Limited monitoring and the essentiality of money

Luis Araujo, Braz Camargo

Abstract

Monetary theory emphasizes that imperfect monitoring is necessary for money to be essential, that is, for money to achieve socially desirable allocations. Little is known about how limited monitoring must be if money is to be essential, though. Understanding sufficient conditions for the essentiality of money is important since monitoring is a natural way in which credit is introduced in monetary models. In this paper, we show that money can fail to be essential even if monitoring is quite limited. This indicates that one must be careful when introducing monitoring in monetary models to allow for the coexistence of money and credit.

1. Introduction

Money is essential if socially desirable allocations can only be achieved with its use. It is well-known that imperfect monitoring, that is, the fact that individual histories are imperfectly observed, is necessary for the essentiality of money. Money is not essential if monitoring is perfect because the information about past histories conveyed by money is redundant if past histories can be perfectly observed (Kocherlakota, 1998). On the other hand, much less is known about how limited monitoring must be if money is to be essential. Wallace (2014) summarizes this state of affairs as follows: “There are, however, no general necessary and sufficient conditions for essentiality of money. In particular, imperfect monitoring is not sufficient to give a role for money. Therefore, it is not surprising that many models contain extreme sufficient conditions to ensure that money is essential” (pp. 259–260).

Understanding the restrictions one needs to impose on monitoring in order to preserve the essentiality of money is important, though, as the introduction of credit in monetary models usually requires a monitoring technology, and one needs to be careful to prevent the monitoring technology that enables credit from making money irrelevant.

In this paper we show that the first-best can be achieved without money if monitoring is such that in every meeting an agent observes only the last period action of his partner and the preferences of the last period partner of his partner. This is much less information than the information contained in the record-keeping notions previously considered in the literature. We conduct our analysis in a class of environments that allows for heterogeneous utility functions and includes the standard random matching, overlapping generations, and turnpike environments as special cases.

There are two messages to be taken from our non-essentiality result. First, since it significantly relaxes the requirements on monitoring needed to implement the first-best, our non-essentiality result suggests that the extreme sufficient conditions usually made in the literature to ensure that money is essential are warranted. This is bad news if one wants to build coherent models where money-like and credit-like instruments coexist and are both relevant. Second, our non-essentiality result calls for a more careful look at the notions of limited monitoring used by monetary theorists. This is good news if one believes that a better understanding of the details of monitoring technologies leads to better models of money and credit.

The rest of the paper is organized as follows. We discuss the related literature in the remainder of this section and describe our setting in the next section. We prove our non-essentiality result in...
Section 3 and discuss its robustness in Section 4. We conclude in Section 5.

Related literature

Kocherlakota (1998) considers a monitoring technology, which he calls memory, that allows agents to observe the past histories of their direct and indirect partners, and shows that any allocation that can be achieved with money can also be achieved with memory. Our monitoring technology is structured as in Kocherlakota (1998) except that it allows agents to observe only a subset of the information they observe in Kocherlakota (1998), namely, their partner’s last period action and partner preferences. Our main result is that this information is enough to implement the first-best. Intuitively, this information allows agents to observe the context of the last period meeting of their partners, which is sufficient for an agent to verify whether his current partner produced the efficient quantity to his previous period partner. Thus, almost all the information contained in Kocherlakota’s (1998) monitoring technology, while necessary to replicate the set of monetary allocations, is not necessary to achieve socially desirable allocations.

The literature on the relationship between monitoring and the essentiality of money has considered other notions of record-keeping besides the notion introduced in Kocherlakota (1998). Kocherlakota and Wallace (1998) consider a monitoring technology in which the private histories of all agents in the economy are publicly revealed with a lag, so that one does not need to meet an agent in order to observe his private history. Cavalcanti and Wallace (1999a,b) consider a variant of this technology in which only a subset of the agents in the population can have their private histories revealed, but the revelation occurs without a lag. The monitoring technologies in both of these papers feature a coordination component that is absent from the monitoring technology that we consider. In fact, public revelation of private histories allows agents to coordinate on extreme punishments, such as global autarky, something that is not possible when agents can observe only the private histories of the agents they meet.

