Price dispersion, information and learning

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Abstract

We consider an economy where trade is decentralized and agents have incomplete information with respect to the value of money. Agents' learning evolves from private experiences and we explore how the formation of prices interacts with learning. We show that multiple equilibria arise, and equilibria with price dispersion entail more learning than equilibria with one price. Price dispersion increases communication about private histories, which in turn increases the overall amount of information in the economy. We also compare ex ante welfare under price dispersion and one price. Our results show that, despite the existence of some meetings where no trade takes place, ex ante welfare under price dispersion may be higher than under one price.

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1. Introduction

We use a random-matching model of money to study the interaction between learning and the real effects of monetary uncertainty in the short-run. It has become an agreement point in the literature that in order to get output fluctuations from monetary shocks one has to assume some type of uncertainty and decentralized markets (see Lucas, 1996). The

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search-theoretic framework, although stylized, is very useful in studying this type of question. It provides a deep theory of money as a medium of exchange and at the same time it allows us to introduce an environment with imperfectly informed agents and decentralized trade. The same frictions that generate an essential role for money make learning very interesting but still tractable.

Since Lucas (1972) there has been a significant number of papers developing models consistent with the observed short-run and long-run relationship between changes in the supply of money and fluctuations in output and prices. The underlying argument in those papers implicitly assumes that it takes time for agents to learn the true state of the economy. However, the informational structure of these models usually does not leave much space for learning. Moreover, it exogenously assumes a particular form of one-sided asymmetric information where consumers are informed but producers are uninformed about the actual value of money. Barro and King (1984) point out that the actual distribution of information can be of crucial importance in the determination of the effects of money on output.

More recently, the original environment of Lucas (1972) has been enriched through the use of more sophisticated informational structures. Examples are Jones and Manuelli (2001) and Katzman et al. (2003). Each one of these papers develops a model with pairwise trade and one-sided asymmetric information. As an important novelty, they allow for a separation between informed–uninformed status and consumer–producer status. Katzman et al. (2003) assume an exogenous informational asymmetry: some agents observe monetary shocks as they occur and others observe them only with a one-period lag. Jones and Manuelli (2001) allow for costly information acquisition and as a result an endogenously chosen information structure arises.

We depart from the previous literature by considering an economy with two-sided asymmetric information. More precisely, instead of assuming informational asymmetry from the beginning, we introduce homogeneous agents with an access to the same information technology. Agents learn from their private experiences in the market and we assume that each agent receives the same amount of information. A result of this learning process is the emergence of two-sided informational asymmetries and correlation of private experiences. These features are not present in models with one-sided asymmetric information and they have important short-run implications in the economy.

In this paper, we explore the interaction between learning and the formation of prices. We obtain that, in addition to equilibria with one price, equilibria with price dispersion can arise. The intuition for this result runs as follows. In a decentralized environment, distinct private experiences lead to distinct beliefs about the state of the economy. This implies that an agent’s behavior in a trade meeting can be contingent on his history. Moreover, since private histories are correlated, buyers with a belief that money has a high value have more incentives to ask for a low price since they believe that there is a higher probability that such an offer is going to be accepted. As a result, prices convey information about the state of the economy.

In what follows, we provide a region of parameters where both one price and price dispersion arises and we show that, despite the existence of trade meetings where no trade takes place, price dispersion equilibrium may entail higher production and higher ex ante welfare than the one-price equilibrium. Positive aspects of price dispersion have not been emphasized in the literature. Usually, price dispersion is Pareto dominated by a single-price equilibrium. This has been suggested either explicitly by Stigler (1961) (see also Rob,
1985) or implicitly (see Burdett and Judd, 1983) by equilibrium price dispersion theory. In this theory, price dispersion arises as a result of costly or noisy search and differential information. The essence of the theory is that a distribution of prices for a homogeneous good causes confusion about the true value of money: “Price dispersion is a manifestation—and, indeed, it is the measure—of ignorance in the market” (Stigler, 1961, p. 214). In our environment exactly the opposite is true: price dispersion provides agents with additional information that helps them learn the true state of the economy.

The rest of the paper proceeds as follows. The next section describes the model, strategies and the equilibrium concept. The section after that characterizes an equilibrium where prices reveal no information. In Section 4, we describe an equilibrium where prices convey information about the actual value of money. Section 5 characterizes the parameter region where both the equilibrium with one price and with price dispersion exist. In Section 6, we derive the welfare results. Section 7 discusses the robustness of the model and Section 8 concludes. Proofs are in the Appendix.

2. The model

We adopt a version of the environment similar to the one studied by Trejos and Wright (1995), Shi (1995) and Wallace (1997). Consider an economy with a continuum of agents and $K$ divisible and perishable goods. Time is discrete and indexed by $t$. Agents can be of $K$ distinct types. A type $i$ agent derives utility from the consumption of good $i$ and is able to produce good $i + 1$ (modulo $K$) and there is a uniform distribution of agents across types. Utility in every period is given by $u(x) - y$, where $x$ is the amount consumed and $y$ is the amount produced. The function $u$ is defined on $[0, 1]$ and $u(.)$ is increasing, twice differentiable, $u(0) = 0$, $u'' < 0$ and $u'(0) = 1$. Agents maximize expected discounted utility with a discount factor $\beta \in (0, 1)$. In the economy there is also a storable, indivisible and intrinsically useless object, which we denote as fiat money. We assume that an agent can hold at most one unit of money at a time.

A key feature of the environment is that trade is decentralized and agents face frictions in the exchange process. We formalize this idea by assuming that there are $K$ distinct sectors, each one specialized in the exchange of one good. Agents can identify the sectors but inside each sector they are anonymously and pairwise matched under a uniform random matching technology. Each agent faces one meeting per period. Moreover, meetings are independent across agents and independent over time. Therefore, if an agent wants money he goes to the sector which trades his endowment and searches for an agent with money. If he has money he goes to the sector that trades the good he likes and searches for an agent with the good. Since there is a upper bound on money holdings, a transaction can happen only when an agent with money (buyer) meets an agent without money (seller). We denote meetings of this sort as trade meetings. We assume a very simple exchange protocol in a trade meeting, namely, take it or leave it offers by the buyer. In particular, we assume that an offer consists of a demand for an amount of production. If the offer is accepted, trade happens. Otherwise, there is no trade.

At date 0, money is randomly distributed in the economy. We assume that all features of the environment are common knowledge across agents with the exception of the amount of money in the economy. In particular, the fraction of agents that receive money is equal to $m$, where $m$ can be either $m_H$ or $m_L$, with $m_H > m_L$. If $m_H$ ($m_L$) is realized, we say that the
economy is in the high (low) state. Before money is distributed agents have a common prior \( p \), indicating the probability that \( m = m_H \).

The economy is divided in a short-run period and a long-run period. In the short-run agents face incomplete information with respect to the value of \( m \). In the long-run the actual state of the economy (\( m_H \) or \( m_L \)) is revealed to all agents. Wallace (1997) assumes that the short-run lasts for one period. However, he states that: “...the natural assumption is that it (the quantity of money) is never revealed.” He continues: “I see two difficulties in working with that specification or even one that lengthens the lag beyond one period. First, priors get revised in accord with experiences. Since experience is diverse, one would have to keep track of groups that are diverse in terms of their posteriors over the realized change in the amount of money. Second, the bargaining would then occur between two people who do not know each other’s posteriors” (1997), pp. 1304 and 1305).\(^1\)

In what follows we add one more period to the short-run. As Wallace anticipates, by doing this we need to keep track of groups of agents with distinct posteriors and we also need to deal with a bargaining process where agents in a match do not know with certainty each other’s posteriors. The benefit of this modification is that it delivers a natural way in which prices and learning interact in decentralized economies.

Finally, note that in random matching models with an upper bound on money holdings, changes in the amount of money have two different margins, extensive and intensive. The extensive margin is related to the rate at which trade meetings take place, which depends on the distribution of money in the economy. The intensive margin corresponds to the actual quantity of goods produced in trade meetings. Our primary objective is to analyze the relation between learning and the formation of prices. Hence, in what follows we restrict attention to the intensive margin effects by assuming that \( m_H + m_L = 1 \). This assumption is meant as a way to control for the extensive margin effects since it implies that the number of trade meetings is independent of the state of the economy. Its robustness is discussed in more detail in Section 7.

### 2.1. Benchmark

We consider first a simple version of the model where the short-run has only one period (date 1). The purpose is to provide a framework that can be more easily compared with previous models. First, consider the economy at date \( t (t > 1) \), after the state (high or low) be revealed to all agents. Let \( V_0(m) \) be the value function of an agent without money and let \( V_1(m) \) be the corresponding value function for an agent with money, when the amount of money is \( m \). We have

\[
V_0(m) = m(-y + \beta V_1(m)) + (1 - m)\beta V_0(m),
\]

\[
V_1(m) = m\beta V_1(m) + (1 - m)(u(y) + \beta V_0(m)).
\]

For example, if an agent has money he goes to the sector that trades the good he likes. In this sector, there is a probability \( m \) that he meets another agent with money and no trade happens. There is also a probability \( 1 - m \) that he meets an agent without money. In this case they trade, the agent obtains utility \( u(y) \), where \( y \) is the amount produced by the seller.

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\(^1\)Among others, Wolinsky (1990) and Araujo and Camargo (2005) analyze a decentralized economy where information is never revealed. However, both papers assume exogenous prices.
and moves to the next period without money. A similar reasoning holds for an agent without money.

