The Race to the Base*

Dan Bernhardt[†] Peter Buisseret[‡] Sinem Hidir[§]

August 25, 2018

Abstract

We study multi-district legislative elections between two office-seeking parties when the election pits a relatively strong party against a weaker party; when each party faces uncertainty about how voter preferences will evolve during the campaign; and, when each party cares not only about winning a majority, but also about its share of seats in the event that it holds majority or minority status. When the initial imbalance favoring one party is small, each party targets the median voter in the median district, in pursuit of a majority. When the imbalance is moderate, the advantaged party continues to hold the centre-ground, but the disadvantaged party retreats to target its core supporters; it does so to fortify its minority share of seats in the likely event that it fails to secure a majority. Finally, when the imbalance is large, the advantaged party advances towards its opponent, raiding its moderate supporters in pursuit of an outsized majority.

*We are grateful to Wiola Dziuda, Pablo Montagnes, Carlo Prato, and Debraj Ray for helpful comments and suggestions.

[†]Department of Economics, University of Illinois and University of Warwick, *Email*: danber@illinois.edu

[‡]Harris School of Public Policy, University of Chicago, *Email*: pbuisseret@uchicago.edu

[§]Department of Economics, University of Warwick, *Email*: S.Hidir@warwick.ac.uk

1. Introduction

A near-axiomatic logic of two-party elections is that to win the contest, a party must carry the support of the median voter. To the extent that political parties care solely about winning the election, their platforms should therefore converge to the median voter's most-preferred policy (e.g., Hotelling 1929, Downs 1957). In legislative elections, however, winning is *not* everything. In fact, winning a majority of legislative seats may be neither necessary nor sufficient for a party to achieve its goals.

Two examples help illustrate this point. In 1992, John Major's Conservative party won a majority of seats in the House of Commons, and the largest number of votes of any party in British electoral history. Nonetheless, Major's parliamentary majority fell from 102 to a mere 21 seats. Despite its victory, Major's government was persistently hampered by its small majority, which contributed to its first legislative defeat just over one year later.

In 2017, by contrast, Jeremy Corbyn's Labour party failed to win a majority of seats. Nonetheless, the party advanced its minority seat share by 26 seats, and successfully denied the Conservative party its previously-held parliamentary majority. Since the Conservatives had enjoyed a 20-point lead in the polls at the moment Theresa May called the election, the press concluded that, despite its failure to achieve outright victory, Labour had triumphed—in particular, over expectations of an electoral rout. The outcome was summarized by one commentator as "the sweetest of defeats", while Labour MP and campaign strategist John Trickett boasted that "every lesson all these politics professors ever learned has been proved wrong".[1]

Labour's election strategy—dubbed by one observer as "baffling"— eschewed centrist voters in favor of its core supporters.[2] The party's manifesto promised to nationalize public utilities, abolish university tuition fees, and levy special taxes on firms with highly-paid staff.[3] To the extent that the party's strategy was calculated to maximize its election performance, why then did it forego the centrist—or even right-leaning—route that led Tony Blair's party to a majority of 179 seats in 1997?

More broadly, we ask: under what circumstances does an office-seeking party in a legislative election prefer to choose its electoral platform to target to its traditional supporters, rather than

[1] "The Jeremy Corbyn factor", *BBC News*, 9 June 2017.

[2] "The baffling world of Labour's election strategy", *The Spectator*, 27 April 2017.

[3] "For the Many, Not the Few", *Labour 2017 Election Manifesto*, https://goo.gl/GZaTbk.

centrist voters? If it targets its traditional supporters, should the opposing party try to maintain its hold on the centre-ground, cater to its own base, or instead try to raid its opponent's more moderate supporters? And, how do the answers to these questions depend on parties' expectations about their popularity, the extent of voters' partisan loyalties, and the relative marginal value that a party derives from winning additional seats below, at or above the majority threshold?

Our Approach. To address these questions, we develop a model of two-party competition between office-motivated parties in a multi-district legislative election. For example, the election could determine control of a legislative chamber such as the U.S. House of Representatives, or the British House of Commons. One of the parties holds an initial net valence advantage—for example, its leadership may perceived as more competent, its opponent may be dogged by scandal or simply worn out by a long period of incumbency. After the parties simultaneously choose platforms, an aggregate net valence payoff shock in favor of one party is realized, and every voter in every district subsequently casts his or her ballot for one of the two parties.

We assume throughout that each party's payoff depends solely on its share of districts, or seats in the legislature. However, this does not imply that parties care solely about *winning* the election. If a party wins more than half of the total districts (seats), it not only derives a large fixed payoff from majority status, i.e., from winning the election, it also receives a strictly increasing payoff from any additional seats that it wins beyond the majority threshold. The fixed office rent reflects the value of majority status *per se*: in a parliamentary democracy, majority status confers the right to form the government regardless of the size of a party's majority. Even in presidential systems, majority status grants a party control over crucial aspects of the legislative process, including scheduling bills and staffing committees. However, additional seats beyond the majority threshold are also valuable: they further insulate the majority party from the threat of confidence votes in a parliamentary context, insure against defections of a few party members on key votes, and mitigate the obstructionist legislative tactics that a minority party can employ.

If, instead, a party holds minority status, i.e., its share of seats falls below one half, its payoff nonetheless strictly increases in its share of seats. This reflects that a stronger minority receives more committee positions, and can more effectively derail the majority party's agenda by use of parliamentary procedures that privilege a more numerous minority. Our key substantive assumption is that, conditional on winning minority status, the minority party cares sufficiently about the number of seats it holds. For example, more seats may secure a larger share of committee assignments, and greater recourse to obstructionist tactics that require supermajorities to override. It may also reflect non-institutional factors: winning any sized majority is sufficient to secure a reprieve for an embattled leader (e.g., John Major), but the strength of a losing performance may be crucial for a party leader's short-term survival (e.g., Jeremy Corbyn).

Results. We obtain a unique equilibrium, in pure strategies, for all levels of the initial popularity imbalance between the parties. The equilibrium characterization can be indexed according to whether the initial imbalance is small, moderate, or large.

If the advantage is *small*, both parties locate at the policy preferred by the median voter in the median district. The reason is that—even with an imbalance—both parties remain competitive for majority status, encouraging them to compete aggressively to win the election, outright. This reflects that while winning isn't everything, it certainly matters a lot.

If the advantage is *moderate*, the disadvantaged party assesses that its prospect of winning an outright majority is distant enough that it no longer finds it worthwhile to single-mindedly pursue outright victory. Instead, its strategy reverts to moving its policy platform away from the median voter in the median district, and in the direction of its core supporters. This choice may seem paradoxical, since this shift in strategy renders the party's prospects of winning even more distant. Nonetheless, it also *increases* its anticipated share of seats in the relatively more likely event that the election consigns the party to minority status. The reason is that the party raises its attractiveness to its core supporters by differentiating itself ideologically from the advantaged party. With further increases in imbalance, the disadvantaged party further retreats to its base, as the prospect of losing the election rises.

By contrast, the advantaged party maintains its strategy of targeting its platform at the policy preferred by the median voter in the median district. While it could chase the disadvantaged party into its own ideological turf in order to push for an even larger share of districts, its advantage is only moderate, and alienating centrist voters would risk its prospects of majority status —a cost that is high relative to the prospective gains from bolstering its seat share in the event that it wins.

Finally, if the imbalance is *large*, the disadvantaged party continues its retreat by locating its platform even further from the centre and towards its core supporters. But now the advantaged party gives chase, moving its platform beyond the median voter in the median district and into the disadvantaged party's ideological territory. This is strategically appealing for three reasons. First, the party's very strong advantage makes it less concerned about the risk of losing the election—i.e., failing to win a majority of seats; instead, its focus shifts to generating a comfortable seat advantage *conditional on winning majority status*. Second, it reduces the policy wedge between the parties, which heightens the salience of the advantaged party's net valence advantage, raising its appeal amongst all voters. Third, it capitalizes on the opportunity created by the disadvantaged party's increasingly extreme lurch to raid its more moderate supporters.

While platforms fully converge when initial imbalances are small, we show how changes in political primitives in the context of either a moderate or large initial imbalance either exacerbate or mitigate the disadvantaged party's incentive to revert to its base, and platform polarization.

When the initial imbalance is moderate, the disadvantaged party increasingly retreats to its base whenever its initial disadvantage grows, whenever the marginal value of seats conditional on minority status rises, or whenever uncertainty about voter preferences increases. It also further retreats when there is a decline in the strength of partisan loyalty amongst its traditional supporters, since these voters are less easily taken for granted. Because the advantaged party maintains its position in the centre, these changes thus trigger increased platform polarization.

Once the imbalance is large enough, however, further increases in the popularity imbalance induce *both* parties to move towards the disadvantaged party's core supporters. And, in contrast with moderate imbalances, the distance between platforms *falls*, reducing platform polarization. Thus, our model predicts that platform divergence is maximized when the initial electoral imbalance is neither very small, nor very large, and especially where partisan loyalties are in flux.

We extend our benchmark model to analyze the dynamics of political campaigns in contexts where some voters cast ballots early, or make up their minds up before a campaign concludes. To wit, we assume that some voters cast their ballots after an initial valence shock that favors one of the parties, but before the parties have communicated their policy commitments, and prior to any other developments—such as leader debates, town hall meetings, or personal revelations—that occur over the course of a campaign. We interpret these voters as 'early deciders', who are relatively insensitive or inattentive to the twists and turns of election campaigns.

If the initial valence shock favoring one of the parties is small, the parties converge on a platform that—rather than targeting the median voter in the median district, as in our benchmark—moves towards the advantaged party's core districts, by an increment that grows with both the magnitude of the initial valence shock and the fraction of early deciders. To see why, notice that the advantaged party enjoys a larger share of support amongst early deciders, and thus gains a starting lead in the polls. In order to win the election, the disadvantaged party therefore needs to offset its disadvantage by carrying strictly more than a majority of supporters amongst the remaining voters. This leads it to move beyond the ideological centre-ground, targeting voters that are ideologically disposed towards its advantaged opponent. Thus, the disadvantaged party designs its policy to appeal to its rival's voters even though ideology is not the source of its disadvantage.

Perhaps counterintuitively, with more early deciders, a disadvantaged party is *slower* to retreat to its base—the initial imbalance beyond which it begins to retreat towards its base *rises*. More early deciders implies a larger initial lead in the polls that is harder to close in pursuit of outright victory, so one might think that—relative to our benchmark setting—an initially weak party would be more prone to revert in favor of its core supporters when more voters make up their minds early in the campaign. However, this reasoning is incorrect. What drives a party to retreat is that it cares disproportionately about marginal seats when it is in the minority. This makes a party risk averse, and it retreats to its base to insure against large popularity swings in favor of its opponent during the campaign. With more early deciders, a greater fraction of voters make their minds up before the campaign; this implies that the disadvantaged party faces less uncertainty, reducing its insurance motives. A similar logic implies that more early deciders cause the advantaged party to begin pursuing its weaker rival at lower levels of initial advantage: a larger fraction of early deciders resolves uncertainty, reducing the risk of an adverse valence shock that might cost its majority.

Contribution. Our premise and results contrast starkly with existing models of party positioning in elections. In the framework developed by Calvert (1985) and Wittman (1983), policy-motivated parties face uncertainty about the preferences of the electorate—specifically, the median voter's most preferred platform. In equilibrium, if a party becomes *more* advantaged, i.e., the expected location of the median voter moves towards its most-preferred policy, *both* parties advance towards the advantaged party's ideal policy.

Our framework predicts the opposite. When the advantaged party's net valence advantage is large enough, an increased electoral imbalance encourages both parties to move in the direction of *disadvantaged* party's base. The advantaged party seeks to invade its opponent's ideological turf in pursuit of a strong majority, while the disadvantaged party retreats to its base in an attempt to rally its core supporters. The first implication seems eminently better suited to interpreting Tony Blair's electoral strategy in 1997 to transition his party to *New Labour*, at a time when the party enjoyed a clear preference advantage amongst British voters. This advantage was so strong that even *The Sun* newspaper, which had supported the Conservatives in every election in the previous twenty years, endorsed Labour, condemning the Conservatives as "tired, divided and rudderless".[4] The second implication more closely corresponds to Bogdanor's summary of the Conservative lurch to the right from 2001 to 2010, in which "three successive Conservative leaders... responded to defeat by seeking to mobilize the Tory 'core' vote".[5] And, consistent with our predictions, Theresa May in 2017 pursued an "aggressive strategy, influenced by her strong lead in the initial polls... parking her tank on Labour's lawn in heartlands such as the North East and the North West of England."[6] Over the course of the campaign, sixty-one percent of her visits were to Labour-held constituencies.

Groseclose (2001) augments the Calvert-Wittman framework by introducing a deterministic valence advantage for one party. However, Groseclose does not establish existence or uniqueness of an equilibrium. Moreover, his main theoretical results are limited to a context with a small valence advantage (specifically, moving from no advantage to an arbitrarily small advantage), and his framework features a single (median) voter—forestalling the question of whom to target that drives our framework. The predictions that he derives when an equilibrium exists differ substantially from our office-motivated context. For example, his framework predicts that an increase in the advantaged party's net valence advantage *always* raises platform differentiation; and, if the advantaged party's net valence advantage is very large, it always adopts more extreme policy positions in the direction of its ideal policy.

Our framework predicts the opposite: the advantaged candidate responds to a large advantage not by adopting more extreme positions favored by its own core supporters, but instead by targeting its opponent's moderate supporters. Moreover, we find that policy divergence is maximized when there is an *intermediate* electoral imbalance in favor of one party. If the imbalance is very small, both parties compete for the support of the median voter in the median district, resulting in complete policy convergence; and if the imbalance is large, the advantaged party chases the disadvantaged party into its own turf, reducing policy divergence. When the imbalance is intermediate, the disadvantaged party retreats to its base, but the advantaged party maintains the centre-ground. Finally, we prove existence and uniqueness of an equilibrium for all levels of the valence advantage.

[4] See Butler and Kavanagh (1997).

[5] "The Conservative Party: From Thatcher to Cameron", *New Statesman*.

[6] "What Theresa May's campaign stops tell us about her failed strategy", *The Telegraph*, 13 June 2017.