Our paper is also related to the literature that studies how social norms can be enforced in random matching environments. Kandori (1992) is the seminal reference in this literature. The papers in this literature most closely related to ours are Araujo (2004), Aliprantis et al. (2007), and Takahashi (2010). Araujo (2004) shows that even without record-keeping, a gift-giving social norm can substitute the use of money as long as the population is finite and agents are patient enough. Aliprantis et al. (2007) show that the presence of centralized trade can help sustain a social norm that leads to efficient non-monetary trade even in large populations and discuss how this result can be overturned by suitably changing the matching process in the centralized market. Takahashi (2010) studies community enforcement in a large population in which in every period individuals are randomly and anonymously matched in pairs to play a prisoner’s dilemma game and shows that if agents are patient enough, cooperation can be sustained by a belief-free equilibrium as long as players can observe their (direct partners’ past play. We extend Takahashi’s analysis to our richer environment, which allows for heterogeneous agents and for a more general matching technology.

2. Setting

We first describe the (physical) environment and preferences. Then we introduce our monitoring technology and define equilibria.

2.1. Environment and preferences

Time is discrete and indexed by $t \geq 1$. There exist a continuum of non-atomic agents, that we identify with the interval $[0, 1]$, and a countable set $\Omega$ of agent types. For each $\omega \in \Omega$, there exists a positive mass of agents who are of type $\omega$. Different types of agents can live for different lengths of time and in different periods. Let $N(\omega)$ be the set of periods in which the agents of type $\omega$ are alive. Preferences are additionally separable over time and all agents have the same discount factor $\beta \in (0, 1)$. We normalize payoffs by $1 - \beta$.

Agents can trade a divisible and perishable good that comes in many varieties. There exists one variety of the good for each type of agent. An agent of type $\omega$ can produce only the variety of type $\omega$ and likes to consume a variety of type $\omega'$ only if $\omega' \in \lambda(\omega)$, where $\lambda : \Omega \rightarrow \Omega$ is a correspondence such that $\omega \notin \lambda(\omega)$ for all $\omega \in \Omega$. Thus, no agent can consume the variety of the good that he produces. An agent of type $\omega \in \Omega$ obtains instantaneous utility $u(x(\omega, \omega'))$ if he consumes $x$ units of the variety $\omega' \in \lambda(\omega)$ and pays an instantaneous production cost $\kappa$ if he produces $x$ units of the variety $\omega$. We let $u(x(\omega, \omega')) = 0$ when $\omega' \notin \lambda(\omega)$ and assume that for each $\omega \in \Omega$ and $\omega' \in \lambda(\omega)$, the function $u(x(\omega, \omega')) - \kappa$ has a unique and positive maximizer $x^*(\omega, \omega')$.

Trade occurs in a decentralized market with pairwise meetings. Let $\Omega_t$ be the set of types of agents who are alive in period $t$. For all $t \geq 1$, there exists a map $M_t : \Omega_t \times \Omega_t \rightarrow [0, 1]$ such that $M_t(\omega, \omega') = M_t(\omega', \omega)$ is the probability that an agent of type $\omega \in \Omega_t$ is matched with an agent of type $\omega' \in \Omega_t$ in period $t$. Meetings are random conditional on the types of agents matched. There are no double-coincidence meetings, that is, if $\omega, \omega' \in \Omega$ are such that $\omega' \in \lambda(\omega)$, then $\omega \notin \lambda(\omega')$. The sequence of events in a pairwise meeting is as follows. First, each agent observes the identity and type of his partner and the information that the monitoring technology makes available in the meeting. Then, each agent simultaneously and independently chooses his action, that is, how much to produce to his partner. Finally, consumption occurs.

Let $T(\omega) = \sup\{t \geq 1 : t \in N(\omega)\}$. An agent of type $\omega$ is finitely lived if $T(\omega) < \infty$, in which case $T(\omega)$ is his last period in the economy. We make the following assumption:

$$ (A1) \quad M_t(\omega, \omega') = 0 \quad \text{for all } \omega, \omega' \in \Omega \text{ such that } T(\omega) < \infty \text{ and } \omega \notin \lambda(\omega'). $$

Thus, as in an overlapping generations framework, a finitely lived agent cannot be a producer in his last period of life. Now, for each $t \geq 1$ and $\omega, \omega' \in \Omega_t$ such that $\omega \in \lambda(\omega')$, let

$$ \Delta_t(\omega, \omega') = -x^*(\omega', \omega) + \sum_{\omega'' \in \lambda(\omega') \cap \Omega_{t+1}} M_{t+1}(\omega, \omega'')u(x^*(\omega, \omega''))(\omega, \omega'). $$

