Information, learning, and the stability of fiat money

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Abstract
We analyze the stability of monetary regimes in an economy where fiat money is endogenously created by the government, information about its value is imperfect, and learning is decentralized. We show that monetary stability depends crucially on the speed of information transmission in the economy. Our model generates a dynamic on the acceptability of fiat money that resembles historical accounts of the rise and eventual collapse of overissued paper money. It also provides an explanation of the fact that, despite its obvious advantages, the widespread use of fiat money is only a recent development.

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All of the governments in China between 1100 and 1500 succumbed to this temptation, and their monetary histories have a strong family resemblance. In each there was a period of inflation, usually quite a long one. Except in the case of the Southern Sung dynasty, which was conquered by the Mongols before the evolution was completed, the use of paper money was, in each case, eventually abandoned. This abandonment of the use of paper money in China is the most interesting feature of the history of paper money in China. (Gordon Tullock, 1957)

1. Introduction

In recent years, a large body of work has been devoted to the study of economies where the value of money arises endogenously. Following Kiyotaki and Wright (1989, 1993), most of the literature takes as a starting point the assumption that the amount of money in circulation is exogenously given. Less attention has been given to the questions that arise when the quantity of money is endogenously determined; i.e., when some of the agents inside the economy are responsible for money creation. Of particular importance is the emergence and stability of fiat money regimes. One of the few contributions in this front is Ritter (1995), where the transition from a barter to a fiat money economy is analyzed. He considers an economy where a coalition of agents, which he identifies as the government, is allowed to issue money, and shows that the size and patience of this coalition must be large for the transition to take place. The government must care about the future if money is to have value, so patience is important. Size plays a role as it allows the government to internalize the costs of overissue.

While providing a framework where the emergence of fiat money occurs endogenously, Ritter does not address the concomitant issue of its stability; i.e., whether money remains in circulation in the long-run or not. In his framework, fiat money always stays in the economy once it is introduced. There is, however, varied evidence that in the past paper money issued by governments was subject to much instability. Tullock (1957) and Yang (1952) describe a succession of failures in the transition to paper money in China due to overissue. A similar pattern of overissue followed by abandonment of paper money also took place in the United States. For an example, see Galbraith’s (1975) account of the monetary experience in the Massachusetts Bay Colony in 1690 and in various American colonies during the mid-18th century. The rise and collapse of paper money seems to be such a common phenomenon throughout history that, according to Friedman and Schwartz (1986), a continuous and widespread use of fiat money can be considered only a 20th-century development.

In what follows, we build a model where monetary stability depends on both exogenous and endogenous factors. We take a simplified version of Kiyotaki and Wright (1989) as our starting point. The main difference is that the amount $m$ of money in circulation is determined by a self-interested agent, the government, and is not known in advance by the other agents in the economy. We refer to the value of $m$ as the monetary regime. These agents can react against the government by not accepting money if they think its value is low. Information about $m$ is obtained from the trade meetings they participate; i.e., from their private experience. Hence, the same technology that governs trade and makes money

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1See also Sik-Kim (2001) and Norwood (2003).
essential in our environment, random and anonymous pairwise meetings, is also used to describe the transmission of information. Related papers always assume that there is a time when the quantity of money in circulation is revealed to all agents.\footnote{That is the case, for example, in Wallace (1997) and Katzman et al. (2003). Outside the money literature, Wolinsky (1990) is the closest to us in how the transmission of information is modeled. A key difference, and central to our analysis, is that we allow the speed of information transmission in our environment to change.}

The exogenous factors affecting the stability of money are the government’s patience and the speed of information transmission. The latter is parameterized by the number of trade meetings per unit of time. The endogenous factor comes from the ability agents have to over time learn the nature of the monetary regime. In an environment where learning happens slowly, an impatient government can exploit the agents’ misinformation and overissue, while maintaining the value of money in the short-run. Agents eventually realize the government’s actual behavior and monetary trade breaks down. However, it takes time until a complete breakdown of trade happens. This result matches with Tullock’s observation on the history of paper money in China. In this case, monetary stability is only feasible with a patient government. On the contrary, when agents accumulate a lot of information in a short period of time, even impatient governments prefer not to overissue in order to avoid a quick breakdown of monetary trade.

Our model then provides an informational rationale for the late emergence of fiat money. Societies’ ability to gather information and learn about the state of the economy increased over time. In modern economies, the dissemination of information is much faster than in the past, and so a government’s temptation to overissue should be less pronounced. Therefore, the late widespread implementation of fiat money is not necessarily a result of exogenous factors like an increase in the government size and patience. It can, instead, be the result of an increase in the society’s ability to monitor the behavior of the money issuer.

This article is structured as follows. In the next section we describe the environment and the equilibrium concept used. We also make some remarks about our modelling assumptions. In Section 3 we discuss the agents’ problem, describe the dynamics of the economy under distinct monetary regimes, and solve the government’s problem. In Section 4 we discuss how changes in the degree of information transmission affect the behavior of the government. In Section 5 the existence of equilibria of the type introduced in Section 2 is established. Their properties are also determined. Section 6 concludes. Two appendices collect several proofs that are omitted from the main text.

2. The model

We start with the basic setup. Before proceeding to a detailed description of strategies, payoffs, and the equilibrium notion employed, we discuss some of our modelling assumptions.

2.1. Basic setup

Time is discrete and indexed by $t$. The economy starts in period zero with one large infinitely lived agent that we call the government. The discount factor of the government, denoted by $\delta$, is its private information. It is determined in $t = 0$ by a draw from a distribution function $F$ with support $[0, 1]$ and a density $f$. The economy is also populated
by a large number of infinitely lived small agents that we describe next. To simplify the exposition, from now on we refer to the small agents simply as agents.

In \( t = 1 \) a continuum of mass one of agents is born. Moreover, in any \( t \geq 2 \) the agents born in \( t - 1 \) give birth to another agent. In this way the mass of agents increases by the same amount in every period. We adopt the following terminology. For any \( t \geq 1 \), an agent born in that period is referred to as newly born, while an agent born before \( t \) is referred to as mature. Beginning in \( t = 2 \) a newly born agent inherits his parent’s private history. We show below that an agent’s private history is what determines his belief about the government’s behavior. This plays an important role in our model, as we explain in the next subsection. All agents have the same discount factor \( \beta \in (0, 1) \).

Besides being newly born or mature, an agent has a type that is determined in his first period of life. There are \( K > 2 \) of them, each one corresponding to one of the \( K \) possible types of goods, all indivisible, that can be produced in the economy. In every period \( t \) the fraction of newly born agents which is of a certain type \( k \in \{1, \ldots, K\} \) is the same, \( 1/K \). Agents of type \( k \) can only consume a type \( k \) good, their preferred good.

Production works in the following way. Each newly born agent receives a non-perishable endowment. For all \( t \geq 1 \) the agents have two options. They can stay in autarky and use their endowment to obtain some flow utility, to be specified below, or they can enter the market and use their endowment in the production of goods. In the market, an agent of type \( k < K \) can only produce a good of type \( k + 1 \), while an agent of type \( K \) can only produce a good of type 1. The cost of production is zero. We refer to the good an agent can produce in the market as his endowment good. Once an agent chooses the market, his endowment becomes useless for production in autarky; i.e., his autarky flow payoff drops to zero. Finally, each agent can hold at most one unit of either goods or money at any point in time.

We show below that in the type of equilibria considered, staying in the market always yields some positive flow utility. Therefore, once an agent enters the market, he never goes back to autarky. A maintained assumption is that if an agent is indifferent between the market and autarky, he chooses the market. The same results are obtained if we assume the opposite. Since no new information about the market is received while in autarky, we then have that an agent who chooses autarky in one period chooses autarky forever after. Therefore, in what follows, each newly born agent effectively makes a once and for all decision between the market and autarky.

