Abstract

It is often observed that politicians “overpay” to attract business investment to their jurisdiction, often despite condemnation by voters and the media. Standard explanations of this phenomenon hinge on the effect of competition among jurisdictions for the investment, leading to a version of the winner’s curse. This paper proposes an alternative explanation that turns on the interaction between informational asymmetries between voters and politicians and electoral incentives.

We show that when business investment is economically impactful it can be used to positively distort the informational inferences of voters regarding incumbent politicians. Our key results show that even when it is common knowledge that the benefits of the investment, while positive, are smaller than the cost of subsidizing investment, that politicians will still offer subsidies, and this will enhance their electoral prospects. Further, we show that even when there is some likelihood the business investment would have occurred in the absence of subsidies, the same equilibrium behavior emerges.
1 Introduction

That governments at all levels regularly provide public funds for or otherwise subsidize private investment projects is hardly news. One often-analyzed instance of this is the subsidization of the construction and sometimes operation of facilities used by professional sports teams. The true amount of public funding of such facilities is difficult to measure accurately, but to cite one attempt, Long (2005) estimates that MLB ballparks have received a mean of $218 million each (in 2001 dollars) in public subsidies that include land, infrastructure, and ongoing operating subsidies\(^1\). That governments - particularly but not exclusively those in developing countries - regularly offer ‘tax incentives’ to firms that site plants within their boundaries is well-known. There is a large literature on these practices focused entirely on determining the circumstances in which such incentives are successful in attracting investment by commercial enterprises (see, for example Klemm and Van Parys (2012)). An important theme in the economic literature on these practices is that the economic case for them is typically hard to make – incentives occur even when any reasonable measurement of their net benefit to the jurisdiction that offers them is negative.

It can be countered that a complete measurement of all the benefits generated by a new NFL football stadium, for example, is difficult to come by, particularly if the possibility of such a facility generating an intangible ‘social value’ is admitted (see, for example, Johnson, Groothuis and Whitehead (2001)). However, it is also true that sports facilities are built and operated both with and without public subsidies and firms locate plants in jurisdictions that offer no subsidies, so a better understanding of the forces that influence governments to offer such incentives (and firms to lobby for and/or accept them) would be useful.

Much of the existing literature on this phenomenon relies heavily on the idea that the firms that receive these subsidies are in the position of entertaining ‘bids’ for their investment projects. Sports franchises seeking a new stadium, for example, often enter into discussions with cities other than that in which they currently operate, apparently looking for the best offer of a subsidized home for the team. Biglaiser and Menzetti (1997) discuss the case

\(^1\)These are calculated as the present discounted value of all government subsidies, grants and tax holidays.
of the location of a Mercedes plant for which some 30 US states competed in the early 90s. This ‘bidding for footloose firms’ phenomenon is at the heart of much of the literature on this topic, and reportedly induced the mayor of the Michigan city that won the ‘auction’ for a Mazda plant to first use the term “industrial blackmail”. However, it is also sometimes the case that firms which appear to have available no serious alternative locations in which to invest are nonetheless provided with tax incentives. Kenyon, Langley and Paquin (2015) cite the example of a property tax abatement awarded to pharmaceutical firm Speracor in 2008. The City of Marlborough, Massachusetts awarded it to the firm after the building of a new facility was underway, despite one city councilman pointing out the absurdity of providing incentives for investment that had already occurred.

In addition the industrial tax competition literature, recent work by Jensen, et al. (2014), Jensen, et al. (2015), and Jensen and Malesky (2018) have examined how electoral incentives encourage the use of industrial incentives by politicians. In particular Jensen, et al. (2015) evaluate the incentive programs of cities under different electoral regimes – namely cities with directly elected mayors versus cities with non-elected city managers. Exploiting exogenous random assignment of political institutions, they find that elected mayors offer incentives with greater frequency, and more generous incentive packages than do their non-elected counterparts. Moreover, the use of incentives increase significantly in election years – they follow a predictable political budget cycle.

In this paper we develop a model of the interaction between a government decision-maker - who we will call a ‘mayor’ - a firm which might invest in that jurisdiction, and voters who will finance and possibly benefit from the investment, and who also determine whether the mayor will remain in office. In our model the key driver of the the mayor’s decision to offer a subsidy to the firm, as well as the firm’s decision to accept it and make a costly (sunk) investment, is informational. The voter is uncertain about the ability of the incumbent mayor, and that ability has a stochastic impact on the voter’s ongoing well-being. The project, if it is undertaken, also has a stochastic impact on the voter, and in equilibrium the mayor offers the subsidy in order to favorably influence the voter’s beliefs about his ability, and thereby increase the probability of re-election. However, the firm’s beliefs about the mayor’s re-election prospects are also important, since the future defeat of
the mayor in an election has a negative impact on the promised subsidy. Further, the project in question, although genuinely productive, is such that the (representative) voter would be unwilling to ratify the subsidy ex-ante. Additionally, depending on its characteristics, the firm may be unwilling to undertake the investment without the subsidy.

Part of the innovation in our model is that although the existence of an alternative jurisdiction in which to make the investment is consistent with our set-up, it is not necessary – jurisdictional competition is not the key mechanism. All we require is that there is some alternative to investing in the proposed project for the firm, and that could be inaction as well as an alternative jurisdiction in which to make the investment. Moreover, we also show how firms who would invest even in the absence of a subsidy, may also be the recipient of a subsidy nonetheless.

The phenomenon has been studied by others. An early literature (See Bond and Samuelson(1986) and Doyle and van Wijnbergen(1994)) was aimed at explaining why the subsidies offered to firms often took the form of ‘tax holidays’ - reductions in the tax liability of the investing firm which have a limited lifespan. Bond and Samuelson’s results point to the fact that a tax holiday can arise as an effective signal of the unknown-to-the-firm productivity of investment in the jurisdiction, while Doyle and van Wijnbergen’s paper points to the front-loading of investment incentives as arising from the improved bargaining position of the jurisdiction once the investment is made. King, McAfee and Welling (1993) develop a dynamic model of intergovernmental competition for a private investment which predicts how the size of the subsidies offered relates to productivity differentials and (firm) sunk costs. When a prior jurisdictional choice of infrastructure is introduced into the model, it is shown that the equilibrium results in differing levels of infrastructure across jurisdictions even when costs are identical. In their model subsidies are always offered and always accepted by the firm, a phenomenon that also occurs in our two primary results.

In the paper closest to ours, Biglaiser and Menzetti (1997) develop a model in which a politician of unknown ability decides whether or not to fund a project which has a stochastic impact on voter well-being. As in our model, the politician’s ability also has such an impact, and voters decide whether to re-elect the politician after observing what happens after the
politician’s decision. They derive the politician’s ‘willingness to pay’ for a project and their main result is to show that it differs, in general, from the voter’s willingness to finance it. Voters who have an inherent bias against the incumbent will be willing to pay less for the project than the incumbent is, and the opposite is true if voters have an inherent bias in favor of the incumbent. In the B&M model there is no Firm, and the only source of information for the voter about the politician’s ability is the first period outcome of the project, but only if the politician funds it. Thus, the politician is in effect determining how much she will pay to have that information generated. This differs from our environment first of all because we have the Firm as a strategic player who must be willing to undertake the project, and who may have private information as to its relative profitability.