At each date $t > 1$, a seller is going to accept an offer from the buyer as long as the value of becoming an agent with money tomorrow minus the production cost today is greater than the value of going to the next period without money, i.e., as long as

$$\beta V_1(m) - y \geq \beta V_0(m).$$

Let $\Delta(y, m) = V_1(m) - V_0(m)$. Note that

$$\Delta(y, m) = (1 - m)u(y) + my.$$

The assumption that buyers make a take it or leave it offer implies

$$y = \beta \Delta(y, m).$$

Condition (1) can be rewritten as

$$y = \frac{\beta(1 - m)u(y)}{1 - \beta m}.$$ 

It is easy to see that, as the amount of money in the economy increases, the value of $y$ decreases. The intuition is simple. If a seller anticipates that a large fraction of agents is holding money, the probability of meeting an agent that has the good he likes goes down. Therefore, he is going to produce less today in exchange for money.

Now, consider the economy at date 1. Let $\theta_1(1)$ ($\theta_1(0)$) indicates the belief that the economy is in the high state for an agent starting with (without) money. Bayesian updating implies

$$\theta_1(1) = \frac{m_H \theta}{m_H \theta + m_L (1 - \theta)}$$

and

$$\theta_1(0) = \frac{(1 - m_H) \theta}{(1 - m_H) \theta + (1 - m_L)(1 - \theta)}.$$

Agents meet in pairs under a uniform random matching technology. These meetings give additional information about the state of the economy and, based on this information, buyers make take it or leave it offers to sellers.

In a trade meeting in the short-run, the information of the buyer can be summarized by the pair $(1, 0)$, where 1 indicates that he had money and 0 indicates that he met an agent without money. Analogously, the information of the seller corresponds to the pair $(0, 1)$. Since meetings are random and the probability of meeting an agent with money is the same probability of starting with money in the economy, we obtain that both the buyer and the seller form the same posterior with respect to the state of the economy. This posterior is equal to

$$\theta_2(1) = \frac{m_H (1 - m_H) \theta}{m_H (1 - m_H) \theta + m_L (1 - m_L)(1 - \theta)},$$

where $\theta_2(1)$ indicates a private history with a length 2 and a cardinality equal to 1. More generally, the belief after a history of length $i$ and cardinality $j$ will be denoted as $\theta_i(j)$. Let $V_2^1(\theta)$ be the value function at the beginning of date 2, i.e., before the state of the economy be revealed to all agents, for an agent with $\tau$ units of money ($\tau = 0, 1$) and a posterior $\theta$. We
obtain
\[ V_0^2(\theta_2(1)) = \theta_2(1)V_0(m_H) + (1 - \theta_2(1))V_0(m_L) \]

and
\[ V_1^2(\theta_2(1)) = \theta_2(1)V_1(m_H) + (1 - \theta_2(1))V_1(m_L). \]

In the short-run, a take it or leave it offer \( y \) by the buyer implies
\[ y = \beta[V_1^2(\theta_2(1)) - V_0^2(\theta_2(1))] = \theta_2(1)y_H + (1 - \theta_2(1))y_L, \]

where \( y_H \) (\( y_L \)) is the amount produced in the high (low) state under complete information, with \( y_L < y_H \).

We observe that, in the short-run, despite the actual realization of the state, the price level \( \frac{1}{y} \) does not change. In the long-run prices reflect the actual state. If the state is \( m_H \), the price is equal to \( 1/y_H \). If it is \( m_L \), price is equal to \( 1/y_L \). Since \( m_H > m_L \), we have that prices in the high state are higher than in the low state. The result that prices in the short-run are independent of the state of the economy is a consequence of the informational structure. Since all agents in trade meetings in the short-run have a common belief, it is natural that only one price arises.\(^2\)

2.2. Strategies and equilibrium concept

We now consider an economy where the short-run has two periods (date 0 and date 1). Assume that agents start at date 0 with a common prior \( \theta \). Over time beliefs are updated according to private experiences and this updating process goes on until date 2, when the information about the state of the economy is then revealed. An agent’s strategy specifies an action at every information set. At any date \( t \), where \( t \geq 2 \), since the state of the economy is revealed to all agents in the economy, information sets are \( \{H\} \) and \( \{L\} \), indicating whether the high or the low state is realized. At dates 0 and 1, an agent’s information set corresponds to his private history. A private history is a complete record of the agent’s initial money holdings, the money holdings of all his partners and the outcome of all his past meetings.

At dates \( t \geq 2 \) the buyer’s behavior is given by
\[ \sigma_B^t : \{H, L\} \rightarrow \mathbb{R}^+ \quad \text{for all } t \geq 2 \]

and the seller’s behavior is given by
\[ \sigma_S^t : \{H, L\} \times \mathbb{R}^+ \rightarrow \{A, R\} \quad \text{for all } t \geq 2, \]

where \( A \) indicates that the offer is accepted and \( R \) indicates that it is rejected. At date 0, since buyers and sellers face private histories which are equivalent from an informational point of view (see previous section), a buyer’s behavior is given by
\[ \sigma_B^0 \in \mathbb{R}^+ \]

\(^2\)Despite some differences in the environment, these results are very similar to the ones obtained by Wallace (1997). The main objective in Wallace is to show how a random matching model can generate short-run effects of changes in the quantity of money which are mainly real, and long-run effects which are mainly nominal.
and a seller’s behavior is given by
\[ \sigma^0_S : \mathbb{R}^+ \to \{A, R\}. \]

Now consider the economy at date 1. At this date, an agent’s private history is only the record of his money holdings and the money holdings of his partner at dates 0 and 1. Clearly, the price observed at date 0 does not add any new information. As a result, the set of private histories of a seller at date 1 are
\[ \Omega^S_3(1) = (0, 0, 1) \quad \text{and} \quad \Omega^S_3(2) = (1, 0, 1). \]

Similarly, the buyer’s private histories at date 1 are
\[ \Omega^B_3(1) = (0, 1, 0) \quad \text{and} \quad \Omega^B_3(2) = (1, 1, 0). \]

Hence, buyer’s behavior at date 1 is given by
\[ \sigma^1_B : \{\Omega^B_3(1), \Omega^B_3(2)\} \to \mathbb{R}^+ \]

and the seller’s behavior is
\[ \sigma^1_S : \{\Omega^S_3(1), \Omega^S_3(2)\} \times \mathbb{R}^+ \to \{A, R\}. \]

Consider the collection \( \sigma = [\sigma^t_B, \sigma^t_S]_{t=0}^\infty \) and a belief function for the seller at date 1, \( \mu : \mathbb{R}^+ \to [0, 1] \), where \( \mu(y) \) indicates the probability that the buyer has a history \( \Omega^B_3(1) \) when the seller observes a demand equal to \( y \). We restrict attention to symmetric strategies, i.e., agents choose the same action if they face the same private history. A perfect Bayesian equilibrium is a pair \( \{\sigma, \mu\} \) such that an agent’s behavior at any information set is optimal given the behavior specified at all other information sets, and beliefs are calculated from Bayes rule whenever possible.

3. An equilibrium where prices are non-informative

We describe the conditions under which in every period only one price emerges in the short-run and all sellers accept this price, irrespective of the state of the economy. We first informally discuss our main result. Proposition 1 provides a formal proof.

First, note that in trade meetings from date 2 onwards, the amount of output is equal to \( y_H \) (\( y_L \)) depending on the state of the economy being high (low). This fact allows us to work backwards and describe the agent’s optimal decision at dates 0 and 1.

3.1. Date 1

Consider a trade meeting at date 1. In this meeting, depending on his belief about the state of the economy, a buyer makes a take it or leave it offer to the seller. The seller, also as a function of his belief, accepts or rejects the offer. We first describe the possible posteriors of buyers and sellers.

Consider the problem faced by a seller in a trade meeting at date 1. Moreover, assume that his private history is \( \Omega^S_3(1) \). This history leads to a posterior
\[ \theta_3(1) = \frac{m_H(1 - m_H)^2 \theta}{m_H(1 - m_H)^2 \theta + m_L(1 - m_L)^2(1 - \theta)}. \]
Analogously, a history $O_S^3(2)$ leads to a posterior

$$\theta_3(2) = \frac{m_H^2(1-m_H)\theta}{m_H^2(1-m_H)\theta + m_L^2(1-m_L)(1-\theta)}.$$ 

Now assume that there is an equilibrium at date 1 where only one price is asked in all meetings and all agents accept this price. This implies that, on the equilibrium path, when a buyer makes a take it or leave it offer to a seller, this offer does not modify the seller’s information set and, as a consequence, does not affect the seller’s posterior. Consider, first, a seller with a history $O_S^3(1)$. If all buyers ask for $y$, a seller accepts the offer as long as

$$y \leq \beta[V^2_1(\theta_3(1)) - V^2_0(\theta_3(1))] = y_l = \theta_3(1)y_H + (1 - \theta_3(1))y_L.$$ 

Alternatively, if all buyers ask for $y$ and the seller has a history $O_S^3(2)$, he accepts the offer as long as

$$y \leq \beta[V^2_1(\theta_3(2)) - V^2_0(\theta_3(2))] = y_h = \theta_3(2)y_H + (1 - \theta_3(2))y_L.$$ 

Note that, since $\theta_3(2) > \theta_3(1)$, we have

$$y_h < y_l.$$ 

Therefore, if we want all buyers to make an offer which is accepted by all sellers, they should ask for $y_h$. The intuition for this result is simple. The seller’s expected gain from accepting an offer is decreasing in his posterior about the state of the economy. As he becomes more pessimistic (that is, as his expectation that the economy is in the high state increases), he is less willing to produce. Therefore, a buyer needs to ask for a production amount which is accepted by the most pessimistic seller. In what follows we say that a buyer (seller) is optimistic if he has a history $O_B^3(1)$ ($O_S^3(1)$) and pessimistic otherwise.

