The essentiality of money in environments with centralized trade

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1. Introduction

Modern monetary theory is based on the notion that one must be explicit about the frictions in the exchange process that generate an essential role for money. The current benchmark in this literature is the model introduced in Lagos and Wright (2005) [LW, hereafter]. The main contribution of LW is in constructing an environment in which, differently from the models of money in the tradition of Kiyotaki and Wright (1993), substantive issues can be analyzed in a tractable manner. The key element of LW is that trade alternates between centralized and decentralized markets.

An important feature of LW and the literature that follows is that they treat trade in the centralized and decentralized markets asymmetrically. While the literature is very explicit about the process of exchange in the decentralized market, the literature remains silent about how trade takes place in the centralized market, modeling it as a Walrasian market where agents take prices as given and trade against their budget constraints. The asymmetric treatment of trade in the different markets, however, is at odds with the emphasis the literature places on taking seriously the process of exchange, and may have important implications for monetary theory in general. In this paper, we model the process of exchange in the centralized market explicitly and ask whether doing so has implications for the essentiality of money.

The starting point of our analysis is the framework of LW modified in two ways. The first departure from LW is that the process of exchange in the centralized market is modeled as a strategic market game along the lines of Shapley and Shubik (1977). The classic reference on strategic market games is Shubik (1973). See also Dubey and Shubik (1978) and Postlewaite and Schmeidler (1978). Green and Zhou (2005), Hayashi and Matsui (1996), and Howitt (2005) are examples of applications of market games in the context of monetary theory.

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1 The classic reference on strategic market games is Shubik (1973). See also Dubey and Shubik (1978) and Postlewaite and Schmeidler (1978). Green and Zhou (2005), Hayashi and Matsui (1996), and Howitt (2005) are examples of applications of market games in the context of monetary theory.
allow for a mapping between the formation of prices and the trading decisions of agents, while retaining the idea of centralized markets as large anonymous markets in which agents only observe prices.5

The second departure from LW is that while LW considers a continuum economy, our environment consists of a large but finite population. In any model, the assumption of a continuum of agents is made for tractability and is only justifiable if it has no substantive economic implications. To put it differently: “The rationale for using the continuum-of-agents model is that it is a useful idealization of a situation with a large finite number of agents, but if equilibria in the continuum model are radically different from equilibria in the model with a finite number of agents, then this idealization makes little sense (Levine and Pesendorfer, 1995, p. 1160).” We want to ensure that the essentiality of money is not an artifact of the continuum population assumption.

Our main message is that providing microfoundations for the process of exchange in the centralized market can have important implications for the essentiality of money. Indeed, when one models the centralized market as a strategic market game, individual actions have an impact on prices, so that prices can be used to infer behavior. The informational role of prices makes it possible to use them to implement desirable non-monetary allocations even in large populations. The key observation is that even if an agent’s behavior has a negligible impact on current aggregate outcomes, the agent can still affect behavior as long as he is informationally relevant, that is, as long as his actions have a measurable impact on prices.3

In order to make our point clear, we first consider a setting in which the mapping from actions in the centralized market into prices is deterministic, and show that if agents are patient enough, then there exists a non-monetary equilibrium that implements the first-best regardless of the population size. The assumption of a perfect correlation between individual behavior and prices is not plausible in large populations, though. So, any non-essentiality result that relies on the existence of a one-to-one map between individual behavior and prices is unsatisfactory. For this reason, we extend our analysis to the case in which there is noise in the map that takes individual actions into prices, which reduces the ability of prices to convey information about individual behavior when there are many agents in the economy. Our main result is that whether agents become informationally negligible or not in large economies critically depends on the ratio between the number of agents who participate in trade and the number of goods that are traded in the centralized market, that is, on how “thick” the centralized market is. Thus, our analysis provides conditions under which money remains essential when one explicitly models the process of exchange in the centralized market as a market game.

This paper is not the first to address the question of whether the presence of centralized trading matters for the essentiality of money. Aliprantis et al. (2007a) [ACP, hereafter] show that money can fail to be essential if individual actions are perfectly observable in the centralized market. Our analysis departs from ACP in the assumption that agents only observe prices in the centralized market and prices can be a noisy function of individual actions. The differences between our approach and ACP’s approach are discussed in more detail at the end of Section 3.

The paper is organized as follows. Section 2 introduces the framework. Section 3 analyzes the benchmark case in which the map taking actions in the centralized market into prices is deterministic. Section 4 considers the main case, the setting in which prices are noisy. Section 5 concludes. All omitted proofs are available in an online appendix, which also contains a number of extensions.

2. The framework

We first describe the environment and preferences. Then we describe the economy as an infinitely repeated game.

2.1. Environment and preferences

Time is discrete and indexed by $t \geq 1$. There are two stages at each date and preferences are additively separable across dates and stages. The population consists of a finite number $N$ of infinitely lived agents. Agents do not discount payoffs between stages in a period and have a common discount factor $\delta \in (0, 1)$ across periods. The two stages of a period differ in terms of the matching process, preferences, and technology. In the first stage, agents are randomly and anonymously matched in pairs. In the second stage, trade takes place in a centralized market.

Agents trade a divisible special good in the decentralized market and a divisible general good in the centralized market. Both the special good and the general good are perishable across stages and dates. There are $S \geq 3$ types of the special good and $G \geq 1$ types of the general good. Each agent is characterized by a pair $(s, g) \in \{1, \ldots, S\} \times \{1, \ldots, G\}$. An agent of type $(s, g)$ can only produce a special good of type $s$ and a general good of type $g$, and only likes to consume a special good of type $s + 1 (\text{mod } S)$ and a general good of type $g + 1 (\text{mod } G)$. Note that $G = 1$ corresponds to the case of a homogeneous general good. There exist an equal number of agents of each type. In particular, $N \geq SG$ and the probability that an agent is a consumer in a meeting in the decentralized market, which equals the probability that he is a producer, is $N/S(N-1).$

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5 A third reason for the non-cooperative approach is that in order to assess the conditions under which money is essential, one must consider whether agents have the incentive to follow alternative credit-like arrangements. The standard Walrasian market does not specify how payoffs are defined off the equilibrium path and thus it is ill-suited to check the feasibility of competing mechanisms of exchange.