The remainder of the paper is organized as follows. The next section presents our baseline model. In the baseline model we assume that, in addition to it being common knowledge that the firm’s local project does not generate sufficient expected benefits for the voter to be willing to pay to subsidize it, is also common knowledge that the project is not privately profitable for the firm. Following the model, we present our main results, namely that the firm’s project enables the Mayor to positively influence (or jam) the signals voters receive about his ability. As a result a (almost) unique equilibrium emerges in which both good and bad mayors offer the subsidy, and the firm accepts it. This enhances the electoral prospects of the incumbent, even though the voter is fully aware that the economic benefits of the project are outweighed by the cost of the subsidy. We then consider a variation on this model in which all that is different is that the firm is privately informed about the profitability of whatever is its best alternative to going ahead with the project. Here our result is that, so long as there is sufficient probability that the local project is unprofitable for the firm, the same behavior emerges in equilibrium. All mayors offer the subsidy, both types of firms take it and this again enhances the re-election prospects of the Mayor. The added feature here is that now the Voter subsidizes projects that would, with positive probability, proceed without any such aid.

[We conclude with a discussion.]
2 Model

The general environment is one in which there is an incumbent politician – the ‘mayor’ – whose ability in office is private information, and who faces a future possibility of re-election. His ability in office has a stochastic influence on the well-being of the citizens in his jurisdiction (henceforth the city). The municipal population consists of a single representative voter, putting our model in the tradition of political agency models that date back to Austen-Smith and Banks (1989) and Rogoff (1990).

The pivotal agent in the model is a firm, which has the option of making a local investment that also has a stochastic influence on the welfare of the voter. Only the firm knows whether undertaking this project is profitable. If it is, then the firm will undertake it, but even if it is not, the project still has a positive, albeit stochastic, impact on the voter; this introduces the possibility that the mayor may be willing to provide a subsidy to the firm that is contingent on the project going forward. It also introduces the possibility that the voter may be willing to pay for this subsidy through the tax system.

As examples of this situation, the firm could locate a distribution center in the city, which will hire some number of its citizens and provide a general boost to economic activity; only the firm knows whether doing this will increase its future profit stream more than would locating it elsewhere, or continuing to operate without it. Similarly, the firm may be a professional sports team contemplating building a new arena or stadium, and only the team knows whether it will generate greater future net returns than would continuing to operate the existing arena, or greater than would moving the team to another city. If indeed one of those alternatives is more profitable for the firm, then a sufficiently large subsidy from the city, contingent on building the stadium there, can increase the relative profitability of the local option and reverse the firm’s decision.

The key informational assumption is that both the Mayor’s ability and the firm’s investment decision have a stochastic impact on the well-being of the voter, who is interested in re-electing the incumbent Mayor only if she is more likely to be able in office than an untested challenger. In our two-period model all the voter has to go on as regards the incumbent’s ability is how well the economy performed in the city during the mayor’s term, but this is
also influenced by what the firm does, which is why the Mayor has reason to subsidize the firm’s local project.

We assume a two-period time frame without discounting, with the mayor in office in period $t = 1$ and facing re-election at the end of that period. In each period there is a state of nature that is realized, which we denote as $\omega_t \in \{0, 1\}$ for $t = 1, 2$. It will be apparent below that state 1 is better for the city’s representative voter.

We refer to the mayor as player $M$, whose ability, $\theta$, is either low or high, so $\theta \in \{l, h\}$ and the mayor of type $\theta$ is referred to as $M_\theta$. We assume that it is common knowledge that $M$ is equally likely to be of either type. The mayor is purely office-motivated, earning a payoff of 1 if re-elected for a second term and 0 if not. Being risk-neutral, $M_\theta$’s payoff function is thus $p^e$, the probability of re-election. The one decision to be made by $M_\theta$ is whether or not to offer an action-contingent subsidy to the firm.

The single voter’s payoff is $\omega_1 + \omega_2 - T$, where $T$ are any incremental tax payments made. The voter has one decision, denoted by $e \in \{0, 1\}$ with 1 being a decision to re-elect $M$ and 0 a decision to replace him with a challenger. The challenger’s ability can also be either $l$ or $h$, and $p_h$ is the exogenously given probability the challenger’s type is $h$.

The third strategic agent is the firm, denoted as $F_\tau$, where $\tau \in \{L, H\}$ is the firm’s type, the meaning of which is detailed below.

The firm has available to it a choice $d \in \{\phi, I\}$, which is a decision to make an initial investment in the city ($I$) or not ($\phi$). Choosing $d = I$ incurs a sunk cost $\gamma$ in period 1; these are any unrecoverable outlays by the firm made prior to the operation of the project in period 2. In period 2, if the Firm incurred $\gamma$ it then decides whether to operate the project ($x = 1$) or not ($x = 0$). If $d = \phi$ we assume $x = 0$ necessarily; there is no project to continue. Choosing $x = 1$ implies the Firm incurs a further period 2 one-time cost of $\eta$ and generates ongoing operating profits with a present value discounted to period 2 of $\pi_t$. If the Firm chooses $x = 0$, then it doesn’t pay $\eta$ in period 2 and generates ongoing operating profits with a period 2 discounted present value of $\pi_t$. It is this, the profitability of the outside option, that depends on the Firm’s type, which is private information to the firm.
We assume that not investing has different implications for the two types of firms, which only the firm knows. We assume in particular, that

$$\pi_H > \pi_I - \gamma - \eta > \pi_L > 0$$

Thus, the $L$-type firm is distinguished by the fact that the best alternative it has to undertaking the local project is inferior to it. Firm $F_L$ thus has every incentive to make the local investment in the city. The $H$ type firm on the other hand has an alternative to the local investment that is more profitable. The Firm’s type, as with that of the Mayor, is private information, but we assume that $q = \Pr \{\tau = H\} \in [0, 1]$ is common knowledge.

We will in fact make the stronger assumption that $\pi_H > \pi_I - \eta$: any extra operating profits generated by the local project do not cover even its continuation costs, which of course implies the $H$ type Firm does not wish to incur the initial $\gamma$, either. In the above example of the sports team considering building a new arena in the city, $\pi_r$ may be the future profit stream generated by moving the team to another city, but it may also be generated by continuing to operate the existing local arena. The key is that only the Firm knows which it is, and the value of $\pi_r$ it generates.

The final element of the model is the subsidy, $s$, which either type of Mayor may offer the Firm. We set aside any attempt to model a possible bargaining process between the Firm and Mayor regarding the size of a subsidy, and limit the Mayor to a stark choice. He can offer no subsidy at all, so $s = 0$, or offer a subsidy $v > 0$ that has the property that $\pi_I - \gamma - \eta + v > \pi_H$. This simplification means that we can model the choice of the Mayor as $\sigma = (\sigma_I, \sigma_H) \in [0, 1]^2$, where $\sigma_\theta$ is the probability the type $\theta$ mayor offers $s = v$. Thus our full assumptions about firm types and the subsidy are as below:

$$\pi_I - \gamma - \eta + v > \pi_H > \pi_I - \gamma - \eta > \pi_L > 0$$

The subsidy is enough to make the local investment more profitable than the $H$ firm’s best alternative, but if it is offered to the $L$ type firm, that firm also has no reason to turn it down. Furthermore, the cost of the subsidy is covered by the voter-as-taxpayer, so that the Voter’s payoff is in fact $\omega_1 + \omega_2 - s$.

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2Note that it can still be that the operating profits from operating the project from period 2 onward are superior to not doing so, so that $\pi_I > \pi_H$. The hockey team will in fact earn greater profits after the land for a larger arena is developed and rezoned ($\gamma$) and the arena is constructed ($\eta$).
While neither the voter nor the firm can observe $\theta$, the voter does see $\omega_1, s$ and $d$ before choosing $e$, her re-election decision. This matters because it is common knowledge that both $\theta$ and $d$ have an impact on the realization of $\omega_1$.