We also need to check the incentives of a buyer to demand a production consistent with all sellers accepting it. In principle, a buyer could ask for a higher production with the hope that an optimistic seller would accept it. First, note that if there is a buyer willing to ask for a higher production it has to be an optimistic buyer. The reason is clear. The success of a demand for high production depends upon the buyer meeting an optimistic seller. Since optimistic sellers are more frequent when the state of the economy is low, an agent with a low value of the posterior has more incentives to ask for a high demand.

Therefore, we only need to check deviations from the one-price rule that come from an optimistic buyer. If this buyer follows the proposed equilibrium and asks for $y_h$, he obtains $u(y_h)$. In this case, he gives the money to the seller and moves to the next period without money. His overall payoff is equal to

$$u(y_h) + \beta V^2_0(\theta_3(1)).$$ 

However, he can deviate and ask for a higher amount. His expected payoff depends on the type of the seller and on the belief that the seller attaches to the buyer’s possible types. We could assume, for example, that all sellers give equal weight to each of the buyer’s type after seeing any deviation from $y_h$. These beliefs are consistent with the definition of a perfect Bayesian equilibrium. For now, we fix the seller’s belief and assume that the

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3 However, they may not pass the intuitive criterion of Cho and Kreps (1987), which imposes further constraints on beliefs out of the equilibrium path. In the proof of Proposition 1, we analyze this issue in detail.
maximum demand of output consistent with partial acceptance (i.e., only optimistic sellers accept the offer) is equal to \( y \), where \( y > y_h \).

Define \( P_{00} \) as the probability that an optimistic buyer meets an optimistic seller. Formally, we have

\[
P_{00} = \theta_2(1)(1 - m_H) + (1 - \theta_2(1))(1 - m_L).
\]

After some algebraic manipulation we obtain that an optimistic buyer is not going to deviate from the proposed one-price equilibrium as long as

\[
u(y_h) \geq P_{00}u(y) + (1 - P_{00})y_m,
\]

\[
y_m = \theta_4(2)y_H + (1 - \theta_4(2))y_L. \tag{2}
\]

For future reference, Fig. 1 shows the relation between the various output levels that we consider throughout the paper. As we are going to see in the next section, \( y_{hh} = \theta_4(3)y_H + (1 - \theta_4(3))y_L \) and \( y_{ll} = \theta_4(1)y_H + (1 - \theta_4(1))y_L \).

Before giving a more precise characterization of the conditions under which inequality (2) holds, we assume the existence of a one-price equilibrium at date 1 with output equal to \( y_h \) and work backwards the solution of the model at date 0.

3.2. Date 0

An agent’s problem at date 0 looks similar to the one faced by an agent in the benchmark model with incomplete information. More precisely, in every trade meeting private histories are common knowledge across agents and imply a unique posterior equal to \( y_2(1) \). Consider an agent with a posterior \( y_2(1) \). Under the assumption that there is a one-price equilibrium at date 1, with output per meeting equal to \( y_h \), a take it or leave it offer, \( y_o \), by the buyer implies

\[
y_o = \beta(V_1^o(\theta_2(1)) - V_0^o(\theta_2(1))),
\]

where \( V_0^o \) (\( V_1^o \)) is the value function at the beginning of date 1 for an agent without (with) money, under the one-price equilibrium. Formally,

\[
V_1^o(\theta_2(1)) = E[m \mid \theta_2(1)]\beta V_1^2(\theta_3(2)) + (1 - E[m \mid \theta_2(1)])u(y_h) + \beta V_0^2(\theta_3(1))
\]

and

\[
V_0^o(\theta_2(1)) = E[m \mid \theta_2(1)](-y_h + \beta V_1^2(\theta_3(2))) + (1 - E[m \mid \theta_2(1)])\beta V_0^2(\theta_3(1)),
\]

where

\[
E[m \mid \theta_2(1)] = \theta_2(1)m_H + (1 - \theta_2(1))m_L.
\]

It is straightforward to compute

\[
y_o = \beta[E[m \mid \theta_2(1)]y_h + (1 - E[m \mid \theta_2(1)])u(y_h)]. \tag{3}
\]

We conclude that in every trade meeting the buyer offers \( y_o \) and the seller accepts the offer.

![Fig. 1. Output levels.](image-url)
Proposition 1 provides a formal characterization of the one-price equilibrium (where \( p_t \) is price at date \( t \)).

**Proposition 1.** There exists \( \eta_0 > 0 \) such that for all \( m_H, m_L \) with \( m_H - m_L < \eta_0 \), there exists an equilibrium with the following features:

(i) In every trade meeting at date 0 \( p_0 = 1/y_o \), where \( y_o \) is given by (3).

(ii) In every trade meeting at date 1 \( p_1 = 1/y_h \), where

\[
y_h = \theta_3(2)y_H + (1 - \theta_3(2))y_L.
\]

(iii) In every trade meeting at date \( t (t \geq 2) \) \( p_t = 1/y_H \) if the economy is in the high state and \( p_t = 1/y_L \) if the economy is in the low state.

**Proof.** See Appendix A.

In the class of equilibria where transaction takes place in all trade meetings at date 1, the above equilibrium induces the highest production. The reason is that there can be no one-price equilibrium with \( y \) above \( y_h \) consistent with all sellers accepting \( y \).

Note that the price level at date 1 is higher than the price level at date 0 as long as

\[
\frac{u(y_h)}{y_h} > \frac{1 - \beta E[m | \theta_2(1)]}{\beta(1 - E[m | \theta_2(1)])}.
\]

We do not have the analytical solution for the conditions under which this inequality holds but we analyzed its validity for a wide range of values of \( \beta \) and \( \theta \) within the interval \((0, 1)\), given the function \( u(y) = y^{1/2} \). In all cases we obtained that the price level at date 1 is higher than the price level at date 0. The main intuition for this result is that, while in date 0 the most pessimistic seller has a posterior \( \theta_2(1) \), at date 1 his belief increases to \( \theta_3(2) \). As a result, for trade to take place in all trade meetings at dates 0 and 1, the seller must be asked a smaller amount (a higher price) at date 1.

An equilibrium with only one price at date 1 exists whenever the difference between \( m_H \) and \( m_L \) is not very large. Note that there are two distinct effects here. First, when \( m_H \) and \( m_L \) are close to each other, posteriors are not going to be very different across distinct private histories. This implies that, after experiencing a history \( Q^H(1) \) and forming a posterior \( \theta_3(1) \), a buyer does not attach a very high probability that the seller faces a history that leads to a similar posterior. Therefore, he has less incentives to deviate and ask for a higher amount of output. This is the informational effect. Second, when \( m_H \) is close to \( m_L \), the distance between \( y_H \) and \( y_L \) and, as a consequence, the distance between \( y_h \) and \( y \) is small (see formula 1). This effect also reduces the buyer’s incentive to deviate.

In Section 5, we show that there exists a region of parameters where both equilibrium with one price and equilibrium with price dispersion exist. As we show in the following sections, this multiple equilibrium result emphasizes the role that the informational effect plays in our economy.

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Note: We look at the following values of the parameters: \( \beta, \theta \in \{0.01, 0.1, 0.5, 0.9, 0.99\} \). The simulations are available from the authors upon request.
4. An equilibrium where prices are informative

In what follows we describe a region of parameters where an equilibrium with price dispersion exists. As in Section 3, we first informally discuss our main result and Proposition 2 provides a formal proof.

Since information about the state of the economy is revealed at date 2, in every trade meeting from date 2 onwards the amount of output equals \( y_H \) (\( y_L \)) if the state of the economy is high (low). We now consider the formation of prices at date 1.

4.1. Date 1

An agent’s decision at date 1 depends on his private history. As in the one-price case, a private history includes the agent’s money holdings and the money holdings of his partner.\(^5\) Our objective is to describe an equilibrium where the choice of the buyer depends on his private history. In particular, we expect that a buyer asks for a higher output (smaller price) if he faces the more optimistic history \( \Omega^B_3(1) \) and for a smaller output (higher price) if he faces the history \( \Omega^B_3(2) \). More formally, the buyer’s strategy at date 1 implies

\[
\sigma^B_1(\Omega^B_3(1)) > \sigma^B_1(\Omega^B_3(2)).
\]

Before the buyer makes an offer the only part of his private history which is not common knowledge in the economy is the value of the first entry (just note that for both \( \Omega^B_3(1) \) and \( \Omega^B_3(2) \) the last two entries are the same). If the seller believes that the buyer follows a strategy which is history-contingent, he can use the buyer’s offer as an additional piece of information which reveals his entire private history. Suppose, for example, that a seller with a history \( \Omega^B_3(1) \) meets a buyer and the buyer asks for \( \sigma^B_1(\Omega^B_3(1)) \). The seller knows that a buyer only asks for \( \sigma^B_1(\Omega^B_3(1)) \) if his private history is \( \Omega^B_3(1) \). Therefore, instead of having a posterior equal to \( \theta_3(1) \), he includes the buyer’s offer into his information set and updates his belief to \( \theta_3(1) \). Analogously, if a seller has a history \( \Omega^B_3(2) \) and meets a buyer who asks for \( \sigma^B_1(\Omega^B_3(1)) \), he updates his belief from \( \theta_3(2) \) to \( \theta_4(2) \).