3 Mills (2007) considers the case in which the private histories of a subset of agents in the population are publicly revealed with a lag. Cavalcanti et al. (1999) consider a monitoring technology which publicly reveals a summary of the private histories of a subset of agents. Jin and Temzelides (2004) consider an economy in which agents interact both locally and globally and assume that private histories are publicly observed only at the local level. Gomis-Péqueras and Sanches (2013) consider a variant of the monitoring technology of Cavalcanti et al. (1999) in an environment based on Lagos and Wright (2005).

4 The non-essentiality result in Aliprantis et al. (2007) depends on the assumption that all actions in the centralized market are publicly observed without noise. Araujo et al. (2012) extend the analysis in Aliprantis et al. (2007) to the case in which only prices are observed (possibly with noise) in the centralized market.

5 Rosenthal (1979), Kalai et al. (1988), Bhaskar (1998), and Olszewski (2007) consider similar equilibrium constructions. Awaysa (2014) extends Takahashi’s analysis to the case in which agents can observe their partner’s past play but only at a (small) cost.

6 Our model corresponds to Takahashi’s model with $g = l$.

7 It is straightforward to extend our analysis to cover the case in which double-coincidences are possible.
We also make the following assumption:

\[(A2) \inf \{ \Delta_i(\omega, \omega') : t \geq 1 \text{ and } M_t(\omega, \omega') > 0 \} > 0.\]

According to assumption (A2), in any period \( t \) the cost to an agent of producing the efficient amount in a meeting in which he is a producer is smaller than the agent’s expected utility in period \( t + 1 \) if he consumes the efficient amount in all the meetings in which he is a consumer. Notice that (A1) is necessary for (A2).

Our environment contains as a special case the environments with decentralized trade typically considered in monetary theory.

- **Random matching**: \( \Omega = \{ \omega_1, \ldots, \omega_K \} \), with \( K \geq 3 \), \( u(x_i(\omega, \omega')) = u(x), N(\omega) = \N, \lambda(\omega') = \{ 0 \} \), and \( M_t(\omega, \omega') = 1/K \). It is easy to see assumption (A2) reduces to \( K < u(x^*)/x^* \), where \( x^* \) is the unique and positive maximizer of \( u(x) - x \).

- **Overlapping generations**: \( \Omega = \mathbb{Z}_+, u(x_i(\omega, \omega')) = u(x), N(0) = \{ 1 \} \) and \( N(t) = \{ t, t + 1 \} \) if \( t \geq 1 \), \( \lambda(t) = \{ t + 1 \} \), and \( M_t(t, t - 1) = 1 \). The agents of type \( t \geq 1 \) are born in period \( t \) and live for 2 periods. The agents of type 0 are born in period 1 and live for one period. As usual, an agent who is born in \( t \) is ‘young’ in \( t \) and ‘old’ in \( t + 1 \), except for the type-0 agents, who are born ‘old’. In every period the ‘young’ agents meet with the ‘old’ agents and are the producers. Assumption (A2) is automatically satisfied, as it reduces to \( u(x^*) > x^* \), where \( x^* \) again is the unique and positive maximizer of \( u(x) - x \).

- **Turnpike**: \( \Omega = \{ 0, 1 \} \times \mathbb{Z}, u(x_i(\omega, \omega')) = u(x), N(\omega) = \N, \lambda((0, z)) = \{ (1, z + 2t + 1) : t \in \mathbb{Z}_+ \} \) and \( M_t((0, z), (1, 1)) = \{ (0, z - 2t) : t \in \mathbb{Z}_+ \} \). The agents of type \( z \) are the ‘stay ers’ and the agents of type \( 1, z \) are the ‘movers’. In every period the ‘movers’ move one step to the left. The ‘stay ers’ (‘movers’) are producers in the odd (even) periods and consumers in the even (odd) periods. Assumption (A2) is also automatically satisfied, as it reduces to the same condition of the previous example. Despite being fairly general, our environment is not as general as the environment in Kocherlakota (1998). First, we allow only for a countable set of types of agents. Second, we consider only bilateral meetings. It is possible, with a substantial cost in notation, to extend our analysis to allow for an uncountable number of types of agents and multilateral meetings.\(^8\)

### 2.2. Monitoring and equilibria

Monitoring works as follows. If agent \( j \) is the partner of agent \( i \) in period \( t \), then agent \( i \) observes the action of agent \( j \) in period \( t - 1 \) and the type of agent \( j \)’s partner in period \( t - 1 \).

