Aspirations and Inequality

Garance Genicot
Georgetown University

Debraj Ray
New York University

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Abstract

This paper develops a theory in which ambient income distributions shape individual *aspirations*, which in turn affect the investment incentives of individuals. Through its impact on investments, aspirations in turn affect the ambient income distribution. Thus aspirations and the distribution of income evolve jointly, and in many situations in a self-reinforcing way. We study the consequences of this theory for income distribution and growth. We argue that distribution of income cannot converge to perfect equality over time. When segments of the population share the same aspiration, there is clustering of steady state income distributions around local poles. Finally, the theory has predictions for the growth rates along the cross-section of income. In particular, the theory captures both the complacency stemming from low aspirations and the frustration resulting from aspirations that are too high.

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1. Introduction

What individuals want for themselves, or what parents want for their children, is conditioned by society in fundamental ways. One such pathway is the creation of individual aspirations. The existing literature views such aspirations as drawn from the past experience of the individual herself. In this paper, we argue that they are also profoundly affected by her *social* environment. The lives of others shape our desires and goals. This is a view of individual preferences that isn't standard in economic theory. But it should be.

At the same time, while social outcomes affect aspirations, those very aspirations influence — via the aggregation of individual decisions — the overall development of a society. As a result, aspirations and income (and its distribution) evolve together. An examination of this relationship is the subject of our paper.

Any such theory must address three issues. First, there is the question of how aspirations are formed. Second, we must describe how individuals react to the aspirations that they do have. Finally, the theory must aggregate individual behavior to derive society-wide outcomes. We emphasize the first two, as they are relatively new.

We define utilities around a "reference point" and interpret that point as an *aspiration*. This much is standard; see, e.g., [24], [25], and [26]. The contribution of the model is to emphasize the dependence of aspirations on the ambient income distribution, thereby linking aggregate outcomes to individual behavior.[1] For instance, individuals may simply use some common function of the income distribution (such as the mean or income at the 75th percentile) to form their aspirations. Or they may look at the conditional mean of all individuals richer than them.

Next, we relate aspirations to an individual's incentives to invest and bequeath. We argue that the "best" aspirations are those that lie at a moderate distance from the individual's current economic situation standards, large enough to incentivize but not so large as to induce frustration. This part of the theory draws on [2], [34] and [35], who make similar arguments in a more informal setting. We also rely on evidence from cognitive psychology, sports, and lab experiments (see, e.g., [5]) and [21])) that goals that lie ahead — but not too far ahead — provide the best incentives. This argument captures both encouragement and frustration, and on its own can be used to create an aspirations-based theory of poverty traps.

Finally we embed the theory of aspirations formation into a simple growth model. In equilibrium, the overall distribution influences individual aspirations which in turn shape the distribution via individual choices.[2] Our main results concern the properties of equilibrium income distributions. We show — even in the absence of any stochastic shocks — that perfect or near-perfect equality is unsustainable. That is, income distributions

[1]See [27] for evidence of the importance of social interactions in the formation of aspirations.

[2]This approach develops the ideas laid down in an earlier working paper [20]. Following that approach, [6] and [12] also develop models of socially determined aspirations that explore related but distinct issues.

cannot converge to a degenerate distribution. Our results are in line with a recent literature on endogenous inequality that models various reason for inequality to arise and persist in a society, including nonconvexities ([18], [28]), occupational choice ([3], [17], [29]) and endogenous risk-taking ([4], [36]). In the case in which aspirations are common or stratified, we show that in any steady state, incomes must cluster into local poles. In the special case of common aspirations, typically two poles emerge, in line with the findings of [32].

We also study the behavior of growth rates along the income distribution. Our propositions attempt to captures the idea that aspirations that are too high can serve to frustrate, while aspirations that are too low might breed complacency. It follows that over a zone of incomes that share the same aspirations, individual growth rates should be inverted U-shaped in income. We also discuss how growth rates along the cross-section of incomes must react to a shift in aspirations brought about, say, through the rise of mass communications media.

Finally, we use the percentile distributions of growth rates available for 43 countries in an empirical exercise. We find the returns to individual investment that match in the model the aggregate growth experiences of these countries, and then employ the model to see which aspirations formation structure appears to fit the data best, in that they come closes to the observed growth incidence curves by percentile. We show that a model of aspirations formation in which individuals use umbrella-shaped weights on incomes in some interval around their own incomes comes closest to replicating the data, and this specification captures 75% of the observed variation in growth.

There is, of course, a large literature which connects social outcomes to individual behavior. In most part, the connection is made by linking aggregate features to an individual's feasible set (and not her preferences). For instance, macroeconomic outcomes might affect an individual's access to the credit market, or what she receives as wages. We emphasize, in contrast, the effect on what an individual *wants* to do. In this sense, the closest literature would be the one which emphasizes the effect of the ambient distribution on status-seeking and therefore behavior (see, e.g., [8], [9], [10], [13], [16], [36], [37], [38], [39], and [41]). This is a direct effect that works through preferences for *relative* wealth or income. However, the structure we place on aspirations formation as a reference point, and on the "nonlinear" way in which individuals react to the gap between their aspirations and their current standards of living, makes this a distinct exercise, with its own novel distributional and growth implications.[3]

2. The Model

2.1. **Preferences and Aspirations.** An economy is populated by a large number of families or dynasties. A dynasty is a sequence of individuals, each of whom lives for a

[3]Our approach is also related to [40] who endogenizes the reference point using the expected outcome of the game.

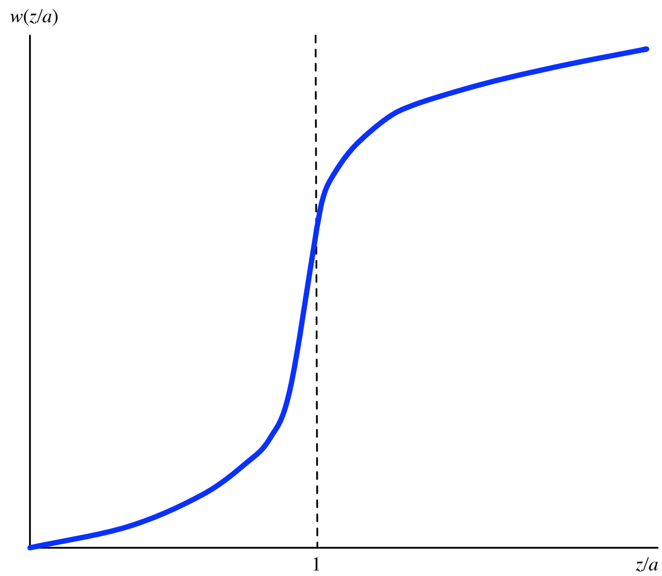

FIGURE 1. THE FUNCTION w

single period. A typical member of generation t cares about her own lifetime consumption, c_t, and the income y_{t+1} that she leaves for her child, who will grow up to be a member of generation $t+1$:

$$u(c_t) + w(y_{t+1}, a_t),$$

where a_t is the *aspiration* that our individual has at time t, and u is a standard utility function satisfying:

[U] u is increasing, smooth and strictly concave, with $u(0) = -\infty$.

We will soon discuss how aspirations might be determined. For now, we focus on the function w, which describes preferences for given aspiration levels. We maintain the following assumption throughout:

[W] (i) w is smooth, with $w_1(z, a) > 0$ and $w_2(z, a) < 0$;
(ii) $w_{11}(z, a) > 0)$ for $z < a$ and $w_{11}(z, a) < 0$ for $z > a$;
(iii) $w_{12}(z, a) < 0$ for $z < a$ and $w_{12}(z, a) > 0$ for $z > a$;
(iv) $w_1(a, a) = \infty$.