Consider the problem of a buyer with a history \( \Omega^B_3(1) \) who asks for \( \sigma^B_1(\Omega^B_3(1)) \). Moreover, assume that he wants to target only optimistic sellers. Since he knows that these sellers are going to have a posterior equal to \( \theta_4(1) \) after receiving the offer, the buyer’s optimal decision is to set

\[
\sigma^B_1(\Omega^B_3(1)) = \beta(V_1^2(\theta_4(1)) - V_0^2(\theta_4(1))) \equiv y_H = \theta_4(1)y_H + (1 - \theta_4(1))y_L.
\]

A similar reasoning applies for a buyer that asks for \( \sigma^B_1(\Omega^B_3(2)) \) after a history \( \Omega^B_3(2) \). In this case, a seller with history \( \Omega^B_3(1) \) updates his belief from \( \theta_3(1) \) to \( \theta_4(2) \). Alternatively, a seller with history \( \Omega^B_3(2) \) updates his belief from \( \theta_3(2) \) to \( \theta_4(3) \). Now assume that a buyer with a history \( \Omega^B_3(2) \) targets all sellers and makes an offer that is always accepted. Under the proposed strategy, the best offer he can make is to ask for

\[
\sigma^B_1(\Omega^B_3(2)) = \beta(V_1^2(\theta_4(3)) - V_0^2(\theta_4(3))) \equiv y_{hh} = \theta_4(3)y_H + (1 - \theta_4(3))y_L.
\]

\(^5\)Note that prices at date 0 are the same across trade meetings. Therefore, we can apply the same reasoning as before and conclude that the only relevant piece of information an agent carries from date 0 to date 1 is his money holdings and the money holdings of the agent he met. This implies that the set of agents’ private histories at the beginning of date 1 are the same as in the one-price case.
The next step is to check the buyer’s incentive to follow the prescribed strategy. Consider, first, a buyer with a history \( O_B^3(1) \). If he asks for \( y_{ll} \), his expected payoff is

\[
P_{00}[u(y_{ll}) + \beta V_0^2(\theta_4(1))] + (1 - P_{00})\beta V_1^2(\theta_4(2)).
\]

There is a probability \( P_{00} \) that he meets a seller with a history \( O_S^3(1) \), in which case the latter accepts the offer and the buyer moves to date 2 without money. There is also a probability that the seller has a history \( O_S^3(2) \). In this case, he rejects the offer, there is no trade and the buyer moves to date 2 with money.

If a buyer deviates from the prescribed strategy and, for example, asks for \( y \), his payoff depends on the seller’s belief after observing \( y \). A necessary condition for our proposed strategy to be an equilibrium is that, for any value of \( y \) such that \( y_{hh} < y \leq y_{ll} \), only optimistic sellers accept the offer. As we show in the proof of Proposition 2, this condition is satisfied as long as the seller’s belief after observing \( y \) gives a sufficiently small probability to the optimistic buyer.\(^6\) Hence, the best possible deviation for an optimistic buyer is to ask for \( y_{hh} \). In this case, he obtains

\[
u(y_{hh}) + \beta V_1^2(\theta_3(1)).
\]

Note that, since all sellers accept the offer, a buyer does not gain any additional information and moves to date 2 with a posterior \( \theta_3(1) \). Hence, a buyer with a history \( O_B^3(1) \) follows the prescribed strategy as long as

\[
P_{00}[u(y_{ll}) + \beta V_0^2(\theta_4(1))] + (1 - P_{00})\beta V_1^2(\theta_4(2)) \geq u(y_{hh}) + \beta V_0^2(\theta_3(1)).
\]

We can rewrite this expression as

\[
P_{00}[u(y_{ll}) + (1 - P_{00})y_{hm}] \geq u(y_{hh}). \tag{4}
\]

Now, consider the incentive constraint faced by a buyer with history \( O_B^3(2) \). A computation similar to the one above shows that a pessimistic buyer does not deviate from the prescribed strategy and asks for \( y_{hh} \) as long as

\[
u(y_{hh}) \geq P_{01}u(y_{ll}) + (1 - P_{01})y_{hh}. \tag{5}
\]

Before providing a formal characterization of the conditions under which the inequalities in (4) and (5) hold, we assume that the equilibrium with price dispersion at date 1 exists and work backwards the solution to the model at date 0.

### 4.2. Date 0

In every trade meeting at date 0, private histories are common knowledge and lead to the posterior \( \theta_2(1) \). Under the assumption that the equilibrium at date 1 exhibits price dispersion, a take it or leave it offer \( y_d \) by the buyer implies

\[
y_d = \beta(V_0^d(\theta_2(1)) - V_0^d(\theta_2(1))),
\]

where \( V_0^d \) (\( V_1^d \)) is the value function at the beginning of date 1 for an agent without (with) money, under the price dispersion equilibrium. Formally,

\[
V_1^d(\theta_2(1)) = E[m \mid \theta_2(1)]\beta V_1^2(\theta_3(2)) + (1 - E[m \mid \theta_2(1)])(P_{00}[u(y_{ll}) + \beta V_0^2(\theta_4(1))] \\
+ (1 - P_{00})\beta V_0^2(\theta_4(2))]
\]

\(^6\)We also prove in the Appendix that this constraint on beliefs satisfies the intuitive criterion.
and
\[
V^d_0(\theta_2(1)) = E[m \mid \theta_2(1)][(1 - P_{01})[-y_{hh} + \beta V_1^2(\theta_4(3))] + P_{01}\beta V_0^2(\theta_4(2))] \\
+ (1 - E[m \mid \theta_2(1)])\beta V_0^2(\theta_3(1)).
\]

It is straightforward to compute
\[
y_d = \beta(E[m \mid \theta_2(1)](1 - P_{01})y_{hh} + P_{01}y_m) + (1 - E[m \mid \theta_2(1)])(P_{00}u(y_H) \\
+ (1 - P_{00})y_m).
\]

Therefore, if there is price dispersion at date 1, in every trade meeting at date 0 a buyer offers \(y_d\) and a seller accepts the offer. We now provide a formal characterization of the equilibrium with price dispersion (where \(p_t\) is price at date \(t\)).

**Proposition 2.** There exists \(\eta_d > v > 0\) such that for all \(m_H, m_L\) with \(v < m_H - m_L < \eta_d\), there exists an equilibrium with the following features:

(i) In every trade meeting at date 0 \(p_0 = 1/y_d\), where \(y_d\) is given by (6).
(ii) At date 1 price dispersion arises. The distribution of prices across trade meetings is

\[
p_{hh} = \frac{1}{y_{hh}} \text{ with probability } m_i,
\]

where \(y_{hh} = \theta_4(3)y_H + (1 - \theta_4(3))y_L\).

\[
p_{ll} = \frac{1}{y_{ll}} \text{ with probability } (1 - m_i)^2,
\]

where \(y_{ll} = \theta_4(1)y_H + (1 - \theta_4(1))y_L\).

Moreover, there is no trade with probability \(m_i(1 - m_i)\)

\[
i = H, L.
\]

(iii) In every trade meeting at date \(t (t \geq 2) p_t = 1/y_H\) if the economy is in the high state and \(p_t = 1/y_L\) if the economy is in the low state.

**Proof.** See Appendix A.

Note that, in the class of equilibria where buyers condition their behavior on private experiences, the above equilibrium induces the highest production. The reason is that a seller does not accept an offer \(y > y_{ll}\), and a pessimistic seller does not accept an offer \(y > y_{hh}\) if he believes that such an offer comes from a pessimistic buyer.

As in the previous section, we can also compare output at date 0 with output at date 1. For example, if the low state of the economy is realized, we can compute the average output across trade meetings at date 1 as

\[
m_Hy_{ll} + m_Ly_{hh}.
\]

\(^7\)Precisely, we have \(((1 - m_L)^2/(1 - m_L)^2 + m_L(1 - m_L))y_H + (m_L(1 - m_L)/(1 - m_L)^2 + m_L(1 - m_L))y_{hh} = m_Hy_{ll} + m_Ly_{hh}.\) For example, \((1 - m_L)^2/(1 - m_L)^2 + m_L(1 - m_L))\) is the probability of a meeting between an optimistic buyer and an optimistic seller given that a trade meeting is realized and the economy is in the low state.
We did not find the analytical solution for these comparisons. However, for all numerical specifications that we considered we obtained that, if there is an equilibrium with price dispersion and the low state is realized, the average level of output at date 1 is always greater than the output at date 0. Intuitively, in the low state there is a high probability of trade meetings with output \( y_{ll} \). If the high state is realized output at date 0 is usually larger than the average output at date 1, which reflects low probability of trade meetings with output \( y_{hh} \) and high probability of trade meetings with no trade or with output \( y_{lh} \).

An equilibrium that exhibits price dispersion arises whenever the difference between \( m_H \) and \( m_L \) is sufficiently large. As in the one-price case, there are two effects in action. First, when \( m_H \) and \( m_L \) are distant from each other, posteriors can be very different across distinct private histories. This implies that, after experiencing a history \( \Theta^B \) and forming a posterior \( \theta_3(1) \), a buyer attaches high probability that the seller is facing a history leading to a similar posterior. Therefore, he has an incentive to ask for a higher amount of output. Alternatively, if the buyer faces a history leading to a posterior that attaches high probability to the high state, he prefers to be more cautious and asks for a smaller amount of output. This is the informational effect. Second, when \( m_H \) is distant from \( m_L \), the difference between \( y_H \) and \( y_L \) is large. As a consequence, the distance between \( y_{hh} \) and \( y_{ll} \) is also large. This increases the incentives for a buyer to demand a higher amount of output.