Since the economy is populated by a continuum of agents and meetings are random conditional on the types of agents matched, an agent’s private information in a meeting is independent of his partner’s private information in the same meeting. Moreover, an agent’s private information in a meeting cannot affect the behavior of his future partners. Hence, there is no loss of generality in assuming that in any meeting the agents matched condition their behavior only on the information that is common in the meeting, which is the information provided by the monitoring technology.

For each \( i \in [0, 1] \), let \( \ell(i) \) be the number of periods in which agent \( i \) is alive. Now, for each \( n \in \{ 1, \ldots, \ell(i) \} \), let \( \mathcal{H}^{i,n} \) be the set of pairs \((j, C)\) where \( j \neq i \) is an agent and \( C \) is the set describing the common information in the meeting between agents \( i \) and \( j \) when agent \( i \) is of age \( n \). A behavior strategy for agent \( i \) is a sequence \( \sigma_i = \{ \sigma_i^{j,n} \}_{j,n=0}^\infty \), where \( \sigma_i^{j,n} : \mathcal{H}^{i,n} \to \Delta(\mathbb{R}_+) \) is a map describing agent \( i \)’s (mixed) action when he is of age \( n \) conditional on the identity of his partner and the common information in their meeting. A strategy profile is a map \( \Psi \) from \([0, 1] \) into the set of all possible behavior strategies with the property that for each \( i \in [0, 1] \), \( \Psi(i) \) is a behavior strategy for agent \( i \).

The record (or history) of an agent in a given period is the list of his past actions, the identity and type of his previous partners, and the actions of his partners in all his previous meetings. To each strategy profile \( \Psi \) there is associated a list \( \mu = \{ \mu_i \}_{i=0}^\infty \) where \( \mu_i = \{ \mu_i^{j,n} \}_{j,n=0}^\infty \) is a sequence such that \( \mu_i^{j,n} \) is a probability distribution on the set of possible records for agent \( i \) when he is of age \( n \). We refer to \( \mu \) as the evolution of records induced by the strategy profile \( \Psi \) and denote by \( \Gamma \) the map that takes a strategy profile into its corresponding evolution of records.

Since agents are non-atomic, their behavior does not affect the evolution of records. However, when an agent computes his expected payoff from following a given strategy, he takes the evolution of records to be independent of his strategy. We assume that agents believe that the evolution of records is also independent of their information. In particular, an agent with an information set that has zero probability under the postulated evolution of records does not change his belief about the evolution of records. This corresponds to the assumption that agents believe that any off-the-equilibrium-path behavior they observe is caused by a deviation initiated by a finite number of agents, which has no impact on aggregate behavior. We can now define an equilibrium.\(^9\)

**Definition 1.** A pair \((\Psi, \mu)\) is an equilibrium if \( \mu = \Gamma(\Psi) \) and \( \Psi(i) \) is sequentially rational given \( \Psi \) and \( \mu \) for all \( i \in [0, 1] \).

### 3. Achieving the first-best

The first-best is achieved if surplus is maximized in every single-coincidence meeting. The surplus in a meeting between a consumer of type \( \omega \) and a producer of type \( \omega' \in \lambda(\omega) \) is maximized if the producer produces \( x^*(\omega, \omega') \) units of the good to the consumer.

An agent who observes the preferences of the last period partner of his partner learns whether his partner was a producer in the previous period and how much he was supposed to produce in case he was a producer. We now show that observing this information and the previous period action of one’s partner is sufficient to achieve the first-best if discounting is small enough.

**Proposition 1.** There exists \( \beta^* \in (0, 1) \) such that if \( \beta > \beta^* \), then there exists an equilibrium that achieves the first-best.