The government derives utility from the consumption of all \( K \) goods in the economy, but cannot produce any of them. It has, however, the technology to print indivisible units of fiat money. These units provide no direct benefit, but can be offered in exchange for goods. In every period after \( t = 0 \) the government approaches a fraction \( m \) of the agents who decided to enter the market in that period and exchanges one unit of fiat money for their corresponding endowment good. In other words, the government derives utility from seigniorage. The value of \( m \) is restricted to the set \( \{m_L, m_H\} \), with \( m_L < m_H \), but no agent in the economy observes the government’s choice. One important assumption that we make is that once the government chooses the value of \( m \) in \( t = 1 \), it cannot change it afterwards. We also assume that when indifferent between \( m_L \) and \( m_H \), the government chooses \( m_L \). Nothing changes if we assume the opposite. We say the monetary regime is soft if \( m = m_H \) and tight if \( m = m_L \).

The market is organized as follows. There are \( K \) distinct sectors, each one specialized in the exchange of one of the \( K \) possible goods. Agents can identify sectors, but inside each
one of them they are pairwise matched under an uniform random matching technology.\footnote{This market structure is adopted for its simplicity. A more general meeting technology yields the same results.} Since $K > 2$, there are no double coincidence of wants meetings. An agent, however, can trade his endowment good for money and use money to buy his preferred good. If an agent has no money, he goes to the sector that trades his endowment good and searches for an agent with money. Once he meets such an agent, he produces in exchange for money. We assume that only one unit of the good can be produced per meeting. If he has money, he goes to the sector that trades his preferred good and searches for an agent with it. Once he meets such an agent, he surrenders his money in exchange for his preferred good. As soon as an agent obtains one unit of his preferred good he consumes it and obtains utility $u > 0$. Any agent going to the market faces $n \geq 2$ rounds of meetings per period, where $n$ is fixed. We assume that agents do not discount within a trading period.\footnote{Notice that despite the cost of production being zero for the agents, gift-giving is not an equilibrium in this environment. The reason for this is the market structure adopted. In fact, if gift-giving were to be an equilibrium, all agents would like to stay in the market where their preferred good is traded. This, however, rules out single-coincide of wants meetings, making gift-giving impossible.}

In the above description of the market, we took the behavior of the agents and the government as given. It is possible, in a natural way, to model the market environment itself as a game involving the agents and the government. As Section 2.4 makes clear, this game has an equilibrium where the agents always exchange their endowment good for one unit of money if approached by the government and their behavior in the market (sectors to visit and trading decisions) is as above. Since we are interested in the agents’ market/autarky decisions, and in how these interact with the choice of monetary regime by the government, we omit the above details for the sake of brevity.

2.2. Discussion

An agent’s experience in a given meeting is important as it allows him to make inferences about the government’s behavior. This experience has two components: the money and good holdings of his partner and the outcome of the trade process between them. When we assume that both money and goods are indivisible, the money holdings of the agent’s partner become the only relevant piece of information for him. Even though this implies a very simple exchange process, it greatly simplifies the updating of beliefs while still capturing the idea that agents learn from their private histories. If goods and/or money are divisible, we have to specify how prices are formed in a trade meeting. Since prices depend on private histories, they also transmit information about the government’s behavior. The updating of beliefs becomes more complex as a result. The interaction between the agents and the government becomes more complex as well. For example, if goods are indivisible but money is divisible, agents with different beliefs about the monetary regime demand different amounts of money in exchange for one unit of their good. Without indivisible goods and money the analysis becomes much harder and, given our goals, we believe it does not generate additional insights.

The assumption of indivisible money comes at a cost, however. In an economy with a constant population, it implies that a continuous increase in the amount of money in circulation is not sustainable. If new money is injected in every period, at some point in time enough agents in the market hold money to make it worse than autarky. This happens
regardless of what was the initial comparison between the two. In other words, it is not possible to make a distinction between a good and a bad monetary regime. By adding population growth, we avoid this difficulty. We need, however, to assume that an agent’s personal experience in the market can be passed to a member of the next generation (his progeny). This assumption is important, since the indivisibility of both money and goods implies that the only way the government can be discouraged from overissuing is through a reduction in the fraction of agents of future generations that accept money.

There are alternative mechanisms to deal with endogenous money creation in the presence of indivisibility. For example, we can assume that: (i) the government buys back the stock of money at the end of each period and re-injects the money in the economy at the beginning of the next period; (ii) the government receives utility from storing goods between periods. This route avoids the need for the inclusion of population growth and, in turn, the need for personal experiences to be passed from one generation to the next. However, it is too stylized to provide a model of seigniorage revenue that maps naturally into the phenomena we want to address. This alternative approach is studied in detail in Araujo and Camargo (2005).

2.3. The government’s payoffs and strategies

Let \( \mu_t(m) \) be the measure of agents that enters the market in period \( t \) as a function of the government’s choice of \( m \) in period 1. Besides their dependence on \( m \), these measures depend on the agents’ behavior. The amount of goods the government consumes in \( t \) is then \( \mu_t(m)m \). In other words, \( \mu_t(m)m \) is its period \( t \) revenue from seigniorage. Consequently, the government’s lifetime utility as a function of its discount factor \( \delta \) is

\[
U(m, \delta) = (1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} \mu_t(m)m
\]

if \( \delta < 1 \). For an infinitely patient government we use the limit of means criterion. The details can be found in Appendix B.

A strategy for the government is a (measurable) function \( m_L : [0, 1] \to [0, 1] \), where \( m_L(\delta) \) is the probability that a government with discount factor \( \delta \) chooses a tight monetary regime.

2.4. Agents’ payoffs and strategies

We now specify the agents’ payoffs and give a description of their possible strategies. We begin by describing how their (lifetime) payoffs from entering the market depend on \( m \). An assumption in all that follows is that there is always a positive measure of agents in the market, otherwise the random matching process described above does not make sense. In Section 5 we show that there are conditions on the model parameters that justify this assumption.

First notice that once an agent enters the market, he never leaves it. This is a direct consequence of two facts. First, an agent’s flow payoff from staying in the market is, in any period, greater than zero. In fact, the number of market meetings in each period is at least two and the fraction of agents holding (one unit of) money is never zero nor one (because \( m \in (0, 1) \)). Therefore any agent in the market has a positive probability of acquiring his preferred good at least once in a period. Second, once an agent enters the market, his
autarky flow payoff is reduced to zero. Notice that this result implies that $m$ is also the fraction of agents in the market that hold money.

Suppose then that the fraction of agents with money in the market is $m$. Let $w^i_{j,t}$ indicate the period $t$ (lifetime) expected payoff of an agent with $j$ units of money right before his $i$th meeting, where $j \in \{0, 1\}$ and $i \in \{1, \ldots, n\}$. Let $w^{i+1}_{j,t}$ indicate the period $t$ expected payoff of an agent that has $j$ units of money at the end of the market meetings. Since the market environment is stationary, the above payoffs are independent of $t$. Then,

$$w^i_1(m) = mw^{i+1}_1(m) + (1 - m)[u + w^{i+1}_0(m)],$$

$$w^i_0(m) = mw^{i+1}_1(m) + (1 - m)w^{i+1}_0(m).$$

An agent with money right before his $i$th trade opportunity in a given period has probability $m$ of meeting another agent with money. In this case no trade occurs and he moves to the next round of meetings with money. With probability $(1 - m)$ he meets an agent without money, obtains utility $u$, and moves to the next round of meetings without money. A similar interpretation holds for the second equation.