Aragones and Palfrey (2002) and Hummel (2010) characterize equilibria in a Downsian setup with purely office-motivated candidates and a deterministic net valence advantage. As in our setting, the advantaged candidate benefits by raising the salience of this valence advantage. This encourages the advantaged candidate to mimic the disadvantaged candidate, and the disadvantaged candidate to try to differentiate itself from the advantaged party. Both papers are limited to characterizing a particular mixed strategy equilibrium, under the restriction either of a small (Aragones and Palfrey, 2002) or large (Hummel, 2010) initial valence advantage.

Our framework offers an explanation for why parties may instrumentally choose relatively extreme policies. In Eguia and Giovannoni (2017), a party that is sufficiently disadvantaged today may give up on a mainstream policy, and instead invest in an extreme policy; it does so not to increase its office-motivated payoffs today, but instead to gamble on a shock to voters' preferences in a subsequent election. Our explanation emphasizes that the instrumental adoption of extreme policies in the face of a likely election defeat arises not only via dynamic office-holding incentives, but also via static office-holding incentives that emphasize the value of a strong minority position.

Our multi-district framework is closest to Callander (2005), in which two parties simultaneously choose national platforms, facing entry by local candidates, generating equilibrium platforms that differ greatly from ours. Other authors—for example, Austen-Smith (1984), Kittsteiner and Eyster (2007), and Krasa and Polborn (2018)—study multi-district competition in which party platforms are an aggregate of decentralized choices by local candidates. Our framework, like Callander's, instead reflects a context in which voters predominantly assess their view of the party on the basis of its national platform.

2. Benchmark Model

Preliminaries. Two parties, L and R, simultaneously choose campaign platforms, x_L and x_R, prior to an election. The policy space is the one-dimensional continuum, $\mathbb{R}$. Competition involves multiple districts, with the winner of each individual district determined by a plurality rule. Each district features a continuum of voters; each voter i is indexed by his or her preferred policy, x_i. There are a continuum of districts: in a district with median ideology m, voters' preferred policies are uniformly distributed on the interval $[m-Z, m+Z]$; and district medians are uniformly distributed on the interval $[-1, 1]$. We assume $Z > 1$ to capture the idea that there is more preference heterogeneity within districts than across different district medians.

Voter Payoffs. If party L implements platform x_L, a voter i with preferred policy x_i derives payoff

$$u(i, x_L) = -\gamma |x_L - x_i| - \theta x_i / 2. \tag{1}$$

If, instead, party R implements its platform x_R, the voter derives the payoff

$$u(i, x_R) = -\gamma |x_R - x_i| + \theta x_i / 2 + \rho_0 + \rho_1. \tag{2}$$

Here, ρ_0 is an initial valence advantage in favor of party R, commonly known by all agents, and ρ_1 is a preference shock, uniformly distributed on the interval $[-\psi, \psi]$.[7] The valence advantage ρ_0 reflects voters' relative assessment of the parties at the outset of a campaign—for example, evaluations of its leadership that are inherited from a party's previous spell in government. The valence shock ρ_1, by contrast, summarizes unanticipated developments that unfold over the course of an election campaign— right up to election day—including performances by party leaders in public debates or town hall meetings, or scandal revelations. If the legislative election coincides with a presidential election, ρ_1 could also capture evaluations of a party arising from its presidential candidate's campaign. Without loss of generality, we assume $\rho_0 \geq 0$.

The policy-related part of voters' preferences has two distinct components. The first component is a linear policy loss that increases with the distance between the party's policy platform and the voter's preferred policy.

The second component, which implies that voter i derives an additional net value $-\theta x_i$ from party L, has multiple interpretations. For example, it could reflect a fixed policy position on another dimension of policy conflict, e.g., social issues such as abortion, gay marriage, or on trade and immigration, and voter preferences on this issue are correlated with their preferences on taxation—over which parties are proposing policies—where the extent to which voters care about this second issue is proportional to θ. Our running interpretation is that it reflects voters' partisanship, or party loyalty that transcends short-term policy platforms that parties adopt from one election to the next. For example, while the British Labour Party has vacillated between centrism and more left-wing policies many times in the twentieth century, its loyalty amongst its core voters in the north of England has remained firm. In the United States, southern support for Republican party is robust to changes in the party's platform across elections.

[7] In our benchmark model, one could alternatively interpret ρ_0 as the mean of the preference shock ρ_1, i.e., $\rho_1 \sim U[\rho_0 - \psi, \rho_0 + \psi]$. In our dynamic extension, the present interpretation is more appropriate.

Party Payoffs. Let $d_P \in [0,1]$ denote the share of districts won by party $P \in \{L, R\}$, and let $M_P = \mathbb{I}[d_P > 1/2]$ denote the event in which party P wins a majority of districts. Then, party P's payoff is

$$u_P(d_P) = M_P[r + \beta(d_P - 1/2)] + (1 - M_P)\alpha d_P. \tag{3}$$

A party receives a fixed payoff of $r > 0$ if it wins the election, i.e., if $d_P > 1/2$. Higher values of r reflect the majoritarian organization of a legislature: winning a majority gives a party agenda-setting authority, and control over committee assignments and leadership. And, in a parliamentary democracy, winning a majority yields formal control over the executive branch.

Parties also value winning additional seats both below and above the majoritarian threshold. Even if a party fails to achieve a majority, i.e., $d_P < 1/2$, it still gains from winning more seats. And, if a party achieves a majority, it values increasing its share of seats above the majority threshold. To capture this idea in the simplest possible way, we let $\alpha > 0$ denote the marginal value of winning districts that nonetheless keep a party's total share of districts less than a majority; similarly, $\beta > 0$ denotes the marginal value of winning districts above and beyond the majority threshold of one half. This piece-wise linear formulation facilitates tractable solutions, and may be viewed as an approximation of more sophisticated payoff schedules.

We impose two assumptions; the first assumption focuses on party preferences, while the second assumption focuses on voter preferences.

Assumption 1: $r > \frac{1}{2}\left(\alpha + \frac{\psi}{\theta}(\alpha - \beta)\right)$, and $\alpha \geq \beta$.

The first restriction states that parties sufficiently value winning majority status. Notice that for majority status to convey a benefit, it *must* be that $r > \alpha/2$.[8]

The second preference restriction, $\alpha \geq \beta$ states that the marginal value of additional seats to a minority party exceeds the marginal value of additional seats to a majority party, above and beyond its gains from majority status that are captured by r. We later describe properties of equilibrium policy platforms under the alternative assumption that $\beta > \alpha$; nonetheless, we view the restriction in Assumption 1 as inherently more plausible. For example, one can view $\alpha > \beta$ as a reduced-form preference assumption that captures the value of extra seats to the majority

[8] The restriction on r is not needed for our qualitative results. We make it to streamline exposition; moreover, we view it as a reasonable description of real-world contexts, in which the value of achieving majority status per se is large relative both to the value of minority status and relative to any incremental gains from an ever-larger share of seats beyond the majority threshold.

party when it obtains additional leeway to move policy outcomes closer to its most-preferred goal. Then, a larger majority allows the majority party to shift policy in its preferred direction. If party leaders have concave utility over policy, these incremental policy movements harm the minority party by more than they benefit the majority party, implying that $\alpha > \beta$.[9]

Assumption 2: $\theta > 2\gamma$.

This assumption eases analysis by ensuring that for any pair of platforms, voter preferences are single-peaked. This implies that in any district, the median voter is decisive for the outcome of that district's election. It further implies that the median voter in the median district determines which party wins the national election.[10]

Timing. The interaction proceeds as follows.

1. The parties simultaneously select platforms x_L and x_R.
2. The preference shock ρ_1 is realized and observed by all agents.
3. Each voter chooses to vote for one of the two parties.
4. The party that wins a majority of districts implements its promised platform, and payoffs are realized.

3. Results

Preliminary Results. We begin by identifying the share of districts won by each party—and thus each party's probability of winning the election—for any platform pair (x_L, x_R) and net valence advantage $\rho_0 + \rho_1$. Under Assumption 2, preferences are single-peaked, so there is a unique voter that is indifferent between the candidates: there is some ideal policy $x_i^*(x_L, x_R, \rho_0 + \rho_1)$ such that a voter prefers party L if and only if her ideal policy lies to the left of x_i^*. Because voters' ideal policies in a district with median m are uniformly distributed on $[m - Z, m + Z]$, and district

[9] We thank Pablo Montagnes for this observation. Alternatively, the reduced-form preference assumption could reflect a party leader's calculation about how the election outcome will affect her risk of being replaced. While an embattled leader who wins an election may secure a reprieve from the threat of replacement, regardless of her margin of victory, her survival if she loses an election may depend very sensitively on just how *badly* she loses.

[10] For tractability, we also assume that ψ is large enough that each party wins with positive probability in all districts, for any platform pair $(x_L, x_R) \in [-1, 1]^2$.

medians are uniformly distributed on $[-1,1]$, party L's share of districts is

$$d_L(x_L, x_R, \rho_0 + \rho_1) = \frac{1}{2} + \frac{x_i^*(x_L, x_R, \rho_0 + \rho_1)}{2}.$$

Party L therefore wins the election if and only if $x_i^* \geq 0$, i.e., if and only if it is preferred by the median voter in the median district, with ideal policy zero. We have:

$$x_i^* \geq 0 \iff -\gamma|x_L| + \gamma|x_R| - \rho_0 \geq \rho_1 \equiv \rho_1^*(x_L, x_R, \rho_0).$$

Substituting into the party payoff function in equation (3) yields party L's expected payoff:

$$\begin{aligned}\pi_L(x_L, x_R) = \int_{-\psi}^{\rho_1^*(x_L,x_R,\rho_0)} &(r + \beta(d_L(x_L, x_R, \rho_0 + \rho_1) - .5))f(\rho_1)d\rho_1 \\ &+ \int_{\rho_1^*(x_L,x_R,\rho_0)}^{\psi} \alpha d_L(x_L, x_R, \rho_0 + \rho_1)f(\rho_1)d\rho_1. \end{aligned} \tag{4}$$

Party R's corresponding expected payoff is:

$$\begin{aligned}\pi_R(x_L, x_R) = \int_{-\psi}^{\rho_1^*(x_L,x_R,\rho_0)} &\alpha(1 - d_L(x_L, x_R, \rho_0 + \rho_1))f(\rho_1)d\rho_1 \\ &+ \int_{\rho_1^*(x_L,x_R,\rho_0)}^{\psi} (r + \beta(.5 - d_L(x_L, x_R, \rho_0 + \rho_1)))f(\rho_1)d\rho_1. \end{aligned} \tag{5}$$

Main Results. We now characterize equilibrium platforms choices and highlight how they depend on R's initial advantage (ρ_0), uncertainty about how preferences will evolve over the course of the election (i.e., uncertainty about ρ_1), the relative value of seats to the minority party (α) versus the majority (β), and the value of winning a legislative majority (r). We first establish that that our framework produces a unique equilibrium, in pure strategies.

Theorem 1. *Under Assumptions 1 and 2, there always exists a unique pure strategy equilibrium.*

A pure strategy equilibrium exists because, in addition to his or her payoffs from the policy dimension on which parties choose platforms, a voter with ideal policy x_i derives an additional net value from party L of $-\theta x_i$, reflecting partisan loyalties or fixed policy platforms on a secondary issue dimension. To understand the role of this assumption, notice that the advantaged party benefits when it chooses the same policy as the disadvantaged party, since this encourages voters to evaluate parties based on non-policy (valence) factors. For the same reason, the disadvantaged party benefits when it chooses a different policy from the advantaged party. These "chase-and-evade" incentives are present, for example, in Aragones and Palfrey (2002). When $\theta = 0$, as in their framework, the advantaged party's electoral return from mimicking the disadvantaged party is the same, *regardless* of where the disadvantaged party locates. Conversely, the disadvantaged party's electoral return from distancing itself is the same, *regardless* of where it locates. These incentives are so powerful that equilibria are in mixed strategies if $\theta = 0$.

In our framework, however, the assumption that $\theta > 0$ implies that the relative sensitivity of a voter with ideal policy x to platform choices diminishes with $|x|$: centrist voters are relatively more sensitive to platforms than extreme voters. So, while it is cheap for the advantaged party to chase the disadvantaged party if the latter adopts centrist policies, chasing its more extreme policies costs the advantaged party support relatively dearly amongst the most policy-responsive (centrist) districts—which are crucial for winning the election, and therefore enjoying the office rent $r > 0$—and is relatively less effective at converting more extreme districts. This limits the advantaged party's value from chasing the disadvantaged party.[11] For the same reason, the relatively-disadvantaged party finds it relatively less attractive to distance itself from the advantaged party by targeting the latter's core supporters. The net effect is to abate the otherwise unfettered chase-and-evade incentives, yielding an equilibrium in pure strategies.

We show that characterization of the unique equilibrium can be indexed according to whether the advantaged party's initial imbalance is small, intermediate, or large. To develop intuition, we highlight the three channels that govern each party's incentives.

1. The parties are initially imbalanced due to the valence advantage ρ_0 in favor of party R. If parties advocate similar policy positions, voters place relatively more emphasis on non-policy differences, which favor party R. This gives party L an incentive to differentiate itself ideologically from R, and conversely encourages party R to emulate L's policies.

2. Majority legislative status is valuable (r). This encourages each party to target its policy to appeal to the median voter in the median district, whose support is necessary and sufficient to win the election. However, each party also cares about its share of seats both below (α) and above (β) the majority threshold. As α increases, each party becomes less concerned about trying to win outright, and instead focuses on bolstering its seat share in the event

[11] Notice that the restriction on r—the value of winning a majority—in Assumption 1 is violated when $\theta = 0$.

that it fails to achieve a majority. And, as β increases, each party becomes relatively more concerned about the size of its majority, rather than winning outright.

3. Parties face uncertainty about the electoral outcome due to the stochastic payoff shock, ρ_1. Thus, even voters who prefer party L's election platform x_L to party R's platform x_R cannot be taken for granted. To see this, suppose that $x_L < x_R$ and observe that while a voter with ideal policy $x_i < x_L$ prefers L's platform, she supports L if and only if:

$$\gamma(x_R - x_L) - \theta x_i - \rho_0 - \rho_1 \geq 0 \iff x_i \leq \frac{\gamma(x_R - x_L) - \rho_0 - \rho_1}{\theta}. \tag{6}$$

   Thus, either party can improve its prospects of winning districts even amongst its traditional supporters by offering them more aligned platforms, i.e., by lowering x_L or increasing x_R, thereby intensifying their core supporters' relative preference.

The trade-offs between these three channels drive all of our subsequent results, which we index according to the initial valence advantage ρ_0 of party R.