3 Green (1980) makes a similar point in the context of repeated Cournot competition.
For convenience, in the remainder of the paper \(g \pm 1\) stands as a shorthand for \(g \pm 1 \pmod{G}\) and we say that an agent is of type \(g\) if he is of type \((s, g)\) for some \(s \in \{1, \ldots, 5\}\).

2.1.1. Decentralized market

An agent who consumes \(q \geq 0\) units of the special good he likes enjoys utility \(u(q)\), while an agent who produces \(q\) units of the special good incurs a cost \(c(q)\). The functions \(u\) and \(c\) are strictly increasing and differentiable, with \(u\) strictly concave and \(c\) convex. Moreover, \(u(0) = c(0) = 0\), \(u'(0) > c'(0)\), and \(u'(q) = c(q)\) for some \(q > 0\). Let \(q^* > 0\) be the unique solution to \(u'(q) = c(q)\).

Trade in the decentralized market takes place as follows. In every single-coincidence meeting the agents in the match simultaneously and independently choose from \((\text{yes}, \text{no})\) after learning whether they are consumers or producers. If either agent in the match says no, then the match is dissolved and no trade occurs. If both agents in the match say yes, that is, if both agents agree to trade, then the producer produces \(q^*\) units of the good to the consumer. Afterwards, the match is dissolved.4

2.1.2. Centralized market

Production in the centralized market occurs in two stages. First, agents exert effort for the production of the general good, with each unit of effort producing one unit of an intermediate good. Following that, for each \(g \in \{1, \ldots, G\}\), the intermediate good produced by the agents of type \(g\) are transformed in general goods of type \(g\). There exists an upper bound \(\bar{x} > 0\) on the amount of effort an agent can exert in a period.

An agent who consumes \(x \geq 0\) units of the general good he likes obtains utility \(U(x)\), while an agent who exerts effort \(x\) incurs disutility \(x\). The function \(U\) is differentiable and strictly increasing, with \(U(0) = 0\), and \(\lim_{x \to \infty} U(x) = \infty\). There are two possibilities: either \(U\) is strictly concave, with \(1 < U'(0) < \infty\) and \(\lim_{x \to \infty} U'(x) = 0\), or \(U\) is linear with slope greater than one.5

Trade in the centralized market takes place as follows. There are \(G\) trading posts, one for each type of general good. In every period \(t\), each agent \(j \in \{1, \ldots, N\}\) simultaneously and independently chooses: (i) the effort \(y_j^t\) he exerts for the production of the general good; (ii) the vector \(b_j^t = (b_j^{1G}, \ldots, b_j^{cG})\) of bids he submits to the trading posts. Bids are expressed in units of the intermediate good and the vector of bids \(b_j^t\) is such that \(\sum_{g=1}^{G} b_j^{gG} \leq y_j^t\). Thus, the sum of an agent’s bids cannot exceed the amount of the intermediate good that he produces.6 Note that agents must trade in the centralized market in order to consume the general good even when \(G = 1.7\)

Let \(N(g)\) be the set of agents of type \(g\). The supply of general good \(g\) in period \(t\) is given by \(\theta^t_0 Y^t_g\), where \(Y^t_g = \sum_{j \in N(g)} y_j^t\) is the amount of the intermediate good produced by the agents of type \(g\) and \(\theta^t_0\) is an unobservable stochastic shock to production. If in period \(t\) the price of general good \(g\) in terms of the intermediate good is \(p^t_g\), then the quantity of general good \(g\) that each agent \(j\) obtains is \(x^t_j = b^j_{gG} / p^t_g\). Hence, the supply of general good \(g\) in period \(t\) equals its demand in the same period if

\[
\theta^t_0 \sum_{j \in N(g)} y_j^t = \sum_{j=1}^{N} x^t_j = \frac{1}{p^t_g} \sum_{j=1}^{N} b^t_{jG},
\]

that is, if \(p^t_g\) is such that

\[
p^t_g = \frac{\sum_{j=1}^{N} b^t_{jG}}{\theta^t_0 \sum_{j \in N(g)} y_j^t},
\]

with the convention that \(0/0 = 0\).

The shocks \(\theta^t_0\) are independently and identically distributed over time and across goods according to a continuously differentiable cumulative density function \(\Omega\) with \(\mathbb{E}(\Theta_0) = 1\). The support of \(\Omega\) is \([\theta_{\min}, \theta_{\max}]\), where \(0 < \theta_{\min} \leq 1 \leq \theta_{\max} < \infty\).

The map between actions and prices is deterministic when \(\theta_{\min} = \theta_{\max} = 1\). The assumption that \(\theta_{\min}\) is positive and \(\theta_{\max}\) is finite is natural. When either \(\theta_{\min} = 0\) or \(\theta_{\max} = \infty\), the shocks to production can be so extreme that one agent of a given type can produce more units of the general good than all agents of a different type no matter the population size.

4 The same results hold if the producer can choose the quantity \(q\) he transfers to the consumer. Just note that \(q^*\) can be identified with yes, while all other quantities \(q\) can be identified with no. Our approach simplifies the description of strategies.

5 Our analysis extends to the case in which \(U(0) = 0\) and \(U'(0) = \infty\) see the discussion at the end of Section 4. Our analysis can also accommodate the case in which \(U(x) = x\), see the Online Appendix. The case in which \(U(x) = x\) differs from the other cases in that there are no gains from trade in the centralized market.