**Proof.** Consider an agent of type \( \omega \in \Omega \) and let \( \lambda^{-1}(\omega) = \{ \omega' : \omega \in \lambda(\omega') \} \) be the set of agents who consume the good the agent produces. We say the agent is in state \( b(\omega) \) if in the previous period he met with an agent of type \( \omega' \in \lambda^{-1}(\omega) \) but did not produce \( x^*(\omega', \omega) \) in the meeting. We say the agent is in state \( g \) if in the previous period either he met with an agent of type \( \omega' \notin \lambda^{-1}(\omega) \) or he met an agent of type \( \omega' \in \lambda^{-1}(\omega) \) and produced \( x^*(\omega', \omega) \) in the

\(^8\) There is one dimension in which our results are more general than the results in Kocherlakota (1998). An important assumption in Kocherlakota (1998) is that the matching technology is such that whenever two agents meet, there exists no possibility of any previous direct or indirect contact between them (see assumption (A2) in p. 236). This also holds in our environment given the continuum population assumption together with the assumption that meetings are random conditional on the types of agents matched. However, as we discuss in Section 4, our non-essentiality result holds even if the population is finite. Hence, the assumption of no direct or indirect contact between any two agents who are matched can be relaxed. We thank an anonymous referee for pointing this out to us.

\(^9\) Our equilibrium notion generalizes the equilibrium notion in Takahashi (2010) to our setting.
meeting. Now for every \( t \geq 1 \) and \( \omega, \omega' \in \Omega_t \) with \( \omega' \in \lambda^{-1}(\omega) \), let \( \xi_t(\omega|\omega') \) be such that
\[
1 - \xi_t(\omega|\omega') = \beta \sum_{\omega'' \in \lambda(\omega) \cap \Omega_t} M_{t+1}(\omega, \omega'') u(x^*(\omega', \omega''))/u(\omega, \omega').
\]
(1)

Assumption (A1) implies that there exists \( \beta^* \in (0, 1) \) such that if \( \beta > \beta^* \), then \( \xi_t(\omega|\omega') \in (0, 1) \) regardless of \( t, \omega, \) and \( \omega' \). Our candidate equilibrium is the strategy profile \( \Psi^* \) such that: (i) all agents start in state \( g \) and state transitions are as described above; (ii) an agent of type \( \omega \) does not produce in a meeting if his partner is of type \( \omega'' \notin \lambda^{-1}(\omega) \); (iii) an agent of type \( \omega \) who meets with an agent of type \( \omega'' \in \lambda^{-1}(\omega) \) in state \( g \) produces \( x^*(\omega', \omega') \) with probability \( q_t(\omega', \omega'') \) = 1 to his partner; and (iv) in period \( t \geq 2 \), an agent of type \( \omega \) who meets with an agent of type \( \omega'' \in \lambda^{-1}(\omega) \) in state \( b_{\omega''} \) produces \( x^*(\omega', \omega') \) to his partner with probability \( q_t(\omega, \omega'') \beta(\omega'') = \xi_{t-1}(\omega'|\omega) \) and produces zero otherwise. By construction, \( \Psi^* \) implements the first-best. In what follows we show that \( \Psi^* \) is an equilibrium if \( \beta > \beta^* \). Suppose that \( \beta > \beta^* \) and consider an agent of type \( \omega \in \Omega_t \), in period \( t \). Let \( V_t(\omega|\theta) \) be the agent’s ex-ante expected present discounted payoff if he is in state \( \theta \); the payoff \( V_t(\omega|\theta) \) is computed before the agent is matched with a partner in period \( t \). Suppose the agent is not a producer. Since his continuation payoff does not depend on his action, it is optimal for him to produce zero. Now suppose the agent’s partner is of type \( \omega'' \in \lambda^{-1}(\omega) \), so that the agent is a producer. The agent must be indifferent between producing zero and producing \( x^*(\omega', \omega') \) otherwise he would have a profitable deviation. Moreover, the agent’s payoff from producing \( 0 < x < x^*(\omega', \omega') \) is smaller than his payoff from producing zero. So, a necessary and sufficient condition for \( \Psi^* \) to be an equilibrium is that
\[
(1 - \beta)x^*(\omega', \omega') + \beta V_{t+1}(\omega|g) = \beta V_{t+1}(\omega|b_{\omega''})
\]
(2)
for all \( t \geq 1 \) and all \( \omega, \omega' \in \Omega_t \) such that \( M_t(\omega, \omega') > 0 \) and \( \omega \in \lambda(\omega') \).