Proposition 1. *If party R's advantage is **small** in the sense that*

$$0 \leq \rho_0 \leq \frac{\theta(2r - \alpha) - \psi(\alpha - \beta)}{\alpha + \beta} \equiv \underline{\rho}_0,$$

then both parties locate at the ideal policy of the median voter in the median district:

$$x_L^*(\rho_0) = 0, \quad x_R^*(\rho_0) = 0.$$

A party wins a majority of districts if and only if it is most-preferred by the median voter in the median district, i.e., with most-preferred policy zero. When the parties are initially balanced, i.e., ρ_0 is zero, each party is competitive for a majority, and since majority status generates a value regardless of a party's seat share, each party aggressively pursues an outright victory.

Increases in ρ_0 reduce L's chances of winning, but do *not* alter the policy platform that maximizes this probability. Thus—and to an extent that is proportional to its value from majority status, r—L's electoral strategy continues to target a legislative majority by way of a centrist policy platform even as its prospects of winning deteriorate. Notice that as $(\alpha - \beta)\psi$ increases—implying a greater relative concern for incremental minority versus majority seat shares, combined with greater electoral risk—the upper bound of imbalance for which the disadvantaged party wants to compete directly with the advantaged party ($\underline{\rho}_0$) *falls*.

When the imbalance between the parties is large enough, however, L no longer prefers unmitigated competition with R for outright victory.

Proposition 2. *If party R's advantage is* ***intermediate*** *in the sense that*

$$\underline{\rho}_0 \leq \rho_0 < \underline{\rho}_0 + \frac{\psi(\alpha-\beta)(2\theta\alpha+(\alpha-\beta)\gamma)}{(\alpha+\beta)(\alpha\theta+(\alpha-\beta)\gamma)} \equiv \overline{\rho}_0,$$

then party L retreats to its base,

$$x_L^*(\rho_0) = \frac{\theta(2r-\alpha)-\alpha(\rho_0+\psi)+\beta(\psi-\rho_0)}{\gamma(\alpha-\beta)+2\alpha\theta} < 0,$$

but R still locates at the ideal policy of the median voter in the median district, choosing $x_R^*(\rho_0) = 0$.

When the electoral imbalance in favor of party R surpasses an initial threshold $\underline{\rho}_0 > 0$, the disadvantaged party L's prospect of winning a majority—even when targeting the pivotal voter, directly—becomes too low. Reflecting the three channels that we highlight above, L's best electoral strategy reverts to galvanizing its base, i.e., by selecting a platform $x_L(\rho_0) < 0$.

By distancing itself from party R, it creates a meaningful ideological alternative to R's centrist platform: policy differentiation partly mitigates L's valence disadvantage amongst voters that value more left-wing policies. While retreating from the political centre further lowers L's prospect of winning a majority of districts, $\rho_0 > \underline{\rho}_0$ implies that party L no longer finds it worthwhile to target an outright victory. That is, acknowledging that is very likely to hold minority status, its priority smoothly reverts from solely pursuing a majority to instead balancing this objective with the need to secure the most advantageous minority share of seats possible.

By contrast, the same primitives encourage party R to maintain its hold on the ideological centre-ground. Its prospect of winning the election is maximized by selecting the policy preferred by the median voter in the median district. Party R *could* chase L into its own ideological turf, in order to increase its seat advantage conditional on holding a majority. However, its initial electoral advantage is still small enough ($\rho_0 < \overline{\rho}_0$) that it does not want to risk its prospect of winning.

To see this more clearly, notice that the size of the interval $[\underline{\rho}_0, \overline{\rho}_0]$ is proportional to $\psi(\alpha-\beta)$, and the interval is empty when $\alpha = \beta$. This reflects the advantaged party's incentive to hold back versus give chase: as it advances on its retreating opponent, it raises its share of districts conditional on winning, which it values at rate β; but by lowering its appeal amongst its core supporters, it risks losing these districts in the event of an unfavorable election outcome that consigns the party to minority status, which it values at rate α. As the wedge $\alpha - \beta$ increases—augmented by the risk surrounding the election outcome ψ—the advantaged party increasingly prefers to 'play it safe', holding back even as its initial advantage increases.

These channels generate natural effects of primitives on party L's platform, and thus the degree of policy divergence between the parties.

Corollary 1. *Party L increasingly retreats to its base—and thus platform divergence* increases*—whenever its initial disadvantage* ρ_0 increases*, if the marginal value of minority seats* α increases*, or when uncertainty about voter preferences* ψ *increases. Conversely, L increasingly targets the median voter in the median district if the value of majority status* r increases*, when party loyalty* θ increases*, or when policy responsiveness* γ increases*.*

If party loyalty θ amongst more ideologically polarized voters rises, party L becomes less worried about losing districts with a larger share of core supporters; this encourages it to target more centrist districts whose support is crucial for the party to win. Conversely, when voters are relatively more responsive to platform choices via γ, the weaker party must make greater concessions to its base to win their support.

Finally, suppose that parties anticipate that a more volatile electorate via higher ψ. This implies that for any pair of platforms, there is a heightened prospect of a large post-election imbalance between the majority and minority party via more extreme realizations of $\rho_1 \sim U[-\psi, \psi]$. If the disadvantaged party competes more aggressively by moving its platform towards its opponent, it could win more seats in the event of a majority, but it may lose more seats in the event of an unfavorable ρ_1 realization that consigns the party minority status. Here, with $\alpha > \beta$, risk-aversion encourages the weaker party to hasten its retreat. Thus, our framework predicts that platform polarization is greater in contexts where party loyalty is weaker (θ smaller) and voters' preferences are more volatile.

Finally, when the imbalance between the parties is very large, party R becomes so emboldened by its initial advantage over L that it abandons the mere pursuit of victory, and instead chases its weaker opponent in an effort to plunder its moderate supporters.

Proposition 3. *If party R's advantage is* ***large****, i.e., $\rho_0 > \overline{\rho}_0$, then party L retreats by more to its base:*

$$x_L^*(\rho_0) = \frac{((\alpha-\beta)\gamma+\beta\theta)(\theta(2r-\alpha)-(\alpha+\beta)\rho_0)-\beta\theta\psi(\alpha-\beta)}{(\alpha^2-\beta^2)\gamma+2\alpha\beta\theta}, \tag{7}$$

and party R advances towards party L's base:

$$x_R^*(\rho_0) = x_L^*(\rho_0) + (\alpha-\beta)\frac{(\alpha+\beta)(\psi-\rho_0)+\theta(2r-\alpha)}{(\alpha^2-\beta^2)\gamma+2\alpha\beta\theta}. \tag{8}$$

When the electoral imbalance in favor of party R are very large, party L overwhelmingly focuses on consolidating support amongst its base. When this advantage is large enough, however, party R also advances into L's ideological territory to win over centre-left districts that are increasingly ill-served by the more extreme L party. It does so for two reasons. First, a sufficiently large advantage ($\rho_0 > \overline{\rho}_0$) makes party R less concerned about its chances of achieving majority status and instead more focused on generating the largest possible legislative majority in the event that it wins. Second, by reducing the policy differentiation between the parties, R intensifies its comparative valence advantage, further increasing its support. Notice that when $\alpha-\beta=0$, the platforms converge, i.e., $x_L^*(\rho_0) = x_R^*(\rho_0)$, reflecting the chase-and-evade logic that is present in Aragones and Palfrey (2002).[12]

We summarize the effect of primitives on the parties' platforms, and their consequences for platform divergence.

Corollary 2. *As R's initial advantage $\rho_0 \geq \overline{\rho}_0$ increases, both party L and party R move towards L's base, and platform divergence* decreases.

As party L becomes further disadvantaged, it faces even greater incentives to target its base; by differentiating itself further from the advantaged party, it increases its appeal to its core supporters, consolidating its minority position. However, party R is also further emboldened to advance into its opponent's home turf. The incentive to do so is two-fold; a higher ρ_0 strengthens R's incentives to chase its increasingly weakened L *and*—independently—it also wants to use its platform to turn centrist districts that L has abandoned, in pursuit of an outsized majority. The net effect is that platforms further converge, with the speed of convergence increasing in β, the marginal value of seats conditional on majority status.

[12] In our framework, $\theta > 0$ still implies that the equilibrium is in pure strategies.

The Corollary highlights that party R's platform moves to the left faster than party L's, so that the net effect is to reduce policy differentiation between the parties. Conversely, if ρ_0 *decreases*, both parties move their platforms towards the median voter in the median district, but party L moves more slowly than party R, *increasing* the degree of platform divergence.

Other changes in primitives may lead to different effects for the ex-ante advantaged versus disadvantaged party, and may further depend on other features of the political environment.

Corollary 3. *When the marginal value of minority seats, α, increases, party L increasingly retreats to its base by an amount that increases in preference volatility, ψ. By contrast, when α increases, there exists $\hat{\rho}_0 \geq \overline{\rho}_0$ such that party R moves towards L's base if and only its initial advantage ρ_0 exceeds $\hat{\rho}_0$.*

As α rises, party L grows more concerned about not losing the election too badly, so it increasingly targets its core supporters. Party R, however, faces two conflicting incentives. First, as α increases, it too has a stronger incentive to consolidate its core support by reverting to the right, i.e., in the direction of its base. This incentive increases with preference volatility, ψ, since more volatility implies a greater risk of a bad election result that consigns the party to minority status. Second, as party L increasingly moves to its base, party R faces a stronger incentive to advance towards party L's platform in order to reduce the policy differentiation between parties, thereby heightening its comparative advantage.

The net effect on party R's equilibrium platform depends on the size of its initial advantage. If this initial advantage is low, party R's unwillingness to abandon its core supporters is the dominant force, encouraging it to move its platform back towards the median voter in the median district. If, instead, its initial advantage is large enough, party R chases party L even more aggressively, in order to reduce platform differentiation and further press its heightened advantage.

Corollary 4. *As the value of majority status r increases, both party L and party R revert towards the ideal policy of the median voter in the median district, but platform divergence* increases.

A party wins a majority if and only if it is preferred by the median voter in the median district. A higher value r of majority status therefore encourages both parties to target this voter. Corollary 4 highlights that party R's platform moves faster than party L's. To see why, recall that party L remains at a competitive disadvantage; moving towards the centre raises its attractiveness to moderate voters, but dampens its relative appeal amongst its base. This represents a trade-off for party L. For party R, however, moving back towards the centre both raises its appeal to centrists *and* its core supporters.

Since both trade-offs are complementary to party R, but opposing for party L, the net effect is to increase platform divergence: L reluctantly abandons its base, while R's increased desire to win implies that its platform choice is governed less by the incentive to chase L, and more by the incentive to maximize its appeal to the decisive voter in a bid for outright victory.

Corollary 5. *As electoral volatility ψ increases, both party L and party R revert towards their respective core supporters, and platform divergence* increases*.*

When there is a large initial wedge in the parties' initial strength, more uncertainty *always* raises platform divergence. This reflects that both parties grow more concerned with insuring themselves against an adverse popularity shock by consolidating their core supporters. Greater volatility raises the prospect that the election will result in large imbalance in favor of one of the two parties. Because $\alpha - \beta > 0$, each risk-averse party resolves in favor of buttressing its seat share in the event that it is consigned to minority status.

We highlight our framework's predictions about the political contexts in which platform divergence is maximized. Maximal platform divergence occurs when $\rho_0 = \overline{\rho}_0$, i.e., when the initial imbalance between the parties is large, but not overwhelming. Platform divergence also rises when parties care substantially about bolstering the minority position (α large), when traditional party loyalties are in flux (θ small) and when there is significant uncertainty about the mood of the electorate, as reflected in uncertainty about ρ_1 (i.e., ψ is large).

What about $\beta > \alpha$? Our analysis focuses on settings where $\alpha \geq \beta$, i.e., where the marginal value of additional seats to the minority exceeds the marginal value of additional seats to the majority (β), above and beyond its *per se* benefit from majority status, r. In the less plausible context in which $\beta > \alpha$, the parties fully converge on the ideal policy of the median voter in the median district when R's advantage is not too large—as in our benchmark setting. As we detail in a Supplemental Appendix, however, with a very large initial advantage, the shape of preferences may induce parties to engage in risk-taking behavior, generating platform separation in which party R gambles on a left-wing platform, leaving the centre-ground to its weaker opponent. Our benchmark presentation, by contrast, reflects the more typical scenario in which parties may court their opponent's core supporters (as detailed in Proposition 3), but never to the extent that their own core voters are better served by their opponent.

4. Early Advantages, and Early Deciders.

In real-world elections, both policy commitments (platforms) and the unforeseen contingencies that are captured in the valence shock ρ_1 unfold over the course of an election campaign. Policies are initially set out in party manifestos, then further substantiated and communicated to the public in the course of candidate debates and journalistic scrutiny.[13] And, campaign developments that shape voters' assessments of the parties on election day—such as personal scandals—may emerge only in the final days running up to polling day. Many voters, nonetheless, make up their minds about which party to support long before the campaign concludes and therefore before these processes fully unfold. These voters may make their choices largely based on the parties' relative standing at the start of the campaign—which is captured in our framework by ρ_0.

The existence of these voters is reflected by the extensive early voting in democracies across the world: in Australia, a ballot can be cast at any time during the course of an election campaign, and in the 2016 federal election, nearly one in three votes were cast early (McAllister and Muller, 2018). In the United States, early votes may be cast as many as 50 days before the election,[14] and in the 2016 presidential election, 36.6% of voters voted early. In New Zealand, advance votes represented 48% of votes cast in the 2017 general election. And, in the academic literature on political campaigns, a *minimal effects* hypothesis contends that most voters make their decisions long before a campaign concludes (e.g., Ansolabehere, 2006).

We next show how even though these votes are in some sense sunk, these early deciders nonetheless exert a disproportionate effect on the course of the subsequent election campaign. When initial valence advantages are small, more early deciders cause both parties to shift towards targetting the supporters of the advantaged party. Moreover, we show that more early deciders make an initially weak party *more reluctant* to retreat to its core supporters, but exacerbate an initially strong party's incentive to give chase. To our knowledge, ours is the first framework to study the consequences of early voting—and early deciders, more broadly—in a formal model of elections.

We suppose that a fraction $\lambda \in (0, 1)$ of voters independent of their district or ideology, are *early*

[13] For example, in the UK 2017 general election, Jeremy Corbyn infamously pledged to write off student debts just seven days before the end of the campaign. After the election, Corbyn clarified that the phrase "deal with it" was not, in fact, an explicit commitment to write off student debt.