Specifically, we assume the probability that $\omega_1 = 1$ for any given $\theta, d$ pair is given by $p_{\theta d} \in (0, 1)$, and has these properties:

$$p_{hd} > p_{ld}, \text{ for } d \in \{\phi, I\}, \text{ and}$$

$$p_{\theta I} > p_{\theta \phi}, \text{ for } \theta \in \{l, h\}.$$ (2)

Thus, a high ability incumbent has a positive impact on expected voter well-being in period 1, as does the firm’s decision to spend the initial $\gamma$ on the project. In this sense, local investment by the Firm is productive. However, we assume that it is insufficiently productive (in expectation) from the voter’s perspective to justify the subsidy. That is, we assume that the stochastic impact of the project going ahead is the same in both periods, and that

$$2p_{\theta \phi} > 2p_{\theta I} - v, \text{ for } \theta = l, h.$$ (3)

The LHS of (3) is the expected value of $\omega_1 + \omega_2$ if the project is not undertaken, while the RHS is the same if the project is undertaken minus the cost of the subsidy. Note also that since $p_{\theta \phi} < p_{\theta I}$ by (2), then (3) also implies that

$$p_{\theta \phi} > p_{\theta I} - v, \text{ for } \theta = l, h.$$ (4a)

That is, once things have moved on to period 2, the voter still does not find the project to be worth the cost of subsidizing it, which will be important in what follows.

The voter does have a say in the subsidization of the firm’s project even if $M$ has offered $s = v$ and the firm has chosen $d = I$ and made its sunk investment. The voter can vote a new politician into office in the second term. If this happens, we assume that the payment of the subsidy to the firm is cancelled. This implies that the $H$ type firm chooses not to continue

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3It is not at all necessary that the stochastic impact of $\theta$ and $d$ on $\omega_t$ be the same for both periods, so long as the inequalities in (2) hold. We maintain the same values of $p_{\theta d}$ for each $t$ for simplicity.
the project in period 2, since \( \pi_H > \pi_I - \eta \). A type \( L \) firm that chose \( d = I \) at period 1 however, will choose \( x = 1 \) even if the subsidy is pulled back or reduced, since the local project was its best alternative without any subsidy.

We assume that the project has no impact on the realization of \( \omega_2 \) unless the firm chooses \( x = 1 \) (which it can only do if it chose \( d = I \) at \( t = 1 \)).

As to the impact of the mayor on \( \omega_2 \), if the incumbent is not re-elected, then his type, \( \theta \), has no impact on the probability that \( \omega_2 = 1 \). Placing a challenger in office whose probability of being high ability is \( p_h \), implies that the probability that \( \omega_2 = 1 \) becomes

\[
p_C = p_h p_{\theta \phi} + (1 - p_h) p_{\theta \phi},
\]

if the Firm is of type \( H \), because the project doesn’t go forward (\( F_H \) chooses \( x = 0 \)). It is

\[
p^d_C = p_h p_{I I} + (1 - p_h) p_{I I},
\]

however, if \( \tau = L \).

Note that \( p^d_C \) is the expected value of \( \omega_2 \) conditional on \( d \) and the challenger being elected. If the incumbent is re-elected, then the probability that \( \omega_2 = 1 \) depends on both \( \theta \) and on whether the project is operated in period 2. Thus, it is assumed to be \( p_{\theta x} \) where

\[
p_{\theta 1} = p_{\theta I}, \text{ and } p_{\theta 0} = p_{\theta \phi}.
\]

Condition (4a) implies the voter finds paying the subsidy unwarranted by the project’s stochastic impact on \( \omega_2 \). There is, however, an (expected) cost to the voter of turfing \( M \) out of office to avoid funding the subsidy. The condition (4a) only implies that the voter is better off in period two without the project and subsidy for a politician of \( \text{given} \) ability. If the voter is sufficiently convinced of the high quality of the incumbent mayor (relative to the expected challenger \( p_h \)), then it may be in the voter’s interest to re-elect her even if she has committed to paying the subsidy \( v \) to the firm. Thus, the voter’s beliefs about the ability of the incumbent are critical, and these depend on three elements: the choice of \( d \) by the firm (because it influences

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4 This assumption, that the subsidy is cancelled in the event the challenger is elected, is stronger than necessary. It is sufficient for our purposes that election of the challenger implies the subsidy is reduced, to \( v' \) with \( \pi_H > \pi_I - \eta + v' \), where \( v' \) could the expected value of a stochastically reduced subsidy.
the realization of $\omega_1$), the realization of $\omega_1$, and - if the equilibrium strategies of $M_l$ and $M_h$ differ - $\sigma$ and the observed $s$.

We will write the voter’s posterior belief that $\theta = h$ as $\alpha_d(\omega_1, \sigma, s)$. In addition, the Firm will have beliefs about $M$’s type which matter to $F$ because they may influence the likelihood $M$ is re-elected, which in turn determines whether $F$ will see a promised subsidy reduced. These will depend only on $(\sigma, s)$ and we write $\delta(\sigma, s)$ as the probability $F$ attaches to $\theta = h$ after seeing $s$, when $\sigma = (\sigma_l, \sigma_h)$.

The sequence of moves in the game is as follows:

1. Nature chooses $M$’s type, $\theta \in \{l, h\}$, which only $M$ observes, and $F$’s type $\tau \in \{L, H\}$, which only $F$ observes.
2. $M_{\theta}$ chooses $\sigma_{\theta} \in [0, 1]$.
3. $F_{\tau}$ observes the value of $s$, and chooses $d_{\tau} \in \{\phi, I\}$.
4. The state $\omega_1 \in \{0, 1\}$ is realized, with $p_{\theta \omega_1} = Pr\{\omega_1 = 1\}$.
5. The voter ($V$) observes $s, d$ and $\omega_1$ and chooses $e \in \{0, 1\}$.
6. $F_{\tau}$ observes $I, s, e$ and chooses $x \in \{0, 1\}$.
7. The state $\omega_2$ is realized according to the following probabilities:
   (i) If $V$ chose $e = 1$, then $\omega_2 = 1$ with probability $p_{\theta x}$,
   (ii) If $V$ chose $e = 0$, then $\omega_2 = 1$ with probability $p_{C}^d$.
8. Player payoffs are realized

The payoffs to each player in this game, expressed so as to explicitly recognize their dependence on all players’ strategies, are:

$$M_{\theta} : U_{\theta} = e$$

$$F_{\tau} : U_{F_{\tau}} = \varphi(d) [x (\pi_l - \eta + se) - \gamma] + (1 - x) \pi_{\tau}$$

$$V : U_{V} = \omega_1 + \omega_2 - esx \varphi(d)$$

where

$$\varphi(I) = 1 - \varphi(\phi)$$
and all players are assumed risk-neutral.

Note that the firm receives the subsidy $v$ (and the voter finances it) only if all of $s = v$, $d = I$ and $e = x = 1$ are chosen.

We will determine the set of perfect Bayesian equilibria (henceforth PBE or just ‘equilibria’) of the model. The derivation will be made simpler by noting the following about stage 6 in the game above. The firm is only in a position to choose $x = 1$ at this stage if it chose $d = I$ at stage 3. If that is the case, the choice of each type of firm is deterministic. For firm $H$, $x = 1$ if and only if all of $d = I, s = v$ and $e = 1$ hold, and we will simply insert this behavior by $F_H$ into stage 6 of the game above. Similarly, Firm $L$ always chooses $x = 1$ if $d = I$, which we insert into the game above.