Since $w^{i+1}_0(m) = \beta w^i_1(m)$, we can solve the above system of equations recursively to obtain that

$$w_1(m) = w^1_1(m) = (1 - \beta)^{-1}[(n - 1)m(1 - m)u + (1 - m)u - \beta(1 - m)^2u],$$

$$w_0(m) = w^1_0(m) = (1 - \beta)^{-1}[(n - 1)m(1 - m)u + \beta m(1 - m)u]$$

are the payoffs to an agent from entering the market with money and without money, respectively. Notice that $w_1(m) = w_0(m) + (1 - m)u > w_0(m)$, and so an agent is always willing to exchange one unit of his endowment good for one unit of money since its cost of production is zero. An agent’s lifetime expected payoff $w(m)$ from going to the market as a function of $m$ is then equal to

$$w(m) = (1 - m)w_0(m) + mw_1(m) = (1 - \beta)^{-1}nm(1 - m)u.$$  

Let us now describe payoffs in autarky. When in autarky, an agent can use his endowment as an input to a production technology. Within each period there are $n$ production possibilities, where $n$ is, as above, the number of market meetings in each period. Each good produced in autarky, the so-called autarky good, yields utility $a < u$.\(^5\)

While all agents derive the same utility from consuming their preferred good in the market, they differ in how much utility they obtain from the consumption of their autarky good. In what follows, a family is the collection of all agents whose genealogy can be traced to a particular agent born in period 1. The value of $a$ for each family in the economy is determined in $t = 1$ by an independent draw from a distribution function $G$ with density $g$ and support $[a, a]$.\(^6\) From now on we refer to $a$ as an autarky value.

The flow payoff from staying in autarky is then equal to $na$, and so the flow payoff gain or loss from going to the market is

$$(1 - \beta)w(m) - na = nm(1 - m)u - na.$$  

\(^5\)We can think that the autarky good produced by an agent is of the type he likes, but there is a disutility $c = u - a$ in producing each unit of it.

\(^6\)This assumption is important for the existence argument of Section 5.
Instead of looking at this difference, we consider the flow payoff gain or loss per unit of market meetings,

\[
\frac{1}{n}[(1 - \beta)w(m) - na] = m(1 - m)u - a. \tag{8}
\]

We denote the above quantity by \(v(m)\). This normalization is useful when we make comparisons between the market and autarky for different values of \(n\). Indeed, this difference is independent of \(n\), the number of market meetings. Therefore, a change in \(n\) only affects the informational content of the market. As we are going to see below, a consequence of this fact is that an agent’s market/autarky decision is independent of \(n\).

We are interested in a situation where an agent’s expected flow payoff from choosing the market decreases when the fraction of people with money increases. Otherwise, there are no trade-offs involved when the government decides on the value of \(m\). Therefore, we restrict attention to the region of parameters where \(v(m)\) decreases with \(m\). This leads to:

**Assumption 1.** \(m_H > m_L \geq \frac{1}{2}\).

Our next assumption states that the values of \(m_H\) and \(m_L\) correspond, respectively, to non-existence and existence of monetary equilibrium under full information. In other words, if agents know that the government chooses \(m_H\), they prefer autarky. On the other hand, if they know that the government chooses \(m_L\) they prefer the market.

**Assumption 2.** \(m_H(1 - m_H) < a/u < m_L(1 - m_L)\) for all \(a \in [a, \bar{a}]\).

Now that we know the full information flow payoffs, we can describe strategies and payoffs in the incomplete information case, the case of interest. Observe that agents who stay in autarky do not accumulate any new information about the nature of the monetary regime. Moreover, the market environment is stationary. Hence, if a newly born agent chooses autarky, he stays there forever after, given that in all subsequent periods he faces the same decision problem.

The only piece of information available to newly born agents when they make their (once and for all) market/autarky decision is the private history they inherit from their parents. Besides, what matters for this decision is the expected lifetime value of entering the market compared with the lifetime value of staying in autarky. Consequently, these private histories can be summarized by \(\theta(h)\), the posterior belief that the monetary regime is tight given a private history \(h\). It may be that \(h\) is empty. That happens for agents in period 1. In this case,

\[
\theta(h) = \theta_0 = \int_{[0,1]} m_L(\delta)f(\delta) \, d\delta, \tag{9}
\]

the ex ante probability that the monetary regime is tight; i.e., the probability of a tight monetary regime prior to the determination of the government’s discount factor.

A behavioral strategy for an agent born in \(k \in \mathbb{N}\) can then be summarized by a (measurable) function \(\tau : [0, 1] \rightarrow [0, 1]\) where \(\tau(\theta)\) is the probability of entering the market when \(\theta\) is his belief that the monetary regime is tight.\(^7\) The payoff to such a strategy is

\[
V(\tau, \theta) = (1 - \beta)^{-1}[(\theta v(m_L) + (1 - \theta) v(m_H))] \tau(\theta). \tag{10}
\]

\(^7\)Rigorously speaking, a behavioral strategy for an agent born in period \(k\) is an infinite sequence \(\{\tau_{k,t}\}_{t=1}^{\infty}\), where \(\tau_{k,t} : [0, 1] \rightarrow [0, 1]\) is the map describing the probability of entering the market in period \(k + t - 1\) as a function of the belief that the monetary regime is tight.
2.5. Equilibrium

Denote by $\tau_{k,a}$ a behavioral strategy for an agent born in $k$ with autarky value $a \in [\underline{a}, \overline{a}]$. Assume that agents use Bayes rule to update their beliefs about the monetary regime. We are going to be precise about how this updating takes place in the next section. A collection $\{\tau_{k,a}\}$ of behavioral strategies together with a choice of $m$ by the government determines the measures $\mu_t(m)$. An equilibrium of this game is then a pair $(m^*_L, \{\tau^*_{k,a}\})$ such that: (i) the government behaves optimally, that is,

$$E_m[U(m, \delta)|m^*_L(\delta)] = \sup_{m_L} E_m[U(m, \delta)|m_L(\delta)]$$

for almost all $\delta \in [0, 1]$, where $E_m$ denotes expectation with respect to $m$; (ii) agents are sequentially rational and behave optimally, that is,

$$V(\tau^*_{k,a}, \theta) = \sup_{\tau} V(\tau, \theta)$$

for all $k \in \mathbb{N}$, all $\theta \in [0, 1]$, and almost all $a \in [\underline{a}, \overline{a}]$. The fact that agents are sequentially rational is what implies that their choice in their first period of life is a once and for all decision. Notice the assumption that agents with the same autarky value and born in the same period follow the same strategy. Notice also the restriction to equilibria where a positive measure of agents is in the market in every period. From above we know that this is achieved if a positive measure of agents enters the market in period 1. This condition rules out the uninteresting non-monetary equilibrium where no agent ever enters the market.\footnote{It also rules out equilibria where a period with a positive measure of agents in the market is followed by a period in which there is no one in the market. Such equilibria, if they exist, are only possible because we assume that the agents’ behavior in the market is exogenously given. If we were to endogenize their market behavior, sequential rationality would rule this out.} To finish notice that our tie braking rules for both the agents and the government imply that we are also restricting attention to pure strategy equilibria.

3. Individual behavior, dynamics, and government’s behavior

We begin by determining each individual agent’s behavior. We then determine how the agents’ aggregate behavior depends on the nature of the monetary regime. This is done by showing that the problem of obtaining the measures $\mu_t(m)$ is the same as computing certain survival probabilities in a related two-armed bandit problem, and using certain results from Banks and Sundaram (1992). We establish that if the monetary regime is soft, then over time the measure of agents entering the market converges to zero. Therefore, the same happens with the fraction of the population with money. If, instead, the monetary regime is tight, the measure of agents entering the market in any period is bounded away from zero. As a consequence, the same holds for the fraction of the population with money. Notice that in both monetary regimes the amount of money in circulation is increasing over time. We finish by determining the government’s choice of monetary regime as a function of its discount factor.
3.1. Individual behavior

Suppose a newly born agent with autarky value $a$ has belief $\theta$ that the monetary regime is tight. His lifetime payoff from entering the market is

$$\left(1 - \beta\right)^{-1}\left[\left(\theta m_L(1 - m_L) + (1 - \theta)m_H(1 - m_H)\right)u - a\right].$$

(13)

Remember that we are considering payoffs per unit of market meetings. Let $\theta^*(a)$ be the value of $\theta$ for which the above expression is equal to zero; that is, let

$$\theta^*(a) = \frac{a - m_H(1 - m_H)u}{m_L(1 - m_L)u - m_H(1 - m_H)u}.$$  

(14)

Notice that $\theta^*(a)$ is increasing in $a$ and that $\theta^*(a) > 0$ by Assumption 2. From the previous section, a newly born agent enters the market if, and only if, $\theta \geq \theta^*(a)$. Summarizing, we have the following result. Notice that it holds even without the symmetry assumption made about the agents’ behavior.