[14] A state-by-state guide as of 2017 can be found at `https://goo.gl/4sKx7V`, and a broader discussion of early voting in the United States can be found at `https://goo.gl/ynM3FG`.

deciders. Early deciders vote based on the initial standing of the parties at the start of a campaign, i.e., based on their partisan attachments θ and the parties' relative standing ρ_0 at the start of the election. In particular, they do not make their decisions on the basis of platforms x_L and x_R, nor on subsequent campaign developments that are encapsulated in the random valence shock, ρ_1.[15] The remaining fraction $1-\lambda$ of voters make their decision on the basis of platforms and campaign developments (i.e., ρ_1), as in our benchmark setting.[16]

Proposition 4. *There always exists a unique pure strategy equilibrium, characterized by threshold values*

$$\underline{\rho}_0 = \frac{2\theta(r-\frac{\alpha}{2}) - (1-\lambda)\psi(\alpha-\beta)}{\alpha+\beta}, \tag{9}$$

and

$$\overline{\rho}_0 = \underline{\rho}_0 + (1-\lambda)\frac{\psi(\alpha-\beta)(2\theta\alpha+(\alpha-\beta)\gamma)}{(\alpha+\beta)(\alpha\theta+(\alpha-\beta)\gamma)}, \tag{10}$$

such that

1. *if $\rho_0 \in (0, \underline{\rho}_0]$, the parties locate at $x_L^*(\rho_0) = x_R^*(\rho_0) = \frac{\lambda}{1-\lambda}\frac{\rho_0}{\theta}$.*
2. *If $\rho_0 \in (\underline{\rho}_0, \overline{\rho}_0]$, the Right party continues to locate at $x_R^*(\rho_0) = \frac{\lambda}{1-\lambda}\frac{\rho_0}{\theta}$, but the Left party retreats to its base, locating at a platform $x_L^*(\rho_0) < \frac{\lambda}{1-\lambda}\frac{\rho_0}{\theta}$.*
3. *If $\rho_0 > \overline{\rho}_0$, the parties locate at platforms $x_L^*(\rho_0) < x_R^*(\rho_0) < \frac{\lambda}{1-\lambda}\frac{\rho_0}{\theta}$, strictly decreasing in ρ_0.*

To understand the result notice that an early decider with ideal policy x_i prefers party L if and only if $-\theta x_i \geq \rho_0$, i.e., $x_i \leq \frac{-\rho_0}{\theta}$. Letting x_i^* denote the indifferent voter amongst the $1-\lambda$ remaining voters, party L's vote share in a district with median m is therefore

$$\frac{1}{2} + \frac{\lambda\frac{-\rho_0}{\theta} + (1-\lambda)x_i^* - m}{2Z},$$

[15] This does not rule out that these voters care about policy: they may already internalize policies on other dimensions that are fixed before the election, on which the parties have converged. For simplicity, early deciders do not incorporate their conjectures about the platforms the parties will choose into their decision. This is not important for our qualitative results, however—what matters is that $\rho_0 > 0$ implies an advantage for the R party that is monotone in λ, a property that extends when early deciders are forward-looking with respect to platform choices.

[16] For analytical convenience, we continue to restrict primitives such that each party wins in every district with positive probability, for any platform pair $(x_L, x_R) \in [-1, 1]^2$.

and its total share of districts is:

$$d_L(\lambda, x_L, x_R, \rho_0 + \rho_1) = \frac{1 + \lambda\frac{-\rho_0}{\theta} + (1-\lambda)x_i^*}{2}.$$

Party L therefore wins the election if and only $d_L \geq \frac{1}{2}$, or

$$x_i^* \geq \frac{\lambda}{1-\lambda}\frac{\rho_0}{\theta}. \tag{11}$$

In the absence of voters who make their voting decision prior to the campaign, i.e., when $\lambda = 0$, the crucial voter is the median voter in the median district. But, as $\lambda > 0$ increases, party R secures a greater share of early votes, since $\rho_0 > 0$. It therefore enjoys an initial lead in the polls. Thus, for the disadvantaged party to win the election, it must offset this early lead by carrying an even larger majority of supporters amongst the remaining voters. It must therefore move beyond the median voter in the median district, instead targeting voters that are more ideologically disposed towards its advantaged opponent. Thus, it looks as if the parties adopt platforms that pander to shifts in voter policy views toward the party with the valence advantage, even though policy preferences are actually unaffected.

Corollary 6. *The threshold $\underline{\rho}_0$ at which the disadvantaged starts to revert to its core supporters* increases *in the share of early deciders λ. In contrast, the threshold $\overline{\rho}_0$ at which the advantaged party begins to move its platform toward its rival's supporters* decreases *in λ.*

Because an increase in early deciders cements an initial polling lead in favor of the advantaged party, one might expect the disadvantaged party to respond by beginning to retreat at lower levels of initial imbalance. Inspection of the threshold $\underline{\rho}_0$ reveals that this intuition is incorrect:

$$\underline{\rho}_0 = \frac{2\theta(r - \frac{\alpha}{2}) - (1-\lambda)\psi(\alpha - \beta)}{\alpha + \beta}. \tag{12}$$

The threshold highlights the disadvantaged party's trade-off between two distinct objectives. First, it values winning a majority of seats: moving from minority to majority status generates a benefit of $r - \frac{\alpha}{2}$. This motive encourages the party to compete directly with its stronger opponent for *higher* levels of ρ_0. Second, the party values insurance against large and possibly adverse popularity swings during the campaign, which are captured by the valence shock ρ_1. This motive—which encourages the party to retreat to its base—becomes more important as the uncertainty surrounding the valence shock, ψ rises. Notice, however, that a larger share λ of early deciders implies that a *smaller* share $1 - \lambda$ of voters are affected by this valence shock—making this channel relatively less salient in the disadvantaged party's calculation. This encourages the disadvantaged party to focus more on its pursuit of outright victory, *raising* the threshold $\underline{\rho}_0$.

The same logic implies that an increase in the share of decided voters *lowers* the threshold $\overline{\rho}_0$, above which the advantaged party begins to pursue its weaker opponent's core supporters—the advantaged party is quicker to pursue its rival's supporters because it faces less risk of losing.

Once the disadvantaged party starts its retreat, i.e., once $\rho_0 \geq \overline{\rho}_0$, a further deterioration in its competitiveness—i.e., a further increase in ρ_0—always strengthens its incentive to distinguish itself from its stronger opponent. In contrast with our benchmark setting, however, this need not imply that its platform x_L moves to the left.

Corollary 7. *There exists a threshold λ^*, such that if and only if $\lambda \leq \lambda^*$, $x_L^*(\rho_0)$* decreases *in $\rho_0 \in (\underline{\rho}_0, \overline{\rho}_0)$. Regardless of the share of early deciders, platform polarization $x_R^*(\rho_0) - x_L^*(\rho_0)$* always increases *in ρ_0.*

As the imbalance between the parties increases, their platforms always diverge; since the strong party continues to locate at the pivotal voter's preferred policy, this implies that its weaker opponent always reverts in favor of its core districts at the expense of targeting the pivotal voter—just as in our benchmark setting. But, this pivotal voter, with ideal policy $\frac{\lambda}{1-\lambda}\frac{\rho_0}{\theta}$, is now a moving target—and moving in the opposite direction of the weaker party's core districts as both the initial imbalance ρ_0 and the share of early deciders λ increase. Thus, as its competitive standing deteriorates, the weaker party's efforts to differentiate itself while nonetheless clinging to a prospect of outright victory imply that it may either move to the left—or simply move to the right at a slower rate than its stronger opponent.

5. Conclusion

We analyze two-party competition in multi-district legislative elections. We ask: how do initial electoral imbalances encourage an office-seeking party to target its traditional supporters, rather than the centrist voters that are crucial for outright victory? If it targets traditional supporters, when should the opposing party maintain its focus on courting centrist voters, and when instead should it chase its opponent, targeting voters that are more ideologically disposed towards its opponent? And, how do the answers to these questions depend on parties' expectations of how voters attitudes might change over the course of the campaign, the strength of pre-existing party loyalty, and the relative marginal value that a party derives from winning additional seats below, at, or even above the majority threshold?

A small initial imbalance does not deter a disadvantaged party from the sole pursuit of outright victory by way of a centrist policy agenda. However, a sufficiently large imbalance induces it to revert in favor of a strategy that consolidates its core supporters, in order to avoid a catastrophic defeat. Similarly, an advantaged party initially prefers to maintain uncontested control of the political centre to further fortify its prospects of a post-election majority. But, if the imbalance is large enough, it chases its opponent to plunder its increasingly ill-served moderate supporters; its goal is not only to win, but to win with a large post-election majority.

Our framework generates novel predictions about the consequences of initial electoral imbalances for platform choices and polarization. In particular, we predict that a very advantaged party uses its strength as an opportunity to expand the frontier of its political support beyond the median voter. This contrasts with Groseclose (2001), who predicts that a very advantaged party instead uses its advantage to revert towards its own base. This logic also implies that polarization between parties is maximized not when the imbalance between parties is very large, but instead when it is intermediate—small enough that the stronger party maintains rather than attempting to expand its support, but large enough that the weak party reverts towards a more defensive strategy. We also find that polarization is greatest *not* when party loyalties are strong, but rather when they are *weak* so that core supporters cannot be taken for granted. Finally, we show that when some voters pay little attention to platforms and campaigns, their presence may paradoxically exert a disproportionate influence on parties' electoral strategies.

References

ANSOLABEHERE, S. (2006): "The Paradox of Minimal Effects," in *Capturing Campaign Effects*, ed. by H. Brady and R. Johnston, University of Michigan Press.

ARAGONES, E. AND T. R. PALFREY (2002): "Mixed equilibrium in a Downsian model with a favored candidate," *Journal of Economic Theory*, 103, 131–161.

AUSTEN-SMITH, D. (1984): "Two-party competition with many constituences," *Mathematical Social Sciences*, 7, 177–198.

BUTLER, D. AND D. KAVANAGH (1997): *The British general election of 1997*, Palgrave Macmillan UK.

CALLANDER, S. (2005): "Electoral Competition in Heterogeneous Districts," *Journal of Political Economy*, 113, 1116–1145.

CALVERT, R. L. (1985): "Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence," *American Journal of Political Science*, 29, 69–95.

DOWNS, A. (1957): *An Economic Theory of Democracy*, New York: Harper and Row.

EGUIA, J. AND F. GIOVANNONI (2017): "Tactical Extremism," Unpublished.

GROSECLOSE, T. (2001): "A Model of Candidate Location when One Candidate Has a Valence Advantage," *American Journal of Political Science*, 45, 862–886.

HOTELLING, H. (1929): "Stability in Competition," *Economic Journal*, XXXIX, 41–57.

HUMMEL, P. (2010): "On the nature of equilibria in a Downsian model with candidate valence," *Games and Economic Behavior*, 70, 425–445.

KITTSTEINER, T. AND E. EYSTER (2007): "Party platforms in electoral competition with heterogeneous constituencies," *Theoretical Economics*, 2, 41–70.

KRASA, S. AND M. POLBORN (2018): "Political Competition in Legislative Elections," *American Political Science Review*, forthcoming.

MCALLISTER, I. AND D. MULLER (2018): "Early voting, election campaigning and party advantage in Australia," *Electoral Studies*, 52, 103–110.

WITTMAN, D. (1983): "Candidate Motivation: A Synthesis of Alternative Theories," *American Political Science Review*, 77, 142–157.

6. Appendix: Proofs of Results

We solve the full model, in which a fraction $\lambda \geq 0$ of voters are *early deciders*, as described in Section 4 of the paper. Our benchmark model—stated in Propositions 1,2 and 3, are obtained by setting $\lambda = 0$. We refer to the remaining fraction $1-\lambda$ of voters as *late deciders*. We maintain our restriction that, for all primitives, each party wins with positive probability in all districts, for any platform pair $(x_L, x_R) \in [-1,1]^2$.

Let $i^1(\rho_0 + \rho_1, x_L, x_R)$ denote the preferred policy of the late decider that is indifferent between the parties L and R given the realized net valence advantage to party R of $\rho_0 + \rho_1$ and the platform choices x_L and x_R. Given $\theta > 2\gamma$, for any pair (x_L, x_R), a late decider j with ideal policy $x_j > i^1$ strictly prefers party R, and a late decider j with ideal policy $x_j < i^1$ strictly prefers party L. Similarly, let $i^0 = \frac{-\rho_0}{\theta}$ denote the preferred policy of the early decider that is indifferent between the parties L and R given the net valence to party R of ρ_0. Party L's total vote share in a district with median $m \in [-1,1]$ is therefore

$$\frac{1}{2} + \frac{\lambda i^0 + (1-\lambda) i^1 - m}{2Z}, \tag{13}$$

and its total share of districts is

$$d_L(\lambda, x_L, x_R, \rho_0, \rho_1) = \frac{1}{2} + \frac{\lambda i^0 + (1-\lambda) i^1}{2}, \tag{14}$$

and therefore wins the election if and only if $d_L \geq \frac{1}{2}$, which is equivalent to $\lambda i^0 + (1-\lambda) i^1 \geq 0$, or:

$$i^1 \geq \frac{\lambda}{1-\lambda} \frac{\rho_0}{\theta}. \tag{15}$$

Proof of Propositions 1, 2, 3 and 4. Define $X = \frac{\lambda}{1-\lambda}\frac{\rho_0}{\theta}$. We first rule out the existence of an equilibrium in which the platform profile (x_L, x_R) fails to satisfy $x_L \leq x_R \leq X$.