Now observe that if condition (2) holds, then
\[
V_t(\omega|\theta) = \sum_{\omega'' \in \Omega_t \setminus \lambda^{-1}(\omega)} M_t(\omega, \omega'') \beta V_{t+1}(\omega|g)
+ \sum_{\omega'' \in \lambda^{-1}(\omega) \setminus \Omega_t} M_t(\omega, \omega'') \beta V_{t+1}(\omega|b_{\omega''})
+ \sum_{\omega'' \in \lambda^{-1}(\omega) \cap \Omega_t} M_t(\omega, \omega'') \left[ (1 - \beta) q_t(\omega, \omega'') \theta | \right] \times u(x^*(\omega', \omega''))/u(\omega, \omega') + \beta V_{t+1}(\omega|g) \right].
\]

Indeed, the agent’s expected present discounted payoff is: (i) \( \beta V_{t+1}(\omega|b_{\omega''}) \) if he meets with an agent of type \( \omega'' \in \lambda^{-1}(\omega) \), and so is a producer; (ii) \( (1 - \beta) q_t(\omega, \omega'') \theta | u(x^*(\omega', \omega''))/u(\omega, \omega') + \beta V_{t+1}(\omega|g) \) if he meets with an agent of type \( \omega'' \notin \lambda^{-1}(\omega) \), and so is a consumer; and (iii) \( \beta V_{t+1}(\omega|g) \) otherwise. Straightforward algebra then shows that (2) is equivalent to
\[
\beta \sum_{\omega'' \in \lambda(\omega) \cap \Omega_{t+1}} M_{t+1}(\omega, \omega'') u(x^*(\omega', \omega''))/u(\omega, \omega')
= [1 - \xi_t(\omega|\omega') \beta \sum_{\omega'' \in \lambda(\omega) \cap \Omega_{t+1}} M_{t+1}(\omega, \omega'') u(x^*(\omega', \omega''))/u(\omega, \omega')]
\times u(x^*(\omega', \omega''))/u(\omega, \omega') = x^*(\omega', \omega'),
\]
which is satisfied by (1). This completes the proof of Proposition 1. □

In the equilibrium of Proposition 1 a producer must be indifferent between producing the surplus maximizing amount and producing zero. So, the punishment to an agent with a bad record, that is, an agent who is not in state \( g \), cannot be too severe. This implies that the discount factor necessary to sustain the equilibrium of Proposition 1 cannot be too low. When monitoring is as in Kocherlakota (1998), it is possible to construct an equilibrium implementing the first-best where producers who fail to produce the efficient amount suffer the worst punishment possible, permanent auctarky. Naturally, the discount factor necessary to sustain the first-best under the threat of permanent auctarky after a deviation is lower than the discount factor of Proposition 1.

4. Robustness

In this section we show that Proposition 1 does not depend on the continuum-population assumption and is robust to the introduction of: (i) noise in the monitoring technology; and (ii) unobservable shocks to preferences.

4.1. Finite population

The continuum population assumption makes it plausible to assume that an agent’s belief about the evolution of records in the population is independent of his private history. This is no longer true in a finite population setting. However, the strategy profile \( \Psi^* \) of Proposition 1 is such that an agent’s behavior is independent of his own history of play. The same argument in the proof of Proposition 5 in Takahashi (2010) then shows that \( \Psi^* \) together with a consistent belief system is a sequential equilibrium of the finite-population version of our model if agents are patient enough.

4.2. Imperfect record-keeping

We now show that Proposition 1 is robust to the introduction of noise in the record-keeping technology. Suppose that in every meeting an agent participates there is a small probability \( \varepsilon \) that the record-keeping technology incorrectly records either the agent’s action or the type of the agent’s partner. To fix things, suppose that there is a map \( \mu: \Omega \rightarrow \Omega \) with \( \mu(\omega) \in \lambda^{-1}(\omega) \) for all \( \omega \in \Omega \) such that in case a mistake occurs for an agent of type \( \omega \), the record-keeping technology records that the agent’s action is zero and the type of the agent’s partner is \( \mu(\omega) \); we can extend our argument to the case in which both the map \( \mu \) and the action that is recorded are random. Proposition 2 shows that if agents are patient enough, then we can approximate the first-best arbitrarily well as \( \varepsilon \) decreases to zero. The discount factor \( \beta^* \in (0, 1) \) is the same of Proposition 1.