In (i), we assume that w is increasing in child income, and decreasing in aspirations. We also presume, in (ii) and (iii), that the utility excess (or shortfall) as one moves away from the aspirations level is *increasing at a decreasing rate in either direction*. If I am far ahead of my aspirations, an extra gain is not going to create much additional satisfaction, and likewise if I am way below my aspirations, an increase or decrease is not going to make much of a difference. It is in the region of the aspiration itself that utility gains are most sensitive to an increase in income. See Figure 1.[4] Part (iv) presumes that this effect is infinite at the inflection point (a, a).

2.2. **The Determination of Aspirations.** We now turn to a discussion of how aspirations might be determined. We follow [2] and [34] in emphasizing the *social* construction of aspirations, in contrast to a view in which aspirations might be determined entirely by one's own personal history (see, e.g., [26], [1], [11]). While we do not ignore this aspect of the aspirations formation process, we emphasize social effects. We can capture a fairly broad range of possibilities under the specification

$$a = \Psi(y, F), \tag{1}$$

where a stands for the aspiration of an individual (and is applied to the income of the next generation), y is income, and F is the relevant society-wide *distribution* of income (more below). It is innocuous to maintain that Ψ is nondecreasing in y; one would not reasonably expect higher incomes to dampen aspirations. If Ψ is strictly increasing in y, then personal histories have an active role to play in the determination of aspirations, something that we find reasonable but do not necessarily insist upon.

We maintain the following assumption on Ψ:

[A] Ψ is continuous in y and F,[5] and takes values within the range of F.

The substantive restriction in Condition A is that aspirations do not wander outside the society-wide range of incomes.

Consider some particular processes of aspirations formation:

COMMON ASPIRATIONS: This is the simplest case. All individuals have exactly the same aspirations, which are given by some *common* function of the income distribution, and do not depend on the specific value of individual income:

$$\Psi(y, F) = \psi(F).$$

STRATIFIED ASPIRATIONS: Aspirations are likely to depend on one's position in the income distribution. Divide up the income distribution into n quantiles. Define

$$\Psi(y, F) = a_i$$

for individuals with income y in quantile i, where a_i is a scalar representing some summary statistic of the distribution in that quantile: e.g., the income of the 75th conditional percentile in that quantile.

UPWARD-LOOKING ASPIRATIONS: Aspirations might always exceed incomes. For instance, say that individuals look "upwards" at all families who are richer than them, and

[4] Notice that (iii) and (ii) are equivalent when $w(z, a)$ is homogenous of degree one.

[5] Continuity in F is with respect to the topology of weak convergence on distributions.

that aspirations are the conditional mean of all such incomes:

$$\Psi(y, F) = \frac{\int_y^\infty x dF(x)}{1 - F(y)}.$$

Local Aspirations With Population Neighborhoods: [34] discusses aspirations "windows", in which people draw upon the experiences of those in some cognitive window around them. For instance, suppose that weight is placed only on the surrounding d (income) percentiles of the population. That is, we consider an income y' only if $|F(y') - F(y)| \leq d$, so that

$$\Psi(y, F) = \frac{1}{d} \int_{L(y)}^{H(y)} x dF(x),$$

where $L(y)$ and $H(y)$ are the appropriately defined edges of the cognitive window for a person situated at y.[6]

Local Aspirations With Income Neighborhoods: Now suppose that weight is placed instead only on incomes within an interval $N(y)$ of the individual's income. Then

$$\Psi(y, F) = \frac{1}{F(N(y))} \int_{N(y)} x dF(x),$$

where $F(N(y))$ has the obvious meaning.

There is an important aspect of aspirations formation which we have not emphasized so far. Recall that a is to be interpreted as the aspiration that an individual holds for her progeny's income in the next generation. In an environment of ongoing intergenerational growth (or decay), it is entirely plausible that the anticipated *future* distribution of income should enter the aspirations formation process. This requires a reinterpretation of F in (1): it is not, then, the current ambient distribution but the anticipated distribution of income in the next generation. In a rational expectations equilibrium, we will take this to be a correct point forecast of the true income distribution that will actually prevail. When future distributions are used, we will reinterpret y as the anticipated future income of all dynasties with the same current income as our individual.

To refer to these two alternatives, we will use the terminology "current aspirations" to describe the case in which current income distributions are used to form aspirations, and "future aspirations" for the case in which aspirations are formed by using the anticipated income distribution for the next generation.

2.3. **Equilibrium.** To describe equilibrium, we embed our model of aspirations formation into a standard growth model. For each individual in generation t, lifetime income y_t is divided between consumption c_t and an investment or bequest for the future, k_t.

$$y_t = c_t + k_t,$$

[6]That is, $L(y)$ is the lowest income in the support of F with $F(y) - F(L(y)) \leq d$, and $H(y)$ is the highest income in the support of F with $F(H(y)) - F(y) \leq d$.

That bequest gives rise to fresh income for the next generation.

$$y_{t+1} = f(k_t),$$

where f is a smooth increasing function. A *policy* maps current income y to bequests k.

An *equilibrium with current aspirations* (ECA) from some initial distribution F_0 is a sequence of policies $\{\phi_t\}$ and income distributions $\{F_t\}$ such that

(i) For every t and y in the support of F_t, $k = \phi_t(y)$ maximizes

$$u(y-k) + \mathbb{E}\left[v\left(f(k)\right) + w\left(f(k), \Psi(y, F_t)\right)\right] \tag{2}$$

over $[0, y]$, and at every date t,

(ii) F_{t+1} is generated in the obvious way, given F_t and the policy ϕ_t.

An *equilibrium with future aspirations* (EFA) is defined exactly in the same way, except that the term F_t in (2) is replaced by F_{t+1}, and y is replaced by future income; that is, by $f(\phi_t(y))$.[7]

Proposition 1. *An equilibrium exists.*

With current aspirations, establishing existence is a simple recursive exercise of no great interest. Existence in the case of future aspirations, in which agents need to be predictive, is much less trivial, but still an exercise of largely technical interest. In what follows, we take Proposition 1 as given, and move on.

3. The Instability of Equality

The purpose of this section is to show that a natural theory of aspirations, such as the one we have outlined here, has the implication that "high" levels of income equality are unattainable. The intuition is simple: when income is excessively bunched around a common value, there is a large gain to be had in accumulating a bit more relative to others. (In our model, this gain comes through the relative ease of meeting and exceeding one's aspirations.) That leads to a race to the top in which all agents accumulate too much. Eventually, the pressure must ease as symmetry is broken by some agents taking present consumption instead and falling behind in accumulation. The resulting outcome separates near-identical agents, thus destroying equality.

3.1. Nonconvergence.

The following proposition formalizes this idea.

Proposition 2. *If $\{F_t\}$ is an equilibrium sequence of income distributions, then it cannot converge to a degenerate distribution.*

[7] This is reminiscent of the literature on psychologic games where agents' utility depends on both outcomes and all agents beliefs and beliefs must be correct in equilibrium [19].