Profile 1: $x_L \leq X < x_R$. There are 3 possible locations for the indifferent late voter:

1. *Location 1*: $i^1 \geq x_R$, i.e. $\rho_1 \leq \gamma(x_L - x_R) - \rho_0 - \theta x_R$:

$$i^1 = \frac{\gamma(x_L - x_R) - \rho_0 - \rho_1}{\theta} \equiv i^1_i.$$

2. *Location 2*: $x_L \leq i^1 \leq x_R$, i.e. $\gamma(x_L - x_R) - \rho_0 - \theta x_R \leq \rho_1 \leq \gamma(x_R - x_L) - \rho_0 - \theta x_L$:

$$i^1 = \frac{\gamma(x_L + x_R) - \rho_0 - \rho_1}{2\gamma + \theta} \equiv i^1_{ii}.$$

3. <u>*Location 2*</u>: $i^1 \leq x_L$, i.e. $\rho_1 \geq \gamma(x_R - x_L) - \rho_0 - \theta x_L$:

$$i^1 = \frac{\gamma(x_R - x_L) - \rho_0 - \rho_1}{\theta} \equiv i^1_{iii}.$$

Party R wins if and only if $i^1_{ii} \leq X$, i.e.,

$$\rho_1 > \gamma(x_L + x_R) - \rho_0 \frac{(\theta + 2\lambda\gamma)}{(1-\lambda)\theta}. \tag{16}$$

Party R's expected payoff is therefore:

$$\begin{aligned}
\pi_R(x_L, x_R) \ &= \frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L - x_R) - \rho_0 - \theta x_R} \left(\frac{\alpha}{2} + \alpha\frac{\lambda\rho_0 - (1-\lambda)(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta}\right) d\rho_1 \\
&+ \frac{1}{2\psi}\int_{\gamma(x_L - x_R) - \rho_0 - \theta x_R}^{\gamma(x_L + x_R) - \rho_0\frac{(\theta+2\lambda\gamma)}{(1-\lambda)\theta}} \left(\frac{\alpha}{2} + \alpha\left(\frac{\lambda\rho_0}{2\theta} - (1-\lambda)\frac{\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(2\gamma + \theta)}\right)\right) d\rho_1 \\
&+ \frac{1}{2\psi}\int_{\gamma(x_L + x_R) - \rho_0\frac{(\theta+2\lambda\gamma)}{(1-\lambda)\theta}}^{\gamma(x_R - x_L) - \rho_0 - \theta x_L} \left(r + \beta\left(\frac{\lambda\rho_0}{2\theta} - (1-\lambda)\frac{\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(2\gamma + \theta)}\right)\right) d\rho_1 \\
&+ \frac{1}{2\psi}\int_{\gamma(x_R - x_L) - \rho_0 - \theta x_L}^{\psi} \left(r + \beta\frac{\lambda\rho_0 - (1-\lambda)(\gamma(x_R - x_L) - \rho_0 - \rho_1)}{2\theta}\right) d\rho_1.
\end{aligned} \tag{17}$$

Under Assumption 1, this expected payoff is strictly concave in x_R; solving the first-order condition yields:

$$\hat{x}_R^{\text{int}}(x_L) = \frac{(2r - \alpha)\theta + x_L(\alpha - \beta)\gamma(-1 + \lambda) + (\alpha + \beta - 2\alpha\lambda)\rho_0 + (\alpha - \beta)(-1 + \lambda)\psi}{(-\beta\gamma + \alpha(\gamma + 2\theta))(-1 + \lambda)},$$

which increases in x_L. Setting $x_L = X$, we find that

$$\hat{x}_R^{\text{int}}(x_L) - X = \frac{(2r - \alpha)\theta - (\alpha - \beta)(1 - \lambda)\psi + (\alpha + \beta)\rho_0}{(-\beta\gamma + \alpha(\gamma + 2\theta))(-1 + \lambda)}$$

which is strictly negative for all $\rho_0 \geq 0$, under Assumption 1. This contradicts the supposition that $x_R > X$ is a best response.

<u>*Profile 2: $X \leq x_L \leq x_R$*</u>*, with at least one strict inequality*. Under this profile, party R wins if and only if $i^1_i \leq X$ which happens if and only if $\rho_1 \geq \gamma(x_R - x_L) - \frac{\rho_0}{(1-\lambda)}$. Party R's expected payoff is

therefore:

$$\begin{aligned}\pi_R(x_L,x_R) &= \frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L-x_R)-\rho_0-\theta x_R}\left(\frac{\alpha}{2}+\alpha\frac{\lambda\rho_0-(1-\lambda)(\gamma(x_L-x_R)-\rho_0-\rho_1)}{2\theta}\right)d\rho_1\\ &+\frac{1}{2\psi}\int_{\gamma(x_L-x_R)-\rho_0-\theta x_R}^{\gamma(x_R-x_L)-\rho_0-\theta x_L}\left(\frac{\alpha}{2}+\alpha\left(\frac{\lambda\rho_0}{2\theta}-(1-\lambda)\frac{(\gamma(x_L+x_R)-\rho_0-\rho_1)}{2(2\gamma+\theta)}\right)\right)d\rho_1\\ &+\frac{1}{2\psi}\int_{\gamma(x_R-x_L)-\rho_0-\theta x_L}^{\gamma(x_R-x_L)-\frac{\rho_0}{1-\lambda}}\left(\frac{\alpha}{2}+\alpha\left(\frac{\lambda\rho_0-(1-\lambda)(\gamma(x_R-x_L)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1\\ &+\frac{1}{2\psi}\int_{\gamma(x_R-x_L)-\frac{\rho_0}{1-\lambda}}^{\psi}\left(r+\beta\frac{\lambda\rho_0-(1-\lambda)(\gamma(x_R-x_L)-\rho_0-\rho_1)}{2\theta}\right)d\rho_1.\end{aligned} \quad (18)$$

Under Assumption 1, this expected payoff is strictly concave in x_R; solving the first-order condition yields:

$$\hat{x}_R^{\text{int}}(x_L)=\frac{(2r-\alpha)\theta+x_L(\alpha-\beta)\gamma(-1+\lambda)+\rho_0(\alpha+\beta-2\alpha\lambda)-(\alpha-\beta)\psi(1-\lambda)}{((\alpha-\beta)\gamma+2\theta\alpha)(-1+\lambda)}, \quad (19)$$

so that:

$$\hat{x}_R^{\text{int}}(x_L)-X=\frac{(2r-\alpha)\beta\theta^2-(\alpha-\beta)\beta\theta(1-\lambda)\psi+\rho_0(\alpha^2\gamma+\alpha\beta\theta-\beta^2\gamma+\beta^2\theta)}{\theta((\alpha^2-\beta^2)\gamma+2\alpha\theta\beta)(-1+\lambda)}<0,$$

which implies $x_L=x_R$; the supposition $X\le x_L\le x_R$ with at least one strict inequality implies $X<x_L=x_R$. Consider, therefore, party R's expected payoff from a choice of $x_R\in[X,x_L]$:

$$\begin{aligned}\pi_R(x_L,x_R) = &\frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L-x_R)-\rho_0-\theta x_L}\left(\frac{\alpha}{2}+\alpha\frac{\lambda\rho_0-(1-\lambda)(\gamma(x_L-x_R)-\rho_0-\rho_1)}{2\theta}\right)d\rho_1\\ &+\frac{1}{2\psi}\int_{\gamma(x_L-x_R)-\rho_0-\theta x_L}^{\gamma(x_R-x_L)-\rho_0-\theta x_R}\left(\frac{\alpha}{2}+\alpha\left(\frac{\lambda\rho_0}{2\theta}-(1-\lambda)\frac{-\gamma(x_L+x_R)-\rho_0-\rho_1}{2(\theta-2\gamma)}\right)\right)d\rho_1\\ &+\frac{1}{2\psi}\int_{\gamma(x_R-x_L)-\rho_0-\theta x_R}^{\gamma(x_R-x_L)-\frac{\rho_0}{1-\lambda}}\left(\frac{\alpha}{2}+\alpha\left(\frac{\lambda\rho_0-(1-\lambda)(\gamma(x_R-x_L)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1\\ &+\frac{1}{2\psi}\int_{\gamma(x_R-x_L)-\frac{\rho_0}{1-\lambda}}^{\psi}\left(r+\beta\frac{\lambda\rho_0-(1-\lambda)(\gamma(x_R-x_L)-\rho_0-\rho_1)}{2\theta}\right)d\rho_1.\end{aligned} \quad (20)$$

Under Assumption 1, this expected payoff is strictly concave; solving the first-order condition yields:

$$\hat{x}_R^{\text{int}} = \frac{(2r-\alpha)\theta + x_L(\alpha-\beta)\gamma(-1+\lambda) + \rho_0(\alpha+\beta-2\alpha\lambda) - (\alpha-\beta)\psi(1-\lambda)}{(-\beta\gamma+\alpha(2\theta+\gamma))(-1+\lambda)}$$

which replicates (19), and thus satisfies $\hat{x}_R^{\text{int}} - X < 0$, contradicting the supposition that $X < x_L = x_R$, in an equilibrium.

Profile 3: $x_R \leq X < x_L$. There are three possible locations for the indifferent late voter:

Location 1: $i^1 \leq x_R$, i.e., $\gamma(x_R - x_L) - \gamma(x_R - x_R) - \rho_0 - \rho_1 - \theta x_R \leq 0$, i.e., $\rho_1 \geq \gamma(x_R - x_L) - \rho_0 - \theta x_R$:

$$\gamma(i^1 - x_L) - \gamma(i^1 - x_R) - \rho_0 - \rho_1 - \theta i^1 = 0 \iff i^1 = \frac{\gamma(x_R - x_L) - \rho_0 - \rho_1}{\theta} \equiv i^1_i. \tag{21}$$

Location 2: $i^1 \in (x_R, x_L)$, i.e., $\gamma(x_R - x_L) - \gamma(x_R - x_R) - \rho_0 - \rho_1 - \theta x_R > 0$ and $\gamma(x_L - x_L) - \gamma(x_R - x_L) - \rho_0 - \rho_1 - \theta x_L < 0$, i.e., $\gamma(x_R - x_L) - \rho_0 - \theta x_R > \rho_1 > \gamma(x_L - x_R) - \rho_0 - \theta x_L$. This implies:

$$\gamma(i^1 - x_L) - \gamma(x_R - i^1) - \theta i^1 - \rho_0 - \rho_1 = 0 \iff i^1 = \frac{-\gamma(x_L + x_R) - \rho_0 - \rho_1}{\theta - 2\gamma} \equiv i^1_{ii}. \tag{22}$$

Location 3: $i^1 \geq x_L$, i.e., $\gamma(x_L - x_L) - \gamma(x_R - x_L) - \rho_0 - \rho_1 - \theta x_L > 0$, i.e., $\rho_1 < \gamma(x_L - x_R) - \rho_0 - \theta x_L$. This implies:

$$\gamma(x_L - i^1) - \gamma(x_R - i^1) - \theta i^1 - \rho_0 - \rho_1 = 0 \iff i^1 = \frac{\gamma(x_L - x_R) - \rho_0 - \rho_1}{\theta} \equiv i^1_{iii}. \tag{23}$$

Party R wins if and only if $i^1_{ii} \leq X$, i.e.,

$$\rho_1 \geq -\gamma(x_L + x_R) - \frac{\theta - 2\gamma\lambda}{(1-\lambda)\theta}\rho_0.$$

Party R's expected payoff from $x_R \leq X$ is therefore:

$$\begin{aligned}
\pi_R(x_L, x_R) = {} & \frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L - x_R) - \rho_0 - \theta x_L} \left(\frac{\alpha}{2} + \alpha\left(\frac{\lambda\rho - (1-\lambda)(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta}\right)\right) d\rho_1 \\
& + \frac{1}{2\psi}\int_{\gamma(x_L - x_R) - \rho_0 - \theta x_L}^{-\gamma(x_L + x_R) - \frac{\theta - 2\gamma\lambda}{(1-\lambda)\theta}\rho_0} \left(\frac{\alpha}{2} + \alpha\left(\frac{\lambda\rho_0}{2\theta} - (1-\lambda)\frac{-\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(\theta - 2\gamma)}\right)\right) d\rho_1 \\
& + \frac{1}{2\psi}\int_{-\gamma(x_L + x_R) - \frac{\theta - 2\gamma\lambda}{(1-\lambda)\theta}\rho_0}^{\gamma(x_R - x_L) - \rho_0 - \theta x_R} \left(r + \beta\left(\lambda\frac{\rho_0}{2\theta} - (1-\lambda)\frac{-\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(\theta - 2\gamma)})\right)\right) d\rho_1 \\
& + \frac{1}{2\psi}\int_{\gamma(x_R - x_L) - \rho_0 - \theta x_R}^{\psi} \left(r + \beta\left(\frac{\lambda\rho_0 - (1-\lambda)(\gamma(x_R - x_L) - \rho_0 - \rho_1)}{2\theta}\right)\right) d\rho_1.
\end{aligned} \tag{24}$$

Under Assumption 1, this expected payoff is strictly concave in x_R; solving the first-order condition yields:

$$\hat{x}_R^{\text{int}}(x_L) = \frac{(-2r+\alpha)\theta + x_L(\alpha-\beta)\gamma(-1+\lambda) + \rho_0(\alpha+\beta-2\alpha\lambda) - (\alpha-\beta)\psi(1-\lambda)}{((\alpha-\beta)\gamma+2\beta\theta)(-1+\lambda)}.$$

Similarly, party L's expected payoff from $x_L \geq X$ is:

$$\begin{aligned}
\pi_L(x_L, x_R) = {} & \frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L-x_R)-\rho_0-\theta x_L} \left(r \; + \beta\left(\frac{-\lambda\rho_0 + (1-\lambda)(\gamma(x_L-x_R)-\rho_0-\rho_1)}{2\theta}\right)\right) d\rho_1 \\
& + \frac{1}{2\psi}\int_{\gamma(x_L-x_R)-\rho_0-\theta x_L}^{-\gamma(x_L+x_R)-\frac{\theta-2\gamma\lambda}{(1-\lambda)\theta}\rho_0} \left(r \; + \beta\left(-\frac{\lambda\rho_0}{2\theta} + (1-\lambda)\frac{-\gamma(x_L+x_R)-\rho_0-\rho_1}{2(\theta-2\gamma)}\right)\right) d\rho_1 \\
& + \frac{1}{2\psi}\int_{-\gamma(x_L+x_R)-\frac{\theta-2\gamma\lambda}{(1-\lambda)\theta}\rho_0}^{\gamma(x_R-x_L)-\rho_0-\theta x_R} \left(\frac{\alpha}{2} + \alpha\left(-\frac{\lambda\rho_0}{2\theta} + (1-\lambda)\frac{-\gamma(x_L+x_R)-\rho_0-\rho_1}{2(\theta-2\gamma)}\right)\right) d\rho_1 \\
& + \frac{1}{2\psi}\int_{\gamma(x_R-x_L)-\rho_0-\theta x_R}^{\psi} \left(\frac{\alpha}{2} + \alpha\left(\frac{-\lambda\rho_0 + (1-\lambda)(\gamma(x_R-x_L)-\rho_0-\rho_1)}{2\theta}\right)\right) d\rho_1. \qquad (25)
\end{aligned}$$

Under Assumption 1, this expected payoff is strictly concave in x_L; solving the first-order condition yields:

$$\hat{x}_L^{\text{int}}(x_R) = \frac{(2r-\alpha)\theta + x_R(\alpha-\beta)\gamma(-1+\lambda) + \rho_0(\alpha+\beta-2\alpha\lambda) - (\alpha-\beta)\psi(1-\lambda)}{((\alpha-\beta)\gamma+2\beta\theta)(-1+\lambda)}.$$

Solving these interior best responses, we obtain a pair (x_L^*, x_R^*) satisfying, under Assumption 1:

$$x_R^* - x_L^* = \frac{-2r\theta - \beta(-1+\lambda)\psi + \alpha(\theta+(-1+\lambda)\psi)}{((\alpha-\beta)\gamma+\beta\theta)(-1+\lambda)} > 0.$$

We conclude that there does not exist an equilibrium in which $x_R < X < x_L$.