6 The assumption that the sum of bids of an agent cannot exceed his production of the intermediate good is similar to one made in Shapley and Shubik (1977). Shapley and Shubik consider a strategic market game in a pure endowment economy with \(m+1\) commodities in which agents use commodity \(m+1\) to bid for the first \(m\) commodities. They assume that total bids by one agent cannot exceed his endowment of commodity \(m+1\). In our production economy, the intermediate good an agent produces performs the role of commodity \(m+1\) in Shapley and Shubik’s environment.

7 Our analysis extends to the case in which an agent can directly consume the general good that he produces when \(G = 1\), see the Online Appendix. As in the case in which \(U(x) = x\), there are no gains from trade in the centralized market when agents can produce the general good for their own consumption.
Let $U_D : \mathbb{R}_+ \to \mathbb{R}_+$ be such that $U_D(x) = \mathbb{E}_D[U(\theta x)]$. Note that $U_D(x) = U(x)$ when there is no noise in prices. Since $U_D(0) = U'(0)E_D[\theta] > 1$, the problem $\max_{x \geq 0} [U_D(x) - x]$ has a unique and positive solution $x^*$. Note that $x^* = x$ when $U$ is linear. Given that the disutility from effort is linear in effort, a necessary condition for welfare in the centralized market to be maximized is that total effort exerted for the production of each type of general good is $(N/G)x^*$. The per capita payoff in the centralized market is $U_D(x^*) - x^*$ when welfare is maximized.

2.2. The game

The economy is an infinitely repeated game in which the stage game consists of one round of trade in the decentralized market followed by one round of trade in the centralized market. We describe strategies in the repeated game by means of automata. Let $A_1 = \{\text{yes, no}\}$ be the action set of an agent in a single-coincidence meeting in the decentralized market and 

$$A_2 = \left\{ a_2 = (y, (b^1, \ldots, b^c)) : y \leq \bar{X} \text{ and } \sum_{g=1}^G b^g \leq y \right\}$$

be the action set of an agent in the centralized market. An automaton for an agent of type $(s, g)$ is a list $(W, w^0(f_1, f_2), (\tau_1, \tau_2))$ where: (i) $W$ is the set of states; (ii) $w^0 \in W$ is the initial state; (iii) $f_1 : W \times \{1, \ldots, S\} \times \{1, \ldots, G\} \to A_1$ and $f_2 : W \to A_2$ are decision rules in the decentralized and centralized markets, respectively; (iv) $\tau_1 : W \times \{1, \ldots, S\} \times \{1, \ldots, G\} \times A_1 \to W$ and $\tau_2 : W \times A_2 \times \mathbb{R}_+^G \to W$ are transition rules in the decentralized and centralized markets, respectively.

If the decision rules for an agent of type $(s, g)$ are given by $(f_1, f_2)$, then the agent’s behavior in state $w$ is as follows: (i) he chooses $f_1(w, s, g')$ in a single-coincidence meeting in the decentralized market if his partner’s type is $(s, g')$; (ii) he chooses $f_2(w)$ in the centralized market. If the transition rules for an agent of type $(s, g)$ are given by $(\tau_1, \tau_2)$, then: (i) $\tau_1(w, s, g, a_1)$ is the agent’s new state when he enters the decentralized market in state $w$ if he chooses $a_1$ and his partner is of type $(s', g')$ and chooses $a_1'$; (ii) $\tau_2(w, a_2, p)$ is the agent’s new state when he enters the centralized market in state $w$, chooses $a_2$, and observes the vector of prices $p$. We restrict attention to strategy profiles in which the set of states is the same for all agents and the same type follow the same strategy.

Given a strategy profile $s$, a profile of states for an agent is a map $\pi : W \times \{1, \ldots, S\} \times \{1, \ldots, G\} \to \{1, \ldots, N-1\}$ such that $\pi(w, s, g)$ is the number of agents of type $(s, g)$ who are in state $w$, excluding the agent himself if he is of type $(s, g)$. Denote the set of all profiles of states by $\Pi$. A belief for an agent is a map $\rho : \Pi \to [0,1]$ such that $\rho(\pi) = 1$, where $\rho(\pi)$ is the probability the agent assigns to the event that the profile of states is $\pi$. Let $\Lambda$ be the set of all possible beliefs. A belief system for an agent is a map $\mu : W \to \Lambda$. In an abuse of notation, $\mu$ also denotes the profile of belief systems in which all agents have the same belief system $\mu$.

We consider sequential equilibria of the repeated game. The first-best is achieved if in each period trade takes place in all single-coincidence meetings in the decentralized market and welfare is maximized in the centralized market.

3. Deterministic prices

In order to make clear that being explicit about the process of exchange in the centralized market has important implications for the essentiality of money, our analysis begins with the benchmark case in which the map that takes individual actions in the centralized market into prices is deterministic. In this case, when agents are patient enough, there exists a sequential equilibrium that sustains the first-best regardless of the population size and the number of trading posts.