Proof. Suppose, contrary to our assertion, that F_t converges weakly to F, where F is degenerate. Then by an obvious continuity argument, the stationary sequence $\{F\}$ centered on y^* is an equilibrium. Because v has unbounded steepness at 0, it must be that $y^* > 0$. By [A], every individual must have a common aspiration, given by $a = y^*$. Thus each individual solves the maximization problem

$$\max_k u(y^* - k) + v(f(k)) + w(f(k), y^*)$$

and does so by choosing k so that $f(k) = y^*$. Using [W], in which $w_1(a, a) = \infty$ and an interior first-order condition for a maximum, which must necessarily hold, we must conclude that $y^* - k = 0$. But then, by [U], payoffs are negative infinity, a contradiction, for any individual can guarantee finite utility by, say, dividing his resources equally across the two periods. ■

Several remarks on this proposition are warranted. First, to prove this result, we make use of the assumption that $w_1(a, a) = \infty$, so that there is an irresistible urge to break the symmetry of perfect equality. It is easy enough to see that w_1 does not *literally* have to be infinite; a large enough slope will suffice. On the other hand, if the slope is relatively flat, it may well be that an equal limit distribution is stable; see the numerical example in Section 3.4.

Second, note that convergence of the equilibrium to some stationary distribution is not needed for this result, provided we restrict ourselves to the case of EFA. When aspirations are formed using future distributions of income, the sequence of distributions cannot approach perfect equality, or anywhere close to it, at *any* date. For if this were to be false, then $z = a$ and the same argument used in the proof above goes through.

The same is not entirely true of ECA. It is possible that perfect equality could be maintained provided that all incomes grow fast enough at the same common rate. For in this way, we will have $a_t = y_t$ and $y_{t+1} = (1 + g_t)y_t$ at every date t, where y_t is the common level of income and g_t the common rate of growth at date t. It is easy enough to provide examples of such a phenomenon, but equality will necessarily break down if the common rate of growth is nonnegative but small; for instance, if it is 0 as in the statement of Proposition 2.

3.2. **The Case of Common Aspirations.** We can examine the nature of the limit distribution more closely in special cases.

Take the case of common aspirations. To make sure that steady state incomes lie on some compact support, suppose that the production function f satisfies $f(k) < k$ for all k sufficiently large.

Suppose that income distributions converge to some limit distribution F^*, with attendant aspirations a^* (common to all) within the support of F^*. Then for each income y in the support of the limit distribution, an individual solves the maximization problem

$$\max_k u(y - k) + v(f(k)) + w(f(k), a^*).$$

A standard single-crossing argument informs us that the optimal choice of k must be nondecreasing in y.[8] It follows that if F^* is a stationary distribution, then y must map into y again. Let $k(y)$ be the capital stock that permits this to happen; then $f(k(y)) = y$, so that $k(y)$ is just the inverse of $f(y)$. Because of the unbounded steepness of both u and v, the solution must be interior for every $y > 0$, and so the (necessary) first-order condition informs us that

$$D(y, a^*) \equiv -u'(y - k(y)) + f'(k(y)) \left[v'(y) + w_1(y, a^*)\right] = 0 \tag{3}$$

for every positive y in the support of the limit distribution. There will be solutions to this equation both to the right and the left of a^*. Under condition [W], there cannot be any solutions *at* a^*.

Generically, there must be finitely many such solutions, so that the steady state distribution develops multiple poles. This clustering of incomes is a robust feature of the common aspirations model.

It goes without saying that the clustering into degenerate poles described above is not to be taken literally. When there are stochastic shocks, the distribution will always be dispersed, but there will be a tendency for it to exhibit local modes: one above the common aspirations level, and one or more modes below it. See the numerical example in Section 3.4 for an illustration.

These observations can be usefully related to different aspects of the literature on evolving income distributions. The closest relationship is to endogenous inequality, in which high levels of equality are destabilized by forces that tend to move the system away from clustering. In [17] and [29], this happens because of imperfect substitutes among factors of productions, so that a variety of occupations with different training costs and returns *must* be populated in equilibrium. Together with imperfect capital markets, this implies that in steady state, there must be persistent inequality, even in the absence of any stochastic shocks. In related work, [4] and [36] argue that endogenous risk-taking can also serve to disrupt equality, as relative status-seeking effectively "convexifies" the utility function at high levels of clustering.

The clustering of incomes into local poles also speaks to the work of [14], [32] and [33].[9] These authors make a strong case for local clustering in the world income distribution and argue that convergence is a local phenomenon "within the cluster" but not globally. Durlauf and Quah [15] summarize by writing that there is an "increase in overall spread together with [a] reduction in intra-distributional inequalities by an emergence of distinct peaks in the distribution". There is also evidence of multimodality in the income distribution of various countries, including the US (see [23], [42] and [30]). This is consistent with a common aspirations model.

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[8]The assumption that u is strictly concave in consumption is sufficient to deliver this result, using a familiar revealed preference argument.

[9]See also [22], [7] and [31].

We now turn to a comparison of inequality across steady states. There is a sense in which steady states with higher aspirations are also associated with higher inequality. To make this precise, we impose a natural restriction on preferences and technology:

[S] The growth model with no aspirations effect ($w \equiv 0$) has a unique limit value of income, which is strictly positive, and independent of initial income as long as that income is strictly positive.

Assumption [S] guarantees that in the absence of an aspirations effect, there would be no inequality at all in steady state.

Proposition 3. *Suppose that* [W] *and* [S] *hold. If there are two steady states with common aspirations, either owing to multiple steady states under the same parameters or because the aspirations formation process is perturbed, then the steady state with the higher aspiration must have a larger maximal income and a lower minimal income. In this sense, higher aspirations are associated with greater inequality.*

Proof. Suppose that there are two steady states with distinct aspiration levels a and a'; let $a < a'$. Figure 2 plots the derivative of the maximand with respect to k, evaluated at stationary points where $f(k) = y$, or equivalently $k = k(y)$ (this is the function $D(y, a^*)$ in equation (3)). The solid line depicts the derivative when the common aspiration is a, and the solid line depicts the derivative when the aspiration level is a'. The zeroes of this function for any given aspiration level are the steady state incomes for that aspiration.

First study the derivative with no aspiration effects: this is just

$$d(y) \equiv -u'(y - k(y)) + f'(k(y))v'(y).$$

By condition [S], there is a unique strictly positive value of y — call it Y — such that the above expression equals zero. Moreover, for all smaller values $y < Y$, $d(y)$ must be strictly positive. For if it is negative, then it is easy to see that starting from such smaller values, income must converge to a still lower limit (possibly zero) over time, which contradicts [S]. With this in mind, consider the lowest steady state income — call it L — at aspiration level a. We claim that

$$D(y, a) = d(y) + f'(k(y))w_1(y, a) > 0 \text{ for all } y < L.$$

The claim is certainly true for all $y < Y$, because f' and w_1 are both positive. So if the claim is false for some $y < L$, then there must exist $y' \leq y < L$ at which $D(y', a) = 0$, which contradicts the presumption that L is the lowest steady state income.

To summarize, then, $D(L, a) = 0$ while $D(y, a) > 0$ for all $y < L$. Moreover, given [W], an increase in a must lower the value of $w_1(y, a)$ whenever $y < a$. It follows right away that the L must decline as aspirations go up, so that the minimal income in the support of the old steady state declines.