10 The equilibrium is as follows. There are two states, \( g \) or \( b \). An agent produces zero if he is not a producer. A producer in state \( b \) produces zero. An agent of type \( \omega \) in state \( g \) who is matched to an agent of type \( \omega'' \in \lambda(\omega) \) produces \( x^*(\omega', \omega') \) to his partner if the partner is in state \( g \) and produces zero otherwise. All agents start in state \( g \) and state transitions are as follows. State \( b \) is absorbing. An agent of type \( \omega \) in state \( g \) stays in state \( g \): (i) if he is not a producer; (ii) he produces \( x^*(\omega', \omega') \) to his partner and his partner is of type \( \omega'' \in \lambda(\omega) \) and is in state \( g \); and (iii) he produces zero to his partner and his partner is of type \( \omega'' \in \lambda(\omega) \) and is in state \( b \). Otherwise, the agent moves to state \( b \).

11 Suppose that \( \mu(\omega) \) is random and denote the expectation with respect to \( \mu(\omega) \) by \( E_{\mu(\omega)} \). Suppose also that the action that is recorded in case of a mistake is drawn from a probability density function. Then the term \( V_{t+1}(b_{\omega''}) \) in Eqs. (4) and (5) in the proof of Proposition 2 becomes \( E_{\mu(\omega)}(V_{t+1}(b_{\omega''})) \) if a mistake occurs, the probability that the action recorded is not the surplus maximizing transfer is one. It is now easy to see that Proposition 2 holds in this more general case.
Proposition 2. Suppose that $\beta^* < \beta^{**} < 1$. If $\beta > \beta^{**}$, then for all $\delta > 0$ there exists an equilibrium in which the agents’ payoffs are within $\delta$ of the first-best payoffs as long as $\varepsilon$ is sufficiently small.

**Proof.** Consider the strategy profile $\Psi$ of Proposition 1 modified so that now

$$1 - \xi_t(\omega|\omega') = \beta(1 - \varepsilon) \sum_{\omega'' \in \Omega_t(\lambda^{-1}(\omega),\lambda(\omega'))} M_{t+1}(\omega, \omega'') u(x'(\omega, \omega'')(\omega, \omega'')).$$  \hspace{1cm} (3)

Notice that under $\Psi^*$ the agents’ payoffs converge to the first-best payoffs as $\varepsilon$ converges to zero. Let $\beta^* < \beta^{**} < 1$ and suppose $\varepsilon$ is small enough that $(1 - \varepsilon)\beta^{**} > \beta^*$. It follows from assumption (A1) that $\xi_t(\omega|\omega')$ is less than one no matter $t$, $\omega$, and $\omega'$ as long $\beta > \beta^{**}$.

We claim that $\Psi^*$ is an equilibrium if $\beta > \beta^{**}$. Indeed, let $\beta > \beta^{**}$ and consider an agent of type $\omega \in \Omega_t$ in period $t$. If, as before, $V_t(\omega|\theta)$ is the agent’s ex-ante expected present discounted payoff if he is in state $\theta$, an argument similar to the one in the proof of Proposition 1 shows that

$$- (1 - \beta)x(\omega, \omega) + (1 - \varepsilon)\beta V_{t+1}(\omega|g) + \varepsilon \beta V_{t+1}(\omega|b_{\mu(\omega)})$$

$$= (1 - \varepsilon)\beta V_{t+1}(\omega|b_{\omega}) + \varepsilon \beta V_{t+1}(\omega|b_{\mu(\omega)})$$ \hspace{1cm} (4)

for all $t \geq 1$ and all $\omega, \omega' \in \Omega_t$ such that $M_t(\omega, \omega') > 0$ and $\omega' \in \lambda(\omega)$ is a necessary and sufficient condition for $\Psi^*$ to be an equilibrium—clearly, the optimal action for an agent who is not a producer in a meeting is to produce zero. Now observe that (4) implies that

$$V_t(\omega|\theta) = \sum_{\omega'' \in \Omega_t(\lambda^{-1}(\omega),\lambda(\omega'))} M_t(\omega, \omega'')$$