5. Price dispersion and learning

In the previous sections, we described the interaction between the informational content of an agent’s private history and the determination of prices. In our discussion the values of the parameters \( m_H \) and \( m_L \) are important in the characterization of the regions where one price and price dispersion arise. Histories are more or less informative depending on the distance between these parameters. However, as we pointed out, \( m_H \) and \( m_L \) have not only informational impact on the economy but also additional impact, which is due to our assumption that money is indivisible. In what follows, we want to control for this indivisibility effect. More precisely, we show that even for fixed values of parameters, a private history can become more informative as long as agents believe that prices are used as a way to convey information about private experiences. When it happens, price dispersion arises. The following proposition can be proved:

**Proposition 3.** There exists \( \eta > 0 \) and \( \nu > 0 \) such that, for all \( \nu < m_H - m_L < \eta \), both the equilibrium with one price and the equilibrium with price dispersion exist. Moreover, prices in these equilibria are the same as the ones described in Propositions 1 and 2.

**Proof.** See Appendix A.

Fig. 2 illustrates the results of Proposition 3. We plot functions \( T_1(q) \), \( T_2(q) \) and \( T_3(q) \), where \( q = m_H - m_L \) and functions \( T_i(q) \) are defined in Appendix A. These functions represent the incentive constraints for one-price and price dispersion equilibria. In particular, we consider \( u(y) = y^{1/2} \), where \( z > 1 \), a functional form which satisfies our general assumptions. For Fig. 2, we assume that \( z = 2 \), \( \beta = 0.9 \) and \( \theta = 0.5 \). As one can see

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8We look at the following values of the parameters: \( \beta, \theta \in \{0.01, 0.1, 0.5, 0.9, 0.99\} \) and \( u(y) = y^{1/2} \). The simulations are available from the authors upon request.

9However, as the difference between \( m_H - m_L \) decreases, output at date 1 can become larger than output at date 0.
from the figure, the one-price equilibrium exists only if \( q \) belongs to the interval \((0, \eta)\). The equilibrium with price dispersion exists if \( q \in (\nu, 1) \) and it coexists with one price in the region \( q \in (\nu, \eta) \). The same results hold for a wide range of parameters \( z, \theta \) and \( \beta \).\(^{10}\)

In what follows we focus on symmetric equilibria where all agents play the same strategy.\(^{11}\) These equilibria are compared in terms of the degree of learning, measured by the agents’ average belief with respect to the value of money at the end of date 1, before information on the state of the economy is disclosed.

Consider first the equilibrium with one price. Assume, for example, that the high state is realized. Let \( \theta_o(H) \) indicates the average belief in state \( H \). At the end of date 1, \( \theta_o(H) \) is given by

\[
\theta_o(H) = \sum_{i=0}^{3} \Pr(\Theta = i)\theta_3(i),
\]

where \( \Theta \) has a binomial distribution with probability of success \( m_H \) and number of trials equal to 3. The intuition for this result is the following. When there is no price dispersion, the set of private histories is equal to \( \{0, 1\}^3 \). The first entry indicates the agent’s money holdings at the beginning of date 0, the second and third entries indicate the money holdings of his partners at dates 0 and 1. Since histories with the same cardinality have the same informational content, we can partition the set of private histories in terms of their cardinality. This induces a random variable \( \Theta \) which has a binomial distribution with a probability of success that depends on the state of the economy. A similar reasoning applies if the low state is realized.

We now look at the equilibrium with price dispersion. First, note that whenever there is no trade meeting at date 1, the agent’s average belief is the same under one price and price dispersion. Therefore, we focus on private histories where buyers and sellers meet at date 1. In this case a seller’s private history is either \((0, 0, 1)\) or \((1, 0, 1)\). Consider a seller with the

\(^{10}\)When \( x \) and \( \theta \) are small, it is possible to have non-existence of price dispersion for high values of \( q \). An important fact is that there always exists a region of parameters where the one-price and price dispersion equilibria coexist.

\(^{11}\)Note that the decisions taken in a given meeting are independent of the behavior taking place in other meetings. They only depend on agent’s beliefs about the state of the economy and on the strategy profile played in the meeting. Therefore, we can observe equilibria where some agents condition their behavior on their private information and other agents adopt a one-price rule. This implies that, at date 1, three distinct prices can potentially be observed, \( P_{hh} > P_h > P_H \).
history (0, 0, 1). Assume that the high state is realized. There is a probability \(1 - m_H\) that the buyer he faces asks for \(y_h\) and a probability \(m_H\) that he asks for \(y_{hh}\). In the former case, the seller updates his belief to \(\theta_4(1)\) and in the latter he updates his belief to \(\theta_4(2)\).

Analogously, a seller with a history \((1, 0, 1)\) updates his belief to \(\theta_4(2)\) with probability \(\frac{1}{C_0} m_H\) and to \(\theta_4(3)\) with probability \(m_H\). Now, consider the problem of the buyer. When he meets a seller at date 1 his private history is either \((0, 1, 0)\) or \((1, 1, 0)\). In the former case, the buyer asks for \(y_{ll}\). There is a probability \(\frac{1}{C_0} m_H\) that his offer is accepted, in which case he updates his belief to \(\theta_4(1)\). Alternatively, with a probability \(m_H\) his offer is rejected and his belief becomes \(\theta_4(2)\). When a buyer has a history \((1, 1, 0)\), he asks for \(y_{hh}\) and all sellers accept the offer. In this case, no additional information is generated and the buyer keeps his initial belief \(\theta_3(2)\). Hence, at the end of date 1, the average belief under price dispersion \(\theta_d(H)\) is given by

\[
\theta_d(H) = \sum_{i=0}^{3} \Pr(\Theta = i)\tilde{\theta}_3(i),
\]

where

\[
\tilde{\theta}_3(0) = \theta_3(0),
\]

\[
\tilde{\theta}_3(1) = \frac{2m_1^2 m_H}{\Pr(\Theta = 1)} \theta_4(1) + \frac{m_1^2 m_H}{\Pr(\Theta = 1)} \theta_3(1) + \frac{m_1^2 m_H}{\Pr(\Theta = 1)} \theta_4(2),
\]

\[
\tilde{\theta}_3(2) = \frac{m_1^2 m_H}{\Pr(\Theta = 2)} \theta_4(2) + \frac{2m_1^2 m_H}{\Pr(\Theta = 2)} \theta_3(2) + \frac{m_1^2 m_H}{\Pr(\Theta = 2)} \theta_4(3),
\]

\[
\tilde{\theta}_3(3) = \theta_3(3).
\]

A similar reasoning holds if the low state is realized. Proposition 4 shows that, irrespective of the state of the economy, the average belief under price dispersion is closer to the belief under full information (which is equal to one in the high state and equal to zero in the low state) than the average belief under one price.

**Proposition 4.** Average beliefs under price dispersion and one price are related as follows:

(i) \(\theta_d(H) > \theta_o(H)\).

(ii) \(\theta_d(L) < \theta_o(L)\).

**Proof.** See Appendix A.

Moreover, the distance between \(\theta_d(i)\) and \(\theta_o(i)\), for \(i = H, L\) is higher for intermediate values of \(m_H - m_L\). The reason is that, when \(m_H\) is close to \(m_L\), private histories do not add much information. Hence, the extra information regarding private experiences obtained through price dispersion does not improve an agent’s posterior in a significant way. Alternatively, if the distance between \(m_H\) and \(m_L\) is large, private histories are very informative. In this case, the information conveyed by prices also does not significantly improve posteriors.

One might also want to inquire as to the short-run effects of the interaction between learning and prices on output. Consider first the economy at date 1. There are four types of
trade meetings, depending on the private histories of buyers and sellers. Under the one-price equilibrium, these histories are not relevant and all trade meetings generate output equal to $y_h$. However, under the price dispersion equilibrium, the following result arises. If a buyer faces the pessimistic history $O^P_3(2)$, he asks for $y_{hh}$ and his offer is accepted. If instead he faces the optimistic history $O^P_3(1)$ and asks for $y_{ll}$, his offer is accepted if the seller faced a similar history and it is rejected if the seller faced the pessimistic history. Since $y_{ll} > y_h > y_{hh}$, there are meetings in which production is higher under one price and meetings where price dispersion is characterized by higher production.

The aggregate effect on output depends on the distribution of trade meetings across private histories, which in turn depends on the actual state of the economy. Formally, if $y_{oi}^1$ ($y_{di}^1$) is the aggregate output at date 1 under one price (price dispersion) when the state of the economy is $i = H, L$, we obtain

$$y_{oi}^1 = m_i(1 - m_i)y_h,$$

$$y_{di}^1 = m_i(1 - m_i)[(1 - m_i)^2 y_{ll} + m_i y_{hh}],$$

$i = H, L$.