## 3 Results

Before presenting the main results, it will be helpful to first lay out some definitions and assumptions that will be important in solving for the equilibria – in particular regarding voter beliefs and updating about $\theta$.

If $M$’s strategy reveals nothing about his type, then conditional on the observed $d$ and $\omega_1$, the voter uses Bayes rule to determine the probability the incumbent is of high ability ($\theta = h$). These posterior beliefs about $\theta$ given any non-revealing $\sigma$ will be denoted as $\beta_d(\omega_1) \equiv \alpha_d(\omega_1, (\sigma', \sigma'), s)$ and are defined by:

$$
\beta_d(0) = \frac{1 - p_{hd}}{(1 - p_{hd}) + (1 - p_{ld})}, \text{ and} $$
$$
\beta_d(1) = \frac{p_{hd}}{p_{hd} + p_{ld}}
$$

These beliefs about $\theta$ when $M$’s strategy conveys no information allow us to state two key conditions regarding the influence of $\theta$ on the state, $\omega_1$, via its effect on the voter’s posterior beliefs.

**C1:** $\beta_d(0) < p_h < \beta_d(1)$ for $d \in \{\phi, I\}$.

This is an assumption that – when the Mayor’s behavior is uninformative – the realization of state 1 contains enough information to make a nontrivial difference to his beliefs about the incumbent in relation to an unknown challenger.
The second assumption is similar, but for an environment in which the Mayor has granted a subsidy and the Firm has invested, but again this behavior has revealed no new information about $\theta$.

**C2:** $\beta_I(0)p_{HI} + [1 - \beta_I(0)]p_{H} < p_C + v < \beta_I(1)p_{HI} + [1 - \beta_I(1)]p_{HI}$

In summary, what these two conditions require is that, in the absence of further information about the quality of their Mayor, voter’s re-election decision is responsive to the realization of the state in period 1. These conditions also imply that the election is expected to be competitive in that the expected challenger quality is not so low or high to render the voter’s observations of period 1 state irrelevant.

Our final condition relates to the type $H$ firm:

**C3:** $\left[\frac{1}{2}(p_{HI} + p_{H})\right] \left[(\pi_I - \eta + v) - \pi_H\right] > \gamma$

The first bracketed term on the LHS is the probability that $\omega_I = 1$ if the firm chooses to invest and maintains its prior about the Mayor. The second bracketed term is the difference in the Firm’s profits from period 2 onward between going forward with the subsidized project and not; by our assumptions above, this difference is positive. Thus, this condition requires these two values to be sufficiently high relative to the sunk cost of the project. It in effect assumes that the $H$ Firm does not need certainty about the subsidy being maintained in period 2 to initiate the local project – it can live with some uncertainty.

We will in what follows consider only equilibria in which the Mayor uses pure strategies; either offering a subsidy or not, with certainty. Given that restriction, in fact the other players will use only pure strategies also, as neither Firm type nor the Voter will find themselves indifferent between two options.

### 3.1 Equilibrium when $q = 1$

Because it provides clear intuition on the behavior of the agents in our model, and because it is of interest in its own right, we first lay out the unique PBE
under the assumption that $q = 1$, which means that it is common knowledge that the Firm does not find the local project profitable. The local sports team will move to another city or continue to operate its existing arena in the absence of a subsidy, or the local manufacturing firm would not find the construction of a new warehouse and distribution centre profitable. This means that the Firm has no interest in the local project and the Voter, who recognizes this fact, but also recognizes that if it does go forward it will have a stochastically positive effect on her well-being, still would not offer the subsidy of $v$ if she were allowed to decide.

Proposition 1  If $q = 1$, and conditions (1), (2) and (3) hold, along with conditions $C1$, $C2$ and $C3$, then the only PBE of the game above in pure strategies is the following:

1. $[i]$]

2. $\sigma^* = (1,1)$; that is, both types of incumbent mayor offer the subsidy, so $s_l = s_h = v$.

3. $d_H^*(v) = I$ and $d_H^*(0) = \phi$; that is, the Firm incurs the sunk project cost, $\gamma$, if and only if the subsidy is offered.

4. $e^*(\omega_1,s,d) \equiv \omega_1$: that is, the voter re-elects the incumbent if and only if state 1 is realized in the first period, independently of the $s,d$ the voter observes.

Beliefs along the equilibrium path are given by Bayes Rule. Off the equilibrium path we assume that: $\delta(0,\sigma^*) = \frac{1}{2}$ and $\alpha_d(\omega_1,0,\sigma^*) = \beta_d(\omega_1)$. Both $F$ and $V$ maintain their priors about $\theta$ when $s = 0$ is observed.

Thus, in the unique equilibrium both types of Mayor offer the subsidy, the Firm accepts it and makes its first-period investment in the local project, but the Mayor is not necessarily re-elected. That happens only if things turn out well for the voter in the first period, and if they do not, the election of a challenger as Mayor halts the project as the subsidy is withdrawn/reduced and the Firm reacts by abandoning it and going to its next-best option in period 2.
That the aforementioned equilibrium is unique also implies there are no equilibria that are fully revealing – that is, different types of Mayor taking different actions (such as $\sigma = (0, 1)$ or $\sigma = (1, 0)$). It is highly intuitive why there are no such separating equilibria. If only the type $l$ Mayor offers the subsidy then the voter, upon observing a subsidy offer, would prefer to elect the challenger than an incumbent Mayor now know to be a low type. Alternatively, if only the type $h$ Mayor offers the subsidy then the voter, upon observing a subsidy offer, would happily elect the incumbent regardless of the realized state (i.e. $\omega_1$). However, anticipating the voter’s inference and subsequent actions, any $l$ type Mayor would have an incentive to deviate by offering a subsidy (and pretend to be a $h$ type). There are also no equilibria wherein both types of Mayor do not offer a subsidy, as both types would have an incentive to deviate in order to induce firm investment and increase the likelihood of a good outcome (i.e. $\omega_1 = 1$) – an opportunity neither type of Mayor would pass up.\footnote{This argument holds generally for any $q < 1$, while a variant of it utilizing the refinement of divinity (Banks and Sobel, 1987) holds when $q = 1$.}

In the equilibrium, as long as the state outcome is positive, the voter electorally rewards the Mayor for taking an action that she ex ante disapproves of. Since the ex ante expected benefit of the firm’s project is less than the cost of the subsidy, voter welfare would be improved by a ‘rule’ that prohibits the granting of subsidies. However, any such rule prohibiting subsidies would necessarily hurt both the Mayor and Firm, in that the Mayor’s ex ante probability of being re-elected would decrease, as would the likelihood of the firm receiving the subsidy. However, even with subsidies allowed, the Mayor is not guaranteed re-election nor is the firm guaranteed the subsidy. The stochastic state realization could be poor resulting in the election of the challenger and the revocation of the subsidy. Note also that if the project does go forward, the chances that the Voter gets $\omega_2 = 1$ as an outcome are higher than they would be if this Rule existed; ex-post, the Voter may be better off than if $s = v$ were not allowed.

### 3.2 Pure strategy PBE for all $q$ values

Not surprisingly, the type of PBE that can arise in the model depends on the value of $q$ as we now show.
Define a critical value of \( q \), by:

\[
q^o = \frac{p_C^I - \Phi(p_{\theta I}, v, I)}{p_C^I - p_C^\phi}
\]

where \( \Phi(p_{\theta I}, v, I) = \beta_I(1)p_{hI} + [1 - \beta_I(1)]p_{lI} - v = \left(\frac{p_{hI}}{p_{hI} + p_{lI}}\right)p_{hI} + \left(\frac{p_{lI}}{p_{hI} + p_{lI}}\right)p_{lI} - v. \)

This means that \( q^o < 1 \), as \( C2 \) implies \( \Phi(p_{\theta I}, v, I) > p_C^\phi \), and \( q^o \) is negative if \( p_C^I < \Phi(p_{\theta I}, v, I) \).