Let $e_g$ with $g = \{1, \ldots, G\}$, be the vector with all entries equal to zero except the $g$th entry, which is equal to one, and $e$ be the vector with all entries equal to one. Moreover, let $\Theta \in \mathbb{R}^D$ denote the vector with all entries equal to zero. Now define $\sigma^*$ to be the strategy profile in which the agents of type $g$ behave according to the following automaton. The set of states is $W = \{C, D, A\}$ and the initial state is $C$. The decision rules are given by

$$f_1(w, s', g) = \begin{cases} \text{yes} & \text{if } w \in \{C, D\} \\ \text{no} & \text{if } w = A \end{cases} \quad \text{and} \quad f_2(w) = \begin{cases} (x^* e^*_g, 1) & \text{if } w = C \\ (\bar{x}, 0) & \text{if } w = D \\ (0, 0) & \text{if } w = A \end{cases}$$

For instance, an agent in state $C$ behaves as follows. In the decentralized market, the agent agrees to trade regardless of his partner’s type. In the centralized market, the agent exerts effort $x^*$, bids $x^*$ at the trading post $g+1$, and bids zero at the
other trading posts. The transition rules are given by

\[ \tau_1(w, s', g', a_1, a_1') = \begin{cases} C & \text{if } w = C \text{ and } (a_1, a_1') = (\text{yes, yes}) \\ D & \text{if } w = C \text{ and } (a_1, a_1') \neq (\text{yes, yes}) \\ w & \text{if } w \in \{D, A\} \end{cases} \]

and

\[ \tau_2(w, a_2, p) = \begin{cases} C & \text{if } w \in \{C, D\} \text{ and } p \in \mathcal{P}_C \cup \mathcal{P}_D \\ A & \text{if } w \in \{C, D\} \text{ and } p \notin \mathcal{P}_C \cup \mathcal{P}_D \text{ or } w = A' \end{cases} \]

where \( \mathcal{P}_C = \{e\} \) is the set of possible price vectors in the centralized market when all agents are in state \( C \) and \( \mathcal{P}_D \) is the same set when \( N - 2 \) agents are in state \( C \) and the two remaining agents are in state \( D \). For instance, an agent in state \( C \) in a single-coincidence meeting in the decentralized market remains in state \( C \) only if trade takes place in his match, otherwise he moves to state \( D \). Likewise, an agent in state \( C \) in the centralized market stays in state \( C \) if the price he observes belongs to \( \mathcal{P}_C \cup \mathcal{P}_D \), otherwise he moves to state \( A \). Note that a necessary condition for a price vector to be an element of \( \mathcal{P}_D \) is that the bidding exceeds the sum of their bids by \( 2X \). It is easy to check that \( \sigma^* \) implements the first-best.

Now let \( \mu^* \) be the belief system such that: (i) an agent in state \( C \) believes that all other agents are in state \( C \); (ii) an agent in state \( A \) believes that all other agents are in state \( A \); (iii) an agent in state \( D \) believes that there exists one other agent in state \( D \) and the remaining agents are in state \( C \).\(^{11}\) Clearly, \((\sigma^*, \mu^*)\) is a consistent assessment. We have the following result.

**Proposition 1.** Suppose that \( X + U(x^*) - x^* \geq c(q^*) \). There exists \( \delta' \in (0, 1) \) independent of \( N \) and \( G \) such that \((\sigma^*, \mu^*)\) is a sequential equilibrium for all \( \delta \geq \delta' \).

A sketch of the proof of Proposition 1 is as follows. Consider first an agent in state \( C \) in a single-coincidence meeting in the decentralized market. If he is a producer, then his flow payoff gain from a one-shot deviation is \( c(q^*) \). However, in the centralized market meeting that immediately follows, he exerts effort \( \pi \) without receiving anything in return, incurring a payoff loss of \( X + U(x^*) - x^* \). Since \( X + U(x^*) - x^* \geq c(q^*) \) by assumption, the one-shot deviation is not profitable regardless of the agent’s discount factor.

Consider now an agent in state \( C \) in the centralized market. First note that any one-shot deviation by the agent leads to a price vector not in \( \mathcal{P}_D \). Indeed, total effort by the other agents is equal to the sum of their bids. Given that the agent can at most exert effort \( \pi \), it is not possible to have total effort exceeding the sum of bids by \( 2X \). Moreover, it turns out that no one-shot deviation with a price vector equal to \( e \) leads to a flow payoff gain. Since any one-shot deviation with a price outside of \( \mathcal{P}_C \cup \mathcal{P}_D \) triggers permanent autarky, and the payoff loss from global autarky is bounded below by \((1 - \delta')^{-1} \delta [U(x^*) - x^*] \), no one-shot deviation is profitable if the agent is patient enough, no matter the population size and the number of trading posts.

To finish, consider behavior off the path of play. First, observe that regardless of his discount factor, no agent in state \( A \) has an incentive to deviate, as autarkic behavior is a best response to autarky. Now, note that no agent is ever in state \( D \) in the decentralized market. Consider then an agent in state \( D \) in the centralized market. For the same reason as in the previous paragraph, one only needs to show that any one-shot deviation by the agent leads to a price vector outside of \( \mathcal{P}_C \cup \mathcal{P}_D \). Since one other agent is in state \( D \), the remaining \( N - 2 \) agents are in state \( C \), and the agent cannot bid more than his effort, any one-shot deviation by the agent implies that total production differs from the sum of bids by an amount \( \eta \in [X, 2X] \), which leads to the desired result.

Summing up, patient agents do not deviate from \( \sigma^* \) in the decentralized market since this entails a within-period punishment in the following round of central trading, which in turn is sustained by a credible threat of global autarky. Moreover, patient agents do not deviate from \( \sigma^* \) in the centralized market since any deviation that leads to a flow payoff gain triggers autarkic behavior. Proposition 1 holds without the condition that \( c(q^*) \leq X + U(x^*) - x^* \). Since in our candidate equilibrium cooperation is restored after agents observe a price vector in \( \mathcal{P}_D \), the only punishment for an agent who defects in the decentralized market is his payoff loss in the subsequent round of trading in the centralized market. In order for such a punishment to be effective, it must be that \( c(q^*) \), the cost of cooperating in the decentralized market, is small enough. The restriction on \( c(q^*) \) can be dropped if a deviation in the decentralized market were to lead to a greater punishment, as it would be the case if a price vector in \( \mathcal{P}_D \) implied a given number of periods of no trade in both markets. The details are in the Online Appendix.