Suppose, instead, $x_R = X < x_L$. Since $\hat{x}_L^{\text{int}}(x_R)$ strictly decreases in ρ_0, we may set $\rho_0 = 0$ and obtain:

$$\hat{x}_L^{\text{int}}(X) = \frac{2r\theta + \beta(-1+\lambda)\psi - \alpha(\theta+(-1+\lambda)\psi)}{((\alpha-\beta)\gamma+2\beta\theta)(-1+\lambda)} < X,$$

which therefore contradicts $x_L > X$.

Profile 4: $x_R < x_L \leq X$. The three possible locations of the critical voter i^1 are given in expressions (21) through (23). In contrast with Profile 3, however, Party R wins under this profile if and only if $i^1_{iii} \leq X$, i.e.,

$$\rho_1 \geq \gamma(x_L - x_R) - \frac{\rho_0}{1-\lambda}.$$

Party R's expected payoff is:

$$\begin{aligned}
\pi_R(x_L, x_R) = {} & \frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L-x_R)-\frac{\rho_0}{(1-\lambda)}} \left(\frac{\alpha}{2} + \alpha\left(\frac{\lambda\rho_0 - (1-\lambda)(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta}\right)\right)d\rho_1 \\
& + \frac{1}{2\psi}\int_{\gamma(x_L-x_R)-\frac{\rho_0}{(1-\lambda)}}^{\gamma(x_L-x_R)-\rho_0-\theta x_L} \left(r \ + \beta\left(\frac{\lambda\rho_0 - (1-\lambda)(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta}\right)\right)d\rho_1 \\
& + \frac{1}{2\psi}\int_{\gamma(x_L-x_R)-\rho_0-\theta x_L}^{\gamma(x_R-x_L)-\rho_0-\theta x_R} \left(r \ + \beta\left(\frac{\lambda\rho_0}{2\theta} - (1-\lambda)\frac{-\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(\theta - 2\gamma)}\right)\right)d\rho_1 \\
& + \frac{1}{2\psi}\int_{\gamma(x_R-x_L)-\rho_0-\theta x_R}^{\psi} \left(r \ + \beta\left(\frac{\lambda\rho_0 - (1-\lambda)(\gamma(x_R - x_L) - \rho_0 - \rho_1)}{2\theta}\right)\right)d\rho_1. \qquad (26)
\end{aligned}$$

Under Assumption 1, this expected payoff is strictly concave in x_R; solving the first-order condition yields:

$$\hat{x}_R^{\text{int}}(x_L) = \frac{(-2r + \alpha)\theta + x_L(\alpha - \beta)\gamma(-1 + \lambda) + \rho_0(\alpha + \beta - 2\alpha\lambda) - (\alpha - \beta)\psi(1 - \lambda)}{((\alpha - \beta)\gamma + 2\beta\theta)(-1 + \lambda)}.$$

Similarly, party L's expected payoff is:

$$\begin{aligned}
\pi_L(x_L, x_R) = {} & \frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L-x_R)-\frac{\rho_0}{(1-\lambda)}} \left(r \ + \beta\left(\frac{-\lambda\rho_0 + (1-\lambda)(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta}\right)\right)d\rho_1 \\
& + \frac{1}{2\psi}\int_{\gamma(x_L-x_R)-\frac{\rho_0}{(1-\lambda)}}^{\gamma(x_L-x_R)-\rho_0-\theta x_L} \left(\frac{\alpha}{2} + \alpha\left(\frac{-\lambda\rho_0 + (1-\lambda)(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta}\right)\right)d\rho_1 \\
& + \frac{1}{2\psi}\int_{\gamma(x_L-x_R)-\rho_0-\theta x_L}^{\gamma(x_R-x_L)-\rho_0-\theta x_R} \left(\frac{\alpha}{2} + \alpha\left(-\frac{\lambda\rho_0}{2\theta} + (1-\lambda)\frac{-\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(\theta - 2\gamma)}\right)\right)d\rho_1 \\
& + \frac{1}{2\psi}\int_{\gamma(x_R-x_L)-\rho_0-\theta x_R}^{\psi} \left(\frac{\alpha}{2} + \alpha\left(\frac{\lambda\rho_0 - (1-\lambda)(\gamma(x_R - x_L) - \rho_0 - \rho_1)}{2\theta}\right)\right)d\rho_1. \qquad (27)
\end{aligned}$$

Under Assumption 1, this expected payoff is strictly concave in x_L; solving the first-order condition yields:

$$\hat{x}_L^{\text{int}}(x_R) = \frac{(-2r + \alpha)\theta + x_R(\alpha - \beta)\gamma(-1 + \lambda) + \rho_0(\alpha + \beta - 2\alpha\lambda) - (\alpha - \beta)\psi(1 - \lambda)}{((\alpha - \beta)\gamma + 2\alpha\theta)(-1 + \lambda)}$$

Suppose, first, $x_R < x_L < X$. Solving the interior best responses simultaneously yields a pair

(x_L^*, x_R^*) satisfying:

$$x_L^* - x_R^* = -\frac{(\alpha-\beta)(-2r\theta+\beta(\rho_0+(-1+\lambda)\psi)+\alpha(\theta+\rho_0+(-1+\lambda)\psi))}{((\alpha^2-\beta^2)\gamma+2\alpha\beta\theta)(-1+\lambda)},$$

which strictly increases in ρ_0. Straightforward algebra establishes

$$x_L^* - x_R^* \geq 0 \iff \rho_0 \geq \psi(1-\lambda) + \frac{\theta(2r-\alpha)}{\alpha+\beta}, \tag{28}$$

which violates our assumption that for all primitives, each party wins in every district with positive probability for any platform pair $[x_L, x_R] \in [-1,1]^2$. To see why, notice that by straightforward algebra, party L wins the district whose median voter has preferred-policy 1 if and only if

$$\rho_0 < \frac{(1-\lambda)\psi\theta}{\lambda+(1-\lambda)\theta} - \frac{\theta}{\lambda+(1-\lambda)\theta} - \frac{(\theta+2\gamma)\theta(1-\lambda)}{\lambda+(1-\lambda)\theta}, \tag{29}$$

and this threshold is strictly smaller than $\psi(1-\lambda)$.

Suppose, instead, $x_R < x_L = X$. We find that $x_R^{\text{int}}(X)$ strictly decreases in ρ_0, and moreover that

$$x_R^{\text{int}}(X) \leq X \iff \rho_0 \geq \frac{\theta((1-\lambda)\psi(\alpha-\beta)-\alpha\theta+2\theta r)}{\alpha\left(\gamma(\lambda-2)\lambda^2+\theta\right)+\beta\left(\theta\left(2(\lambda-2)\lambda^2+1\right)-\gamma(\lambda-2)\lambda^2\right)} \equiv \hat{\rho}_0. \tag{30}$$

Straightforward algebra reveals that

$$\alpha \geq \beta \Rightarrow \left.\frac{\partial \pi_L(x_L, x_R)}{\partial x_L}\right|_{x_L=X, x_R=\hat{x}_R(X), \rho_0=\hat{\rho}_0} < 0, \tag{31}$$

which implies that a deviation by party L to a platform $x_L' < X$ is profitable.

Profile 5: $X < x_R < x_L$. The three possible locations of the critical voter i^1 are given in expressions (21) through (23). In contrast with Profiles 3 and 4, however, Party R wins under this profile if and only if $i_i^1 \leq X$, i.e.,

$$\rho_1 \geq \gamma(x_R - x_L) - \frac{\rho_0}{(1-\lambda)}.$$

Party R's expected payoff is:

$$\pi_R(x_L, x_R) = \frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L-x_R)-\rho_0-\theta x_L}\left(\frac{\alpha}{2}+\alpha\left(\frac{\lambda\rho-(1-\lambda)(\gamma(x_L-x_R)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1$$

$$
\begin{aligned}
&+ \frac{1}{2\psi}\int_{\gamma(x_L-x_R)-\rho_0-\theta x_L}^{\gamma(x_R-x_L)-\rho_0-\theta x_R} \left(\frac{\alpha}{2} + \alpha\left(\frac{\lambda\rho_0}{2\theta} - (1-\lambda)\frac{-\gamma(x_L+x_R)-\rho_0-\rho_1}{2(\theta-2\gamma)}\right)\right)d\rho_1 \\
&+ \frac{1}{2\psi}\int_{\gamma(x_R-x_L)-\rho_0-\theta x_R}^{\gamma(x_R-x_L)-\frac{\rho_0}{1-\lambda}} \left(\frac{\alpha}{2} + \alpha\left(\frac{\lambda\rho_0 - (1-\lambda)\gamma(x_R-x_L)-\rho_0-\rho_1}{2\theta}\right)\right)d\rho_1 \\
&+ \frac{1}{2\psi}\int_{\gamma(x_R-x_L)-\frac{\rho_0}{1-\lambda}}^{\psi} \left(r + \beta\left(\frac{\lambda\rho_0 - (1-\lambda)(\gamma(x_R-x_L)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1. \qquad (32)
\end{aligned}
$$

Under Assumption 1, this expected payoff is strictly concave in x_R; solving the first-order condition yields:

$$
\hat{x}_R^{\text{int}}(x_L) = \frac{(2r-\alpha)\theta + x_L(\alpha-\beta)\gamma(-1+\lambda) + \rho_0(\alpha+\beta-2\alpha\lambda) - (\alpha-\beta)\psi(1-\lambda)}{((\alpha-\beta)\gamma+2\alpha\theta)(-1+\lambda)}
$$

Similarly, party L's expected payoff is:

$$
\begin{aligned}
\pi_L(x_L,x_R) = &\frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L-x_R)-\rho_0-\theta x_L} \left(r + \beta\left(\frac{-\lambda\rho_0 + (1-\lambda)(\gamma(x_L-x_R)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1 \\
&+ \frac{1}{2\psi}\int_{\gamma(x_L-x_R)-\rho_0-\theta x_L}^{\gamma(x_R-x_L)-\rho_0-\theta x_R} \left(r + \beta\left(-\frac{\lambda\rho_0}{2\theta} + (1-\lambda)\frac{(-\gamma(x_L+x_R)-\rho_0-\rho_1)}{2(\theta-2\gamma)}\right)\right)d\rho_1 \\
&+ \frac{1}{2\psi}\int_{\gamma(x_R-x_L)-\rho_0-\theta x_R}^{\gamma(x_R-x_L)-\frac{\rho_0}{1-\lambda}} \left(r + \beta\left(\frac{-\lambda\rho_0 + (1-\lambda)(\gamma(x_R-x_L)-\rho_0-\rho_1)}{2\theta}\right)d\rho_1 \\
&+ \frac{1}{2\psi}\int_{\gamma(x_R-x_L)-\frac{\rho_0}{1-\lambda}}^{\psi} \left(\frac{\alpha}{2} + \alpha\left(\frac{-\lambda\rho_0 + (1-\lambda)(\gamma(x_R-x_L)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1. \qquad (33)
\end{aligned}
$$

Under Assumption 1, this expected payoff is strictly concave in x_R; solving the first-order condition yields:

$$
\hat{x}_L^{\text{int}}(x_R) = \frac{(2r-\alpha)\theta + x_R(\alpha-\beta)\gamma(-1+\lambda) + \rho_0(\alpha+\beta-2\beta\lambda) - (\alpha-\beta)\psi(1-\lambda)}{((\alpha-\beta)\gamma+2\beta\theta)(-1+\lambda)}.
$$

Solving the interior best responses simultaneously yields a pair (x_L^*, x_R^*) which, under Assumption 1, satisfy:

$$
x_L^* - x_R^* = \frac{2r(\alpha-\beta)\theta - \alpha\theta(\alpha-\beta) + (\alpha^2-\beta^2)(\rho_0+\psi(1-\lambda))}{((\alpha^2-\beta^2)\gamma+2\alpha\beta\theta)(-1+\lambda)} < 0
$$

and which therefore contradicts $x_R < x_L$. $\square$

We now verify that there exists an equilibrium in which $x_L \leq x_R \leq X$, and moreover that it is unique under Assumptions 1, 2 and 3. There are 3 possible locations for the indifferent voter among the late deciders.