**Proposition 2** If conditions (1), (2) and (3) hold, along with conditions \( C1, C2 \) and \( C3 \), then the following hold:

1. \([i]\)
2. There are no PBE in which the strategy of \( M \) is fully revealing for any value of \( q \).
3. There is a PBE in pure strategies in which the Mayor chooses \( \sigma = (0, 0) \) if and only if \( q < q^o \). For this PBE, the full strategies are:
   - Mayor, \( \sigma = (0, 0) \); Firm, \( d_H(s) \equiv \phi, d_L(s) \equiv I \); and Voter, \( e(\omega_1, v, \phi) = \omega_1, e(\omega_1, v, I) \equiv 0 \), and \( e(\omega_1, 0, d) \equiv \omega_1 \).
4. There is a PBE in pure strategies in which the Mayor chooses \( \sigma = (1, 1) \) if and only if \( q > q^o \). For this PBE, the full strategies are:
   - Mayor, \( \sigma = (1, 1) \); Firm, \( d_H(0) \equiv \phi, d_H(v) = I, d_L(s) \equiv I \); and Voter, \( e(\omega_1, s, d) \equiv \omega_1 \).

This is our central result, and it has a number of implications. First, under our assumptions, the Voter’s behavior is always responsive to the realization of the period 1 state, \( \omega_1 \), along the equilibrium path. If things go well, the incumbent Mayor is re-elected, and he is not if \( \omega_1 = 0 \).

When it is relatively likely that the Firm has a good alternative to the local project, so \( q > q^o \), then in fact the Voter is responsive to only \( \omega_1 \): the
incumbent mayor is re-elected if and only if $\omega_1 = 1$ no matter the Mayor’s or Firm’s action. This induces the $H$ firm as well as the $L$ firm to invest in response to being offered a subsidy, and implies it is optimal for the Mayor to do so. However, when it is relatively unlikely that the firm has a good alternative to the local project ($q < q^*$), then no Mayor will offer the firm a subsidy. This is equilibrium behavior because if he did, only the $L$ firm would invest, and that happens without the subsidy. The Voter again responds only to the realization of $\omega_1$ when no subsidy is offered, but responds to whether or not the Firm invests when a subsidy is offered, throwing the incumbent out of office with certainty if a subsidy is offered and the Firm invests. That ‘threat’ is enough to prevent the $H$ firm from responding to the offer of a subsidy, and so the Mayor does not offer it, as doing so gets him thrown out of office with certainty if the firm is of type $L$, which is relatively likely.

4 Discussion

In both propositions, in order to obtain the PBE discussed, a number of conditions needed to be met. Each condition was essential to observing ‘industrial blackmail’ – the overpayment of subsidies – in equilibrium, so they provide essentially some boundary conditions on the phenomenon. Since industrial subsidies are not used by all politicians or in all places, it is important to derive some predictions about the circumstances that are most likely to give rise to industrial blackmail.

Our first prediction is that industrial blackmail is more likely to occur in electorally competitive jurisdictions. If an incumbent is certain to win re-election, then they have little incentive to strike a welfare reducing deal with a firm in exchange for an electoral benefit. The same holds if the incumbent is expected to lose to a challenger, in which circumstance the firm may also be unwilling to commit resources to a project whose subsidies and payouts are tied to a politician who is unlikely to be re-elected. Work by Bertrand et al. (2007), Carvalho (2014), and Bandiera-de-Mello (2018) show, in both France and Brazil, how investment and job creation by firms and incentivized by governments, were systematically targeted at districts that were more electorally contested.

A second prediction is that industrial blackmail is more likely to occur in mid-sized jurisdictions. Jurisdictions that are too small may lack the re-
sources to offer sufficiently appealing incentives to firms – perhaps a reason why small towns rarely offer incentives unless through their state governments. On the other hand, for politicians of much larger jurisdictions, the economic benefits induced by the firm’s investment may not affect enough voters to carry a political benefit. Jensen and Malesky (2015) provide support for this prediction, showing that suburbs, rather than metro areas, spend more on incentives, and mid-sized jurisdictions systematically spend larger portions of their budgets on incentives.

A third prediction is that industrial blackmail is more likely to occur in jurisdictions where the economy an important political issue relative to other non-economic issues. In the model, the key signal being used by the voter to judge the mayor is the performance of the economy. This creates the opportunity for a firm to engage in a project that is politically beneficial precisely because it improves the economy. However, in places where the key political debates are centered on social issues, crime, or environmental issues, such economically oriented projects will carry smaller benefits for politicians.

A fourth prediction concerns the high type firm’s profit condition (i.e. C3) – namely that the subsidy must be large enough to compensate the firm for not taking its outside option (from which it would earn $\pi_H$) and instead undertaking the project in the mayor’s district (for which it earns $\pi_I - \eta - \gamma + v$). However if the outside option is considerably better, then the cost of enticing the firm to forgo the outside option may be prohibitive. The reasoning is also similar for type $L$ firms, whose best option is to undertake the project. This may account for the propensity of firms, in particular sports teams, to speak publicly about the threat of leaving and investing elsewhere – a firm is more likely to obtain subsidies if the perception of $q$ is higher. Additionally, it is industries, firms, and projects with easily perceived outside options, as well as flexibility in location, that are more likely to obtain subsidies.

5 References


6 Appendix

Proof of Propositions:

Although we state it separately, Proposition 1 actually follows from Proposition 2, so we prove only the latter here.

First, define the period 2 expected payoff to the voter as a function of his choice of $e$, and of the history she has seen, $(s,d,\omega_1)$ and of the strategy $\sigma$ of the Mayor. Write this as

$$U^2(e,\omega_1,s,d|\sigma) = \Pr\{\omega_2 = 1|e,\omega_1,s,d|\sigma\} \cdot 1 - \varphi(d)e$$

$$\equiv \chi(e,\omega_1,s,d|\sigma) - \varphi(d)e$$

And note the following:

$$\chi(0,\omega_1,s,d|\sigma) = \xi(s,d,e) [p_h p_{hI} + (1 - p_h) p_{II}] + [1 - \xi(s,d,e)] [p_h p_{h\phi} + (1 - p_h) p_{I\phi}]$$

where $\xi(s,d,e)$ is the probability that the project continues in period 2, given $s,d$ and the voter’s choice of $e$.

and analogously,

$$\chi(1,\omega_1,s,d|\sigma) = \xi(s,d,e) [\alpha(\omega_1,s,d|\sigma) p_{hI} + (1 - \alpha(\omega_1,s,d|\sigma)) p_{II}]$$

$$+ [1 - \xi(s,d,e)] [\alpha(\omega_1,s,d|\sigma) p_{h\phi} + (1 - \alpha(\omega_1,s,d|\sigma)) p_{I\phi}]$$

Note further that $\xi(s,\phi,e) \equiv 0$ and $\xi(0,I,e) \equiv 1$: if there is no initial investment there can be no continuation of the project in period 2, and if there is investment in period 1 with no subsidy, it must be that $\alpha = l$ and
so the project will continue whatever the voter does. Finally, $\xi(v, I, 1) = 1$. If a subsidy was offered and investment made, and the voter re-elects the incumbent, the project continues with certainty, as both types of firm will continue it.