This section concludes with a discussion of how our work and ACP are related. ACP is the first paper to show that money can fail to be essential in the presence of centralized trading. However, as Lagos and Wright (2008) point out, while in LW agents can only observe prices in the centralized market, in ACP agents can observe individual actions.\(^{12}\) We take account of this criticism by formalizing the process of exchange in the centralized market as a strategic market game in which only prices are publicly observable.

\(^{11}\) Note that the continuation payoff of an agent in state \( D \) does not depend on the type of the other agent who is in state \( D \). Thus, there is no loss of generality in describing the belief of an agent in state \( D \) as we do.

\(^{12}\) There exists no explicit price formation mechanism in ACP. They assume that in any centralized meeting agents simultaneously make production decisions and individual consumption is given by average production.
Another limitation of ACP, which they acknowledge, is that the assumption that individual actions in the centralized market are observed without noise regardless of the population size is important for their non-essentiality result.\textsuperscript{13} We allow for noise in the price formation process, which is a natural assumption in the presence of many agents. The next section provides economically meaningful conditions on the market structure under which the essentiality of money can be restored in large populations when prices are a noisy function of individual actions.

Finally, note that our strategy of proof is quite different from the strategy of proof in ACP. Their environment is very much like a standard repeated prisoner’s dilemma in the sense that communicating a defection in the decentralized market to the population involves taking an action (no production) in a subsequent round of trading in the centralized market that is myopically optimal. In our setting, communicating such a defection is costly in terms of flow payoffs as it entails production but no consumption. What sustains the threat of punishment is that if an agent deviates off the path of play, then this leads to an even greater punishment, namely global autarky. The same logic applies in the environment with noisy prices.

4. Noisy prices

The analysis with deterministic prices shows that providing microfoundations for the process of exchange in the centralized market matters for the essentiality of money. However, the assumption that prices are perfectly correlated with actions, so that agents are always informationally relevant, and thus money is not essential for large enough discount factors, is not plausible in large economies. This section analyzes the case in which the price formation process is not deterministic. The main result is that whether agents are informationally relevant in large economies depends on the structure of the centralized market.

The presence of noise in prices implies that the probability that an agent’s actions can affect prices in a noticeable way is small if in each trading post total effort and total bids by the other agents are large. Then, a natural conjecture is that agents are informationally relevant if total activity in each trading post is small. A sufficient condition for total activity in each trading post to be small is that the number of trading posts is not small relative to the population size, that is, the centralized market is not thick. In what follows, we show that market thickness, as measured by the ratio $N/G$ of agents to trading posts, is indeed a key determinant of the agents’ informational relevance.

4.1. Thick centralized market

Our first result is that if the number of trading posts is fixed, so that market thickness increases with the population size, then the first-best is not a Nash equilibrium outcome when the population is large enough no matter how patient agents are.

**Proposition 2.** Fix $G \geq 1$. For every strategy profile $\sigma$ that implements the first-best and for all $\delta \in (0,1)$, there exists $N' \geq 1$ such that $\sigma$ is not a Nash equilibrium if $N \geq N'$.

A sketch of the proof of Proposition 2 is as follows. Fix $G$ and consider a strategy profile $\sigma$ that implements the first-best. A necessary condition for the first-best is that in every period total effort exerted for the production of each type of general good is $(N/G)\bar{x}^*$.\textsuperscript{13} which converges uniformly to infinity as the population size grows. As it turns out, if $\sigma$ is to be a Nash equilibrium, then total bids $\sum_{j=1}^{N} b_{t}^{g}$ for each type of general good must also converge uniformly to infinity as $N$ increases. Indeed, the set of possible prices for general good $g$ in period $t$ on the path of play is $P_{t}^{\sigma} = \{\psi_{t}^{g} / \theta_{\text{max}}, \psi_{t}^{g} / \theta_{\text{min}}\}$, where

$$
\psi_{t}^{g} = \frac{1}{N} \sum_{j=1}^{N} b_{t}^{jg}.
$$

Hence, if $\sum_{j=1}^{N} b_{t}^{jg}$ remains bounded as $N$ increases, then the expected price of good $g$ in period $t$ converges to zero as the population size increases. Given that $U_{G}$ is unbounded above (as $U$ is) and an option for an agent of type $g-1$ in the centralized market is to exert effort $x^{*}$ and bid the same amount for good $g$, it must be that in equilibrium the payoff in period $t$ of any such agent grows without bound as $N$ increases. This, however, is not possible since in the first-best the per capita payoff in the centralized market is $U_{G}(x^{*}) - \bar{x}^* - x^*$.

The next step consists in showing that if total effort and total bids for each type of general good converge to infinity as $N$ increases, then the impact of any single agent’s behavior on prices is negligible when the population size is large. Consider an arbitrary agent $k$ and suppose he changes his action in the centralized market in period $t$ from $(y_{1}^{k}, (b_{1}^{k1}, \ldots, b_{1}^{kL}))$ to

\textsuperscript{13} Aliprantis et al. (2007b) show that one can restore the essentiality of money when individual actions in the centralized market are perfectly observable if instead of one centralized market there are many centralized markets and the matching of agents in these markets is such that the diffusion of information about defections in the decentralized market does not take place.
Agent $k$'s change of behavior changes the price of general good $g$ in period $t$ from $\frac{\varepsilon^{k}_{g}}{\theta^{k}_{l}}$ to $\frac{\varepsilon^{k}_{g}}{\theta^{k}_{l}}$, where

$$\frac{\varepsilon^{k}_{g}}{\theta^{k}_{l}} = \begin{cases} 
\frac{N}{C} \left[ \frac{x^*}{t} + \frac{b_{1}^{k,g} - b_{2}^{k,g}}{t} \right] & \text{if } k \text{'s type is not } g \\
\frac{N}{C} \left[ x^* + \frac{\hat{y}^{k}_t - \tilde{y}^{k}_t}{t} \right] & \text{if } k \text{'s type is } g
\end{cases}$$