**Proposition 1.** Suppose there is always a positive measure of agents in the market. A newly born agent enters the market if, and only if, his belief $\theta$ is greater than or equal to $\theta^*(a)$, where $a$ is his autarky flow payoff per unit of market meetings. Once an agent enters the market he never leaves it. The same is true if he chooses autarky.

In order to have a positive measure of agents in the market at all points in time it is necessary and sufficient that a positive measure of agents enters the market in $t = 1$. Therefore, it must be that $\theta^*(a) < \theta_0$. We also need $\theta_0 < 1$, as there is no equilibrium of this game with $\theta_0 = 1$ and a positive measure of agents in the market in every period. If this were the case, then in every period all newly born agents enter the market independently of the monetary regime. Hence, the government has an incentive to choose $m_H$ regardless of its discount factor, contradicting the assumption that $\theta_0 = 1$.

3.2. Aggregate behavior

We introduce the following auxiliary decision problem. To each family in the economy there is associated an infinitely lived myopic agent who is in charge of the decision making for this family. Call him the family lawyer. In each period he makes a once and for all market/autarky decision for the newly born agent of the family he is in charge. Moreover, in each period he only observes the market experience of the newly born agent of his family. This means that for all $t$ the family lawyer and the family member born in $t$ have the same private history, and so the same belief $\theta$ about the monetary regime. Finally, in each period the family lawyer’s payoff is equal to the flow payoff of the newly born agent of his family. In other words, his payoff is zero if he sends the newly born agent to autarky and $[\theta m_L(1 - m_L) + (1 - \theta)m_H(1 - m_H)]u - a$ otherwise, where $a$ is the autarky value of the family.

Consider then a lawyer to a family with autarky value $a$. Let $\eta_t(m, a)$ be the probability, as a function of $m$ and $a$, that in $t$ he sends to the market the newly born member of this family. This probability is also a function of $\theta_0$, but for now we omit this dependence. We claim that the measure of agents who, in the original problem, enter the market in $t$ when
the government chooses $m$ is

$$
\mu_t(m) = \int \eta_t(m, a) g(a) \, da.
$$

(15)

To see why, denote by $L$ the family lawyer and by $A_t$ the member of this family that is born in $t$. Then, in the original problem, the choice of $A_t$ in $t$ is the same as the choice of $L$ in $t$. They have the same private history and the same flow payoffs. The fact that there is no aggregate uncertainty plays an important role. It implies that the probability that an agent born in $t$ has a given private history is the same as the measure of newly born agents having, in $t$, this same history. Another factor that plays an important role is that these probabilities are independent of the choices of the other agents. This is true because, by assumption, there is a positive measure of agents in the market in each period.

The family’s lawyer problem is an example of a two-armed bandit with one known arm. The known arm is autarky, while the unknown arm is the market. Its value is determined by the government’s choice of monetary regime. It is natural to look at this problem as a Markovian Decision Problem where the state is the belief $\theta$ and it changes through Bayesian updating. Let us now be precise about how this belief updating takes place.

Fix a family and refer to its member born in $k$ as the generation $k$ member. Consider now a family member that is born in some period $t$. Since money and goods are indivisible, they can only be exchanged on a one-to-one basis. Moreover, agents hold either one unit of money or one unit of their endowment good at any point in time. Therefore, the only relevant piece of information for the family lawyer in $t$ is the record of money holdings of the partners of the generation $k$ members in their respective first periods of life, where $k \in \{1, \ldots, t - 1\}$. This includes their meetings with the government, where exchanging their endowment good for money is interpreted as the government having one unit of money. This is also the only relevant piece of information in the original problem for a generation $t$ member when he makes his period $t$ once and for all market/autarky decision. The way $\theta$ changes in both problems is then as follows. If a newly born agent goes to autarky, his belief $\theta$ (and the family lawyer's belief in the modified problem) does not change. If, instead, he enters the market, his updated belief is

$$
B(c, \theta) = \frac{\theta m_G(1 - m_L) \theta_c \theta_c^{n+1-c} + (1 - \theta) m_H(1 - m_H) \theta_c \theta_c^{n+1-c}}{\theta m_L(1 - m_L) \theta_c \theta_c^{n+1-c} + (1 - \theta) m_H(1 - m_H) \theta_c \theta_c^{n+1-c}},
$$

(16)

where $c \in \{0, \ldots, n + 1\}$ is the number of meetings with money he faces. If we exclude the meeting with the government, beliefs for mature agents change in exactly the same way in both problems.

We are now ready to prove the main result of this subsection, namely, that

$$
\lim_{t \to \infty} \mu_t(m_H) = 0 \quad \text{and} \quad \lim_{t \to \infty} \mu_t(m_L) > 0
$$

(17)

if $\theta_0 \in (\theta^*(q), 1)$. For this, let $\Omega = \{0, \ldots, n + 1\} \cup \{A\}$. As argued above, an element of $\Omega$ contains the relevant part of a family lawyer’s private history in any given period. If he sends to autarky a newly born member of the family he is in charge, he observes nothing. If, instead, he sends this agent to the market, he observes $c \in \{0, \ldots, n + 1\}$ meetings with money. The set of period $t$ histories for a family lawyer is then $\Omega^t = \times_{t=1}^{t-1} \Omega$ if $t > 1$ and $\Omega^1 = \emptyset$ if $t = 1$. The set of all infinite histories a family lawyer can face in this environment is then $\Omega^\infty = \times_{t=1}^\infty \Omega$. 


For each \( m \) and \( h' \in \Omega' \), a family lawyer has a belief \( \theta(h', m) \) about the value of \( m \). It is possible to construct, for each \( t \geq 1 \), a random variable \( \theta_t(m) : \Omega^\infty \to [0, 1] \) describing the distribution of period \( t \) beliefs for an arbitrary family lawyer as a function of the monetary regime. Therefore

\[
\eta_t(m, a) = \Pr[\theta_t \geq \theta^*(a) | m].
\]

It is straightforward to show that \( \eta_t(m, a) \) is a measurable function of \( a \), so that the integral in (15) is well-defined. As autarky is an absorbing state, both \( \{\eta_t(m_H, a)\} \) and \( \{\eta_t(m_L, a)\} \) are non-increasing sequences. Hence the same is true of \( \{\mu_t(m_L)\} \) and \( \{\mu_t(m_H)\} \), and so these sequences must converge, as they are bounded above and below.

Let us first establish that for almost all \( a \in [\underline{a}, \overline{a}] \), \( \lim_{t \to \infty} \eta_t(m_H, a) = 0 \). Suppose not, so that a positive measure of family lawyers sends an infinite number of members of their respective families to the market. Any lawyer that does so learns the true value of \( m \) by the consistency of Bayes estimates for the multinomial distribution, see DeGroot (1970). In particular, when \( m = m_H \), the belief of this lawyer converges to zero with probability one. However, any lawyer always sends newly born agents of the family he is in charge to autarky if his belief is smaller than \( \theta^*(a) \). Since \( \theta^*(a) > 0 \) by assumption, we have a contradiction.