Proof of i):

Suppose that $\sigma = (0, 1)$ so that seeing $s = v$ reveals that $\theta = h$. We know that $d_L \equiv I$ and $d_H(0) = \phi$ in any PBE. $d_H(v)$ may be either $I$ or $\phi$ in such a PBE.

We first determine the strategies of $V$ that must follow if this is to be a PBE.

If the voter sees $(s, d) = (0, \phi)$,

Then $V$ knows that $\theta = l$, and $\tau = H$, and so $U^2(0, \omega_1, 0, \phi|\sigma) = p^\phi = p_h p_{h^\phi} + (1 - p_h) p_{l^\phi} > p_{l^\phi} = U^2(1, \omega_1, 0, \phi|\sigma)$, so $e^*(\omega_1, 0, \phi|\sigma) \equiv 0$.

If the voter sees $(s, d) = (0, I)$

Then $V$ knows that $\theta = l$ and $\tau = L$, so the project will continue, so $U^2(0, \omega_1, 0, I|\sigma) = p_h p_{H^l} + (1 - p_h) p_{l^l} = p_C^I > p_{l^l} = U^2(1, \omega_1, 0, I|\sigma)$, and again $e(\omega_1, 0, I|\sigma) \equiv 0$

This means that $M_l$’s payoff in such a PBE must be $U^*_{l^*} = 0$, no matter how $F_H$ reacts to $s = v$.

We now show that $M_h$ must get a positive expected payoff in such a PBE.

i) Suppose first that $d_H(v) = \phi$.

If the voter sees $(s, d) = (v, \phi)$,

Then $V$ knows that $\theta = h$ and $\tau = H$, and so $U^2(0, \omega_1, v, \phi|\sigma) = p^\phi < p_h p_{h^\phi} = U^2(1, \omega_1, v, \phi|\sigma)$ so $e^*(\omega_1, v, \phi) \equiv 1$, and therefore $M_h$’s PBE payoff is $U^*_{h^*} = 1$.

ii) Now suppose that $d_H(v) = I$

If the voter sees $(s, d) = (v, I)$, then she knows that $\theta = h$ and $U^2(0, \omega_1, v, I|\sigma) = \xi(v, I, 0) [p^C_{I^l}] + [1 - \xi(v, I, 0)] p^\phi$ and $U^2(1, \omega_1, v, I|\sigma) = p_{hI} - v$.

As $e = 0$ implies the project continues iff $\tau = L$, it follows that $\xi(v, I, 0) = 1 - q$, so $U^2(0, \omega_1, v, I|\sigma) = (1 - q)p^C_{I^l} + q p^\phi_{I^l} \equiv Q > 0$

This means that, independently of the $\omega_1$ realization again,
\[ e = 1 \iff (1 - q)p_C^L + qp_C^\phi < p_{hl} - v \]

This inequality can go either way under our assumptions so far. [C2 implies \( p_{hl} - v > p_C^\phi \), but \( p_{hl} - v \) could be less than \( p_C^L = p_{hl} + (1 - p_l)p_{hl} \).]

Suppose that \( e^*(\omega_1, v, I) \equiv 0 \) because \( Q < p_{hl} - v \). This would mean that in fact \( d_H(v) = \phi \) in PBE, and we showed above that in that case, \( U_h^* = 1 \) and this cannot be a PBE because then \( M_l \) would deviate to \( s = v \).

Suppose instead that \( e^*(\omega_1, v, I) \equiv 1 \), as \( Q > p_{hl} - v \). Then indeed it will be that \( d_H(v) = I \) in a PBE, and this implies again that \( U_h^* = 1 \) so this cannot be a PBE again, for the same reason.

We now show there cannot be a PBE in which \( M \) uses \( \sigma = (1, 0) \).

First we claim that in a PBE with this \( \sigma \), it must be that \( d_H(v) = \phi \) - the \( H \) firm type does not invest when offered a subsidy.

Suppose not, b.w.o. c., so \( d_H(v) = I \).

Then when the Voter sees \( (v, I) \) his beliefs are that \( \theta = l \) and his beliefs about \( \tau \) remain at \( q \), since both Firm types respond to \( v \) with \( d = I \).

If \( V \) then responds to this with \( e = 0 \) her payoff is \( U(0, v, I|\sigma, d) = qp_C^\phi + (1 - q)p_C^L = Q \) as only the \( L \) firm will continue the project.

If, however, \( V \) responds with \( e = 1 \) her payoff is \( U(1, v, I|\sigma, d) = p_{hl} - v \), as the project continues for certain but with a type \( l \) Mayor and the voter must pay for the subsidy. However, \( p_C^\phi > p_{hl} - v \) by C2 and \( p_C^L > p_{hl} > p_{hl} - v \) so that the Voter will choose \( e = 0 \) independently of the realization of \( \omega_1 \). That, however, implies that the type \( h \) firm’s strategy must be that \( d_H(v) \equiv \phi \) if this is to be a PBE.

So, if the Voter sees \( (s, d) = (v, I) \) she knows that \( \theta = l \) and \( \tau = L \) and so \( U(0, v, I) = p_C^L > p_{hl} - v = U(1, v, I) \), and so chooses \( e = 0 \).

If the Voter sees \( (s, d) = (v, \phi) \) she knows that \( \theta = l \) and \( \tau = H \) and so \( U(0, v, \phi) = p_C^\phi > p_l\phi = U(1, v, \phi) \) and so chooses \( e = 0 \) again.

This implies the PBE payoff to \( M_l \) will be \( U_l = 0 \)

When the voter sees \( (s, d) = (0, I) \) she knows \( \theta = h \) and \( \tau = L \) and so \( U(0, 0, I) = p_C^L < p_{hl} = U(1, 0, I) \) so \( e = 1 \).

When the voter sees \( (s, d) = (0, \phi) \), she knows \( \theta = h \) and \( \tau = H \) and so \( U(0, 0, \phi) = p_C^\phi < p_h\phi = U(1, 0, \phi) \) so again \( e = 1 \) and this implies the payoff
to $M_h$ in such a PBE will be $U_h = 1$, and this implies it cannot be a PBE, as $M_l$ will prefer to play $s = 0$.

This proves i).

Proof of ii)

Suppose there is a PBE in which $M$ plays $\sigma = (0, 0)$. Then it must be that the firm responses in equilibrium are $d_L(0) \equiv I$ and $d_H(0) = \phi$.

This in turn means that if the voter sees $(s, d) = (0, \phi)$ then she knows that $\tau = H$ and so $U^2(1, \omega_1, 0, \phi) = \beta_\phi(\omega_1)p_{h\phi} + (1 - \beta_\phi(\omega_1)) p_{\phi}$ and $U^2(0, \omega_1, 0, \phi) = p_\phi^C$ and C1 then implies that $e(\omega_1, 0, \phi) = \omega_1$.

If the Voter sees instead $(s, d) = (0, I)$ then it knows that $\tau = L$ and the project will continue, so the voter’s possible payoffs are:

If $e = 1$, then $U^2(1, \omega_1, 0, I) = \beta_I(\omega_1)p_{hI} + [1 - \beta_I(\omega_1)] p_{I}$ and if $e = 0$ then $U^2(1, \omega_1, 0, I) = p_C^L = p_{h}p_{hI} + (1 - p_{h}) p_{I} > p_C^\phi$.

This implies $e = 1$ is optimal iff $\beta_I(\omega_1) > p_h$ and so our C1 implies that $e(\omega_1, 0, I) = \omega_1$.