Location 1: $i^1 \geq x_R$, i.e. $\rho_1 \leq \gamma(x_L - x_R) - \rho_0 - \theta x_R$:

$$i^1 = \frac{\gamma(x_L - x_R) - \rho_0 - \rho_1}{\theta} = i^1_{iii}.$$

Location 2: $x_L \leq i^1 \leq x_R$, i.e. $\gamma(x_L - x_R) - \rho_0 - \theta x_R \leq \rho_1 \leq \gamma(x_R - x_L) - \rho_0 - \theta x_L$:

$$i^1 = \frac{\gamma(x_L + x_R) - \rho_0 - \rho_1}{2\gamma + \theta} = i^1_{ii}.$$

Location 3: $i^1 \leq x_L$, i.e. $\rho_1 \geq \gamma(x_R - x_L) - \rho_0 - \theta x_L$:

$$i^1 = \frac{\gamma(x_R - x_L) - \rho_0 - \rho_1}{\theta} \equiv i^1_{i}.$$

Party R wins if and only if $i^1_{iii} \leq \frac{\lambda}{1-\lambda}\frac{\rho_0}{\theta}$, i.e.,

$$\rho_1 \geq \gamma(x_L - x_R) - \frac{\rho_0}{1-\lambda}.$$

Party R's expected payoff from $x_R \in [x_L, X]$ is therefore:

$$\begin{aligned}
\pi_R(x_L, x_R) \; &= \frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L - x_R) - \frac{\rho_0}{1-\lambda}} \left(\frac{\alpha}{2} + \alpha\left(\frac{\lambda\rho_0 - (1-\lambda)(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta}\right)\right) d\rho_1 \\
&+ \frac{1}{2\psi}\int_{\gamma(x_L - x_R) - \frac{\rho_0}{1-\lambda}}^{\gamma(x_L - x_R) - \rho_0 - \theta x_R} \left(r \; + \beta\left(\frac{\lambda\rho_0 - (1-\lambda)(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta}\right)\right) d\rho_1 \\
&+ \frac{1}{2\psi}\int_{\gamma(x_L - x_R) - \rho_0 - \theta x_R}^{\gamma(x_R - x_L) - \rho_0 - \theta x_L} \left(r \; + \beta\left(\frac{\lambda\rho_0}{2\theta} - (1-\lambda)\frac{\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(2\gamma + \theta)}\right)\right) d\rho_1 \\
&+ \frac{1}{2\psi}\int_{\gamma(x_R - x_L) - \rho_0 - \theta x_L}^{\psi} \left(r \; + \beta\left(\frac{\lambda\rho_0 - (1-\lambda)(\gamma(x_R - x_L) - \rho_0 - \rho_1)}{2\theta}\right)\right) d\rho_1.
\end{aligned} \tag{34}$$

Under Assumption 1, this expected payoff is strictly concave in x_R; solving the first-order condition yields:

$$\hat{x}_R^{\text{int}}(x_L) = \frac{(\alpha - \beta)(1-\lambda)(\gamma x_L + \psi) + \theta(\alpha - 2r) + \rho_0(\alpha + \beta - 2\lambda\beta)}{((\alpha - \beta)\gamma + 2\beta\theta)(1-\lambda)}. \tag{35}$$

Similarly, party L's expected payoff from $x_L \leq x_R$ is:

$$\begin{aligned}
\pi_L(x_L, x_R) \; &= \frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L-x_R)-\frac{\rho_0}{1-\lambda}} \left(r \; + \beta\left(\frac{-\lambda\rho_0 + (1-\lambda)(\gamma(x_L-x_R)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1 \\
&+ \frac{1}{2\psi}\int_{\gamma(x_L-x_R)-\frac{\rho_0}{1-\lambda}}^{\gamma(x_L-x_R)-\rho_0-\theta x_R} \left(\frac{\alpha}{2} + \alpha\left(\frac{-\lambda\rho_0 + (1-\lambda)(\gamma(x_L-x_R)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1 \\
&+ \frac{1}{2\psi}\int_{\gamma(x_L-x_R)-\rho_0-\theta x_R}^{\gamma(x_R-x_L)-\rho_0-\theta x_L} \left(\frac{\alpha}{2} + \alpha\left(\frac{-\lambda\rho_0}{2\theta} + (1-\lambda)\frac{\gamma(x_L+x_R)-\rho_0-\rho_1}{2(2\gamma+\theta)}\right)\right)d\rho_1 \\
&+ \frac{1}{2\psi}\int_{\gamma(x_R-x_L)-\rho_0-\theta x_L}^{\psi} \left(\frac{\alpha}{2} + \alpha\left(\frac{-\lambda\rho_0 + (1-\lambda)(\gamma(x_R-x_L)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1. \qquad (36)
\end{aligned}$$

Under Assumption 1, this expected payoff is strictly concave in x_L; solving the first-order condition yields:

$$\hat{x}_L^{\text{int}}(x_R) = \frac{(\alpha-\beta)(1-\lambda)(\gamma x_R+\psi) + \theta(2r-\alpha) + \rho_0(2\alpha\lambda-\alpha-\beta)}{((\alpha-\beta)\gamma + 2\alpha\theta)(1-\lambda)}. \qquad (37)$$

Let (x_L^*, x_R^*) denote a pair that satisfies $x_L^* \leq x_R^* \leq X$ and that solves the system of best responses.

First, we identify conditions for $x_L^* = x_R^* = X$. We observe that $\hat{x}_L^{\text{int}}(X) - X$ strictly decreases in ρ_0, and also that $\hat{x}_R^{\text{int}}(X) - X$ strictly decreases in ρ_0. We find that:

$$\hat{x}_L^{\text{int}}(X) - X \geq 0 \iff \rho_0 \leq \frac{-(1-\lambda)\psi(\alpha-\beta) - \alpha\theta + 2\theta r}{\alpha+\beta} \equiv \underline{\rho}_0 \qquad (38)$$

and

$$\hat{x}_R^{\text{int}}(X) - X \geq 0 \iff \rho_0 \leq \frac{(1-\lambda)\psi(\alpha-\beta) - \alpha\theta + 2\theta r}{\alpha+\beta} = \rho_0', \qquad (39)$$

where Assumption 1 that $\alpha \geq \beta$ implies that $\rho_0' \geq \underline{\rho}_0$. We conclude that $x_L^* = x_R^* = X$ if $\rho_0 \leq \underline{\rho}_0$.

Second, we identify conditions for $x_L^* < x_R^* = X$. In that case, we have

$$x_L^* = \hat{x}_L^{\text{int}}(X) = \frac{\alpha\left(-\gamma\lambda\rho_0 + \theta^2 + \theta(-2\lambda\rho_0 - \lambda\psi + \rho_0 + \psi)\right) + \beta(\gamma\lambda\rho_0 + \theta((\lambda-1)\psi + \rho_0)) - 2\theta^2 r}{\theta(\lambda-1)(\alpha(\gamma+2\theta) - \beta\gamma)}, \qquad (40)$$

and further require that $\hat{x}_R^{\text{int}}(\hat{x}_L^{\text{int}}(X)) \geq X$. Tedious but straightforward algebra establishes that

$$\hat{x}_L^{\text{int}}(X) < X \iff \rho_0 > \underline{\rho}_0, \qquad (41)$$

and

$$\hat{x}_R^{\text{int}}(\hat{x}_L^{\text{int}}(X)) \geq X \iff \rho_0 \leq \frac{\theta\left(\alpha\left(\frac{(1-\lambda)\psi(\alpha-\beta)}{\alpha(\gamma+\theta)-\beta\gamma}-1\right)+2r\right)}{\alpha+\beta} \equiv \overline{\rho}_0. \tag{42}$$

We conclude that $x_L^* < x_R^* = X$ if $\rho \in (\underline{\rho}_0, \overline{\rho}_0]$.

Third, we identify conditions for $x_L^* < x_R^* < X$. In that case, we may solve the system of interior solutions, directly, to obtain:

$$x_L^* = \frac{(\alpha-\beta)(\gamma\theta(2r-\alpha)-\theta\psi\beta(1-\lambda))+\beta\theta^2(2r-\alpha)-\rho_0[(1-\lambda)\gamma(\alpha^2-\beta^2)+\beta\theta(\alpha(1-2\lambda))+\beta^2\theta]}{\theta((\alpha^2-\beta^2)\gamma+2\alpha\beta\theta)(1-\lambda)},$$
$$x_R^* = x_L^* + \frac{(\alpha-\beta)(\alpha(\theta+(\lambda-1)\psi+\rho_0)+\beta((\lambda-1)\psi+\rho_0)-2\theta r)}{(\lambda-1)\left(\alpha^2\gamma+2\alpha\beta\theta-\beta^2\gamma\right)}, \tag{43}$$

where it is easily verified that indeed $x_L^* < x_R^* < X$ for all $\rho_0 > \overline{\rho}_0$.

We now verify that for all $\rho_0 \geq 0$, the solution (x_L^*, x_R^*) is an equilibrium. To establish this, it is necessary and sufficient to verify that there are no profitable deviations for party L to an alternative platform $x_L > x_R^*$, and no profitable deviations for party R to an alternative platform $x_R < x_L^*$.

No profitable deviation by party L to $x_L' > x_R^*$. Consider a deviation by party L to a platform $x_L' \in (x_R^*, X]$. For any pair (x_L, x_R) satisfying $x_R < x_L$, there are three possible locations for the indifferent voter among the late deciders.

Location 1: $i^1 \leq x_R$, i.e., $\rho_1 \geq \gamma(x_R - x_L) - \rho_0 - \theta x_R$.

$$\gamma(i^1 - x_L) - \gamma(i^1 - x_R) - \rho_0 - \rho_1 - \theta i^1 = 0 \iff i^1 = \frac{\gamma(x_R - x_L) - \rho_0 - \rho_1}{\theta} \equiv i_i^1. \tag{44}$$

Location 2: $i^1 \in (x_R, x_L)$, i.e., $\gamma(x_R - x_L) - \rho_0 - \theta x_R > \rho_1 > \gamma(x_L - x_R) - \rho_0 - \theta x_L$. This implies:

$$\gamma(i^1 - x_L) - \gamma(x_R - i^1) - \theta i^1 - \rho_0 - \rho_1 = 0 \iff i^1 = \frac{-\gamma(x_L + x_R) - \rho_0 - \rho_1}{\theta - 2\gamma} \equiv i_{ii}^1. \tag{45}$$

Location 3: $i^1 \geq x_L$, i.e., $\rho_1 < \gamma(x_L - x_R) - \rho_0 - \theta x_L$. This implies:

$$\gamma(x_L - i^1) - \gamma(x_R - i^1) - \theta i^1 - \rho_0 - \rho_1 = 0 \iff i^1 = \frac{\gamma(x_L - x_R) - \rho_0 - \rho_1}{\theta} \equiv i_{iii}^1. \tag{46}$$

If party L locates at a platform $x_L \in [x_R^*, X]$, it wins if and only if $i_{iii}^1 \geq X$, which occurs if and only if $\rho_1 < \gamma(x_L - x_R^*) - \frac{\rho_0}{1-\lambda}$. Party L's expected payoff is then:

$$\begin{aligned}
\pi_L(x_L, x_R) = \frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L - x_R) - \frac{\rho_0}{(1-\lambda)}} & \left(r + \beta\left(\frac{-\lambda\rho + (1-\lambda)(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta}\right)\right)d\rho_1 \\
+ \frac{1}{2\psi}\int_{\gamma(x_L - x_R) - \frac{\rho_0}{(1-\lambda)}}^{\gamma(x_L - x_R) - \rho_0 - \theta x_L} & \left(\frac{\alpha}{2} + \alpha\left(\frac{-\lambda\rho_0 + (1-\lambda)(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta}\right)\right)d\rho_1 \\
+ \frac{1}{2\psi}\int_{\gamma(x_L - x_R) - \rho_0 - \theta x_L}^{\gamma(x_R - x_L) - \rho_0 - \theta x_R} & \left(\frac{\alpha}{2} + \alpha\left(-\lambda\frac{\rho_0}{2\theta} + (1-\lambda)\frac{-\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(\theta - 2\gamma)}\right)\right)d\rho_1 \\
+ \frac{1}{2\psi}\int_{\gamma(x_R - x_L) - \rho_0 - \theta x_R}^{\psi} & \left(\frac{\alpha}{2} + \alpha\left(\frac{-\lambda\rho_0 + (1-\lambda)(\gamma(x_R - x_L) - \rho_0 - \rho_1)}{2\theta}\right)\right)d\rho_1. \qquad (47)
\end{aligned}$$

Under Assumption 1, party L's expected payoff is strictly concave in x_L; solving the first-order condition yields:

$$x'_L(x_R) = -\frac{\gamma(-2r\theta + x_R(\alpha - \beta)\gamma(-1+\lambda) + \beta(\rho_0 + (-1+\lambda)\psi) + \alpha(\theta + \rho_0 - 2\lambda\rho_0 + \psi(1-\lambda)))}{(-1+\lambda)(\alpha\theta(-2\gamma + \theta) - ((\alpha - \beta)\gamma^2 + \alpha\theta^2))}. \qquad (48)$$

Inserting $x_R^* < X$ into this expression, we obtain $x'_L(x_R^*) < x_R^*$, which establishes that a deviation to $x'_L > x_R^*$ is not profitable.

Consider, instead, a deviation by party L to $x_L > X$. The locations of the indifferent voter are given in expressions (44) through (46). Moreover, party L wins if and only if $i_{ii}^1 \geq X$. Party L's expected payoff from $x_L \geq X$ is:

$$\begin{aligned}
\pi_L(x_L, x_R) = \frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L - x_R) - \rho_0 - \theta x_L} & \left(r + \beta\left(\frac{-\lambda\rho_0 + (1-\lambda)(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta}\right)\right)d\rho_1 \\
+ \frac{1}{2\psi}\int_{\gamma(x_L - x_R) - \rho_0 - \theta x_L}^{-\gamma(x_L + x_R) - \rho_0 - \frac{\lambda(\theta - 2\gamma)}{(1-\lambda)\theta}\rho_0} & \left(r + \beta\left(\frac{-\lambda\rho_0}{2\theta} + (1-\lambda)\frac{(-\gamma(x_L + x_R) - \rho_0 - \rho_1)}{2(\theta - 2\gamma)}\right)\right)d\rho_1 \\
+ \frac{1}{2\psi}\int_{-\gamma(x_L + x_R) - \rho_0 - \frac{\lambda(\theta - 2\gamma)}{(1-\lambda)\theta}}^{\gamma(x_R - x_L) - \rho_0 - \theta x_R} & \left(\frac{\alpha}{2} + \alpha\left(-\lambda\frac{\rho_0}{2\theta} + (1-\lambda)\frac{-\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(\theta - 2\gamma)}\right)\right)d\rho_1 \\
+ \frac{1}{2\psi}\int_{\gamma(x_R - x_L) - \rho_0 - \theta x_R}^{\psi} & \left(\frac{\alpha}{2} + \alpha\left(\frac{-\lambda\rho_0 + (1-\lambda)(\gamma(x_R - x_L) - \rho_0 - \rho_1)}{2\theta}\right)\right)d\rho_1. \qquad (49)
\end{aligned}$$

Under Assumption 1, party L's expected payoff is strictly concave in x_L; solving the first-order condition yields:

$$x'_L(x_R) = \frac{2r\theta + x_R(\alpha-\beta)\gamma(-1+\lambda) + \beta(\rho_0 - 2\lambda\rho_0 + (-1+\lambda)\psi) + \alpha(-\theta+\rho_0+\psi(1-\lambda))}{(\gamma(\alpha-\beta)+2\beta\theta)(-1+\lambda)}, \quad (50)$$

which strictly increases in x_R. Straightforward algebra yields:

$$x'_L(X) - X = \frac{2r\theta - \alpha\theta + (\alpha-\beta)(1-\lambda)\psi + \rho_0(\alpha+\beta)}{((\alpha-\beta)\gamma+2\beta\theta)(-1+\lambda)} < 0,$$

which establishes that a deviation by party L to $x'_L > X$ is not profitable.