Now let $\lambda^{k}_t = \frac{\varepsilon^{k}_{g}}{\theta^{k}_{l}}$. Then,

$$\lambda^{k}_t = \left( \frac{N}{C} \right) x^* \left[ 1 + \frac{b_{1}^{k,g} - b_{2}^{k,g}}{\sum_{j=1}^{N} b^{k,g}_j} \right].$$

where $s_g = 1$ if $k$'s type is $g$ and $s_g = 0$ otherwise. Since $\lambda^{k}_t < 1$ implies that $\frac{\varepsilon^{k}_{g}}{\theta^{k}_{l}} \in \mathcal{P}^{k}_t$ if, and only if, $\theta^{k}_{l} \leq \lambda^{k}_t \theta_{lmax}$. and since $\lambda^{k}_t > 1$ implies that $\frac{\varepsilon^{k}_{g}}{\theta^{k}_{l}} \in \mathcal{P}^{k}_t$ if, and only if, $\theta^{k}_{l} \geq \lambda^{k}_t \theta_{lmin}$, the probability that agent $k$'s change of behavior in period $t$ leads to a price vector in the centralized market that cannot be observed on the path of play is then given by

$$\pi^{k}_t = 1 - \prod_{g=1}^{G} \left( \Omega (\lambda^{k}_t \theta_{lmax}) || (\lambda^{k}_t > 1) + [1 - \Omega (\lambda^{k}_t \theta_{lmin}) || (\lambda^{k}_t < 1)] \right).$$

where $\Omega$ is the indicator function. In particular, $\pi^{k}_t$ is small if $\lambda^{k}_t$ is close to one for all $g$. Now observe that $\lim_{N \rightarrow \infty} \sum_{j=1}^{N} b^{k,g}_j = +\infty$ uniformly in $g$ and $t$ implies that $\lim_{N \rightarrow \infty} \lambda^{k}_t = 1$ uniformly in $g$ and $t$. Hence, $\lim_{N \rightarrow \infty} \pi^{k}_t = 0$ uniformly in $t$, and so the probability that any single agent can affect prices in a noticeable way converges to zero as the population size grows. From this, it follows that the probability that a change of behavior by any finite group of agents can lead to a price vector that cannot be observed on the path of play also converges to zero as $N$ increases.

To finish the argument, notice that since the informational content of prices is negligible when $N$ is large, the profile $\sigma$ cannot be a Nash equilibrium when there are many agents. Indeed, there are two channels through which an agent can be punished for not producing in the decentralized market. The first is through a standard contagion process that is limited to the single-coincidence meetings in the decentralized market as long as no noticeable change of prices occurs in the centralized market. The second is through a noticeable change of prices in the centralized market that can lead to global autarky. It is well-known that the first channel is not effective in disciplining behavior when there are many agents, see Kandori (1992) and Araujo (2004). Moreover, since only a finite number of agents can deviate from the behavior prescribed by $\sigma$ before an observable change of prices takes place in the centralized market, the argument above shows that the expected amount of time it takes for a deflection in the decentralized market to affect prices in the centralized market in a substantial way increases to infinity as the population size grows. Thus, when the population is large, the second channel is also not effective in disciplining behavior in the decentralized market. Therefore, one cannot sustain the first-best as a Nash equilibrium outcome in large populations.

Proposition 2 shows that in large populations, efficient trading in the centralized market is incompatible with efficient trading in the decentralized market when the number of trading posts is fixed. A natural question to ask is whether in this case one can sustain some trade in the decentralized market. As the preceding discussion suggests, this seems possible if one sacrifices efficiency in the centralized market by keeping the volume of trade in some trading posts small enough for agents to be informationally relevant. We conjecture that if agents are patient enough, then efficient trade in the decentralized market is a sequential equilibrium outcome regardless of the population size.

A key element in the proof of Proposition 2 is that in the first-best the set of possible price vectors one can observe in the centralized market has a nonempty interior. This is the case if in each period the shocks to production are independent across posts, as we have assumed. However, this is also the case if the shocks to production in each trading post are the sum of a common component and an idiosyncratic component. Thus, Proposition 2 holds under more general specifications of the shocks to production.

4.2. Thin centralized market

The discussion in the preceding subsection suggests that if $N/G$ is bounded above, which is the case, for instance, if each trading post has finite capacity, then the first-best can be a sequential equilibrium outcome regardless of the population size as long as agents are patient enough. This turns out to be true, as we now discuss.

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14 An agent who deviates in the decentralized market can also be punished for changes in the distribution of prices in the centralized market even if they do not lead to a price vector that cannot be observed on the path of play. When the population is large, this third channel is not relevant for the same reason that the second channel is not relevant.
The analysis in this subsection has two parts. First, we show that under a mild assumption on the density of \( \Omega \), the equilibrium of Proposition 1 is robust to the introduction of small amounts of noise in prices when \( N/G \) is bounded above. Afterwards, we provide conditions under which the first-best can be a sequential equilibrium outcome regardless of the amount of noise. Since the case of interest is the case in which \( N \) is large, for simplicity we let \( G \geq 3 \) in what follows.

### 4.2.1. Small noise

The first result in this subsection concerns the robustness of the equilibrium of Proposition 1 to the introduction of small amounts of noise in prices when the centralized market is thin.