This in turn means that the PBE payoff for $M_\emptyset$ is $U^*_\emptyset = qp_{\emptyset} + (1 - q) p_{\emptyset}$

The above arguments demonstrate that the Firm and voter PBE strategies in response to the Mayor’s strategy are:

$$F_L(s) \equiv I, F_H(0) = \phi$$
$$e(\omega_1, 0, \phi) = \omega_1 = e(\omega_1, 0, I)$$

. For this to be a PBE it is only necessary to show that neither Mayor type can increase their payoff by deviating to $s = v$. Assume as before that if such a deviation occurs, both the Firm and Voter assume that it is equally likely to be either $M_\emptyset$; it will be shown below that either both Mayor types have an incentive to make this deviation, or both types do not.
Now define $Q(q) \equiv qp_C^\phi + (1-q)p_I^I$ and notice that since $p_C^\phi < p_I^I$, this is decreasing in $q$.

$Q$ is in fact the expected payoff to the Voter if she chooses $e = 0$ after seeing $(s,d) = (v,I)$, if the voter maintains her prior beliefs ($q$) about $\tau$. Since $F_H$ will not invest in the project without the subsidy, the payoff to the Voter in period 2 will be $p_C^\phi$, the likelihood the challenger is of type $h$ given $x = 0$.

Further, $U^2(1, \omega_1, v, I) = \beta_I (\omega_1) p_{hI} + [1 - \beta_I (\omega_1)] p_{II} - v$, which is the expected payoff to a Voter who has seen $(v,I, \omega_1)$ if she chooses $e = 1$, as the project will continue in period two independent of the firm type. Note further that C2 implies that $U^2(1, 0, v, I) = \beta_I (0) p_{hI} + [1 - \beta_I (0)] p_{II} - v < p_C^\phi \leq Q(q)$ for any $q \in [0, 1]$.

However, $U^2(1, 1, v, I) = \beta_I (1) p_{hI} + [1 - \beta_I (1)] p_{II} - v > p_C^\phi$ by C2, but this implies nothing about $U^2(1, 1, v, I)$ relative to $Q(q)$. One possibility is that $U^2(1, 1, v, I) > p_I^I$, in which case $U^2(1, 1, v, I) > Q(q)$ for all $q \in [0, 1]$.

However, since $Q(q)$ is decreasing in $q$ and $Q(1) = p_C^\phi < U^2(1, 1, v, I)$, it follows that there is a value of $q < 1$, call it $q^o$, such that $U^2(1, 1, v, I) = Q(q^o)$ and

$$U^2(1, 1, v, I) < Q(q) \iff q < q^o$$

We thus define a critical value for $q$, which is

$$q^o = \frac{p_I^I - U^2(1, 1, v, I)}{p_C^I - p_C^\phi}$$

which is less than 1, as C2 implies $U^2(1, 1, v, I) > p_C^\phi$, and is negative (and irrelevant) if in fact $p_C^I < U^2(1, 1, v, I)$.

To determine whether or not the Mayor can deviate profitably to $s = v$, and so determine if this is a PBE, we consider two cases.

A. Assume $q^o > 0$ and consider values of $q < q^o$

Suppose now that either $M_\theta$ deviates to $s = v$.
First we ask whether it can be that $F_H$ responds with $d_H(v) = I$.  

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Suppose so. Then upon seeing $v, I$ the Voter is left with its priors about both the firm and the mayor, and so its payoffs are:

If the Voter chooses $e = 0$ then $U^2 = Q(q)$, but if she chooses $e = 1$ then $U^2 = U^2(1, \omega_1, v, I)$, and $q < q^o$ implies that $U^2(1, 0, v, I) < U^2(1, 1, v, I) < Q(q)$ and so $e \equiv 0$ is optimal. That in turn means that it cannot be sequentially rational for $F_H$ to choose $I$ in response to this deviation.

Given that the $H$ Firm responds to a deviation to $s = v$ with $d_H(v) = \phi$, the payoffs to $V$ in each possible node are as follows.

When $(s, d) = (v, I)$ the voter knows $\tau = L$ and so choosing $e = 0$ implies a voter payoff of $U^2 = p_C^I$ and $e = 1$ implies $U^2(1, \omega_1, v, I) = \beta_1(\omega_1)p_{hI} + [1 - \beta_1(\omega_1)]p_{HI} - v$

C.2 implies that $p_C^I > p_C^H > U^2(1, 0, v, I)$, so the best payoff the deviating mayor can get if the Firm is of type $L$ is if $e(\omega_1, v, I) = \omega_1$, which is $p_{hI}$

If the voter sees $(s, d) = (v, I)$ then she knows $\tau = H$ and so choosing $e = 0$ implies $U^2 = p_C^\phi$ and $e = 1$ implies $U^2(1, \omega_1, v, \phi) = \beta_\phi(\omega_1)p_{h\phi} + [1 - \beta_\phi(\omega_1)]p_{h\phi} - v$ so that in fact $e(\omega_1, v, \phi) = \omega_1$ by C1.

This means that the maximum payoff $M_\theta$ can expect from this deviation is $q p_{h\phi} + (1 - q) p_{hI}$, which is equal to the PBE payoff, so this deviation is not profitable for either Mayor type, and we conclude that if $q < q^o$ then this is a PBE. Further, the reasoning above has shown that the PBE strategies of the Voter and Firm off the equilibrium path (assuming they maintain their priors) are as follows:

\[
\begin{align*}
d_H(v) & = \phi \\
e(\omega_1, v, \phi) & = \omega_1, \text{ and} \\
e(\omega_1, v, I) & = \omega_1 \text{ or } e(\omega_1, v, I) \equiv 0, \text{ depending on whether } \beta_1(1)p_{hI} + [1 - \beta_1(1)]p_{HI} - v > \text{ or } < 0.
\end{align*}
\]

B. Now assume $q > q^o$ and again we determine if either $M_\theta$ can deviate profitably to $s = v$.

Now we claim that $F_H$ will respond to a deviation to $s = v$ with $d_H(v) = I$.

To see this, note that if $H$ chooses $d = \phi$ then his payoff is $\pi_h$ for certain. Suppose instead he chooses $I$, as claimed.
Then the Voter, seeing \( v, I \), is left with her priors about both \( \theta \) and \( \tau \), and so her payoff from each action is as follows:

Choosing \( e = 0 \) implies \( U^2 = p^I_C \), and choosing \( e = 1 \) implies \( U^2 = \beta_I(\omega_1) p_{hI} + [1 - \beta_I(\omega_1)] p_{lI} - v = U^2(1, \omega_1, v, I) \) and since \( q > q^o \), it follows that \( e(\omega_1, v, I) = \omega_1 \) and so \( F_H \)'s payoff is greater than \( \pi_h \), by our assumption C3.

This in turn means the payoff to \( M_\theta \) from deviating is \( p_{hI} > U^*_0 = q p_{\theta \phi} + (1 - q) p_{\theta l} \), and we have shown that it is profitable for both Mayor types to deviate to \( s = v \) when \( q > q^o \), which proves that \( \sigma = (0, 0) \) can be part of a PBE only if \( q < q^o \), thus completing the proof of ii).

Proof of iii)

Suppose then that \( \sigma = (1, 1) \) is part of a PBE.

There are two possible responses of \( F_H \) to \( s = v \) to consider.

i) \( d_H(v) = \phi \)

Then when the Voter sees \( v, I \) she knows that \( \tau = L \) and so the payoffs for the Voter are:

If she chooses \( e = 0 \) then \( U^2 = p^I_C \), as only \( F_L \) will choose \( x = 1 \) and if she chooses \( e = 1 \) then \( U^2(1, \omega_1, v, I) = \beta_I(\omega_1) p_{hI} + [1 - \beta_I(\omega_1)] p_{lI} - v \) because both types of \( F \) will choose \( d = I \).