No profitable deviation by party R to $x'_R > X$ or $x'_R < x^*_L$. Consider a deviation by party R to a platform $x_R < x_L$. The locations of the indifferent voter are given in expressions (44) through (46). In this case, party R wins if and only if $i^1_{iii} \leq X$. Party R's expected payoff is:

$$\begin{aligned}
\pi_L(x_L, x_R) = &\frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L-x_R)-\frac{\rho_0}{(1-\lambda)}} \left(\frac{\alpha}{2} + \alpha\left(\frac{-\lambda\rho_0 + (1-\lambda)(\gamma(x_L-x_R)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1 \\
&+ \frac{1}{2\psi}\int_{\gamma(x_L-x_R)-\frac{\rho_0}{(1-\lambda)}}^{\gamma(x_L-x_R)-\rho_0-\theta x_L} \left(r + \beta\left(\frac{-\lambda\rho_0 + (1-\lambda)(\gamma(x_L-x_R)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1 \\
&+ \frac{1}{2\psi}\int_{\gamma(x_L-x_R)-\rho_0-\theta x_L}^{\gamma(x_R-x_L)-\rho_0-\theta x_R} \left(r + \beta\left(-\lambda\frac{\rho_0}{2\theta} + (1-\lambda)\frac{-\gamma(x_L+x_R)-\rho_0-\rho_1}{2(\theta-2\gamma)}\right)\right)d\rho_1 \\
&+ \frac{1}{2\psi}\int_{\gamma(x_R-x_L)-\rho_0-\theta x_R}^{\psi} \left(r + \beta\left(\frac{\lambda\rho_0 - (1-\lambda)(\gamma(x_R-x_L)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1. \quad (51)
\end{aligned}$$

Under Assumption 1, party R's expected payoff is strictly concave in x_R; solving the first-order condition yields:

$$\hat{x}'_R(x_L) = \frac{(\alpha-2r)\theta + x_L(\alpha-\beta)\gamma(-1+\lambda) + \rho_0(\alpha+\beta-2\beta\lambda) - (\alpha-\beta)\psi(1-\lambda)}{(-1+\lambda)(2\beta\theta+(\alpha-\beta)\gamma}, \quad (52)$$

which replicates expression (35), and thus we must have $\hat{x}'_R(x^*_L) \geq x_L$. This establishes that a deviation to $x_R < x^*_L$ cannot be profitable.

Consider, instead, a deviation by party R to a platform $x_R > X$. The locations of the indifferent voter are given in expressions (44) through (46). In this case, party R wins if and only if $i^1_{ii} \leq X$.

Party R's expected payoff is:

$$\begin{aligned}\pi_R(x_L,x_R) \;&= \frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L-x_R)-\rho_0-\theta x_R} \left(\frac{\alpha}{2}+\alpha\left(\frac{\lambda\rho_0-(1-\lambda)(\gamma(x_L-x_R)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1\\ &+\frac{1}{2\psi}\int_{\gamma(x_L-x_R)-\rho_0-\theta x_R}^{\gamma(x_L+x_R)-\rho_0-\frac{\lambda(2\gamma+\theta)}{(1-\lambda)\theta}\rho_0} \left(\frac{\alpha}{2}-\alpha\left(\frac{\lambda\rho_0}{2\theta}-(1-\lambda)\frac{(\gamma(x_L+x_R)-\rho_0-\rho_1)}{2(\theta+2\gamma)}\right)\right)d\rho_1\\ &+\frac{1}{2\psi}\int_{\gamma(x_L+x_R)-\rho_0-\frac{\lambda(2\gamma+\theta)}{(1-\lambda)\theta}\rho_0}^{\gamma(x_R-x_L)-\rho_0-\theta x_L} \left(r\;+\beta\left(\frac{\lambda\rho_0}{2\theta}-(1-\lambda)\frac{\gamma(x_L+x_R)-\rho_0-\rho_1}{2(2\gamma+\theta)}\right)\right)d\rho_1\\ &+\frac{1}{2\psi}\int_{\gamma(x_R-x_L)-\rho_0-\theta x_L}^{\psi} \left(r\;+\beta\left(\frac{\lambda\rho_0-(1-\lambda)(\gamma(x_R-x_L)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1.\end{aligned} \tag{53}$$

Under Assumption 1, party R's expected payoff is strictly concave in x_R; solving the first-order condition yields:

$$\hat{x}'_R(x_L)=\frac{(2r-\alpha)\theta+x_L(\alpha-\beta)\gamma(-1+\lambda)+\rho_0(\alpha+\beta-2\beta\lambda)-(\alpha-\beta)\psi(1-\lambda)}{(-1+\lambda)(2\alpha\theta+(\alpha-\beta)\gamma)}, \tag{54}$$

which strictly increases in $x_L \le X$. Straightforward algebra establishes:

$$\hat{x}'_R(X)-X=\frac{(2r-\alpha)\theta-(\alpha-\beta)(1-\lambda)\psi+(\alpha+\beta)\rho_0}{(-\beta\gamma+\alpha(\gamma+2\theta))(-1+\lambda)}<0, \tag{55}$$

which establishes that a deviation to $x_R > X$ is not profitable. $\square$

Proof of Corollary 1. In this case, we have $\lambda=0$ and $x^*_R(\rho_0)=0$, so that

$$x^*_L(\rho_0)=\frac{\alpha(\theta+\rho_0+\psi)+\beta(\rho_0-\psi)-2\theta r}{\beta\gamma-\alpha(\gamma+2\theta)}. \tag{56}$$

We obtain comparative statics for each of the primitives, in turn.

Higher ρ_0. We have $\frac{\partial x^*_L}{\rho_0}=\frac{\alpha+\beta}{\beta\gamma-\alpha(\gamma+2\theta)}<0$. We conclude that x^*_L *decreases* in ρ_0.

Higher θ. We have

$$\frac{\partial x^*_L}{\partial\theta}=\frac{\alpha(-\alpha\gamma+2\alpha(\rho_0+\psi)+\beta(\gamma+2\rho_0-2\psi))+2\gamma r(\alpha-\beta)}{(\beta\gamma-\alpha(\gamma+2\theta))^2}. \tag{57}$$

The numerator of this expression strictly increases in ρ_0, and is therefore positive if and only if

$\rho_0 \geq \frac{(\alpha-\beta)(\alpha(\gamma-2\psi)-2\gamma r)}{2\alpha(\alpha+\beta)}$. This threshold is strictly negative (and thus vacuously satisfied) if $r > \frac{\alpha}{2} - \frac{\alpha\psi}{\gamma}$, which is true. We conclude that x_L^* *increases* in θ.

Higher α*.* We have

$$\frac{\partial x_L^*}{\partial \alpha} = \frac{\beta\gamma(\theta+2\rho_0)+2\beta\theta(\rho_0-\psi)-2\theta r(\gamma+2\theta)}{(\beta\gamma-\alpha(\gamma+2\theta))^2}. \tag{58}$$

Calling $\nu(\rho_0)$t he numerator of this expression, we find that $\nu(\rho_0)$ strictly increases in ρ_0, and that $\nu(\underline{\rho}_0) = \frac{(\beta\gamma-\alpha(\gamma+2\theta))(\beta(\theta+2\psi)+2\theta r)}{\alpha+\beta} < 0$. We conclude that x_L^* strictly *decreases* in α.

Higher γ*.* We have

$$\frac{\partial x_L^*}{\partial \gamma} = \frac{(\alpha-\beta)(\alpha(\theta+\rho_0+\psi)+\beta(\rho_0-\psi)-2\theta r)}{(\beta\gamma-\alpha(\gamma+2\theta))^2}. \tag{59}$$

The numerator of this expression strictly increases in ρ_0, and is strictly positive when evaluated at $\rho_0 = \underline{\rho}_0$. We conclude that x_L^* strictly *increases* in γ.

Higher ψ*.* We have $\frac{\partial x_L^*}{\partial \psi} = \frac{\beta-\alpha}{\alpha(\gamma+2\theta)-\beta\gamma} < 0$.

Higher r*.* We have $\frac{\partial x_L^*}{\partial r} = -\frac{2\theta}{\beta\gamma-\alpha(\gamma+2\theta)} > 0$. $\square$

Proof of Corollaries 2, 3 and 4 and 5. In this case, we have $\lambda = 0$, and:

$$\begin{aligned} x_L^*(\rho_0) = &\ \frac{\beta\theta\psi(\beta-\alpha)-(\alpha\gamma+\beta(\theta-\gamma))(\alpha(\theta+\rho_0)+\beta\rho_0-2\theta r)}{\theta\left(\alpha^2\gamma+2\alpha\beta\theta-\beta^2\gamma\right)}. \\ x_R^*(\rho_0) = &\ x_L^*(\rho_0) + \frac{(\alpha-\beta)((\alpha+\beta)(\psi-\rho_0)+\theta(2r-\alpha))}{\alpha^2\gamma+2\alpha\beta\theta-\beta^2\gamma}. \end{aligned} \tag{60}$$

We obtain comparative statics for each of the primitives, in turn.

Higher ρ_0*.* We find that $\frac{\partial x_L^*}{\partial \rho_0} = \frac{\beta(\alpha-\beta)}{\alpha^2\gamma+2\alpha\beta\theta-\beta^2\gamma} - \frac{1}{\theta}$, which is strictly negative if and only if $\theta > \frac{\beta\gamma-\alpha\gamma}{\beta}$, which is true. Moreover, $\frac{\partial[x_L^*-x_R^*]}{\partial \rho_0} = \frac{\beta^2-\alpha^2}{\alpha^2\gamma+2\alpha\beta\theta-\beta^2\gamma} < 0$, which implies that x_R^* also decreases in ρ_0, and faster than x_L^*. $\square$

Higher α*.* We start with the platform x_L^*. We find that $\frac{\partial x_L^*}{\partial \alpha}$ can be written as a quotient with a strictly positive denominator, and a numerator that we call $\nu(r,\psi)$, which strictly decreases in r. Recalling our Assumption 1 $r > \frac{1}{2}\left(\alpha + \frac{\psi}{\theta}(\alpha-\beta)\right)$, we find:

$$\left.\frac{\partial \nu(r,\psi)}{\partial \psi}\right|_{r=\frac{1}{2}\left(\alpha+\frac{\psi}{\theta}(\alpha-\beta)\right)} = -2\alpha\beta^2\theta - \frac{\gamma^2(\alpha-\beta)^3}{\theta} - \beta\gamma(\alpha-\beta)^2 < 0. \quad \square \tag{61}$$

Since $\psi > \rho_0$, it is then sufficient to observe that:

$$\nu\left(\frac{1}{2}\left(\alpha + \frac{\psi}{\theta}(\alpha - \beta)\right), \rho_0\right) = - \left(\alpha^2\gamma + 2\alpha\beta\theta - \beta^2\gamma\right)(\alpha\gamma + \beta(\theta - \gamma)) + \frac{\rho_0(\alpha - \beta)\left(\gamma^2(\alpha - \beta)^2 + 2\beta\gamma\theta(\alpha - \beta) + 2\beta^2\theta^2\right)}{\theta}, \quad (62)$$

which is strictly negative under Assumptions 1 and 2. We conclude that $\frac{\partial x_L^*}{\partial \alpha} < 0$.

We next consider the platform x_R^*. We find that $\frac{\partial x_R^*}{\partial \alpha}$ can be written as a quotient with a strictly positive denominator, and a numerator that we call $\mu(\rho_0, \psi)$, which strictly decreases in ρ_0, and that there exists $\hat{\rho}_0$ such that $\mu(\rho_0, \psi) \geq 0$ if and only if $\rho_0 \leq \hat{\rho}_0$. Thus, $\rho > \hat{\rho}_0$ implies that x_R^* decreases in α, while $\rho > \hat{\rho}_0$ implies that x_R^* increases in α, and that $\hat{\rho}_0$ strictly *decreases* in α. It is straightforward to verify that there are parameter configurations for which $\hat{\rho}_0 > \overline{\rho}_0$. $\square$

Higher r. We have that $\frac{\partial x_L^*}{\partial r} = \frac{2(\alpha\gamma + \beta(\theta - \gamma))}{\alpha^2\gamma + 2\alpha\beta\theta - \beta^2\gamma} > 0$, and $\frac{\partial x_R^*}{\partial r} = \frac{2\alpha(\gamma + \theta) - 2\beta\gamma}{\alpha^2\gamma + 2\alpha\beta\theta - \beta^2\gamma} > 0$, and finally that $\frac{\partial[x_L^* - x_R^*]}{\partial r} = \frac{2\theta(\alpha - \beta)}{\alpha^2\gamma + 2\alpha\beta\theta - \beta^2\gamma} > 0$. $\square$

Higher ψ. We have that $\frac{\partial x_L^*}{\partial \psi} = \frac{\beta(\beta - \alpha)}{\alpha^2\gamma + 2\alpha\beta\theta - \beta^2\gamma} < 0$, and $\frac{\partial x_R^*}{\partial \psi} = \frac{\alpha(\alpha - \beta)}{\alpha^2\gamma + 2\alpha\beta\theta - \beta^2\gamma} > 0$. $\square$

Proof of Corollary 6. Straightforward algebra yields:

$$\frac{\partial \underline{\rho}_0}{\partial \lambda} = \frac{(\alpha - \beta)\psi}{\alpha + \beta} > 0 \quad (63)$$

and

$$\frac{\partial \overline{\rho}_0}{\partial \lambda} = -\frac{\alpha(\alpha - \beta)\psi\theta}{(\alpha + \beta)((\alpha - \beta)\gamma + \alpha\theta)} < 0. \quad \square \quad (64)$$

Proof of Corollary 7. When $\rho \in (\underline{\rho}_0, \overline{\rho}_0)$, we have

$$\frac{\partial x_L^*}{\partial \rho_0} = \frac{\alpha(\theta - \lambda(\gamma + 2\theta)) + \beta(\gamma\lambda + \theta)}{\theta(\lambda - 1)(\alpha(\gamma + 2\theta) - \beta\gamma)}, \quad (65)$$

and thus $\frac{\partial x_L^*}{\partial \rho_0} > 0$ if and only if $\lambda \geq \frac{\theta(\alpha + \beta)}{\alpha\gamma + 2\alpha\theta - \beta\gamma} \equiv \lambda^*$. $\square$