If the economy is in the high state, aggregate output under one price is greater than output under price dispersion. The reason is that, under price dispersion, meetings with no trade and meetings with low production ($y_{hh}$) occur with high probability. However, if the economy is in the low state, the aggregate effect on output is ambiguous. For example, for $z = 2$, $\beta = 0.9$ and $\theta = 0.5$ the one-price equilibrium dominates the price dispersion equilibrium: $y_{di}^1 < y_{oi}^1$. But if we assume $z = 2$, $\beta = 0.9$ and $\theta = 0.75$, then we get the opposite result. In general, output under price dispersion is higher as long as the utility function is sufficiently concave and/or the prior $\theta$ is sufficiently high. The reason is that, if one of these two conditions holds, one price and price dispersion coexist at intermediate values of $m_H - m_L$. In this case, the extra information regarding private experiences obtained through price dispersion significantly improves the accuracy of an agent’s posterior (see the discussion right after Proposition 4). As a result, output produced in a meeting between two optimistic agents ($y_{ll}$) also increases. Since these meetings occur with a relatively high probability when the economy is in the low state, aggregate output is high.\textsuperscript{12}

It is important to emphasize that price dispersion can generate more production even though it involves trade meetings where no trade takes place. The idea is that the possibility of no trade acts as a punishment device against buyers who demand an output inconsistent with their posteriors. In turn, this punishment allows optimistic buyers to ask for (and receive) a relatively high level of output. This result contrasts, for example, with Jones and Manuelli (2001). They analyze an environment with asymmetric information with respect to the value of money and describe how a lemon’s problem arises leading to the possibility of no trade. However, they do not consider how the possibility of no trade in a given meeting can have a positive external effect by allowing higher production in other meetings.

\textsuperscript{12}For completeness, we also considered numerically the behavior of the economy at date 0, in the region where both one price and price dispersion equilibrium exist. For all values of $\beta, \theta \in [0.01, 0.1, 0.5, 0.9, 0.99]$ and for $u(y) = y^{1/2}$ we obtain that output under the one-price equilibrium is always greater than output under price dispersion.
Finally, one could ask whether there exists another equilibrium where prices are informative, besides the price dispersion equilibrium. It turns out that, if the agents’ prior \( \theta \) is sufficiently small, there may exist another equilibrium of this form. In this equilibrium, buyers target only optimistic sellers, and ask for \( y_l \). We define and discuss this equilibrium in Appendix B. We also compare the beliefs under one price and full acceptability to that under one price and partial acceptability and show that in the latter the average belief is closer to the one under full information.

6. Welfare

In this section we compare equilibrium with one price and equilibrium with price dispersion in terms of their ex ante welfare, calculated as the agent’s expected payoff before money is distributed at date 0.\(^{13}\) As a benchmark, let \( W \) indicates welfare in the presence of complete information. We have

\[
W = \frac{m(1 - m)[u(y) - y]}{1 - \beta}.
\]

Welfare is maximized when the production in every trade meeting equates the marginal utility to the marginal cost (\( u'(y^*) = 1 \)). In environments with indivisible money and an upper bound on money holdings, a well-known result is that welfare in equilibrium usually differs from the optimal value: both underproduction and overproduction are possible. However, as argued in Trejos and Wright (1995), it is natural to expect that \( y \) is less than \( y^* \) in a monetary economy. In our environment output in equilibrium increases with the discount factor \( \beta \) and decreases with the amount of money \( m \). As a result, when the discount factor is high and/or the amount of money in the economy is small, output can be above the optimal value \( y^* \). While the possibility of overproduction is not important for any of the results obtained in the previous sections, one should be careful studying the welfare implications of the model.

One way to rule out overproduction is to introduce lotteries over money, as shown in Berentsen et al. (2002). The intuition is that a buyer can improve his expected payoff by asking for the efficient production and offering a lottery with a probability \( \tau \) that money changes hands. Berentsen et al. (2002) also show that lotteries do not eliminate underproduction. In this case, money is always transferred with probability 1 and production is determined by Eq. (1). Our strategy is instead to focus on the case of the parameter values that guarantee underproduction in the region where one-price equilibrium and price dispersion equilibrium coexist. In this case, money always changes hands with probability 1 and the results are the same as if lotteries were not present. This strategy is used by Katzman et al. (2003). From Eq. (1), for any value of \( m_H \), there exists \( \beta' \) such that, for all \( \beta < \beta' \), \( y_L < y^* \).

Let ex ante welfare be given by

\[
W_f = \theta W_{fH} + (1 - \theta) W_{fL},
\]

\(^{13}\)As in the previous section, we restrict attention to efficient symmetric equilibria.
where \( W_{jH} \) (\( W_{jL} \)) indicates welfare in the high (low) state under the one-price (\( j = o \)) and price dispersion (\( j = d \)) equilibrium. One can show that

\[
W_{oi} = m_i(1 - m_i) \left[ U(y_o) + \beta U(y_h) + \frac{\beta^2}{1 - \beta} U(y_i) \right]
\]

and

\[
W_{di} = m_i(1 - m_i) \left\{ U(y_d) + \beta[(1 - m_i)^2 U(y_{hh}) + m_i U(y_{hh})] + \frac{\beta^2}{1 - \beta} U(y_i) \right\},
\]

where

\[
U(y) = u(y) - y.
\]

Welfare is equal to the product between the number of trade meetings in every period \((m_i(1 - m_i))\) and the present value of the average surplus across those meetings. For example, consider the average surplus at date 1 under the price dispersion equilibrium. It is equal to the probability of a meeting between an optimistic buyer and an optimistic seller \(((1 - m_i)^2)\) times the surplus \(u(y_{hh}) - y_{hh}\), plus the probability of a meeting between a pessimistic buyer and a seller \((m_i)\) times the surplus \(u(y_{hh}) - y_{hh}\). Note that, since information about the state of the economy is revealed at date 2, \( W_{oi} \) and \( W_{di} \), only differ at dates 0 and 1. Moreover, due to our assumption that \( m_H + m_L = 1 \), the number of trade meetings does not depend on the actual state of the economy. This implies that \( W_{oi} > W_{di} \) if and only if

\[
U(y_o) - U(y_d) > \beta[(1 - m_i)^2 U(y_{hh}) + m_i U(y_{hh}) - U(y_h)].
\]

Since \( y_L < y^* \),

\[
U(y_{hh}) > U(y_h) > U(y_{hh}).
\]

Therefore, depending on which state of the economy is realized, \( W_{ji} \) may be higher under the one-price equilibrium or under the price dispersion equilibrium. We obtain that, if the high state is realized, \( W_{jH} > W_{dH} \). This result is not surprising and is related to the fact that price dispersion involves many meetings with no trade and meetings with a very small production. However, if the low state is realized, the result is ambiguous. We discuss this assertion by the way of a numerical exercise. Figs. 3 and 4 describe welfare across equilibria if the economy is in the low state. In Fig. 3 we assumed \( \beta = 0.4, \theta = 0.5 \) and \( z = 2 \), and in Fig. 4 we assumed \( \beta = 0.4, \theta = 0.8, z = 2 \).

![Fig. 3. Welfare comparison: \( W_{dL} < W_{oL} \).](image-url)
When the low state is realized, \( W_{dl} > W_{ol} \) as long as the prior \( \theta \) is sufficiently high. The reason is similar to the one in the previous section. When \( \theta \) is high, one price and price dispersion coexist at intermediate values of \( m_H - m_L \) and the extra information regarding private experiences obtained through price dispersion significantly improves the accuracy of an agent’s posterior. As a result, \( U(y_H) \) is significantly higher than \( U(y_h) \). Since meetings generating \( U(y_H) \) happen with high probability when the economy is in the low state, welfare is higher under price dispersion. Finally, there exist parameter values under which ex ante welfare is higher under the price dispersion equilibrium as compared to the one-price equilibrium. This is the case, for example, when \( \beta = 0.4, \theta = 0.8, z = 2.7 \).

7. Robustness

The search-theoretic framework is a very useful approach (compared to other monetary models, e.g., Lucas’ misperceptions model) when analyzing the learning process in a monetary economy. It allows us to introduce private histories in a tractable way and completely characterize all possible equilibria.

At the same time our model contains many assumptions and some of them deserve special attention. First, we consider an economy with indivisible money and an upper bound on individual money holdings. Our objective is to provide a tractable model which is still rich enough to capture the effects of monetary uncertainty over learning and prices. If money is divisible and there is no upper bound on money holdings private experiences become a much less tractable object and, as a result, the interaction between learning and prices is a far more complicated phenomenon. In principle, the indivisibility of money can generate effects on output and prices which are not related to the informational aspect that we want to address. We control for these effects by considering the region of parameters where the one-price and price dispersion equilibria coexist.

We also assumed that the state of the economy is revealed at date 2. We believe that this two-period lag formulation combines all necessary features in a tractable framework. This allows us to illustrate the main point of the paper: prices, as a part of private histories, play very important role in learning the true state of the economy. In particular, agents learn faster under price dispersion.

A third key assumption is that \( m_H + m_L = 1 \). This assumption is restrictive if one is interested in analyzing extensive margin effects of monetary shocks, since within this range the probability of a trade meeting is independent of the quantity of money. Our interest is
instead on the intensive margin effects of the interaction between prices and learning. Another advantage of the specification where \( m_H + m_L = 1 \) is that it delivers a natural way in which private histories reveal information about the state of the economy. Precisely, we obtain that, as private histories become more informative, i.e., as the difference \( m_H - m_L \) increases, an optimistic agent increases his belief that the economy is in the low state and a pessimistic agent increases his belief that the economy is in the high state. Under alternative specifications, where \( m_H + m_L \neq 1 \), it can happen that optimistic agents become more pessimistic as the distance between \( m_H \) and \( m_L \) increases. We conjecture that the discrepancy between the results under \( m_H + m_L = 1 \) and \( m_H + m_L \neq 1 \) can be eliminated if we consider a more general model where the short-run lasts for more than two periods and private histories become longer and more informative.