Going in the other direction, recall that $f(k) < k$ for all k large enough. Let K be the point at which $f(K) = K$. Then as y converges up to K, $k(y) \to y$, which means that $D(y, a) < 0$ for all y sufficiently close to a. In particular, if we let $H > a$ stand for the

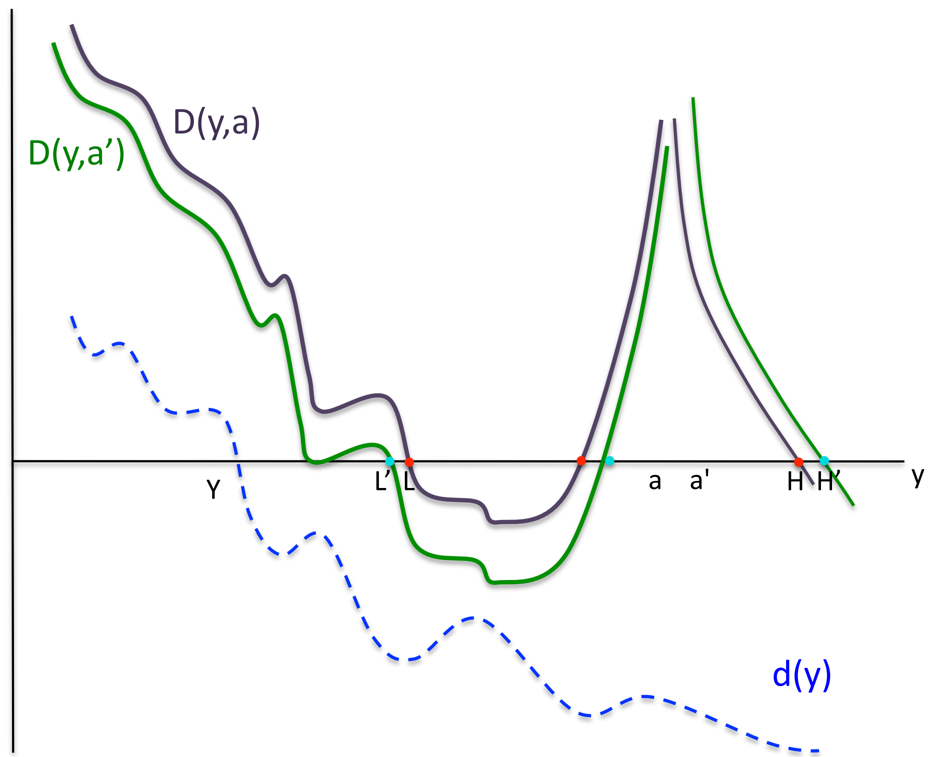

FIGURE 2. FIRST ORDER CONDITIONS IN STEADY STATE.

largest steady state income, it must be the case that $D(H, a) = 0$ while $D(y, a) > 0$ for all $y > H$.

Now consider H', the highest income at a new steady state with aspiration $a' > a$. If $a' \geq H$, we are automatically done. Otherwise $a < a' < H$. But in this case it is easy to see (again using [W]) that the increase from a to a' must raise the value of $w_1(H, a)$. It follows right away that $H' > H$, and the proof is complete. ■

3.3. **Stratified Aspirations.** While the failure to converge to equality is endemic in all the models of aspirations formation (at least under the assumptions that we've imposed), the emergence of clustering — and the number of such clusters or poles — deserves further investigation. One extension to the case of stratified aspirations, in which different segments of the economy each harbor common aspirations, but those aspirations vary across segments. For instance, we might think that the economy is divided among the "poor", the "middle class", and the "rich", and each inhabitant of this coarse classification has common aspirations drawn from the going (or anticipated) income distribution.

Recall that under stratified aspirations, the income distribution is segmented into n quantiles with aspirations

$$\Psi(y, F) = a_i$$

for individuals with income y in quantile i, where a_i is a scalar representing some summary statistic of the distribution in that quantile.

Proposition 4. *Under stratified aspirations, if $\{F_t\}$ is an equilibrium sequence of income distributions converging to some limit distribution F^*, then F^* must generically be concentrated on a finite set of points, at least two in number.*

The proof of this proposition is a direct extension of the argument made for common aspirations, and we omit it.

The only way to get away from clustering is to have aspirations that are fine-tuned to one's own personal circumstances. Such is the case with models of aspirations formation in which *individual* income enters in a highly sensitive way. One example is upward-looking aspirations, in which the aspirations of an individual are given by the conditional expected value of all higher incomes in the distribution. In some circumstances, this specification is compatible with a non-clustered steady state distribution of income. (An example is provided in the Appendix.)

An interesting observation is that, as we will see in an example below, the steady state distribution under stratified aspirations can exhibit relative mobility

3.4. **A Numerical Illustration.** We illustrate the observations above with some numerical simulations. In the examples that follow, we suppose that the production function exhibits the constant elasticity form $f(k, \theta) = \frac{4}{\alpha}\theta k^{\alpha}$, where $\alpha = 0.8$ and θ is a stochastic shock with mean 1.[10] Preferences are as follows: $u(c) = \ln(c)$ and $v(z) = 0.8\ln(z)$, while w is taken to have the logistic structure

$$w(a, z) = \frac{1.6}{1 + \exp\left[-\kappa(\frac{z}{a} - 1)\right]}.$$

Note that the parameter κ controls the steepness of w at the point $z = a$, where aspirations are met, while leaving the level of utility w at $z = a$ unaffected.

Our first exercise illustrates our results on polarization and common aspirations, which we take to be at median income (but any other specification would do just as well). We begin with an initial distribution of income that is uniform over a population of 800 individuals, and iterate the distribution over time. The simulated distributions converge to a steady state (where the only mobility is due to the noise in the production function).

When κ is large so that w is suitably steep at $z = a$, equality is impossible and the distribution converges to a bimodal limit, in line with the discussion above. Over time,

[10]Specifically, we suppose that θ follows a lognormal distribution. The results are not particularly dependent on a specific magnitude of the noise term, though, to be sure, the degree of clustering must fall with the variance of the shock.

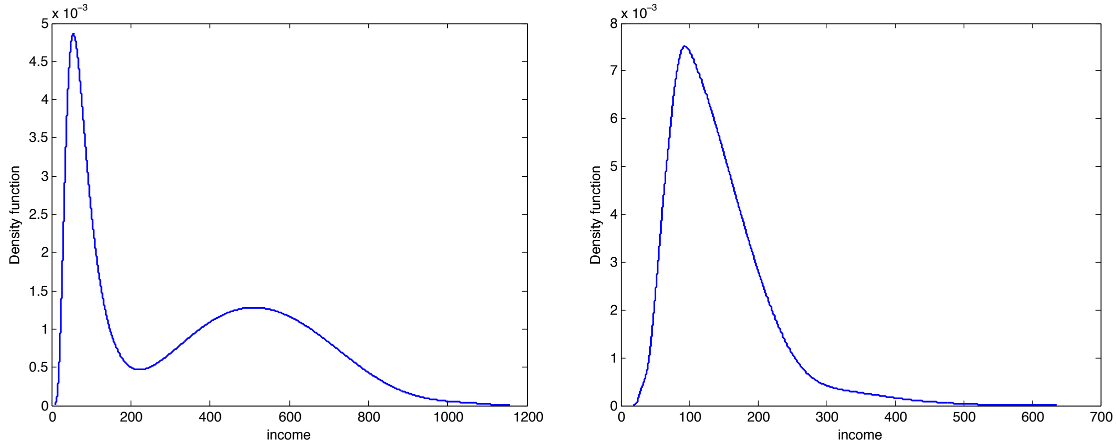

FIGURE 3. POLARIZATION AND COMMON ASPIRATIONS.

the distribution of income clusters around two poles. The first panel of Figure 3 illustrates this outcome for $\kappa = 5$.[11]

If the value of κ is lower, then w is relatively flat at $z = a$ and aspirations are less focal as a goal. Now full equality is possible. In the second panel of Figure 3, constructed for $\kappa = 0.5$, we see convergence to a perfectly equal income distribution.