We now make use of Theorem 5.1 in Banks and Sundaram (1992). It says that the following holds for any multi-armed bandit with independent arms and finite type spaces. Suppose that at some point in time an arm is selected by an optimal strategy. In this case there exists at least one type \( T \) of this arm such that if this arm’s type is indeed \( T \) then, with non-zero probability, this arm remains an optimal choice forever after. In the family lawyer’s problem, the choices of \( m \) by the government determine the type of market arm. Hence, if a lawyer in charge of a family with autarky value \( a \) sends the generation 1 member of this family to the market, there exists \( \pi(a) > 0 \) and \( m \in [m_L, m_H] \) such that \( \eta_t(m, a) > \pi(a) \) for all \( t \geq 1 \). By hypothesis, there exists \( a' \in (\underline{a}, \overline{a}) \) such that if \( a \in [\underline{a}, a'] \), then \( \theta^*(a) < \theta_0 \). Moreover, we know from above that \( \eta_t(m_H, a) \to 0 \) for all \( a \in [\underline{a}, \overline{a}] \). Therefore, if \( a \in [\underline{a}, a'] \), then \( \eta_t(m_L, a) > \pi(a) > 0 \) for all \( t \geq 1 \). Consequently, \( \lim_{t \to \infty} \eta_t(m_L, a) \geq \pi(a) > 0 \). The desired result follows immediately from the Lebesgue dominated convergence theorem, see Kolmogorov and Fomin (1970).

**Proposition 2.** Suppose \( \theta_0 \in (\theta^*(a), 1) \). Then \( \lim_{t \to \infty} \mu_t(m_H) = 0 \) and \( \lim_{t \to \infty} \mu_t(m_L) = \mu_L > 0 \).

Therefore, even though it is true that overissued money circulates in the economy, the fraction of the population that uses it in period \( t \),

\[
\frac{1}{t} \sum_{k=1}^{t} \mu_k(m_H),
\]

converges to zero as \( t \to \infty \). Within the Kiyotaki–Wright framework, due to the indivisibility assumptions, this is a very natural way to describe a monetary collapse. Note that this process of overissue and abandonment of paper money may take a long time if information transmission in the market is low; i.e., if the number of market meetings per unit of time is small.

---


10This is a consequence of Toeplitz’s Lemma, see Shiryaev (1996).
3.3. Government’s behavior

In the previous subsection we determined how the measures \( \mu_i(m) \) evolve when all agents in the economy follow the decision rule described by Proposition 1. We now determine how the government’s decision is affected by these dynamics.

First observe that an infinitely patient agent only cares about what happens in the long-run. Therefore, from the previous proposition, it prefers a tight monetary regime to a soft one. Suppose then that \( d_o = 1 \), so that

\[
U(m; d) = \left( \frac{1}{C_0} \right)^\frac{1}{X_1} \frac{1}{d} \sum_{t=1}^\infty \delta^{t-1} \mu_i(m)m.
\]  

(20)

By choosing a soft monetary regime, the government enjoys a higher flow of utility in the first periods, when the agents are still relatively uninformed. The government knows, however, that \( m_t(m_H) \neq 0 \), and so its revenue from seigniorage converges to zero. The alternative is to choose \( m_L \). In this case the revenue from seigniorage, though smaller at the beginning, is bounded away from zero. Therefore, a sufficiently patient government should choose \( m_L \), while an impatient one should go with \( m_H \) despite the fact that monetary trade collapses. We have the following result. Its proof can be found in Appendix A.

**Proposition 3.** Suppose \( y_0^2/\beta(a) > y_0 \). There exists a unique \( \delta^* \in (0, 1) \) such that

\[
U(m_L; d) < U(m_H; d) \text{ if } \delta > \delta^* \quad \text{and} \quad U(m_L; d) > U(m_H; d) \text{ if } \delta < \delta^*.
\]

In other words, if the agents follow the cutoff belief rule described by Proposition 1, the government should follow a cutoff strategy as well: Choose \( m_H \) if \( \delta > \delta^* \) and \( m_L \) if \( \delta < \delta^* \), where \( \delta^* \in (0, 1) \). Notice that our tie braking assumption about the government’s behavior implies that it chooses \( m_L \) when \( \delta = \delta^* \). Also notice that \( \delta^* \) depends on \( \theta_0 \) and \( n \).

4. Information transmission

The purpose of this section is to study how \( n \), the number of market meetings per period, affects the behavior of the government. This requires a slight change of notation, as we now need to make explicit the dependence of both the agents’ and the government’s problem on \( n \) and \( \theta_0 \). For this reason, the measures of agents entering the market in a given period \( t \) are now denoted by \( \mu_i(m; n; \theta_0) \) and the government’s payoff is now denoted by \( U(m, \delta; n; \theta_0) \).

We know that a change in \( n \) only affects the degree of information transmission in the market, not the market’s gain or loss relative to autarky. This is reflected in the fact that the cutoff beliefs \( \theta^*(a) \) are independent of \( n \). Specifically, as \( n \) increases, the market becomes more informative, and so the agents there learn faster about the nature of the monetary regime. Our intuition then suggests that a soft monetary regime breaks down in a shorter amount of time, thus reducing the incentives of any government to overissue. The next proposition formalizes this. Its proof can be found in Appendix A.

**Proposition 4.** Let \( \delta = (m_H - m_L)/m_H \) and suppose that \( \theta_0 \) is such that \( \theta_0 > \theta^*(a) \) and \( \overline{\theta}_0 < 1 \). Then, for all \( \delta > \delta \) there exists \( n(\delta) \) such that \( U(m_L; \delta; n; \theta_0) > U(m_H; \overline{\delta}; n; \theta_0) \) if \( \theta_0 \in [\theta_0, \overline{\theta}_0] \) and \( n \geq n(\delta) \).
Observe that $(1 - \delta)m_H$ is a lower bound on the payoff a government with discount factor $\delta$ obtains when it chooses a soft monetary regime. If the same government chooses a tight monetary regime, an upper bound for its payoff is $m_L$. Therefore, no matter $n$, a government with discount factor no greater than $\hat{\delta}$ always chooses $m = m_H$. Notice also that $n(\delta)$ is independent of $\theta_0$, but it does depend on the choice of the interval $[\theta_0, \overline{\theta}_0]$.

5. Equilibrium

We now bring together the results from the previous sections and establish our two main results. First, for all $n \geq 2$ there are equilibria of the type introduced in Section 2 where the agents and the government follow the cutoff strategies of Propositions 1 and 3, respectively. Recall the restriction, in the definition of equilibrium in Section 2.5, to strategy profiles where in every period a positive measure of agents enters the market. In fact, this restriction implies that the only behavior possible in any such equilibrium is the one described by the above propositions. Second, the cutoff discount factor of the government in all such equilibria approaches the lower bound $\hat{\delta}$ as $n$ increases.

Fix $n$ and let $\theta_0$ be the ex ante probability of a tight monetary regime. Consider now a single agent and let $\sigma$ be a strategy profile for all other agents in the economy with the property that a positive measure of agents enters the market in each period. We know from Section 3 that the cutoff belief strategy of Proposition 1 is the unique best response of the agent under consideration to $\sigma$. It is, moreover, sequentially rational, as argued previously. We also know, still from Section 3, that if $\theta_0 \in (\theta^*(a), 1)$ and all agents follow the strategy of Proposition 1, then the unique best response for the government is a cutoff discount factor strategy. Let $\delta^*(\theta_0, n)$ denote this cutoff discount factor. Unfortunately this is not enough to guarantee the existence of an equilibrium with the desired properties. We need to deal with the following consistency problem:

Consistency problem. If the government follows a cutoff discount factor strategy with cutoff $\delta^*(\theta_0, n)$, the agents’ common prior that the monetary regime is tight is $1 - F(\delta^*(\theta_0, n))$. How do we know that $\theta_0 = 1 - F(\delta^*(\theta_0, n))$? In other words, how do we know that the agents’ common prior belief about the nature of the monetary regime is justified. We need to determine if the maps $\Theta_n$ that take $\theta_0$ into $1 - F(\delta^*(\theta_0, n))$ have, for all $n \geq 2$, a fixed point. Moreover, we need to ensure that $\Theta_n$ has, for each $n \geq 2$, at least one fixed point in the interval $(\theta^*(a), 1)$, for otherwise the analysis conducted so far breaks down.

