family's and government's period-by-period budget constraints, thus feasibility is satisfied. (2) Premultiply the family's budget constraint in period t with the associated Lagrangian multiplier $\beta^t\lambda_t$ and sum over all periods $t \geq 0$. We proceed by solving for taxes and prices as a function of allocations using the family's and firm's first order conditions. This results in the implementability constraint, equation (11). (3) Since, by definition, the labor tax rate τ^l satisfies both (33) and (34), the allocations also satisfy the intertemporal constraint on labor taxes, equation (12). ■

Ramsey's Problem First Order Conditions

Let η, $\beta^t\mu_t$, $\beta^t\phi_t$ be the Lagrange multipliers on the implementability constraint (11), condition (12) and resource constraint (7), respectively. After some manipulation, the first-order conditions for c_t, k_{t+1}, l_t and e_t are, respectively:

$$0 = (1+\eta)u_1(t) + \eta u_{11}(t)(c_t - b_t) - u_{21}(t)\left[\eta n(e_{t-1})(l_t - e_t) - \frac{(1-n(e_{t-1}))}{n'(e_t)}\left(\eta n(e_t) - \mu_t\right)\right] - \phi_t \tag{36}$$

$$0 = -\phi_t + \beta\phi_{t+1}\left[F_k(t) + (1-\delta)\right] \tag{37}$$

$$\begin{aligned} 0 &= -(1+\eta)u_2(t) - \eta u_{12}(t)(c_t - b_t) \\ &- u_{22}(t)n(e_{t-1})\left[\eta n(e_{t-1})(l_t - e_t) - \frac{(1-n(e_{t-1}))}{n'(e_t)}\left(\eta n(e_t) - \mu_t\right)\right] + \phi_t F_l(t) \end{aligned} \tag{38}$$

$$\begin{aligned} 0 &= -u_2(t)(1-n(e_{t-1}))\left[1 - \eta\frac{1}{(1-n(e_{t-1}))} + \eta\frac{n(e_t)n''(e_t)}{(n'(e_t))^2} - \mu_t\frac{n''(e_t)}{(n'(e_t))^2}\right] - \eta u_{12}(t)(c_t - b_t) \\ &- u_{22}(t)(1-n(e_{t-1}))^2\left[\eta n(e_{t-1})(l_t - e_t) - \frac{(1-n(e_{t-1}))}{n'(e_t)}\left(\eta n(e_t) - \mu_t\right)\right] \\ &- \beta u_2(t+1)n'(e_t)\left[(1+\eta)\left(l_{t+1} - e_{t+1}\right) + \eta\frac{n(e_{t+1})}{n'(e_{t+1})} - \mu_{t+1}\frac{1}{n'(e_t)}\right] \\ &- u_{12}(t+1)n'(e_t)(l_{t+1} - e_{t+1})\left[\beta\eta(c_t - b_t) + \mu_t b_{t+1}\right] \\ &- u_{22}(t+1)n'(e_t)^2(l_{t+1} - e_{t+1})\left[\beta\left(\eta n'(e_t)(l_{t+1} - e_{t+1}) - \frac{(1-n(e_t))}{n'(e_{t+1})}\left[\eta n(e_{t+1}) - \mu_{t+1}\right]\right) + \mu_t\right] \\ &+ \beta\phi_{t+1}F_l(t+1)n'(e_t)l_{t+1} \end{aligned} \tag{39}$$

References

[1] Addison, John T., and Pedro Portugal (2001) 'Job search methods and outcomes.' IZA Discussion Papers.

[2] Barro, R. (1979). On the determination of the public debt. Journal of Political Economy. 87, 940-971.

[3] Basu, P. and T. Renström. (2007). Optimal Dynamic Labor Taxation. Macroeconomic Dynamics 11, 567-588.

[4] Calvó-Armengol, Antoni, and Matthew Jackson (2004) 'The effects of social networks on employment and inequality.' American Economics Review 94, 426–54.

[5] Chari V.V., Christiano L., Kehoe P., 1991. Optimal fiscal and monetary policy: Some recent results. Journal of Money, Credit and Banking 23, 519-539.

[6] Chari V.V., Kehoe P., 1999. Optimal fiscal and monetary policy. In: Taylor J.B., Woodford M. (Eds.) Handbook of Macroeconomics, vol. 1C. Amsterdam: North-Holland; pp. 1671-1745.

[7] Domeij, D. (2005). Optimal capital taxation and labor market search. Review of Economic Dynamics 8, 623-650.

[8] Faia, E. (2008). Optimal monetary policy rules with labor market frictions. Journal of Economic Dynamic and Control 32, 1600-1621.

[9] Granovetter, Mark (1995) Getting a job: A study of contacts and careers, second edition ed. (University of Chicago Press).

[10] Gregg, P, and J Wadsworth (1996) 'It takes two: Employment polarisation in the OECD.' Technical Report, Centre for Economic Performance, LSE, September

[11] Holzer, Harry J. (1988) 'Search method use by unemployed youth.' Journal of Labor Economics 6(1), 1–20

[12] Ioannides, Yannis, and Linda Loury (2004) 'Job information networks, neighborhood effects, and inequality.' Journal of Economic Literature 43, 1056–1093.

[13] Jackson, Matthew (2008). Social and Economic Networks. Princeton University Press.

[14] Krauth, B. (2004). A dynamic model of job networking and social influences on employment. Journal of Economic Dynamic and Control 28, 1185-1204.

[15] Kyndland, F. and E. Prescott (1980). A competitive theory of fluctuations and the feasibility of stabilization policy, in "Rational Expectations and Economic Policy" (Stanley Fischer, Ed.), University of Chicago Press, Chicago, IL.

[16] Lucas R., Stokey N., 1983. Optimal fiscal and monetary policy in an economy without capital. Journal of Monetary Economics 12, 55-93

[17] Montgomery, James D. (1991) 'Social networks and Labor-Market outcomes: Toward an economic analysis.' The American Economic Review 81(5), 1408–1418.

[18] Scott, A. (2007). Optimal taxation and OECD labor taxes. Journal of Monetary Economics 54, 925-944.

[19] Topa, Giorgio (2001) 'Social interactions, local spillovers and unemployment.' The Review of Economic Studies 68(2), 261–295

[20] Vega-Redondo, Fernando (2007). Complex Social Networks. Econometric Society Monographs. ESM 44. Cambridge University Press.

[21] Zhu, X. (1992). Optimal fiscal policy in a stochastic growth model. Journal of Economic Theory 58, 250-289.