In our second exercise, we contrast stratified and unstratified aspirations. We illustrate our point in the "steep" case — $\kappa = 5$ — but without any noise. The thought experiment goes as follows. Consider a society in which individuals are "cognitively stratified" into two income classes, perhaps as a result of social or spatial segregation by income. The poorer half of the population draws his aspiration from the median income *among the poor* while, in similar fashion, the richer half use the conditional median among their group. In this case, the distribution develops multiple poles and exhibits some mobility (despite the absence of noise). In the numerical example illustrated in Figure 4, a group of poor individuals cluster around an income of just below 100, while a group of rich individuals earn around 860. Both experience hardly any mobility. In the middle, groups of individuals earn between 200 and 500 and experience some mobility with their dynasties switching regularly from one class to the other.

It is possible to contrast this outcome with that in a similar society with common aspirations. Say that owing to less segregation or higher media exposure, individuals learn more about the incomes of the entire population and aspirations are commonly tagged at the median income. In this society, the distribution becomes much more polarized and converges to one with twin peaks around 80 and 870 with zero mobility (there is no production noise, in contrast with the previous example). With stratified aspirations,

[11]In the figures, we smoothed the simulated distribution using the density estimator 'ksdensity' for Matlab.

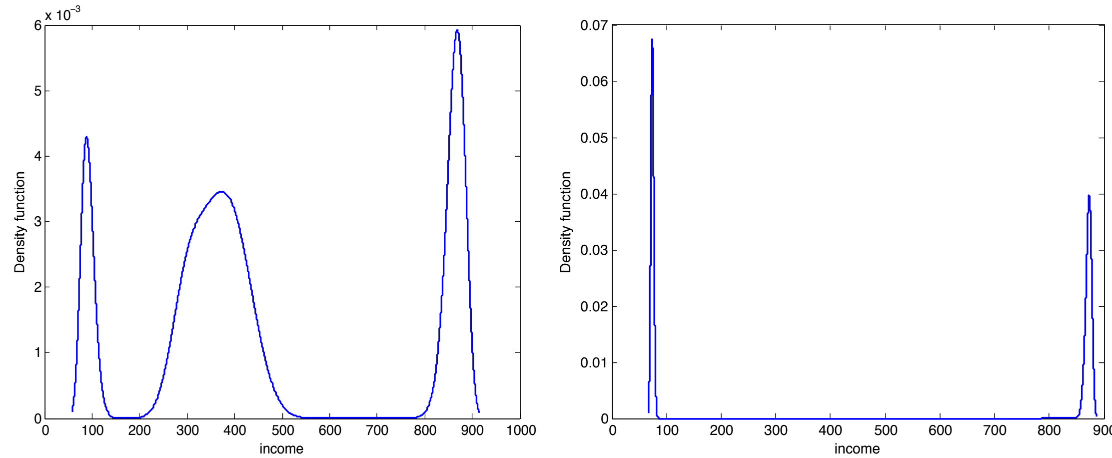

FIGURE 4. STRATIFIED ASPIRATIONS.

aspirations windows are smaller and aspirations consequently more attainable. This permits the emergence of a middle class and generates an income distribution with less inequality.

4. ASPIRATIONS AND GROWTH ON INCOME CROSS-SECTIONS

Moving away from steady state analysis, we want to allow for growth and look more closely at how growth rates vary along the cross-section of incomes. To do so, it will be useful to provide some more structure on the utility functions u and v, as well as the production function f. Let us suppose that u and v have the same constant-elasticity functional form; i.e., $u(c) = c^{1-\sigma}/(1-\sigma)$ and $v(z) = \rho z^{1-\sigma}/(1-\sigma)$ for some $\sigma > 0$ and $\rho > 0$. Suppose, moreover, that the production function is linear: $f(k) = (1+r)k$, where r is the rate of return on investment.

Then individuals maximize

$$u(c) + v(z) + w(z, a) = \frac{1}{1-\sigma} c^{1-\sigma} + \rho \frac{1}{1-\sigma} z^{1-\sigma} + w(z, a), \tag{4}$$

subject to the constraint

$$z = (1 + r)(y - c). \tag{5}$$

for a given starting income level y and aspiration level a.

The expositional advantage of constant-elasticity utility with linear production is that, in the absence of an aspirations effect, bequests are proportional to incomes and therefore growth rates are constant across the cross-section of current incomes. We can therefore be sure that any cross-sectional variation in the presence of aspirations stems entirely from aspirations alone. Without restricting individuals to be at steady state, we would like to describe the *growth incidence curve*, a relationship that links baseline income to subsequent rates of growth.

Rewriting (4) in terms of the growth rate $g = (z/y) - 1$, we get

$$\text{(6)} \qquad \frac{1}{1-\sigma}\left(y\left[\frac{r-g}{1+r}\right]\right)^{1-\sigma} + \rho\frac{1}{1-\sigma}\left([1+g]y\right)^{1-\sigma} + w([1+g]y, a).$$

The problem is nonconvex and may exhibit more than one solution. However, any solution to problem (6) is obviously interior in the choice of g and is therefore described by the first-order condition

$$\text{(7)} \qquad (r-g)^{-\sigma}(1+r)^{\sigma-1} - \rho(1+g)^{-\sigma} = w_1\left([1+g]y, a\right)y^{\sigma}.$$

The chosen growth rate will lie between a minimum of -1 and a maximum of r.

4.1. **Baseline Income and Growth.** The condition (7) permits us to study the effect of income on the (chosen) rate of growth g. To gain intuition, Figure 5 describes how the rate of growth g is determined by this first-order condition. The upward-sloping bold line is the left hand side of the first-order condition, it is obviously increasing in g.[12] The right-hand side (which is the other bold line) is *also* increasing in g, at least as long as aspirations are *unattained*, in the sense of having $(1+g)y < a$. Once aspirations are *exceeded*, so that $(1+g)y > a$, the right-hand side will decline in g.[13]

There could, in principle, be several intersections between the two lines. The second-order condition, however, assures us that we only need to consider those intersections in which the right-hand side cuts the left-hand side "from above". (Even that isn't enough to fully pin the solution down, but it is certainly necessary.) For ease of exposition, the diagram only has the two lines intersecting once, at g_1.

Suppose aspirations are high enough so that they remain unattained. Figure 5 carries out the exercise of raising y in such a context. The right-hand side of (7) is unambiguously shifted upwards, and we see that the new growth rate is higher, at g_2. While this intuitive argument in quite far from a formal proof,[14] it motivates

Proposition 5. *Suppose that aspirations are commonly held. Consider any income level y at which aspirations are unattained. Then growth rates decline as incomes decline below y.*

For a proof, see the Appendix.

What of income levels for which aspirations are exceeded? As before, the answer hangs on what happens to the right-hand side of the first-order condition (7) as income rises:

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[12] Indeed, the left-hand side tends to minus (or plus) infinity when g tends to -1 (or r), with curvature switching from concavity to convexity in between.

[13] It is easy to check that under our assumptions, it will not be the case that $(1+g)y = a$, so these definitions are exhaustive.

[14] In particular, local second-order conditions are not sufficient for optimality, and a change in y could move the optimal choice to an entirely different location instead of simply precipitating a local change. The proofs of Propositions 5 and 6 therefore employ revealed-preference arguments that are not based on local conditions.