In Appendix B we show that there are conditions on the distribution function $F$ ensuring the existence of $\theta_0, \overline{\theta}_0$, with $\theta^*(a) < \theta_0 < \overline{\theta}_0 < 1$, such that the maps $\Theta_n$ all have fixed points in the interval $[\theta_0, \overline{\theta}_0]$. This establishes our first main result. Since the argument of Appendix B relies on Brouwer’s fixed point theorem, there is no reason to expect uniqueness for fixed $F$ and $n$. An interesting possibility in this case is that there may be a range of discount factors for the government where its choice of monetary regime differs across equilibria. The verification of this fact, however, is beyond the scope of this article.

**Proposition 5.** For all $n \geq 2$ there exist equilibria where a positive measure of agents enters the market in all periods. Moreover, all such equilibria have the government following the cutoff strategy of Proposition 3 and the agents following the cutoff strategy of Proposition 1. The self-fulfilling priors $\theta_0(n)$ corresponding to these equilibria lie in $[\theta_0, \overline{\theta}_0] \subset (\theta^*(a), 1)$. 
This proposition describes how a utility maximizing government acts in an environment where agents can only form beliefs from their private experience. A crucial aspect is that the government has a degree of freedom to choose the monetary regime that does not exist when there is full information. If the agents know with certainty what is the government’s choice of monetary regime, the only equilibrium involving monetary trade has $m = m_L$. However, with partial information, the government can overissue money and still operate for some time.

A consequence of this result is that incomplete information can bring instability to trade involving money. For the monetary system to be stable, the government must have incentives to keep money valuable in the long-run. These incentives depend on the government’s discount factor $\delta$ and the number $n$ of per-period market meetings. Since the economy has no mechanism that fully reveals the government’s policy, society has to rely on the possibility that the government is sufficiently patient. Our second main result shows that the government’s incentives to overissue decrease when $n$ increases. It is an immediate consequence of Propositions 4 and 5.

**Proposition 6.** For all $\delta > \frac{1}{2}$ there exists $n(\delta)$ such that if $n \geq n(\delta)$, then the equilibria of Proposition 5 are such that the cutoff discount factor of the government is less than $\delta$.

It seems reasonable to suggest that in modern economies the dissemination of information about decisions made by the government is much faster than in the past. We model this change as an increment in the number of opportunities an agent has to gather information about the state of the economy. This increment is interpreted here as an increase in $n$. Therefore, as a result of the above proposition, we expect that in modern societies the incentives of any government to overissue are less pronounced than in the past.

6. Conclusion

This article addresses in a formal way the determinants of monetary stability in a decentralized economy where fiat money is endogenously created, information about its value is imperfect, and learning is decentralized. To the best of our knowledge, ours is the first one to put together these features. We believe they constitute a reasonable model if one wants to analyze the instability of fiat money as observed throughout history. In particular, by assuming that learning only takes place over time, we are able to generate a dynamics on the acceptability of fiat money that resembles the historical accounts of the rise and eventual demise of overissued paper money.

We show that the government’s temptation to overissue is limited in two different ways. First, it depends on its willingness to maintain the long-run value of money, here given by its patience. This result matches with Ritter (1995), who emphasizes that patience is a key condition in the transition from a barter to a fiat money economy. Second, it depends on society’s ability to monitor the government’s behavior, modelled by the number of transactions an agent faces in a given period. The ability to collect information, and the consequent stability it induces, has changed over time. In modern economies, information flows much faster than in the past. This offers an explanation for the late widespread use of fiat money, despite its clear advantages.

We take the government to be the sole provider of money in our analysis. We believe it also offers insights in scenarios where a private agent issues notes that circulate...
in the economy. Consider, for example the U.S. banking experience in the 19th century. King (1983) explains the overissue of circulating notes during this period by arguing that “(...) holders of circulating notes are unlikely to closely monitor the activities of a note issuer, because notes represent a small fraction of an individual’s wealth and are held only for a brief period” (p. 136). This article offers a potential environment where the likelihood of an overissue of circulating notes can be formally addressed. For instance, assume that the reasons underlying the demand for information (the ratio of money holdings to wealth, or the length of time an individual holds a note, for example) are relatively invariant, but the technology that supplies information improves over time. Our analysis then suggests that the probability of an overissue, and the consequent necessity of imposing stringent controls over the creation of notes, reduces as the economy evolves.

Appendix A. Propositions 3 and 4

**Proposition 3.** Suppose \( \theta_0 \in (\theta^*(\alpha), 1) \). There exists a unique \( \delta^* \in (0, 1) \) such that \( U(m_L, \delta) > U(m_H, \delta) \) if \( \delta > \delta^* \) and \( U(m_L, \delta) < U(m_H, \delta) \) if \( \delta < \delta^* \).

**Proof.** Let \( H(\delta) \) be given by

\[
H(\delta) = U(m_L, \delta) - U(m_H, \delta) = \sum_{t=1}^{\infty} \delta^{t-1} d_t, \tag{A.1}
\]

where \( d_t = \mu_t(m_L)m_L - \mu_t(m_H)m_H \). We want to show that there exists \( \delta^* \in (0, 1) \) such that \( H(\delta) < 0 \) if \( \delta < \delta^* \) and \( H(\delta) > 0 \) if \( \delta > \delta^* \). First observe that

\[
H^{(k)}(\delta) := \frac{d^k H}{d\delta^k}(\delta) = \sum_{t=k+1}^{\infty} (t-1)(t-2)\ldots(t-k)\delta^{t-k-1} d_t. \tag{A.2}
\]

By Proposition 2, \( \mu_t(m_L) \) converges to some \( \mu_L > 0 \), while \( \mu_t(m_H) \) converges to zero. Hence there exists \( t' \geq k+1 \) such that if \( t \geq t' \), then \( d_t \geq \frac{1}{4} \mu_L m_L \), and so

\[
H^{(k)}(\delta) \geq \sum_{t=k+1}^{t'-1} (t-1)\ldots(t-k)\delta^{t-k-1} d_t + \frac{H_t}{4} \sum_{t=t'}^{\infty} (t-1)\ldots(t-k)\delta^{t-k-1}. \tag{A.3}
\]

Since the first term on the right-hand side of the above inequality is finite for all \( \delta \),

\[
\lim_{\delta \rightarrow 1^-} H^{(k)}(\delta) = +\infty \text{ for all } k \geq 0.
\]

Let now \( \bar{t} \) be the smallest integer such that \( d_t \geq 0 \) for all \( t \geq \bar{t} \). Such a \( \bar{t} \) exists because of the asymptotic behavior of the population measures in the two monetary regimes. Since

\[
H^{(\bar{t}-1)}(\delta) = \sum_{t=\bar{t}}^{\infty} (t-1)\ldots(t-\bar{t}+1)\delta^{t-\bar{t}} d_t, \tag{A.4}
\]

we have that \( H^{(\bar{t}-1)}(\delta) > 0 \) for all \( \delta \). Notice that we cannot have \( \bar{t} = 1 \), since this would imply that \( U(m_L, \delta) > U(m_H, \delta) \) for all \( \delta \geq 0 \), which is not true for \( \delta \) sufficiently close to
zero. So, suppose that $\bar{t} > 1$. Now observe that

$$H^{(\bar{t}-2)}(\delta) = \sum_{t=\bar{t}-1}^{\infty} (t-1) \ldots (t-\bar{t}+2)\delta^{t-\bar{t}+1}d_t,$$

$$= (\bar{t}-2) \ldots 2d_{\bar{t}-1} + \sum_{t=\bar{t}}^{\infty} (t-1) \ldots (t-3)d_t,$$

(A.5)

and so $H^{(\bar{t}-2)}(0)<0$. Since $H^{(\bar{t}-2)}$ is strictly increasing and $H^{(\bar{t}-2)}(1^-) = +\infty$, we have that there exists a unique $\delta_1$ such that $H^{(\bar{t}-2)}(\delta)<0$, and only if, $\delta<\delta_1$. If $\bar{t} = 2$, we are done, just set $\hat{\delta} = \delta_1$. So, suppose now that $\bar{t} > 2$. The same reasoning as above shows that $H^{(\bar{t}-3)}(0)<0$. Since $H^{(\bar{t}-3)}(\delta)$ decreases until $\delta_1$, and after this point it increases strictly to $+\infty$, we have that there is a unique $\delta_2 > \delta_1$ such that $H^{(\bar{t}-3)}(\delta)<0$, and only if, $\delta<\delta_2$. If $\bar{t} = 3$, we are again done. Otherwise, we continue with this process. Since $\bar{t}$ is finite, it eventually reaches an end. □

**Proposition 4.** Let $\hat{\delta} = (m_H - m_L)/m_H$ and suppose that $\hat{\theta}_0 < \bar{\theta}_0$ are such that $\hat{\theta}_0 > \hat{\theta}(a)$ and $\bar{\theta}_0 < 1$. Then, for all $\delta > \hat{\delta}$ there exists $n(\delta)$ such that $U(m_L, \delta, n; \hat{\theta}_0) > U(m_H, \delta, n; \bar{\theta}_0)$ if $\theta_0 \in [\hat{\theta}_0, \bar{\theta}_0]$ and $n \geq n(\delta)$.