Finally, the main experiment in our model involves analyzing the short- and long-run effects of varying the initial endowment of money stock. This approach may sound too stylized since what we observe in the real world are changes in the quantity of money through monetary injections. In what follows we discuss some differences between these two approaches and claim that our main results are robust to either specification.

First, note that the introduction of monetary injections\(^{14}\) implies that at date 0 all sellers still form the same posterior regarding the state of the economy. This is so since they all faced the same history, i.e., they started with no money, received no money through a monetary injection and met an agent with money. As a result, the take it or leave it offer assumption implies that in all trade meetings at date 0 trade occurs at the same price. Hence, this price has no informational content, as in the current framework. At date 1 analysis becomes a bit harder. In particular, if there are two possible monetary injections \((m_H \text{ and } m_L)\) the set of private histories for the buyer (seller) becomes \( \{O_B^4(1), O_B^4(2), O_B^3(2)\} \) \( (\{O_S^4(1), O_S^4(2), O_S^3(2)\}) \). Clearly, in the environment with monetary injections, a one-price equilibrium with full acceptability still exists at a price consistent with acceptance by the most pessimistic sellers, i.e., the ones with a belief \( \theta_3(2) \). This happens as long as the distance \( m_H - m_L \) is not too large and the intuition is the same as the one presented in Section 3. It also seems reasonable to expect that equilibria with price dispersion exist under the new specification. Since the set of private histories is larger, we believe that there may be more than one such equilibrium. However, the main intuition from these results should be the same, i.e., price dispersion increases communication and improves the precision of the agents’ average beliefs. We also expect that, as long as the low state is realized, there will be regions of the parameter space where price dispersion will entail a higher ex ante welfare than the one-price equilibrium.

8. Concluding remarks

The literature on the misperceptions theory of money has always emphasized learning as a mechanism through which an economy reaches the long-run. But the modeling of a learning process has been missing in this literature, mostly because of the technical difficulties that arise in a dynamic framework. The main objective of this article has been to characterize equilibria in a dynamic monetary economy with uncertainty where agents use their private experiences to update their knowledge about the actual state of the economy.

\(^{14}\)This can be done in the same way as in Wallace (1997) with a restriction to only two possible levels of monetary injection, high \((m_H)\) and low \((m_L)\).
Several results have been found. First, we have shown the impact of the volatility of monetary policy on prices. Given \( m_L \) is close to \( m_H \), only the one-price equilibrium emerges in the short-run. Otherwise, price dispersion may arise. Second, we have characterized a region of parameters where the one-price equilibrium and the equilibrium with price dispersion coexist. The most interesting finding is that in this region price dispersion provides a mechanism through which agents obtain more precise information about the purchasing power of money. The intuition is that price dispersion allows agents to indirectly communicate their private experiences, which increases the overall amount of information. Despite the loss of output in some of the trade meetings, the aggregate output in the economy with price dispersion may be closer to the one under complete information. As a result the price dispersion equilibrium can entail higher ex ante welfare than the one-price equilibrium. A common perception, that volatile monetary regimes may cause price dispersion and as a result confusion about the value of money, can be misleading. Exactly the opposite may happen. If one price emerges as an equilibrium outcome there is no way for agents to communicate their private histories while they trade with each other. Therefore, it may take much longer for the actual state of the economy to be revealed.

Appendix A

Proof of Proposition 1. First, since the state of the economy is revealed at date 2, we must have

\[
\sigma^t_B(j) = y_j \quad \text{where } j = H, L, \text{ for all } t \geq 2,
\]

\[
\sigma^t_S(j, y) = \begin{cases} A & \text{if } y \leq y_j, \\ R & \text{otherwise} \end{cases} \quad \text{for all } t \geq 2.
\]

Now consider the economy at date 1. Consider the following pair of functions:

\[
\sigma^1_B(\Omega^B_3(1)) = \sigma^1_B(\Omega^B_3(2)) = y_h \quad \text{and}
\]

\[
\sigma^1_S(\Omega^S_3(1), y) = \begin{cases} A & \text{if } y \leq y_{h}, \\ R & \text{otherwise} \end{cases}, \quad \sigma^1_S(\Omega^S_3(2), y) = \begin{cases} A & \text{if } y \leq y_{h}, \\ R & \text{otherwise} \end{cases}
\]

and the following belief function

\[
\mu(y) = \begin{cases} 1 & \text{if } y > y_m, \\ \frac{1}{2} & \text{if } y \leq y_m. \end{cases}
\]

Given \((\sigma^t_B, \sigma^t_S)\) and \(\mu(y)\), an equilibrium with one price exists as long as

\[
u(y_h) \geq P_{00} u(y_{h}) + (1 - P_{00}) y_m, \tag{A}
\]

\[
u(y_h) \geq P_{01} u(y_{h}) + (1 - P_{01}) y_{hh}. \tag{B}
\]

Since the right-hand side of (A) is bigger than the right-hand side of (B), it suffices to consider inequality (A). First, notice that in any one-price equilibrium where trade happens in all meetings, \( y_h \) is the maximum output consistent with acceptance by all sellers. For any \( y \) above \( y_h \), acceptance by a seller with a history \( \Omega^S_3(2) \) implies that this seller has a posterior below \( \theta_3(2) \). However, in an equilibrium where all buyers ask for the same price,
prices do not affect posteriors. Hence, on the equilibrium path, a posterior of a seller with history \( \Theta^S_3(2) \) must be equal to \( \theta_3(2) \). Moreover, notice that \( y_{ll} \) is the maximum output that a seller with a history \( \Theta^S_3(1) \) is willing to accept. This output can be reached if sellers, after seeing a deviation to \( y_{ll} \), attach a probability 1 that such a deviation is coming from a buyer with a history \( \Theta^B_3(1) \). Therefore, \( y_{ll} \) is the best possible gain that a buyer can get by deviating from the equilibrium path. Clearly, by attaching the value 1 to the belief that the optimistic buyer deviated when \( y \geq y_m \), we automatically satisfy the intuitive criterion.

First, we define a function \( T_1(q) \):

\[
T_1(q) = u(y_h) - P_{00}u(y_{ll}) - (1 - P_{00})y_m,
\]

where \( q = m_H - m_L \). Simple calculations show that at \( q = 0 \),
\( y_h = y_{ll} = y_m = y_H = y_L = y_0 \), where \( y_0 \) is the solution to the following equation:

\[
u(y) = \frac{2 - \beta}{\beta} y.
\]

Then, \( T_1(0) = \frac{1}{2}[u(y_0) - y_0] \), and as a result \( T_1(0) > 0 \) is satisfied iff \( u(y_0) > y_0 \), which is always true. Next, notice that \( T_1(q) \rightarrow [-u(y_1)]<0 \) as \( q \rightarrow 1 \), where \( y_1 \) is the solution to \( \beta u(y) = y \). It means that the one-price equilibrium exists if \( q < \eta_o < 1 \), where

\[
\eta_o = \min\{q \text{ s.t. } T_1(q) = 0, T_1'(q) < 0\}.
\]

Finally, since buyers and sellers share the same posterior in a trade meeting at date 0, only one price can emerge in equilibrium. We have

\[
s^B_0(\Theta^B_2(1)) = y_o \quad \text{and} \quad s^S_0(\Theta^S_2(1), y) = \begin{cases} A & \text{if } y \leq y_o, \\ R & \text{otherwise,} \end{cases}
\]

where

\[
y_o = \beta\{E[m | \theta_2(1)]y_h + (1 - E[m | \theta_2(1)])u(y_h)\}.
\]

This concludes our proof. \( \square \)

**Proof of Proposition 2.** First, since the state of the economy is revealed at date 2, we must have

\[
s^B_t(j) = y_j \quad \text{where } j = H, L, \text{ for all } t \geq 2 \quad \text{and} \quad s^B_t(j, y) = \begin{cases} A & \text{if } y \leq y_j, \\ R & \text{otherwise} \end{cases} \quad \text{for all } t \geq 2.
\]

Now consider the economy at date 1. We need to show that the pair of functions

\[
s^B_1(\Theta^B_3(1)) = y_{ll}, \quad s^B_1(\Theta^B_3(2)) = y_{hh}, \quad s^S(\Theta^S_3(1), y) = \begin{cases} A & \text{if } y \leq y_{ll}, \\ R & \text{otherwise,} \end{cases} \quad s^S(\Theta^S_3(2), y) = \begin{cases} A & \text{if } y \leq y_{hh}, \\ R & \text{otherwise} \end{cases}
\]
and the belief function

\[ \mu(y) = \begin{cases} 
1 & \text{if } y > y_m, \\
0 & \text{if } y \leq y_m.
\end{cases} \]

These beliefs guarantee that a seller with a history \( \Omega_S^2(2) \) does not accept any offer above \( y_{hh} \). Note again that, by attaching the value 1 to the belief that the optimistic buyer deviated when \( y \geq y_m \), we automatically satisfy the intuitive criterion. As a result, an equilibrium with price dispersion exists if the following two inequalities are satisfied:

\[
\begin{align*}
\mu(y_{hh}) & \leq \mathbb{P}[U(y_B) + (1 - \mathbb{P}[U])]y_m, \\
\mu(y_{hh}) & \geq \mathbb{P}[U(y_B) + (1 - \mathbb{P}[U])]y_{hh}.
\end{align*}
\]