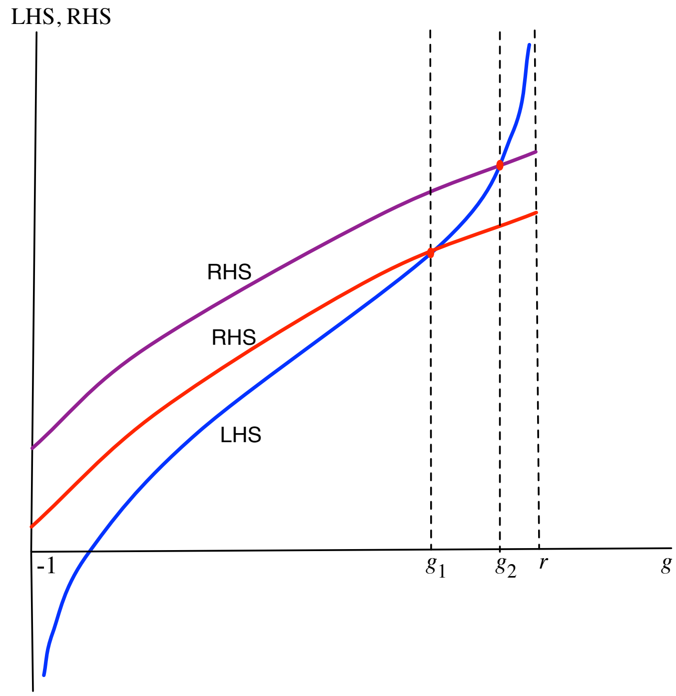

Figure 5. The Optimum at Unattained Aspirations

Proposition 6. *Suppose that aspirations are commonly held, and assume that*

[W′] $w_1(z, a)z^\sigma$ *is declining in* z *when* $z > a$.

Then growth rates decline as incomes increase, once aspirations are attained.

See the Appendix for a proof.

Proposition 5 captures the idea of frustration, and Proposition 6 the notion of complacency. In the former case, as incomes fall further and further below a commonly held aspiration, the incentivizing effects of aspirations weakens: this comes naturally from the S-shaped specification that we've borrowed from the behavioral economics literature. There is, therefore, less of an attempt to save when baseline incomes are lower. This observation is in line with the arguments in [2] and [34].

A different effect comes into play as outcomes exceed aspirations. Now individuals turn complacent. Under the additional condition [W′] in the statement of Proposition 6, an increase in y lowers growth.

How reasonable is [W′]? If w has unbounded steepness at $z = a$, the condition certainly holds in some region above $z = a$. Whether it holds more globally will depend on the value of σ, as well as the specific form of w, and in particular on the degree of concavity exhibited by it when $z > a$. Specifically, it can be shown that the curvature elasticity σ

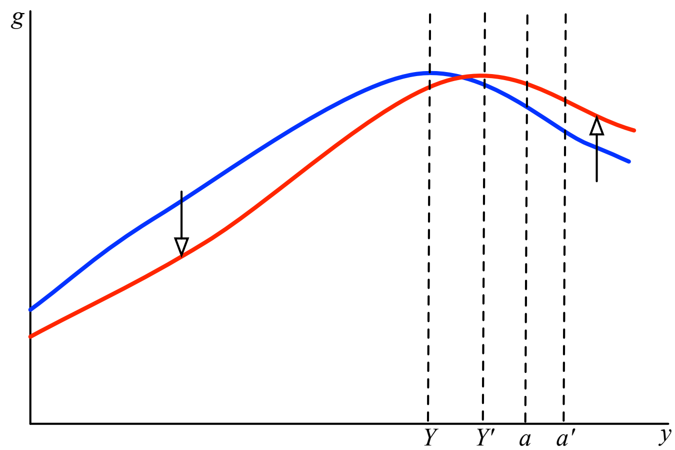

Figure 6. An Upward Shift in Aspirations

of the conventional one-period utility indicators u and v should not be too high, and in particular should not exceed that of w.[15]

It is easy to see that [W′] is equivalent to the requirement that the curvature elasticity of w, given by $-w_{11}(z, a)z/w_1(z)$ exceed those of the conventional utility indicators (σ) when $z > a$.

The two propositions together represent a formal statement of our informal assertion that "attainable" aspirations, which can be closed by a round of sustained growth, are the most conducive to investment. It is tempting to conclude that growth should be maximized at "intermediate" levels of income, but that will depend on more restrictions.

Certainly, when aspirations are commonly held and condition [W′] is applicable, the equilibrium rates of growth rise and then fall on the cross-section of incomes, though the relative sizes of the two segments will depend on just where the common aspirations are placed. More generally, the theory makes no particular prediction regarding the shape of growth rates in income, once aspirations are also suitably endogenized. For instance, in the case of upward-looking aspirations, it is possible that every income grows at exactly the same rate, once the initial distribution is suitably chosen (details available on request).

At the same time, it is intriguing to observe — as we do in Section 4.2 below — that in actual contexts, much of the variation in observed cross-sectional growth rates can be accommodated with simple models of aspiration formation; more on this below.

The theory is also particularly well-suited to assess what might happen to cross-sectional growth rates when some mechanism that predictably shifts aspirations at each level of income comes into play (an upward shift in Ψ). For instance, it is possible to consider

[15] For instance, suppose that w has constant curvature elasticity μ to the right of a; i.e., $-w_{11}(x + a, a)x/w_1(x + a) = \mu$. Then it is easy to check that $\sigma \leq \mu$, along with the already-assumed strict concavity of w to the right of a, is sufficient for [W′].

the rise of mass media in developing countries, in which advertisements and television programs showcase the lives of the rich, thus raising aspirations across the board. Figure 6 illustrates the outcome for the case of common aspirations.

Up to income level Y, aspirations are not met at the (common) aspiration a. When a increases to a', income Y can now be identified with a lower income in the earlier situation, so the growth rate associated with Y — and indeed, with all income levels below Y — fall. We can do the same argument in reverse by defining Y' as the income level after which aspirations are exceeded, in the situation with common aspirations a'. Assume that growth rates are declining after that point; this will automatically be satisfied when condition [W′] is met. When a' is brought back to a, all income levels to the right of Y' can be identified with still higher incomes, so growth rates are lower relative to a than they are relative to a'. These "swivels" in the growth incidence curve translate into predictions about both the aggregate growth rate as well as the evolution of income inequality over time.

4.2. **An Illustrative Empirical Exercise.** This section uses percentile distributions available for 43 countries over at least two distinct years to illustrate the growth incidence curves predicted by different models of aspirations formation.[16] Throughout, we take $w(z, a)$ to have a CARA shape in z/a above 1 and be symmetric around 1,[17] and set $\rho = 0.8$.

In our first exercise, we assume common aspirations tagged to societal mean income. For any distribution, a given return to capital r generates specific individual growth rates as a function of income. These incomes and growth rates imply a specific aggregate growth rate for the country. In this exercise, we find the return to capital r that generate in our model the *actual* aggregate annual rate growth observed. Figure 7 shows the resulting growth incidence curves for nine Latin American countries in the nineties.[18] As seen in Propositions 5 and 6, our model predicts that with common aspirations these growth rates follow an inverted U-shape.

Clearly, the actual growth pattern by percentiles observed in these countries has, in most case, a different shape. This is not surprising as our simple model does not even come close to capture the many factors that drive percentile growth, but on top of that there is no reason to believe that aspirations are commonly held. That suggests the following thought experiment: using these data and our model, we can study the characteristics of the process of aspirations formation that *best fit* the actual growth incidence curves. Our second exercise does just this.

[16]Special thanks to Claudio Montenegro at the Development Research Group, Poverty Unit, The World Bank.

[17]

$$\begin{aligned} w(z,a) &= 10 - e^{-20(z/a-1)} \text{ for all } z \geq a \\ &= 8 + e^{-20*(1-z/a))} \text{ for all } z < a; \end{aligned}$$

[18]We would be happy to provide anyone interested with the graphs for the remaining countries.