**Proof.** Suppose that $\delta > \hat{\delta}$. Because the cutoff beliefs $\hat{\theta}(a)$ are independent of $n$, $\mu_t(m_L, n; \hat{\theta}_0) \to 1$ and $\mu_t(m_H, n; \bar{\theta}_0) \to 0$ as $n \to \infty$ for all $t \geq 2$. Let $\varepsilon = m_L - (1 - \delta)m_H > 0$. Then there exists $n_1(\delta)$ such that

$$\mu_t(m_H, n, \bar{\theta}_0) < \frac{\varepsilon(1 - \delta)}{4m_H\delta},$$

(A.6)

if $n \geq n_1(\delta)$. Since $\mu_{t+1}(m_H, n; \bar{\theta}_0) = \mu_t(m_H, n, \bar{\theta}_0)$ for all $t$ and $n$, we have, in fact, that the above inequality holds for all $t \geq 2$. Therefore,

$$U(m_H, \delta, n; \theta_0) \leq U(m_H, \delta, n; \bar{\theta}_0) = (1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} \mu_t(m_H, n; \bar{\theta}_0)m_H < (1 - \delta)m_H + \frac{\varepsilon}{4}$$

(A.7)

whenever $n \geq n_1(\delta)$.

We now need to find an appropriate lower bound for $U(m_L, \delta, n; \theta_0)$. For this, let $n_2(\delta, t)$ be such that if $n \geq n_2(\delta, t)$, then

$$\mu_t(m_L, n; \hat{\theta}_0) > 1 - \frac{\varepsilon}{4N\delta}\delta^{t-1},$$

(A.8)

where $N$ is such that $(1 - \delta^{N+1})m_L - (1 - \delta)m_H + \varepsilon/2 > 0$. We know that such an $N$ exists by hypothesis. Now let $n_2(\delta) = \max\{n_2(\delta, t) | t = 2, \ldots, N\}$. If $n \geq n_2(\delta)$, then

$$U(m_L, \delta, n; \theta_0) \geq U(m_L, \delta, n; \hat{\theta}_0) = (1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} \mu_t(m_L, n; \hat{\theta}_0)m_L$$

$$> (1 - \delta^{N+1})m_L - (1 - \delta)\frac{\varepsilon}{4},$$

(A.9)
Consequently, if \( n \geq n(\delta) \), where \( n(\delta) = \max\{n_1(\delta), n_2(\delta)\} \), we have that
\[
U(m_L, \delta, n; \theta_0) - U(m_H, \delta, n; \theta_0) > (1 - \delta^{N+1})m_L - (1 - \delta)m_H - \frac{\epsilon}{2} > 0
\]
for all \( \theta_0 \in [\theta_0, \overline{\theta}_0] \). Our claim is indeed true. \( \square \)

**Appendix B. Equilibrium**

Here we show that there are conditions on \( F \) that ensure that the maps \( \Theta_n \) defined in Section 5 have, for all \( n \geq 2 \), a fixed point in the \((\theta^*(a), 1)\). The key part of the argument is to find bounds on the possible cutoff discount factors for the government that are uniform in \( n \). In what follows we need to consider infinitely patient governments. The limit of means criterion implies that
\[
U(m, 1) = \liminf T \frac{1}{T} \sum_{t=1}^{T} \mu_t(m)m.
\]
A convenient feature of this utility specification is that if \( \mu_t(m) \) converges as \( t \to \infty \), then:
(a) \( U(m, 1) = \mu_\infty(m) \), where \( \mu_\infty(m) \) is the limit of \( \mu_t(m) \); (b) \( U(m, \delta) \) converges to \( U(m, 1) \) as \( \delta \to 1 \).

We know, from Section 4, that when \( \delta < \check{\delta} \), where \( \check{\delta} = (m_H - m_L)/m_H \), the government always chooses \( m_H \) no matter \( n \). In other words, no cutoff discount factor \( \delta^* \) can be smaller than \( \check{\delta} \). Therefore, an upper bound for the ex ante probability of a tight monetary regime is \( 1 - F(\check{\delta}) \), and for this reason we set \( \overline{\theta}_0 \) to be equal to \( 1 - F(\check{\delta}) \). Consequently, our first condition is
\[
F_1: F(\check{\delta}) > 0,
\]
which implies that the ex ante probability of a soft monetary regime is positive.

The above probability, however, cannot be too high, otherwise it discourages even the newly born agents that have the lowest value of autarky from entering the market. In order to avoid this we need an upper bound \( \overline{\delta} < 1 \) for the possible cutoff discount factors of the government that is independent of \( n \). In this case, \( \theta_0 \geq \overline{\theta}_0 = 1 - F(\check{\delta}) \) regardless of \( n \), and our second condition is
\[
F_2: F(\check{\delta}) < 1 - \theta^*(a).
\]
A necessary condition for \( F_2 \) is that \( \overline{\theta}_0 = 1 - F(\check{\delta}) > \theta^*(a) \), a maintained assumption from now on. The following three auxiliary results are needed. The first one is a technical result, the proof of which can be found in Araujo and Camargo (2005).

**Lemma 1.** Let \( \{x_{nt}\} \) be an infinite double sequence such that: (i) for all \( n \in \mathbb{N} \), there exists \( x_\infty(n) \) such that \( \{x_{nt}\}_{t=1}^\infty \) converges monotonically to \( x_\infty(n) \); (ii) there exists \( x_\infty \) such that \( \lim_{n \to \infty} x_{nt} = x_\infty \) for all \( t \in \mathbb{N} \); (iii) \( x_\infty(n) \to x_\infty \) as \( n \to \infty \). Then \( \lim_{t \to \infty} \sup_{n \in \mathbb{N}} |x_{nt} - x_\infty(n)| = 0 \).