Since \( p^I_C > Q(q) \) for all \( q \) and \( Q(q) > U^2(0) \), it follows that \( e(0, v, I) = 0 \).

By definition of \( q^o \), it also follows that if \( q < q^o \) then \( U^2(1, \omega_1, v, I) < Q(q) < p^I_C \) so \( e(1, v, I) = 0 \) also.

If the Voter sees \( v, \phi \) she knows that \( \tau = H \) and so her payoffs are:

From \( e = 0 \) she gets \( U^2 = p^\phi_C \) and from \( e = 1 \) it is \( U^2(\omega_1) = \beta_\phi(\omega_1) p_{h\phi} + [1 - \beta_\phi(\omega_1)] p_{l\phi} \) so that C1 implies that \( e(\omega_1, v, \phi) = \omega_1 \)

ii) Suppose instead that \( d_H(v) = I \)

Then the Voter will only see \( v, I \) and will maintain her priors about \( \theta \) and \( \tau \) so her payoffs will be:
If she chooses \( e = 0 \) then \( U_2 = q p_C^\phi + (1 - q) p_C^I = Q(q) \) and if \( e = 1 \) then \( U_2(\omega_1) = \beta_I(\omega_1) p_{hI} + [1 - \beta_I(\omega_1)] p_{II} - v \)

From the definition of \( q^o \) then, we have that if \( q > q^o \) then \( e(\omega_1, v, I) = \omega_1 \) and if \( q < q^o \) then \( e(\omega_1, v, I) \equiv 0 \)

This allows us to prove the following claim:

**Claim F: In any PBE with \( \sigma = (1, 1) \) the strategy of \( F_H \) must be:**

\[
d_H(v) = \begin{cases} 
I, & \text{if } q > q^o \\
\phi, & \text{if } q < q^o
\end{cases}
\]

**Proof of Claim F:** The payoff to \( F_H \) from \( d_H = \phi \) is always the same, \( \pi_H \).

Suppose then that \( q > q^o \). If \( F_H \) chooses \( I \), then the analysis above shows that \( e(\omega_1, v, I) = \omega_1 \) and so the payoff to \( F_H \) is greater than \( \pi_H \) by condition C.3, so \( d_H(v) = I \) as claimed.

If instead \( q < q^o \), then the analysis above implies that if \( F_H \) chooses \( I \), then \( e(\omega_1, v, I) \equiv 0 \) and the payoff to \( F_H \) is \( \pi_H - \gamma \), so in fact \( d_H(v) = \phi \).

This proves Claim F. ■

We now prove iii) in two cases.

**A. Assume \( q > q^o \)**

Claim F implies that \( d_+(v) = I \) for both firm types, and that means the voter sees only \( v, I \) in equilibrium.

This means the payoffs to each possible voter strategy are:

If \( e = 0 \) then \( U_2(0, \omega_1, v, I) = q p_C^\phi + (1 - q) p_C^I = Q(q) \)

If \( e = 1 \) then \( U_V(1, \omega_1, v, I) = \beta_I(\omega_1) p_{hI} + (1 - \beta_I(\omega_1)) p_{II} - v \), so that \( q > q^o \) implies that \( e(\omega_1, v, I) = \omega_1 \)

This in turn implies that the equilibrium payoff to \( M_\theta \) is \( p_{hI} \).

The only thing left to determine whether this is a PBE is whether \( M_\theta \) can get a higher payoff by deviating to \( s = 0 \). We assume \( F \) and \( V \) regard the deviation as being equally likely to come from either \( \theta \) and so maintain their
1/2 prior, and we know the firms will respond with \( d_H(0) = \phi \) and \( d_L(0) = I \), and that leads to the following Voter behavior

If the Voter sees 0, I she knows that \( \tau = L \) and the project will continue, so that

Choosing \( e = 0 \) gives her \( U^2 = p_C^\phi \) and choosing \( e = 1 \) results in \( U^2(\omega_1) = \beta_\phi(\omega_1) p_{h\phi} + [1 - \beta_\phi(\omega_1)] p_{l\phi} \) and therefore \( e(\omega_1, 0, I) = \omega_1 \) by C.1.

If the Voter sees 0, \( \phi \) she knows \( \tau = H \) so that

If she chooses \( e = 0 \) then \( U^2 = p_C^\phi \) and if \( e = 1 \) then \( U^2(\omega_1) = \beta_\phi(\omega_1) p_{h\phi} + [1 - \beta_\phi(\omega_1)] p_{l\phi} \) and so \( e(\omega_1, 0, I) = \omega_1 \) by C.1 again.

This in turn means the payoff to \( M_\theta \) from deviating to \( s = 0 \) is \( U_\theta(0) = q p_{\theta\phi} + (1 - q) p_{\theta I} < p_{\theta I} \) and so this is not a profitable deviation. This proves that in fact the strategies laid out in iii) are a PBE when \( q > q^o \).

B. \( q < q^o \)

Claim F now implies that \( d_H(v) = \phi \) and \( d_L(v) = I \) in any PBE, so we have the following for the Voter’s reactions:

If the voter sees \( v, \phi \) she knows \( \tau = H \) and the project will not happen, so \( e = 0 \) implies a payoff of \( U^2 = p_C^\phi \) and if \( e = 1 \) her payoff is \( U^2(\omega_1) = \beta_\phi(\omega_1) p_{h\phi} + [1 - \beta_\phi(\omega_1)] p_{l\phi} \) and the fact that \( q < q^o \) means that \( e(\omega_1, v, \phi) = \omega_1 \).

If the voter sees \( v, I \) she knows \( \tau = L \) and the project will happen no matter what she does, so the payoff to choosing \( e = 0 \) is \( U^2 = p_C^I \) and the payoff to choosing \( e = 1 \) is \( U^2(\omega_1) = \beta_I(\omega_1) p_{hI} + [1 - \beta_I(\omega_1)] p_{lI} - v \) and \( q < q^o \) means that \( p_C^I > Q(q) > \beta_I(1) p_{hI} + [1 - \beta_I(1)] p_{lI} - v > \beta_I(0) p_{hI} + [1 - \beta_I(0)] p_{lI} - v \) so that \( e(\omega_1, v, I) \equiv 0 \).

Taken together this implies the payoff to \( M_\theta \) from this potential PBE is \( U^*_{\theta} = q p_{\theta\phi} \).

For this to be a PBE it must be that neither \( M_\theta \) can do better by deviating to \( s = 0 \). If either type does so we have that \( d_H(0) = \phi \) and \( d_L(0) = I \).
If the Voter sees $0, \phi$ she knows that $\tau = H$ and so her payoffs are $p^\phi_C$ from choosing $e = 0$ and $\beta_\phi(\omega_1) p_{h\phi} + [1 - \beta_\phi(\omega_1)] p_{l\phi}$ from choosing $e = 1$, so that C1 implies that $e(\omega_1, 0, \phi) = \omega_1$.

If the Voter sees $0, I$ she knows that $\tau = L$ and so her payoffs are $p^I_C$ from choosing $e = 0$ and $\beta_I(\omega_1) p_{hI} + [1 - \beta_I(\omega_1)] p_{lI}$ if she chooses $e = 1$, and so C.1 now implies that $e(\omega_1, 0, I) = \omega_1$, also. This in turn implies that the payoff to $M_\theta$ from this deviation is $q p_{\theta\phi} + (1 - q) p_{\theta I} > U^*_\theta$ since $q < q^o < 1$, and so there can be no PBE with $\sigma = (1, 1)$ when $q < q^o$, proving the rest of iii), and completing the proof of Proposition 2.■