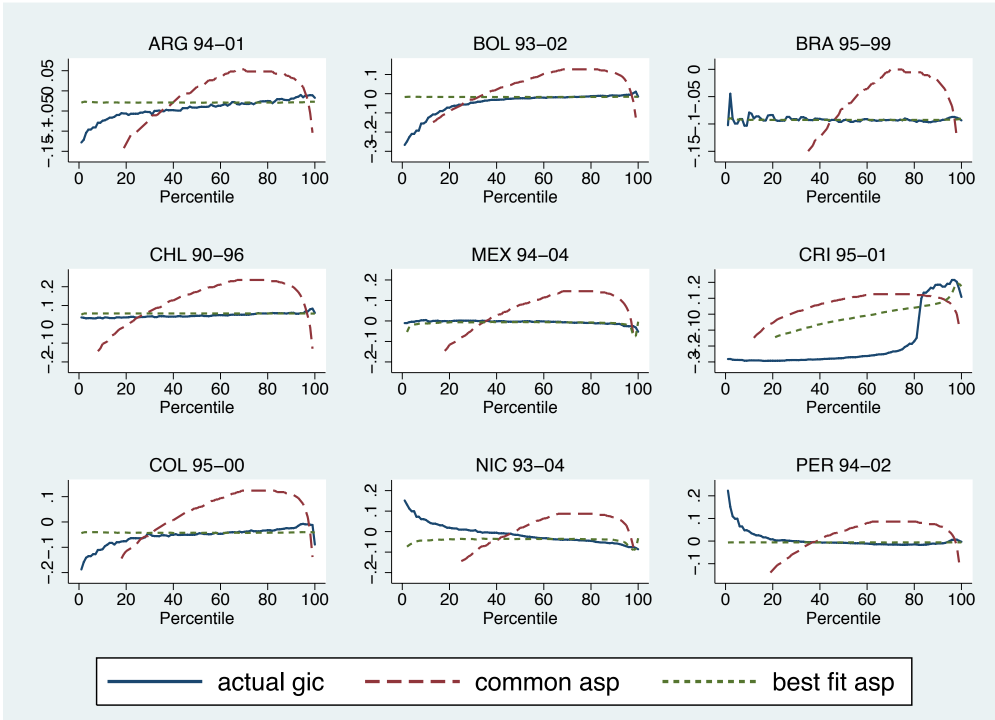

Figure 7. Growth incidence curves

Our data consists in 55 percentile distribution and growth incidence curves. For each of these, we search among a class of aspiration formation processes where 1) the weight put by percentile i on another percentile income j only depends on $(i-j)$ (the percentile distance and whether i is richer or poorer than j); 2) these weights have a quadratic shape in $|i-j|$ on either side of i (and are not necessarily symmetric around i). This class includes among others common and upward aspirations processes. As before, the return to capital r in any country is selected to match *actual* aggregate annual rate growth observed.

The growth incidence curve predicted by our "best fit" aspirations are illustrated in Figure 7. Although they are not perfect match, we see that they come much closer to the actual percentile growth. This specification captures 75% of the observed variation in growth (as opposed to 3% for the common aspirations model).

What we find is *umbrella*-shaped aspirations that a. are centered: for 85% of the countries (47 out of 55) individuals put the most weight in forming their aspirations on the income in their own percentile; and b. have narrow aspirations windows: in more than half the countries individuals put no weight in forming their aspirations on the incomes that are more than two percentiles away from themselves. This is shown in Figure 8 for the same nine countries studied earlier. On the left-hand-side of the picture, we see the weights that the median percentile puts on the neighboring percentiles when forming its

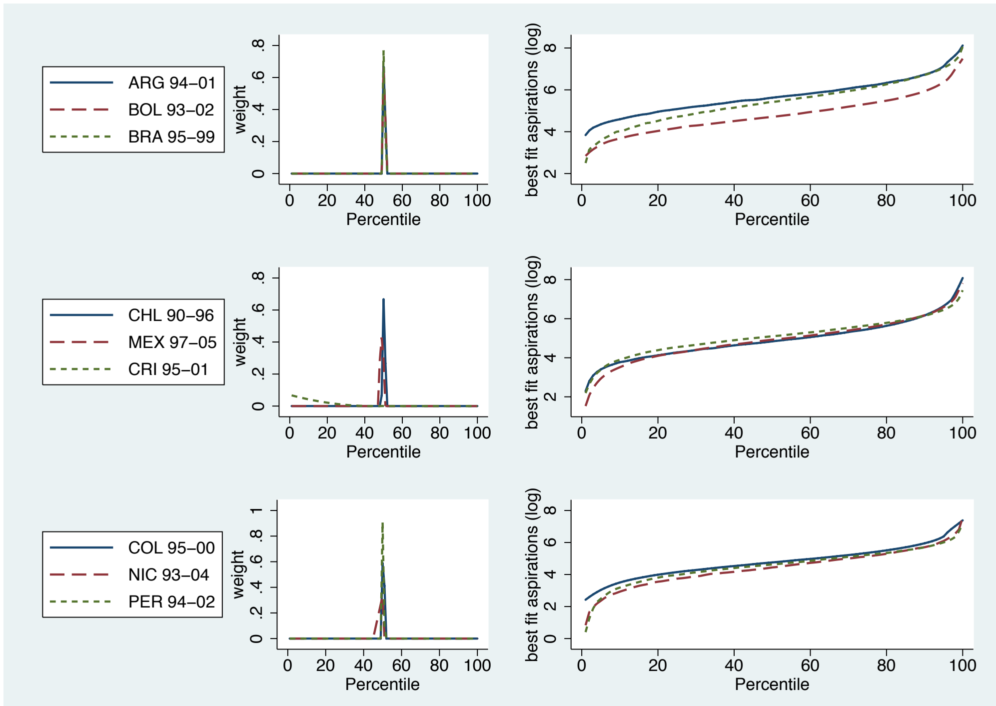

Figure 8. Weights and Aspirations Levels

aspirations. On the right-hand-side of the pictures we see the resulting level of aspirations (in log) for the various percentiles.

Although the limitations of this exercise are obvious, it suggests a fair amount of stratification in the aspiration formation process. This raises the growth rates of the poorest percentiles but reduces the growth rates in the upper middle range of the distribution.

5. Conclusion

This paper builds a theory of aspirations formation that emphasizes the social foundations of individual aspirations, and relates those aspirations in turn to investment and growth. Following a familiar lead from behavioral economics (see, e.g., [24], [25], and [26]), we define utilities around a "reference point", and interpret that reference point as an *aspiration*. Our main departure from this literature is in the determination of aspirations: rather than emphasizing the past experiences of the individual herself in shaping aspirations, we stress the social basis of aspirations formation. We argue that aspirations are likely to depend not only on one's own historical living standards, as commonly assumed, but also on the experience and lifestyle of others.

The theory we propose has three segments. First, individual aspirations determine one's to incentives to invest, accumulate, and bequeath. Second, aspirations are determined by the going distribution of income. Finally, individual behavior is aggregated to derive the social distribution of income, thus closing the model.

A central ingredient of our setup is that aspirations can serve both to incentivize and to frustrate. We argue that aspirations that are above — but not too far — from current incomes can encourage high investment, while aspirations that are too high may discourage it. Formally, this is well-captured by the reference-based utility function, in which departures of income from the reference point in *either* direction lead to diminishing returns in utility (or disutility).

Our main results concern stable equilibrium income distributions. We show that perfect or near-perfect equality is unstable: income distributions cannot converge to a degenerate distribution. Indeed, if aspirations are common or stratified, we show that in any steady state, incomes must polarize over time. We also study the behavior of growth rates along the income distribution.