**Lemma 2.** The following two facts hold:

(i) For all \( \epsilon > 0 \) there exists \( \gamma = \gamma(\epsilon) > 0 \) such that \( U(m_L, \delta, n; \theta_0) \geq \gamma \) for all \( \theta \in [\theta^*_0(a) + \epsilon, \overline{\theta}_0] \) and all \( n \geq 2 \);
(ii) \( \lim_{\delta \to 1} \sup \{U(m_H, \delta, n; \theta_0)|n \geq 2 \} \) and \( \theta_0 \in [\theta^*_0(a) + \epsilon, \overline{\theta}_0] \) = 0 for all \( \epsilon > 0 \).
Proof. (i) Fix $\varepsilon > 0$ and write $\eta_t(m, a, n; \theta_0)$ to denote the dependence of the probabilities $\eta_t$ of Section 3 on both $n$ and the prior $\theta_0$. Let $\hat{\theta}_0 = \theta^*(a) + \varepsilon$. Since $\eta_t$ is non-decreasing in $\theta_0$ and non-increasing in $a$,

$$
\mu_t(m_L, n; \theta_0) \geq \int \eta_t(m_L, a, n; \hat{\theta}_0) g(a) \, da \geq \int_a \eta_t(m_L, a', n; \hat{\theta}_0) g(a) \, da = \varepsilon \eta_t(m_L, a', n; \hat{\theta}_0),
$$

where $a'$ is the unique $a \in [a, \bar{a}]$ such that $\theta^*(a') = \hat{\theta}_0$ and $\varepsilon = \int_{[a, \bar{a}]} g(a) \, da > 0$. From Section 3, $\inf \eta_t(m_L, a', n, \hat{\theta}_0) = \eta_\infty(n) > 0$ for all $n \geq 2$. Therefore, $\mu_t(m_L, n; \theta_0) \geq \varepsilon \eta_\infty(n) > 0$ for all $n \geq n_0$ and $t \in \mathbb{N}$. Notice that this lower bound depends on our choice of $\hat{\theta}_0$, but the latter is independent of $n$. Moreover, since there exists $n_0 \in \mathbb{N}$ such that $\eta_\infty(n) > \eta_\infty(2)$ for all $n \geq n_0$, we have that $\eta = \inf_{n \geq 2} \eta_\infty(n) > 0$. Therefore, for all $\theta_0 \in [\bar{\theta}_0, \hat{\theta}_0]$ and all $\delta \in [0, 1)$,

$$
U(m_L, \delta, n; \theta_0) = (1 - \delta) \sum_{t=1}^\infty \delta^{t-1} \mu_t(m_L, n, \theta_0) \geq \gamma > 0,
$$

where $\gamma = \varepsilon \eta$ is independent of $n$. The same lower bound holds for $U(m_L, 1)$ from (b) in the first paragraph of this appendix.

(ii) Now observe that $\mu_t(m_H, n; \theta_0) \leq \eta_t(m_H, a, n; \bar{\theta}_0)$ for all $n \geq 2$, $t \in \mathbb{N}$, and $\theta_0 \in [\hat{\theta}_0, \bar{\theta}_0]$. Hence,

$$
\frac{U(m_H, \delta, n; \theta_0)}{1 - \delta} \leq \sup_n \left\{ \sum_{t=1}^\infty \delta^{t-1} \eta_t(m_H, a, n; \bar{\theta}_0) m_H \right\} \leq \sum_{t=1}^\infty \delta^{t-1} \sup_n \eta_t(m_H, a, n; \bar{\theta}_0) m_H
$$

for all $[\hat{\theta}_0, \bar{\theta}_0]$ and all $n \geq 2$. We know that $\lim_n \eta_t(m_H, a, n; \bar{\theta}_0) = 0$ for all $t \in \mathbb{N}$. Therefore, as a consequence of Lemma 1, we have that $\sup_n \eta_t(m_H, a, n; \bar{\theta}_0)$ converges to zero as $t \to \infty$. Therefore, the right-hand side of the above inequality converges to zero as $\delta$ goes to one. We can then conclude that $U(m_H, \delta, n; \theta_0)$ converges to zero as $\delta \to 1$ uniformly in $n$ and $\theta_0$, the desired result. This last statement makes use of (a) and (b) in the first paragraph of this appendix. □

Lemma 3. Suppose $\theta_0 \in (\theta^*(a), 1)$. The utility function $U(m, \delta, n; \theta_0)$ is jointly continuous in $\delta$ and $\theta_0$ if the measures $\mu_t(m, n; \theta_0)$ are continuous functions of $\theta_0$.

Proof. Suppose $\mu_t(m, n; \theta_0)$ is a continuous function of $\theta_0$ for all $t \in \mathbb{N}$ when $\theta_0 \in (\theta^*(a), 1)$. Then $U(m, \delta, n; \theta_0)$ is, for all $\delta \in [0, 1]$, a continuous function of $\theta_0$. Since each $\mu_t$ is monotonic in $\theta_0$, $U$ is monotonic in $\theta_0$ as well. Moreover, $U$ is continuous in $\delta$ for all $\theta_0 \in (\theta^*(a), 1)$. We can then apply Lemma 2 in Dutta et al. (1994) to conclude that $U$ is indeed jointly continuous in $\theta_0$ and $\delta$. □

Fix $\varepsilon \in (0, \bar{\theta}_0 - \theta^*(a))$ and let $\theta_0 = \theta^*(a) + \varepsilon$. Lemma 2 implies that there exists $\delta \in (\delta, 1)$ such that if $\delta \geq \delta$, then $U(m_L, \delta, n; \theta_0) > U(m_H, \delta, n; \theta_0)$ for all $\theta_0 \in [\theta_0, \bar{\theta}_0]$ and all $n \geq 2$. In other words, $\delta$ is the desired upper bound for the possible cutoff discount factors for the government. Notice that $\delta$ is independent of $n$ and $F$ (but it does depend on $\varepsilon$). Consequently, the only possible cutoff discount factors for the government lie in the interval $[\delta, \delta]$, regardless of the number $n$ of market meetings per period. Conditions $F_1$ and $F_2$ above ensure that if $\theta_n$ has a fixed point, then it must lie in $[\theta_0, \bar{\theta}_0] \subset (\theta^*(a), 1)$. The final result in this appendix shows that this is indeed the case.
Proposition 7. Suppose $F$ satisfies conditions $F_1$ and $F_2$. Then, for all $n \geq 2$, the maps $\Theta_n$ have a fixed point in $[\theta_0, \bar{\theta}_0]$.

Proof. We know that if $\theta_0 \in [\theta_0, \bar{\theta}_0]$, then $\delta^*(\theta_0, n)$ is the unique solution to the equation $U(m_L, \delta; n; \theta_0) = U(m_H, \delta; n; \theta_0)$ for each $n \geq 2$. Hence, if we show that both $U(m_L, \delta; n; \theta_0)$ and $U(m_H, \delta; n; \theta_0)$ are jointly continuous in $\delta$ and $\theta_0$ for all $n \geq 2$, the same is true of $\delta^*(\theta_0, n)$. Because $F$ is continuous by assumption, the maps $\Theta_n$ are all continuous. The existence of a fixed point for each $\Theta_n$ is now a consequence of Brouwer’s fixed point theorem together with the fact that they all map $[\theta_0, \bar{\theta}_0]$ into itself.

By Lemma 3, we only need to show that both $\mu(m_L, n; \theta_0)$ and $\mu(m_H, n; \theta_0)$ are continuous functions of $\theta_0$ for all $n \geq 2$ and $t \in \mathbb{N}$. For this, suppose that $\{\theta_{0k}\}$ converges to $\theta_0$. Observe that the fractions $\eta_t(m, a; n; \theta_{0k})$ do not necessarily converge to $\eta_t(m, a; n; \theta_0)$ as $k \to \infty$ for a given $a$ in $[a, \bar{a}]$. In fact, for fixed $a$ and $\theta_0$, there may exist private histories that leave an agent with autarky value $a$ indifferent between the market and autarky for at least one $t' \leq t$. In this case, perturbing $\theta_0$ induces a discontinuous change in this fraction. However, for each $t, n$, and $\theta_0$, the subset of $[a, \bar{a}]$ where convergence fails has Lebesgue measure zero. Hence, for each $n \geq 2$ and $t \in \mathbb{N}$, $\eta_t(m, a; n; \theta_{0k}) \to \eta_t(m, a; n; \theta_0)$ for almost all $a \in [a, \bar{a}]$. By Egorov’s theorem, see Kolmogorov and Fomin (1970), almost everywhere pointwise convergence implies convergence in $L_1$. Therefore

$$\mu_t(m, n; \theta_{0k}) = \int \eta_t(m, a; n; \theta_{0k}) g(a) \, da \to \mu_t(m, n; \theta_0) = \int \eta_t(m, a; n; \theta_0) g(a) \, da$$

for all $n \geq 2$ and $t \in \mathbb{N}$, and so the desired continuity result holds. \qed

References