$$\times \left[ (1 - \varepsilon)\beta V_{t+1}(\omega|g) + \varepsilon \beta V_{t+1}(\omega|b_{\mu(\omega)}) \right]$$

$$+ \sum_{\omega'' \in \lambda(\omega)} M_t(\omega, \omega'')$$

$$\times \left[ (1 - \varepsilon)\beta V_{t+1}(\omega|b_{\omega'}) + \varepsilon \beta V_{t+1}(\omega|b_{\mu(\omega)}) \right]$$

$$+ \sum_{\omega'' \in \lambda(\omega)} M_t(\omega, \omega'')$$

$$\times \left[ (1 - \beta)q_t(\omega, \omega'') u(x'(\omega, \omega'')(\omega, \omega'')) + (1 - \varepsilon)\beta V_{t+1}(\omega|g) + \varepsilon \beta V_{t+1}(\omega|b_{\mu(\omega)}) \right].$$ \hspace{1cm} (5)

Straightforward algebra then shows that (5) implies that (4) is equivalent to (3). This proves the desired result. \hfill \Box

4.3. Preference shocks

To finish, we show that Proposition 1 is also robust to the introduction of unobservables shocks to preferences. Suppose now that in every single-coincidence meeting a consumer receives a private shock to his utility function that changes the surplus maximizing transfer in the meeting by a random amount. More precisely, suppose that for all $t \geq 1$ and all $\omega, \omega' \in \Omega_t$, $\omega' \in \lambda(\omega)$, the surplus maximizing transfer is $x'(\omega, \omega') + z$, where $z$ is private to the consumer, independent across agents, and such that $\mathbb{E}[z] = 0$. The efficiency loss in a meeting between an agent of type $\omega$ and an agent of type $\omega' \in \lambda(\omega)$ when the latter produces $x$ units of the good to the former is $\mathbb{E}[u(x'(\omega, \omega') + z|\omega, \omega') - u(x'|\omega, \omega')]$. Let $[-\xi, \xi]$ be the common support of the random variables $z$. Proposition 3 shows that if agents are patient enough, then we can approximate the first-best arbitrarily well as the magnitude of the idiosyncratic shocks to preferences decreases to zero. The discount factor $\beta^* \in (0, 1)$ is the same of Proposition 1.

Proposition 3. Suppose that $\beta > \beta^*$. For all $\varepsilon > 0$ there exists $\tilde{E}$ such that if $\xi < \tilde{E}$, then an equilibrium exists in which the efficiency loss in every single-coincidence meeting is less than $\varepsilon$.

**Proof.** Suppose that $\beta > \beta^*$. Since the shocks are independent across agents and meetings are random conditional on the types of agents matched, the strategy profile $\Psi^*$ of Proposition 1 is still an equilibrium. \hfill \Box

5. Concluding remarks

In this paper, we show that money can fail to be essential even if monitoring is quite limited. This indicates that one must be careful when introducing monitoring in monetary models to allow for the coexistence of money and credit. In order to make our analysis directly comparable to Kocherlakota (1998), we do not allow for centralized trade. Thus, our environment does not include a more recent class of monetary models, based on Lagos and Wright (2005), in which trade alternates between centralized and decentralized markets. Extending our analysis to allow for centralized trade does not alter our results. Since in models with centralized trading agents can produce and consume the efficient amount in the centralized market, a strategy similar to the one we consider also achieves the first-best in the presence of centralized trading if discounting is sufficiently small.

Although our monitoring technology is much less demanding than the one considered in Kocherlakota (1998), it is still quite demanding if one is looking for a parallel with what we observe in actual markets. Indeed, while one can make the argument that information about past actions and preferences of an agent can be accessed in real economies (e.g., past purchasing histories recorded by credit bureaus), the same cannot be said about information on the preferences of previous trading partners. The study of a notion of monitoring which matches what one sees in reality is an interesting direction for future research.

Acknowledgments

We thank Satoru Takahashi for comments on an early draft of this paper and three anonymous referees for their comments and suggestions. We also thank seminar participants at Illinois, Paris X, and PUC-Rio, and audience members at the SAET meetings in Italy and the Workshop on Money, Banking, and Payments at the Federal Reserve Bank of Chicago for their feedback. The authors gratefully acknowledge financial support from CNPq.

References


---

This is true even if the shocks are serially correlated.