The theoretical results are complemented by an empirical exercise, which uses percentile distributions of growth rates available for 43 countries. We use aggregate growth experiences to estimate a rate of return to individual investment, and then employ the model to see which aspirations formation structure appears to fit the data best, in that they come closes to the observed growth incidence curves by percentile. We show that a model of aspirations formation in which individuals use umbrella-shaped weights on incomes in some interval around their own incomes comes closest to replicating the data, and that a large fraction of the observed variation is indeed "explained" by our specification.

The goal of this paper has been to take a modest step towards thinking about the social determinants of aspirations or reference points. As in the case of any model with social effects on individual behavior, which are then aggregated to yield those social outcomes, there are difficulties in undertaking a full-blown dynamic analysis, and this paper is no exception. It would be of great interest to fully describe income-distribution dynamics for different models of aspirations formation. In the same spirit, one might ask for a more comprehensive structural exercise which would allow us to exploit the model to uncover more fully the process of aspirations formation. We believe that this approach will shed new and complementary light on the endogenous emergence of inequality.

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Appendix: Proofs Not in the Main Text

Define a function L(g) by

$$L(g) \equiv \frac{1}{1-\sigma}\left(\frac{r-g}{1+r}\right)^{1-\sigma} + \rho\frac{1}{1-\sigma}(1+g)^{1-\sigma}.$$

Then it should be clear from (6) that for each y, the optimal choice of g maximizes

$$L(g) + y^{\sigma-1}w([1+g]y, a).$$

Let g_1 and g_2 be optimal choices at y_1 and y_2 respectively. For additional simplicity of notation, let $G_1 \equiv (1+g_1)$ and $G_2 \equiv (1+g_2)$, and let $h(z) \equiv w(z,a)$. A standard revealed preference argument tells us that

$$L(g_1) + y_1^{\sigma-1}h(G_1y_1) \geq L(g_2) + y_1^{\sigma-1}h(G_2y_1)$$

and

$$L(g_2) + y_2^{\sigma-1}h(G_2y_2) \geq L(g_1) + y_2^{\sigma-1}h(G_1y_2).$$

Combing these two inequalities, we must conclude that

$$\Phi(y_1) \geq L(g_2) - L(g_1) \geq \Phi(y_2), \tag{8}$$

where the function Φ is defined by

$$\Phi(y) \equiv y^{\sigma-1}\left[h(G_1y) - h(G_2y)\right].$$

Proof of Proposition 5. Suppose that $y_1 < y$, and that aspirations are unattained at y: $(1+g)y < a$. Suppose, contrary to our assertion, that $g_1 > g$. Then, with a little work, we can find $y_2 > y_1$ such that $g_1 > g_2$ *and* $(1+g_1)y_2 < a$.[19]

Now, simple differentiation of Φ tells us that

$$\begin{aligned}\Phi'(y) &= y^{\sigma-2}\left[\left\{h'(G_1y)G_1y - h'(G_2y)G_2y\right\} + (\sigma-1)\left\{h(G_1y) - h(G_2y)\right\}\right] \\ &\geq y^{\sigma-2}\left[\left\{h'(G_1y)G_1y - h(G_1y)\right\} - \left\{h'(G_2y)G_2y - h(G_2y)\right\}\right]. \end{aligned} \tag{9}$$

At the same time, the function $h'(z)z - h(z)$ is strictly increasing in z for $z \in [G_2y_1, G_1y_2]$, because $z < a$ over this entire range.[20] Using this information in (9), and recalling that $G_1 > G_2$ by assumption, we must conclude that $\Phi'(y) > 0$ for all $y \in [y_1, y_2]$, which contradicts (8). ■

Proof of Proposition 6. Suppose that $y_1 < y_2$, and that aspirations are exceeded at y_1: $(1+g_1)y_1 > a$. Suppose, contrary to our assertion, that $g_2 > g_1$. Just as in the proof of Proposition 5, we know that

$$\Phi'(y) = y^{\sigma-2}\left[\left\{h'(G_1y)G_1y + (\sigma-1)h(G_1y)\right\} - \left\{h'(G_2y)G_2y + (\sigma-1)h(G_2y)\right\}\right].$$

---

[19] Suppose that $y_1 < y$ and $g < g_1$. A standard argument establishes the monotonicity of aggregate investment in initial income, so that aspirations are unattained for all $y' < y$. Define $y^* \equiv \inf\{y' < y | g' < g_1\}$. If $y^* = y_1$, we are done by choosing y_2 slightly above y_1: because $(1+g_1)y_1 < a$, we will have $(1+g_1)y_2 < a$ and $g_1 > g_2$, as desired. If $y^* > y_1$, then $g' \geq g_1$ for all $y' \in [y_1, y^*)$, so that once again, y choosing y_2 slightly above y^*, we have $(1+g_1)y_2 < a$ and $g_1 > g_2$.

[20] To see this, differentiate to see that $\frac{d}{dz}[f'(z)z - f(z)] = zf''(z) > 0$.

But condition [W′] informs us that the function $h'(z)z+(\sigma-1)h(z)$ is strictly decreasing in z for $z \in [G_1y_1, G_2y_2]$, because $z > a$ over this entire range.[21] We must therefore conclude that $\Phi'(y) > 0$ for all $y \in [y_1, y_2]$, which contradicts (8). ■

Appendix: An Example of a Non-Clustered Steady State When Aspirations are Fine-Tuned to Personal Income.

In the case of upward-looking aspirations, it is possible to have continuous distribution F in steady state. Let

$$a(y) = \frac{\int_y^\infty xdF(x)}{1-F(y)}.$$

Assume that there is no uncertainty and that $w(y,a) = \omega(y/a)$. In steady state, $k = k(y) \equiv f^{-1}(y)$ and the first order condition (3)

$$u'(y-k(y)) = f'(k(y))\left[v'(y) + w'(\gamma(y))\right]$$

where $\gamma(y) = y/a(y)$.

Assume that $\frac{u'(y-k(y))}{f'(k(y))} - v'(y)$ is constant in y, to have a continuous distribution of relative income in steady state, it must be the case that $\gamma(y)$ is constant over the support of the distribution. Lemma 7 tells us that this is the case if and only if F is a Pareto distribution.

Lemma 7. *$\gamma(y)$ is constant if and only if y follows a Pareto distribution.*

Proof.[22] Let $\phi(y) = [(1+g^*)\gamma(y)]^{-1}$.

$$\phi(y) = \frac{1}{y(1-F(y))}\int_y^\infty xdF(x).$$

$\gamma(y)$ constant means that $\phi(y) = k$ and $\phi'(y) = 0$. Hence,

$$\phi'(y) \equiv -h(y) - \frac{1}{y}(1-yh(y))\phi(y) = 0,$$

where $h(y) = \frac{f(y)}{1-F(y)}$. This condition can be rewritten as

$$\frac{\partial log(1-F(y))}{\partial y}(1-\phi(y)) - \frac{\partial logy}{\partial y}\phi(y).$$

Substituting in $\phi(y) = k$ yields

$$\frac{\partial}{\partial y}\left(log(1-F(y))\right). = \frac{\partial}{\partial y}\left(logy^{\frac{k}{1-k}}\right).$$

Therefore,

$$1-F(y) = Ay^{\frac{k}{1-k}},$$

a Pareto distribution. ■

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[21] To see this, differentiate to see that $\frac{d}{dz}[h'(z)z+(\sigma-1)h(z)] = zh''(z)+\sigma h'(z) < 0$, by [W′].

[22] We are thankful to Joan Esteban for this proof.