**Proposition 3.** Suppose that \( \Gamma = \sup N N/G < \infty \) and that there exists \( \omega > 0 \) such that \( \Omega(\theta) \geq \omega \) for all \( \theta \in [\theta_{\min}, \theta_{\max}] \). Moreover, assume that \( x + U_d(x^*) - x^* \geq c(q^*) \). There exist \( \kappa > 1 \) and \( \delta' \in (0, 1) \) independent of \( N \) and \( G \) such that if \( \theta_{\max}/\theta_{\min} < \kappa \) and \( \delta \geq \delta' \), then the assessment \((\sigma^*, \mu^*)\) of Proposition 1 is a sequential equilibrium.

The argument that follows shows that if players are patient enough and \( \theta_{\max}/\theta_{\min} \) is sufficiently close to one, then \((\sigma^*, \mu^*)\) admits no profitable one-shot deviation on the path of play regardless of the population size and the number of trading posts. The proof that the same holds off the path of play is similar, and thus is left for the Online Appendix.

The first step of the argument consists of describing the sets \( P_C \) and \( P_D \) in detail. Let \( \Theta = [\theta_{\min}, \theta_{\max}]^G \) be the set of the possible vectors of shocks to production in the centralized market. It is immediate to see that

\[
P_C = \left\{ p \in \mathbb{R}_+^G : p = \left( \frac{1}{\theta^1}, \ldots, \frac{1}{\theta^G} \right) \text{ with } (\theta^1, \ldots, \theta^G) \in \Theta \right\}.
\]

In order to describe \( P_D \), let

\[
\xi^g(n_1, \ldots, n_G) = \begin{cases} 
\frac{(N-G)}{G} x^* + (2 - \sum_{g}^G n_g) x^* & \text{if } g = 1 \\
\frac{(N-G)}{G} x^* + n_g x^* + (2 - \sum_{g}^G n_g) x^* & \text{if } g \neq 1
\end{cases}
\]

where \( n_g \in [0, 1, 2] \) is the number of agents of type \( g \) who are in state \( D \) in the centralized market. Moreover, let \( \Xi \) be the subset of \( \mathbb{R}_+^G \) such that \( \xi = (\xi^1, \ldots, \xi^G) \) belongs to \( \Xi \) if, and only if, \( \xi^g = \xi^g(n_1, \ldots, n_G) \) with \( \sum_{g}^G n_g = 2 \). Then

\[
P_D = \left\{ p \in \mathbb{R}_+^G : p = \left( \frac{\xi^1}{\theta^1}, \ldots, \frac{\xi^G}{\theta^G} \right) \text{ with } (\xi^1, \ldots, \xi^G) \in \Xi \text{ and } (\theta^1, \ldots, \theta^G) \in \Theta \right\}.
\]

As in Proposition 1, the assumption that \( c(q^*) \leq x + U_d(x^*) - x^* \) implies that an agent in state \( C \) in the decentralized market has no profitable one-shot deviation no matter his discount factor.

Consider then an agent in state \( C \) in the centralized market and assume, without loss of generality, that he is of type \( g = 1 \). Suppose the agent changes his action from \((x^*, x^* e_2)\) to \( a_2 = (y, (b^1, \ldots, b^G))\). The price vector resulting from the deviation is \( \tilde{p} = (\tilde{\xi}^1/\tilde{\theta}^1, \ldots, \tilde{\xi}^G/\tilde{\theta}^G) \), where \( \tilde{\xi}^g \geq 1 \) for \( g \geq 3 \) and

\[
\tilde{\xi}^g = \begin{cases} 
\frac{(N-G)}{G} x^* + b^1 & \text{if } g = 1 \\
\frac{(N-G)}{G} x^* + y - x^* & \text{if } g = 2
\end{cases}
\]

Hence, the change in the agent’s flow payoff is

\[
\Delta = U_d(y, b^2) - U_d \left( b^2, \frac{(N-G)}{G} x^* \right) - y - [U_d(x^*) - x^*].
\]

Since \( \Delta(y, b^2) \) is maximized when \( b^2 = y \), choosing \( y > x^* \) implies that

\[
\Delta(y, y) < U_d(y) - y - [U_d(x^*) - x^*] < 0,
\]

in which case the one-shot deviation is not profitable. Suppose then that \( y < x^* \) in the remainder of the argument. The same algebra as in the proof of Proposition 1 shows that \( \Delta \leq x^* - y \) when \( y < x^* \).

The next step consists in showing that if the noise in prices is sufficiently small, then no one-shot deviation leads to a price vector in \( P_D \) regardless of the number of agents and the number of trading posts. Note that \( b^1 + b^2 \leq y < x^* \) implies that \( \tilde{\xi}^1 > 1 \) and \( \tilde{\xi}^g \in [1 - (N/G)^{-1}], 1 \). Also observe (since \( G > 2 \)) that \( \tilde{\xi} \in \Xi \) implies that either one component of \( \tilde{\xi} \) is equal to \( 1 - 2(N/G)^{-1} \) or two components of \( \tilde{\xi} \) are bounded above by \( 1 - (N/G)^{-1} \). Consequently, \( \max_g |\tilde{\xi}^g - \xi^g| \geq (N/G)^{-1} \geq 1/\Gamma \),
where $\Gamma = \sup_N N/G < \infty$. Hence, there exists $\kappa_1 > 1$ such that if $\theta_{\max}/\theta_{\min} < \kappa_1$, then $\tilde{P} \not\in P_D$ for all $N$ and $G$ regardless of the realization of the vector $(\theta^1, \ldots, \theta^N)$ of shocks to production.