To prove the statement we define functions \( T_2(q) \) and \( T_3(q) \):

\[
\begin{align*}
T_2(q) &= \mu(y_{hh}) - \mathbb{P}[U(y_B)] - (1 - \mathbb{P}[U])y_m, \\
T_3(q) &= \mu(y_{hh}) - \mathbb{P}[U(y_B)] - (1 - \mathbb{P}[U])y_{hh},
\end{align*}
\]

where \( q \equiv m_H - m_L \). Similar to the proof of Proposition 1, at \( q = 0 \), \( y_{hh} = y_B = y_m = y_L = y_0 \). Then, \( T_2(0) = T_3(0) = \frac{1}{2}[\mu(y_0) - y_0] > 0 \) since \( \mu(y_0) > y_0 \). Moreover, \( T_2(q) \rightarrow [-\mu(y_1)] < 0 \), and \( T_3(q) \rightarrow 0 \) as \( q \rightarrow 1 \). Notice that \( T_3(q) - T_2(q) > \mathbb{P}[U(y_B)] - \mu(y_{hh}) > 0 \) for \( q \in (0, 1) \). It means that equilibrium with price dispersion exists if \( v < q < \eta_d \), where

\[ v = \min\{q \text{ s.t. } T_2(q) = 0, T'_2(q) < 0\} \]

and

\[ \eta_d = \min\{q \text{ s.t. } T_3(q) = 0, T'_3(q) < 0\}, \]

where

\[
\begin{align*}
y_d &= \beta[E[m | \theta_2(1)]((1 - \mathbb{P}[U])y_{hh} + \mathbb{P}[U])m + (1 - E[m | \theta_2(1)])(\mathbb{P}[U]y_B \\
&+ (1 - \mathbb{P}[U])y_m)].
\end{align*}
\]

This concludes our proof. \( \Box \)

**Proof of Proposition 3.** The result follows immediately after observing that \( T_1(q) - T_2(q) = [\mu(y_B) - \mu(y_{hh})] > 0 \) for all \( q \in (0, 1) \), \( T_1(0) = T_2(0) \) and \( T_1(1) = T_2(1) \). We know from the proof of Proposition 2 that \( T_3(q) > T_2(q) \), while the relationship between \( T_3(q) \) and \( T_1(q) \) is ambiguous. Therefore, equilibria with one price and equilibria with price dispersion coexist if \( v < q < \eta \), where \( \eta = \min[\eta_o, \eta_d] \).

This concludes our proof. \( \Box \)
Proof of Proposition 4. (i) We need to show that
\[ \sum_{i=0}^{3} \Pr(\Theta_H = i) \tilde{\theta}_3(x) - \sum_{i=0}^{3} \Pr(\Theta_H = i) \theta_3(x) > 0. \]
Applying the definition of \( \tilde{\theta}_3(x) \), we obtain that this expression is equivalent to
\[ 2(1 - m_H)((1 - m_H)\theta_4(1) + m_H\theta_4(2) - \theta_3(1)) + m_H[(1 - m_H)\theta_4(2) \]
\[ + m_H\theta_4(3) - \theta_3(2)] > 0. \]
After simple algebraic manipulations one can show that
\( (1 - m_H)\theta_4(1) + m_H\theta_4(2) - \theta_3(1) > 0, \)
\( (1 - m_H)\theta_4(2) + m_H\theta_4(3) - \theta_3(2) > 0 \)
as long as
\( (m_H - m_L)^2 > 0. \)
(ii) We need to show that
\[ \sum_{i=0}^{3} \Pr(\Theta_L = i) \tilde{\theta}_3(x) - \sum_{i=0}^{3} \Pr(\Theta_L = i) \theta_3(x) < 0. \]
Applying the definition of \( \tilde{\theta}_3(x) \), we obtain that this expression is equivalent to
\[ 2(1 - m_L)((1 - m_L)\theta_4(1) + m_L\theta_4(2) - \theta_3(1)) + m_H[(1 - m_H)\theta_4(2) \]
\[ + m_H\theta_4(3) - \theta_3(2)] < 0. \]
Again, it is straightforward to show that
\( (1 - m_L)\theta_4(1) + m_L\theta_4(2) - \theta_3(1) < 0, \)
\( (1 - m_L)\theta_4(2) + m_L\theta_4(3) - \theta_3(2) < 0 \)
as long as
\( (m_H - m_L)^2 > 0. \)
This concludes our proof. \( \square \)

Appendix B

In this appendix, we provide the conditions under which there exists an equilibrium with one price and partial acceptability by sellers. First, from date 2 on, price in equilibrium equals \( \frac{1}{y_H} \) \( (1/y_L) \) depending on whether the high (low) state is realized. Now consider the economy at date 1. We need to show that the pair of functions
\[ \sigma_B^1(\Omega^B_1(1)) = \sigma_B^1(\Omega^B_1(2)) = y_l, \sigma_S(\Omega^S_1(1), y) = \begin{cases} A & \text{if } y \leq y_l, \\ R & \text{otherwise} \end{cases}, \]
\[ \sigma_S(\Omega^S_1(2), y) = \begin{cases} A & \text{if } y \leq y_h, \\ R & \text{otherwise} \end{cases} \]
together with the belief function

\[ \mu(y) = \begin{cases} 
\frac{1}{2} & \text{if } y \geq y_l, \\
0 & \text{otherwise}
\end{cases} \]

are part of an equilibrium. Notice that, according to \( \mu \), after seeing a deviation \( y \) below \( y_l \), sellers attach probability 1 that the deviant buyer has history \( O_B(2) \). Since pessimistic buyers have more incentive to deviate and ask for \( y_0 < y_l \), the proposed beliefs are consistent with the intuitive criterion. We obtain the pair \( \{\sigma_B^1, \sigma_S^1\} \) and the belief function \( \mu \) are part of an equilibrium iff

\[ P_{01}u(y_l) + (1 - P_{01})y_{hh} \geq u(y_{hh}) \]

and

\[ P_{00}u(y_l) + (1 - P_{00})y_m \geq u(y_{hh}). \]

Finally, since buyers and sellers share the same posterior in a trade meeting at date 0, only one price can emerge in equilibrium. Notice that the above equilibrium induces the highest production in the class of equilibria involving one price and partial acceptability by sellers. Moreover, it can be easily shown that the equilibrium with one price and partial acceptability cannot coexist with the equilibrium with price dispersion. Formally, in the equilibrium with price dispersion the following inequality must hold:

\[ u(y_{hh}) \geq P_{01}u(y_l) + (1 - P_{01})y_{hh}. \]

However, since

\[ P_{01}u(y_l) + (1 - P_{01})y_{hh} > P_{01}u(y_l) + (1 - P_{01})y_{hh}, \]

and an equilibrium with one price and partial acceptability implies

\[ P_{01}u(y_l) + (1 - P_{01})y_{hh} \geq u(y_{hh}) \]

we have

\[ u(y_{hh}) > u(y_{hh}), \]

a contradiction. We conclude that there exists a region where both the equilibrium with price dispersion and the equilibrium with one price and full acceptability coexist, but the equilibrium with one price and partial acceptability does not exist.

We can also compute the average belief under one price and partial acceptability. We proceed in a way similar to the analysis for the equilibrium under price dispersion. Assume that the high state is realized. First, we can focus on private histories where buyers and sellers meet at date 1. In this case a buyer’s private history is either \((0,1,0)\) or \((1,1,0)\). Consider a buyer with the history \((0,1,0)\), hence a belief \( \theta_3(1) \). Assume that the high state is realized. There is a probability \( (1 - m_H) \) that the seller he faces accepts the offer \( y_I \) and a probability \( m_H \) that the seller rejects the offer. In the former case, the buyer updates his belief to \( \theta_4(1) \) and in the latter case he updates his belief to \( \theta_4(2) \). A similar analysis implies that a buyer with a history \((1,1,0)\) updates his belief to \( \theta_4(2) \) upon meeting a seller that accepts the offer \( y_I \). Moreover, he updates his belief to \( \theta_4(3) \) after meeting a seller that rejects the offer \( y_I \). Hence, at the end of date 1, the average belief under one price and partial acceptability \( \theta_p(H) \) is given by

\[ \theta_p(H) = \sum_{i=0}^{3} \Pr(\Theta = i)\theta_3(i), \]
where
\[
\hat{\theta}_3(0) = \theta_3(0),
\]
\[
\hat{\theta}_3(1) = \frac{m^2_H m^2_L}{\Pr(\Theta = 1)} \theta_4(1) + \frac{2 m^2_L m^2_H}{\Pr(\Theta = 1)} \theta_3(1) + \frac{m^2_L m^2_H}{\Pr(\Theta = 1)} \theta_4(2),
\]
\[
\hat{\theta}_3(2) = \frac{m^2_H m^2_L}{\Pr(\Theta = 2)} \theta_4(2) + \frac{2 m_L m^2_H}{\Pr(\Theta = 2)} \theta_3(2) + \frac{m_L m^2_H}{\Pr(\Theta = 2)} \theta_4(3),
\]
\[
\hat{\theta}_3(3) = \theta_3(3).
\]
A similar reasoning applies if the low state is realized. It is straightforward to show that, irrespective of the state of the economy, the average belief under one price and partial acceptability is closer to the belief under full information than the average belief under one price and full acceptability. The proof is essentially the same as the proof of Proposition 4, replacing \( \hat{\theta}_3(1) \) with \( \hat{\theta}_3(1) \) and \( \theta_3(2) \) with \( \hat{\theta}_3(2) \).

References