Suppose now that $\theta_{\max}/\theta_{\min} < \kappa_1$. Since $\frac{\kappa_1}{\theta^1} > 1$, this implies that a one-shot deviation triggers global autarky if $\theta^1 > 1/\theta_{\min}$, which is equivalent to

$$\theta^1 < \theta_{\min} \left[ 1 + \frac{b^1 + x^* - y}{N} \right].$$

Given that $\Omega(\theta)$ is bounded below by $\omega$, a lower bound on the probability of punishment is

$$\omega \theta_{\min} \frac{b^1 + x^* - y}{N} \geq \omega \theta_{\min} \frac{x^* - y}{N} \geq \alpha \omega,$$

where $\alpha = \omega \theta_{\min}/\Gamma x^*$. Hence, regardless of $N$ and $G$, the expected payoff loss from a one-shot deviation is bounded below by $(1-\delta)^{-1} \delta[U_0(x^*)-x^*] \omega \alpha$, which is greater than the gain $\Delta$ from the one-shot deviation as long as agents are patient enough.

Therefore, as claimed above, if the support of $\Omega$ is sufficiently small and players are patient enough, then $(\sigma^*, \mu^*)$ admits no profitable one-shot deviation on the path of play regardless of $N$ and $G$. The assumption that the density of $\Omega$ is bounded away from zero implies that the probability that a one-shot deviation in the centralized market leads to punishment is always proportional to the size of the deviation.

As in the case with deterministic prices, the restriction that $c(q^*) \leq x + U_D(x^*) - x^*$ can be relaxed. See the Online Appendix for a discussion. The key fact is that if the support of $\Omega$ is small enough, then the sets $P_C$ and $P_D$ do not coincide, in which case we can proceed as before and change the assessment $(\sigma^*, \mu^*)$ so that a price vector in $P_D$ leads to a number of periods of no trade that is high enough to induce cooperation in the decentralized market. Also note that Proposition 3 holds even if the density of $\Omega$ is not bounded away from zero as long as $\Omega(\theta_{\min})$ is positive. See the Online Appendix as well. The reason for why the weaker condition on $\Omega$ is sufficient is that one needs the probability of a one-shot deviation in the centralized market leading to punishment proportional to the size of the deviation only for small deviations. A large one-shot deviation implies a probability of punishment that is bounded away from zero, which is sufficient to discourage any such deviation when agents are patient enough.

The only instance in which the assumption that $U'(0)$ is finite when $U$ is strictly concave plays a role in our analysis is in the proof of Proposition 3. This assumption implies that the marginal benefit to an agent in state $D$ in the decentralized market who defects by increasing his bid is bounded. Our analysis extends to the case in which $U'(0) = +\infty$ when $U$ is strictly concave if we modify $\sigma^*$ so that now an agent in state $D$ in the centralized market bids $\varepsilon \approx 0$ for the good he likes. When prices are deterministic, the modified assessment $(\sigma^*, \mu^*)$ remains a sequential equilibrium regardless of $N$ and $G$ if agents are patient enough, and this equilibrium is robust to the introduction of small amounts of noise if $N/G$ is bounded above as long as the density of $\Omega$ is bounded away from zero.

4.2.2. Large noise

We now show if $N/G$ is bounded above, then the first-best is a sequential equilibrium outcome regardless of the population size and the amount of noise in prices when agents are patient enough as long as the disutility of production in the decentralized market is sufficiently small.

Proposition 4. Suppose that $\Gamma < \infty$ and there exists $\omega > 0$ such that $\Omega(\theta) \geq \omega$ for all $\theta \in [\theta_{\min}, \theta_{\max}]$. Moreover, suppose there exists $0 < \kappa < \min \{x^*, x - x^*\}$ such that

$$-c(q^*) + U_D(x^*) - x^* \geq U_D(x^* - \kappa) - (x^* - \kappa).$$

Then there exists $\delta' \in (0, 1)$ independent of $N$ such that the first-best is a sequential equilibrium outcome for all $\delta \geq \delta'$ regardless of the population size.

Note that the above restriction on $c(q^*)$ can only be satisfied if $U$ (and thus $U_D$) is strictly concave. The proof of Proposition 4 is in Appendix. The idea of the proof is as follows. If $c(q^*)$ is small enough, then punishing an agent who defects in the decentralized market in the subsequent round of trading in the centralized market is enough to induce cooperation in the decentralized market. Hence, since no further punishment is required, one can set the prices obtained in the round of centralized trading that follows a defection in the decentralized market to be the same as the prices one obtains on the path of play. In this case, the assumption that the density of $\Omega$ is bounded away from zero implies that one-shot deviations in the centralized market can be detected regardless of the support of $\Omega$, since now it is not a problem that a price vector on the path of play be confused with a price vector off the path of play. Proposition 4 can be extended to the case in which the density of $\Omega$ is not bounded away from zero but both $\Omega(\theta_{\min})$ and $\Omega(\theta_{\max})$ are greater than zero. This is possible for the same reason why Proposition 3 can be extended to the case in which $\Omega(\theta_{\min})$ is positive.
5. Concluding remarks

An important concern in monetary theory is the relationship between the essentiality of money and centralized trading. This paper investigates this relationship. A key feature of our analysis is that we model the process of exchange in the centralized market explicitly. Doing so means that one has to introduce a map between individual actions in the centralized market and prices. Even though we restrict attention to a particular map, the one derived from a strategic market game, our message is quite general. Namely, that the essentiality of money is tied to the informational relevance of agents, which in turn depends on the market structure. Thus, modeling the centralized market as a Walrasian market, where agents are informationally irrelevant by assumption, needs to be justified. Our analysis shows that if we take the centralized market to be organized as a market game, then money is essential as long the ratio between the number of agents who participate in trade and the number of goods that are traded in the centralized market is sufficiently large.

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Appendix A. Supplementary material

Supplementary material (electronics diagrams, optical and mechanical drawings) associated with this paper can be found in the online version at http://dx.doi.org/10.1016/j.jmoneco.2012.10.007.

References


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15 Another trading mechanism would be a double auction. Large double auctions have also been used to provide non-cooperative foundations for competitive markets. See Rustichini et al. (1994) and Cripps and Swinkels (2006).